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M3 Challenge Runner Up, Magna Cum Laude Team Prize: $15,000

***Note: This cover sheet has been added by SIAM to identify the winning team after judging was completed. Any identifying information other than team # on an M3 Challenge submission is a rules violation.

*****Note: This paper underwent a light edit by SIAM staff prior to posting.**

Executive Summary

To high school administrators across the country:

There has been a recent call for an increase in STEM education not only across the United States, but also around the world. Our youth need to be on the edge of progress to keep up with these advancements and remain competitive in the global job market. We understand that, as educators, your top priority is to prepare your students for success in the future, and education in STEM fields is the best way to do so in the 21st century. As mathematical modelers, it is our top priority to provide you with tools to accomplish this goal.

We have developed a model to help your students understand the costs, benefits, and value of a higher education in STEM fields. One of the major obstacles many students face in pursuing an advanced education is the increasing cost of college. Therefore, we have created a simple linear equation for determining the expected cost a student will pay for a 4-year Bachelor's degree. Our model factors in the expected costs of a private or public college education as well as payment methods such as financial grants, work-study, scholarships, and loans. This model comprehensively examines several family situations; it is thus well-tailored to individual students and households.

It is important to remember that a college education is a tool to be used. With any tool, though, one must conduct a cost-benefit analysis, especially when the tool has as large a price tag as college's. To appeal to many different students, we analyzed multiple academic paths, not just the traditional undergraduate education in a STEM field. We found that although paying for college may cause more short-term financial deficits, individuals with Bachelor's degrees in STEM fields eventually make more money, with their net financial benefit eclipsing those of working high school graduates about 10 years after high school.

Finally, we have created a personal preference survey that students can take which will alert them to academic paths that may be of interest to them. Even after examining the costs of attending college and the economic advantages of various education plans, we felt that the human experience amounted to more than just dollars and cents. As rational as we can be, all of us have some inner drive that is fundamentally irrational. We attempt to capture and quantify this through the personalized surveys, which determine which traits a student possesses and which plans match their traits. The hope is that we can help personalize educational paths so that everyone gets the most of their experience.

Please note that the part of our model that determines the value of a STEM education based on the student's preferences is not yet fully automated and thus somewhat tedious to use. It is also important to remind ourselves that change is the only truly constant thing in this world, and that an individual's preferences may change if they are exposed to new ideas, such as they would be in college. No model is accurate enough to account for this. On the whole, however, we have great confidence in our model, and we hope that you find these tools useful in informing your students about STEM education.

STEM Sells: What is higher education really worth?

Team #4904

March 1, 2015

Contents

1 Introduction

In this constantly evolving world, Science, Technology, Engineering, and Mathematics (STEM) fields are widely regarded as the academic and professional careers of the future. New innovations in computer science, medicine, engineering, and industry continue to appear. As history has shown time and time again, the most successful countries are those that utilize these innovations to launch their economies, infrastructure, and communicative abilities into a new era. Therefore, in order to remain competitive at the global level, the United States needs to encourage more students to pursue a higher education with STEM majors.

However, regardless of how beneficial STEM fields are, students cannot pursue STEM careers without higher education. The costs of college continue to increase year after year, leaving some families to wonder if investing in the benefits of attending college is worth the skyrocketing price tag. The complexity of paying for college does not simplify the process either: What kinds of scholarship and financial aid are available? Will a student be better off working out of high school or attending college for several thousand dollars? Why should students choose to pursue STEM over other fields, such as business? All of these factors need to be considered.

1.1 Restatement of the Problem

We believe that looking into this issue is a worthwhile endeavor both for the United States and for individual students. President Obama's Council of Advisors on Science and Technology call for over one million more STEM professionals in the next decade, and school administrators all across the country need a tool to encourage students to pursue STEM fields in college. Hence we have developed a mathematical model to address the following questions:

1. What is the price of college? Based on the factors such as household income, scholarships, and financing options (like grants and work study), how much can a student expect to pay for his/her college education?
2. What is the benefit of a STEM degree? There are many possible career paths—such as entering the workforce straight out of high school, participating in 2-year programs, obtaining a non-STEM degree, etc. What are the short term and long term rewards of a STEM degree compared to these other options?
3. What is the best education and career path? Some students may enjoy STEM fields and wish to pursue higher education including graduate schools, but this may not be the best option for all students. How can this metric be determined quantitatively and qualitatively?

