

# Judge's Commentary

## MathWorks Math Modeling Challenge 2020

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### Introduction

The problem posed in this year's MathWorks Math Modeling (M3) Challenge examined the transition from internal combustion to electric motors in freight trucks. The first of the three questions required teams to predict what percentage of the truck fleet will be converted to electric vehicles. The second question required teams to determine the number of stations and chargers per station required for each of five specific routes if every truck were converted to an electric vehicle. The third and final question required teams to rank the five routes.

The quality of the very top papers this year is similar to that of the top papers in recent years, however the quality of the papers *overall* continues to improve. This overall improvement in quality has made it increasingly more difficult for judges to rank the papers. The coaches and teams continue their impressive diligence in preparing for the event. The quality of the writing continues to improve, and a larger percentage of teams is taking more care in the fundamental parts of the modeling process. For example, this year it seemed to be much more common for papers to include a basic sensitivity analysis as well as citations within the narrative and the corresponding details in a separate reference section.

In this commentary we provide some details and observations from this year's event. The observations represent insights into what some teams did based on a relatively small sample of the submitted reports. The observations are divided into separate sections for questions one, two, and three, and a separate section on some general modeling issues is included as well.

### Question One

The first question required teams to predict the percentage of electric trucks in the overall fleet of semi-trucks in the immediate future. The question requires teams to determine the relationship between electric and internal combustion vehicles, and teams had to determine how to balance the number of trucks entering and leaving the fleet. Conservation of a quantity is a fundamental principle of modeling, and the way this idea is expressed mathematically is that the change in the number of trucks is equal to the number of new trucks entering service minus the number of older trucks removed over a specific time period.

Three basic aspects of question one are examined. First, many teams used regression techniques to build an approximation, and some observations of how this approach was employed are discussed first. The second aspect discussed is the basic question of trying to balance the total number of trucks with respect to the two given types, internal combustion and electric. Finally, a brief discussion is given about the decision process of how trucks are replaced over time.

Rather than start with the idea of how the distribution of trucks changes, a number of teams took a different approach and simply used the existing data and found an expression by using regression. This is problematic since currently there is not a significant production source of electric trucks, and the past data has little bearing on future trends. To make up for this many teams made use of the data for electric cars and assumed that truck production would be comparable. This is a good first step in trying to get a feel for the situation, but once production increases, some of the resources required for electric trucks overlaps those of electric cars, and there is likely to be some kind of tension between the production limits for the two types of vehicles.

An additional issue is that simply finding a function that matches the past data does not necessarily lead to appropriate predictions for the future. For example, fitting the future production of electric semi-trucks based on production prior to 2018 will predict zero electric semi-trucks for the future. A basic model should be developed based on sound principles, and then the data can be used to determine a specific function.

In terms of the specific forms for the models developed, a number of teams made use of a linear model, some made use of sine and cosine functions, and a logistic function was another common model. A specific timeline of 2020 through 2040 is specified in the problem statement. There are potential problems with using a linear model, but in this particular instance the long term growth is not necessarily a problem because of the fixed end date. Those that used the sine and cosine models stated that it was due to the cyclical nature of the economic cycle. The teams that used a logistic function (some teams referred to it as a Bass diffusion model) argued that the production levels should eventually plateau, and the number of electric trucks entering service would eventually balance with the number of trucks leaving service.

For all of the models employed, a key insight is the balance that must exist between internal combustion and electric trucks. The total amount of freight that must be moved is not likely to change substantially in the given time frame. The number of new trucks entering service is likely to be limited to balance the total movement of freight. At the same time, the amount of new vehicles available cannot exceed production capacity. Few teams considered both of these two limitations. Given the short time allotted to the teams to address the problem it would be difficult to find a way to combine and balance the two considerations, and the judges did not attempt to impose a priority over which approach is more important. Instead, it was expected that teams state their own priorities and provide a reasonable rationale for their choice. Teams that mentioned combining the two competing considerations as a potential improvement, though, demonstrated an important insight into the problem and showed an understanding of an important part of the process of critiquing and improving a mathematical model.