2 Part 1: Looking Beyond Sticker Prices

2.1 Analysis of Problem

The tuition that a college advertises is almost never an accurate representation of what an undergraduate education actually costs for a student and his or her family. In developing our model for Part 1 of the problem, we seek to provide an accurate dollar amount that households are expected to pay in order to get a student through college. Our model should incorporate tuition, room and board, fees, books, transportation, and other miscellaneous costs. It must also include the level of financial aid (including scholarships, work-study, and federal grants) that the student can expect, which is dependent on such factors as household income, tuition, and size of family.

2.2 Assumptions and Justifications

- **Assumption 1:** There are no significantly impactful economic crises.

  Justification: While an economic crisis like a recession can drastically impact costs, they are often unpredictable, so we have excluded such extenuating circumstances in order to simplify our model.

- **Assumption 2:** The cost of school over the course of four years does not change, and the student loan interest rate does not change over time.

  Justification: These are simplifying assumptions, because the cost of school over the course of 4 years does not drastically change, and the refinancing of loans is unpredictable based on individual circumstances.

- **Assumption 3:** The college students will pursue one major, and will not have external sources of income while in school (i.e., part-time jobs).

  Justification: Double majors requires more classes and will cost more for the student. Having a major in one STEM field is sufficient. In addition, the students are full-time students and will be too busy in their studies to take on a job.

- **Assumption 4:** All community college students in our model transfer over to a 4-year institution for their final two years of the Bachelor's degree.

  Justification: It would be pointless to determine the cost of an undergraduate education if it were completely free, and under President Obama's plan, only two years of an undergraduate education are paid for.

- **Assumption 5:** A student who receives two free years of education under President Obama's plan will not also receive other aid such as scholarships for those two years.

  Justification: If the student is already receiving a free education, they do not need any other compensation.

- **Assumption 6:** Students otherwise attempt to minimize costs (i.e., they will not waste money on buying gourmet food, cars, luxuries, etc.); similarly, students accept all offers of financial aid.

Justification: College students typically do not want to put themselves in a worse financial situation than is necessary. Moreover, most people would accept any opportunities to save more money.

- **Assumption 7:** Need-based financial aid comes solely from the government. The typical college does not provide need-based financial aid to students.

  Justification: Nearly all public institutions do not provide financial aid. The amount of aid provided by private institutions is highly college-dependent—they either meet 100 percent of demonstrated need (making the cost trivially zero) or provide some nonzero amount that varies too widely per college for us to generally determine a value. This is thus a necessary simplifying assumption for our model.

- **Assumption 8:** Students pay off their government loans according to the 10-year plan for repayment as proposed by the U.S. Department of Education.

  Justification: Since students are taking out loans from the the government, and therefore must pay the government back, it is a natural assumption that they will negotiate to terms similar to those published by the U.S. Department of Education.

2.3 Design of the Model

We begin by assessing the cost of a typical student's college education before taking any loans; we will later proceed to consider the final cost after calculating interest on loans. The amount Z that a household will have to pay for a 4-year college education, either out of pocket or with loans, can be expressed as the difference between total cost and total aid:

$$Z = \sum_{i=1}^{4} (C_i - F_i), \tag{1}$$

where C_i is total annual cost of college (tuition, room and board, fees, books, transportation, and miscellaneous costs) and F_i is the total financial aid received (scholarships, work-study, federal grants).

We obtain C_i from published data from the College Board for different types of institutions—specifically, \$46,000 a year for private colleges, \$23,000 for public in-state colleges, and \$37,000 a year for public out-of-state colleges (COLLEGEdata). F_i depends on the specifications of the household—in particular, when calculating the amount of federal grant money received, we must account for the financial situation of the household, as given in the problem. We compute F_i as follows:

$$F_i = G(I, E, D) + W_i + S_i, \tag{2}$$

where

- I = annual income
- E = the number of earners in the family
- D = the number of dependents in the family

- G = amount of federal grant money received
- W = the annual money provided by the work-study option, and
- S = the amount of scholarships received.

The values of I, E, and D are given in the problem. We determine $G(I, E, D)$ using the Department of Education's FAFSA4caster calculator, which computes the amount of federal grant money that a student will receive, given his/her household situation. It is important to note that federal grant aid is calculated independently of costs of attending college. Additionally, per Assumption 6, each of our households used as much federal aid as available.

We obtain W and S from Sallie Mae's report on college spending, which provides values for low-income (35,000 or less), middle-income (35,000 to 100,000), and high-income households (100,000 or above) for private and public colleges. Again, we invoked Assumption 6 and had each of our households use as much federal aid as possible.

If $F_i < C_i$, then the household will have to cover the remaining costs (Z). These funds can come from either savings or student loans. We used Sallie Mae's report to determine the annual average out-of-pocket payment for families in each income bracket. We call this O_{avg}. The remaining money, $P = Z - O_{avg}$, is the amount taken out in loans.