With respect to the adoption of new trucks, a company must balance a number of considerations. First, a fleet is composed of different kinds of trucks: short haul, long haul, and regional haul. This can be a subtle and difficult aspect to incorporate into a model. Incorporating it into a model is not something that is necessary in a report, but how a model can be adapted and improved should be mentioned.

Another, more important, aspect is to consider the decision process for a company that is replacing a truck. In the scenario posed this year, a company has two choices, an internal combustion vehicle or an electric vehicle. A number of teams simply assumed every internal combustion vehicle would be replaced with an electric vehicle. It is not an obvious choice though. Teams that made the case as

to why a company would choose an electric vehicle based on the costs tended to make a better impression. A smaller number of teams made the case for a percentage of new vehicles to be of a certain type which may be a more likely pattern.

In deciding the replacement rate, a number of teams discussed the life cycle of a truck. A common assumption was that a truck is replaced after twelve years. This is reasonable given the data and the very short time to work on the problem. Providing a note that this is a simplification that can be improved in a future model was likely to make a better impression. It was uncommon for teams to note that trucks will cycle out of service due to a number of causes, such as accidents, but this is a difficult phenomenon to model given adequate time, and this is an aspect that was important to note as a way to demonstrate how a model can be improved.

## Question Two

One of the questions for the second part required teams to estimate the number of facilities that would be required along five different trucking corridors to service a complete fleet of electric trucks. An additional question required teams to estimate the number of chargers required in each facility. A wide range of estimates were produced for this part of the event, and the teams made a wide range of assumptions. It was vital for teams to make their assumptions clear, and as long as the team provided a good rationale for their assumptions the judges accepted them. Furthermore, given the wide variation in assumptions, teams that provided citations associated with their rationale tended to make a more positive impression.

Providing a response to the second question was a difficult task for many teams. As has occurred in past years, it was not uncommon to find teams that provided a strong response to the second question yet struggled on the third question. One difference from past years, though, is that it was just as common to find other teams that provided a strong response to the third question but struggled on the second question. With this in mind, we will provide an overview of some of the major issues and struggles associated with the second question. First, we briefly discuss the importance of simplicity and examining the final results. Next, an overview of the interplay between battery capacities, time to charge the batteries, and distance traveled is discussed. Finally, the subtleties associated with which factors impact efficiencies is discussed.

First, the teams that tried a relatively simple approach and then provided a good analysis of their model tended to make a strong impression. In the end, this question lent itself well to a basic “sniff test.” The final predictions of a model should be checked to see if they are consistent with some kind of common sense. For example, if a team predicted a small number of facilities with a huge number of chargers (more than 700 chargers per facility, for example) then it was expected they should provide more insight into why their model produced such results and extra care was expected to demonstrate why the results are reasonable. Compounding matters, an additional burden was to balance different kinds of traffic. The traffic at this level includes short haul, long haul, and regional haul trucks.

It is important to note that the judges try to look at a whole paper and recognize that the students have a very short amount of time to provide a response. Many teams spent more time on other parts of their reports, and made simplifications about the traffic distributions. This is completely understandable, but a team should provide some recognition of this and indicate that it should be an aspect that should be incorporated in a future update to the model.

Another important aspect is the amount of time and charge level required by a vehicle after a single stop. The estimates varied widely. A number of teams assumed that an operator would simply charge the batteries on a vehicle to a full charge. This is a good first step, but a more likely scenario is to only wait for a partial charge. The time to charge a battery to a given percentage is not a linear relationship, and it may not be worthwhile to wait the extra time to charge up to the final 15-20% of a battery's capacity. This is a key insight, and teams that recognized this demonstrated that they understood an important part of the situation.

An additional aspect of the problem is to recognize that the distance that an electric vehicle can move on a given charge depends on the circumstances of a trip and not just the amount of charge in the batteries. For example, the traffic density can impact the range of a vehicle. Also, the outside temperature impacts the efficiency of the battery, but it can also result in a different pattern of usage. Finally, a number of teams also recognized that elevation changes matter, although they tended to focus on the total altitude change based on the start point and end point rather than the cumulative changes in the road between the start and end points.