We must account for loans separately because they accrue simple interest over the time taken to pay them off (typically 10 years (Interest Rates and Fees)), which adds to the total cost. Using an amortization formula for simple interest (HSFPP), we can express the monthly payment A on a student loan as follows:

$$A = P * \frac{r(1+r)^n}{(1+r)^n - 1}, \tag{3}$$

where r is the monthly rate (which we determine from APR calculations), and n is the number of times that part of the loan is payed off. We can then calculate the total payment by the household as the sum of the out-of-pocket payment and all of the monthly payments of 10 years (120 months). Our overall equation is thus

$$\begin{aligned} Total &= O_{avg} + 120 * A \\ &= O_{avg} + 120 * \left(\sum_{i=1}^{4} (C_i - G(I, E, D) - W_i - S_i) - O_{avg} \right) * \frac{r(1+r)^n}{(1+r)^n - 1}. \end{aligned} \tag{4}$$

2.4 Considering Two Years of Free Community College

To consider Obama's recommendation of two years of government-funded community college, we simply evaluate the summation in (4) starting from $i = 3$ instead of $i = 1$, since no costs are incurred and no aid is awarded in the first two years.

2.5 Testing and Justification

Using our design, we tested the six different household situations as given in the problem: one parent/one child households with annual incomes of $35k, $75k, and $125k, and two parents/three children households with annual incomes of $35k, $75k, and $125k.

In order to test our model, we have created a simple spreadsheet document. By manipulating the variables within the spreadsheet, we can change the values within our equation to find the total cost of attending college over the course of 4 years, taking into account all the above variables.

Total Costs for 1 Parent / 1 Child Households

Total Income of Household	Type of College	Total Payment	Total w/ free 2 years
$35K	Private	$ 154,200	$ 73,300
	In-state public	$ 75,200	$ 33,700
	Out-of-state public	$ 146,500	$ 69,400
$75K	Private	$ 171,900	$ 81,900
	In-state public	$ 88,800	$ 40,300
	Out-of-state public	$ 160,200	$ 76,000
$125K	Private	$ 173,800	$ 78,700
	In-state public	$ 87,100	$ 60,000
	Out-of-state public	$ 158,400	$ 71,000

Figure 1: *This table shows the total cost for a student from a one parent, one child household to attend college for four years. Variables tested include the total income of the household, the type of college, and whether or not the first two years were free at a community college.*

Total Costs for 2 Parent / 3 Children Households

Total Income of Household	Type of College	Total Payment	Total w/ free 2 years
$35K	Private	$ 143,000	$ 67,700
	In-state public	$ 63,900	$ 28,100
	Out-of-state public	$ 135,300	$ 63,800
$75K	Private	$ 167,400	$ 79,600
	In-state public	$ 84,300	$ 38,000
	Out-of-state public	$ 155,600	$ 73,700
$125K	Private	$ 173,800	$ 78,700
	In-state public	$ 87,100	$ 60,000
	Out-of-state public	$ 158,400	$ 71,000

Figure 2: *This table shows the total cost for a student from a two parent, three child household to attend college for four years. Variables tested include the total income of the household, the type of college, and whether or not the first two years were free at a community college.*

As is evident from the tables above, a family with a smaller income is expected to pay less for a college education. This makes sense because they would receive more money in financial aid, thus leaving fewer costs to be covered by the family.

Another trend to observe is that when two years of community college are free, the costs are less than half of having to pay for all four years. This is most likely because of the interest on the loan. The smaller the loan is, the less time it takes to pay off the loan, and the less total interest is owed. These two factors compound, causing the total payment to be less than proportional to the principal amount of the loan.

To examine how accurate our model is, we compared the average cost data, as calculated in our model, with the average cost as calculated by the U.S. Census (SavingsforCollege.com). We note that the Census's average cost does not incorporate room and board, books, and miscellaneous costs, so we add those to the expected average cost.

Error Analysis

College Type	Average total cost from our model	Average Cost from online source	Percent Error
Private college	$164,000	$193,800	15%
In-state public college	$81,000	$96,000	16%
Free 2 years of community college + 2 years of private college	$76,600	$107,400	29%

Figure 3: *Error analysis table. We added room and board costs (obtained from CollegeBoard's COLLEGEdata set) to make the data reporting more consistent.*

As the table shows, our model matched expected results relatively well. The percent error for two years of community college followed by two years of private college is relatively high, but this makes sense, since our online source assumed that community college was cheap but not free; our percent error would be much lower had the online source assumed free community college. Our model is otherwise quite close to the expected results. Additional sources of error include: (1) our model does not account for inflation, and (2) our model's values of W, S, O_{avg}, and G are income-dependent means, unlike those of the U.S. Census study, which are overall medians; thus, our final results are justifiably different from those of the other study.