No team was able to incorporate all of these different aspects, and it is generally not expected. Question two turned out to be a difficult part of the Challenge, and it is not reasonable to expect a team to be able to address so many subtleties in such a short time. However, it is important for a team to clearly state which parts they feel are most important and provide a justification. A team that can also identify a wide array of aspects and note that they should be incorporated in a future update tends to make a more positive impact, and it demonstrates that the team understands the process of modeling as a cycle of continual improvements. Being able to identify what parts of a model should be kept or thrown out as well as which new parts to add is a vital part of the process.

Surprisingly, there were two important factors that received minimal consideration. The first is the cost of retooling facilities, and the second is the cost of long-term maintenance. A small number of teams referred to this as a weakness in their model, but only a few teams incorporated it into their model. Furthermore, few teams examined more than one economic motivator that will influence a corporation to make a change in its fleet. Without economic incentives it is unlikely for a shift in the *status quo* to occur, and the incentives should eventually be considered in a follow-up model.

### **Question Three**

The third part required teams to consider a variety of factors to determine and prioritize which routes in the previous question should have their infrastructure converted so that the routes can accommodate a truck fleet that is entirely electric. The teams were required to determine factors that can predict how receptive a region might be to the change as well as the economic considerations for each route. We first discuss how judges approached the task of comparing papers with differing emphases. Next we discuss what factors teams considered in deciding how to develop a ranking system. Finally, we discuss how teams brought together the different factors to make a final decision.

A number of teams were able to provide a good response to the third question. What was most surprising was that some teams that struggled with the second question were able to provide creative responses to the third question. As judges, we try to accept a team's previous results and then ask if they are able to build on their conclusions in an appropriate and imaginative way. This year many teams were able to build on a response to the second question that was not as well

developed as some other teams' efforts, but they were able to create an outstanding response to the third question.

One of the difficulties associated with the third part is that teams had to determine which factors to consider and then determine how to balance the different factors. One of the factors to consider is how people within each region view the transition from internal combustion vehicles to electric vehicles. Teams considered numerous aspects that can be used as a proxy for how people might react. For example, a number of teams looked at party affiliation or recent voting trends. While this is a rough guide, it is a clever alternative given the very short time span teams have to work on the Challenge.

Once teams determined which factors to consider, they had to decide how to combine them to make a decision. The most common approach was to add the different factors together in some predetermined combination. Teams have to be extremely careful when adopting this approach. If it seems like an arbitrary sum then a judge's initial reaction likely will not be positive. For example, if the team adds up three terms for which the units are money, proportions of people in a given group, and number of charging stations, then it is difficult to assign meaning to the result. On the other hand, if all three terms are converted to equivalent units, such as money, then it is much easier to understand the calculation.

When teams try to combine different terms it is vital that the coefficients not be arbitrary. For example, a common decision is to simply use the value 1 for every coefficient and then determine the sum of a set of numbers. Some care and thought should be given about how to determine relevant weights of the different factors, and the units should be consistent. Given the short time that the teams have, judges understand that it is not reasonable to demand rigorous derivations of the weights, but it is vital that a team recognize the shortcoming of their approach. A team should explicitly identify the problem, and they should provide some guidance and insight into what they would do to address the problem given more time to do so.

## **Writing and Modeling**

Some of the unique approaches and issues associated with this year's event are given in the previous sections. Here, we provide some insights and discussion to more general observations that arise each year. These topics are things that teams should consider in many contexts. In particular, two broad categories are discussed below. The first topic is about writing and how to present results in a written form. The second topic is a broad range of general modeling issues that should always be considered.

First we discuss the presentation of results from simple calculations. It is often the case that some factors are calculated by combining a number of different terms. For example, a team may want to calculate the total number of miles driven by all trucks on a given stretch of highway over a given time period knowing the number of trucks for each day of the week, the number of miles for a given stretch of highway, and the length of the time spans. The team may decide to use the products of a number of different terms and then sum the results to get a final number.

The approach is sensible and appropriate, and the team has many options in how to decide to present the calculation of a result. It is a delicate balance between being too verbose and not providing enough information. A long-winded discussion of the calculation without a formula will likely be difficult to follow, and at the same time, a brief statement without details can make it

impossible to understand what the team did. For these kinds of calculations a simple formula and a brief discussion of what the terms mean is likely the easiest to read. The discussion should be brief, but at the same time the reader should be able to repeat and verify a team's calculation. A team should state its results, but should avoid leading the reader through all of the algebra as it can be tedious to read.

Another important practice is how a team documents the sources for their data. The organizers of the event have been taking extra steps to provide data to reduce the time required by teams to search and evaluate data sources. Teams are improving in how they provide both citations within their narrative as well as a references section, but many teams struggle to be consistent in documenting the data sources they are using. Even if the data is provided to the team, the report should include full documentation for the original source and the team should state which part of the data was used for each context.

In this year's event most teams made use of different data sources for different parts of their efforts, and it can be difficult for a reader to understand which data source was used for a given result. To compound matters, some teams were able to find alternate data sources to supplement their efforts. When reading many papers it could be unclear what sources a particular team used, even for those teams that worked strictly with the information that was provided.

In addition to the writing practices discussed above, a number of basic modeling issues took on greater importance this year. One practice is self-reflection and identifying the strengths and weaknesses of a model. This year a large number of teams were able to provide an outstanding response to either question two or to question three but struggled with the other question. One option in trying to balance and compare papers with disparate strengths is to examine whether or not a team recognized areas in which its model can be improved.

Modeling is a process in which the approximations are continually refined and improved. Given the very short time period and difficult task we do not expect a team to produce an outstanding response for all three questions. (Very few of the judges would be able to do so in the short time allotted.) We do expect, though, that a team provide a critical review of their model, identify the parts that are good and identify the parts that can be improved. This is one of the most important parts of the modeling process, and teams that are able to demonstrate critical self-reflection that will lead to improvements in the next iteration are more likely to make a positive impression on the person reading their work.

Another aspect of modeling is how to present results. A team should present their results in multiple formats such as tables, graphs, and written descriptions. Every graph should be properly annotated including labels, units, a title, and a caption. Every graph and table should also be described in the narrative so that the reader knows what to look for.

One important improvement that was seen this year was how teams present the work of numerical simulations. A number of teams constructed computational models that include a stochastic component, and each time a simulation is run a different result is likely. In such cases it is not appropriate to look at the results of a single run. This year we saw a number of teams conduct multiple runs and then present their work in the form of histograms as well as present statistical results in terms of confidence intervals. If this positive trend continues it will be difficult in the future for a team to be highly ranked if they make use of simulations and do not make use of appropriate statistical methods.

One final thing to note is the continued use of a sensitivity analysis as a way to gauge the robustness of a model. Every year this practice is more common, and it will likely be a “must have” for the higher ranked papers in the near future. While this practice is more common than in previous years, the majority of teams only look at how their results change when one parameter undergoes a small change. A sensitivity analysis should examine multiple parameters. For each parameter, the question is how do the end results change if a parameter is changed by a small amount. The second step is to then compare the impact across multiple parameters and determine which parameter makes the biggest difference when changed.

## **Conclusions**

We continue to see overall improvements in how students respond to the Challenge. This year the overall quality of the papers improved, making it harder for the judges to differentiate between the entries. Moreover, in past years it was more common for a majority of teams to provide stronger responses to question two and struggle with question three. This year, however, different teams tended to provide better responses to one or the other with no clear pattern.

When it came down to trying to compare papers with very different relative strengths, judges fell back on examining the fundamental practices of modeling as a way to compare papers. Practices that emphasize the roles of introspection and critical analysis of a model are vital steps in the process to adjust and improve a model. Additionally, the quality of the writing, consistency of documenting sources, and the basic practices associated with describing a model also make an immediate impact on a reader.

Given the narrower differences in the quality of papers and the varied way in which teams respond, it is more important than ever to pay attention to the most basic practices. These practices arise from the broader patterns of modeling that cannot be fully implemented in a short event such as MathWorks Math Modeling Challenge. The practices are also motivated by the desire to communicate and share complex results given a context of uncertainty in how robust the results may be. In the end, a simple model that includes an extensive, detailed analysis and a healthy dose of skepticism is more valuable than a more complex model that lacks insights or guidance on how to proceed in future refinements.

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