2.6 Sensitivity Analysis

To test the sensitivity of our model, we changed various input parameters by 10% and evaluated the effect on the output for four years of paid education and for two years of free education. The results of this analysis are shown in the table below (we only display the most sensitive scenarios):

Independent Variable and Percent Increase	Greatest % change in T (all four years)	Greatest % change in T (free community college)
10% increase in tuitions	**19%** (2 parent 3 children household, public in-state, $35K)	**21%** (2 parent 3 children household, public in-state, $35K)
10% decrease in tuitions	**18%** (2 parent 3 children household, public in-state, $35K)	**21%** (2 parent 3 children household, public in-state, $35K)
10% increase in scholarship money	**3.7%** (2 parent 3 children household, private, $35K)	**3.9%** (1P1C household, private, $35K)
10% decrease in scholarship money	**3.7%** (2 parent 3 children household, private, $35K)	**4.0%** (1P1C household, private, $35K)
10% increase in time taken to repay loans	**1.9%** (2 parent 3 children household, private, $75K)	**1.5%** (1P1C household, public, $75K)
10% decrease in time taken to repay loans	**2.5%** (2 parent 3 children household, public in-state, $35K)	**4.0%** (2 parent 3 children household, public in-state, $35K)

Figure 4: *Worst-case effects of varying key parameters by* 10% *on the overall total price.*

As apparent in the table, our model is relatively robust, with 10% changes in variables generally producing small percent changes in total cost. Our model was most sensitive to changes in tuition (around 20% resulting change in T); this is attributable to the relatively large value of C_i and its presence in the quadruply (or doubly) iterated summation in (4).

3 Part 2: Show Me the Money!

3.1 Analysis of Problem

In order to assess the short- and long-term rewards and liabilities of an undergraduate STEM degree, it is necessary to compare this option with other academic paths. Some examples are: entering the workforce directly with only a high school diploma, a 4-year non-STEM degree, a 2-year degree (such as an Associate degree from a community college), or attending a community college for two years before transferring and receiving a degree from a 4-year university. The amount of money these students need to pay back in loans, and the time it takes to enter the workforce, changes depending on each academic path. In determining the "best" course of action, it is important to note that this portion of the model accounts only for the earning potential and financial stability offered by a given degree, not the qualitative satisfaction that one may derive from their achievements.

3.2 Assumptions and Justifications

- **Assumption 1:** Personal finances are budgeted according to the 50/30/20 rule of thumb (WSJ).

Justification: The 50/30/20 plan refers to a common rule of thumb in financial planning: 50% of one's income should be reserved for necessities, 30% of one's income should be reserved for everyday spending in a checking account, and 20% of one's income should be reserved for a savings account as well as for investments. This is a simplifying assumption, which aims to account for the majority of the population. Every person will manage their finances differently, but following this rule is standard advice in financial planning courses, and allows us to assess the population at large.

- **Assumption 2:** Unemployment rate is kept constant.

  Justification: In recent years, unemployment has decreased, especially in STEM careers. By using the current rate, we are implementing a realistic worst-case scenario, so that students and workers can plan for the worst.

- **Assumption 3:** The rate at which income increases is constant.

  Justification: Due to limited data, we were unable to find exact numbers for this value. However, we did see that income was nearly directly related to experience, so a linear approximation could be used for the rate.

3.3 Design of the Model

To determine the rewards and drawbacks of different academic paths, a method for determining the overall benefit is needed. This benefit must account for not only the earnings potential of the job, but also the financial security offered by a job. For example, acquiring a job immediately after receiving a high school diploma allows one to begin earning money at an earlier age, but the job market for these workers is more volatile, meaning that savings often need to be used if the job is lost. To address this, we created a variable called "Financial Stability" (FS). This variable addresses the two major concerns of financial stability: the volatility of the market and the resistance of an individual's finances to economic shock, such as the loss of a job. FS is defined to be the product of employment rate (an estimate of how likely one is to keep their job) and total assets (an estimate of the wealth accumulated by an individual). "Assets" (α) is a measure of the time (in years) that one could continue to live their lifestyle without working; it is given by the recursive formula below:

$$\alpha_{n+1} = .2\left(I_n - \frac{T_n}{10}\right) + \alpha_n, \tag{5}$$

where

- I refers to an individual's income,
- n is the number of years since initial employment,
- T_n is the remaining debt to be paid (calculated by subtracting the amount of loan repaid after n years from the initial total payment/cost T_0, which was determined in Section 2.5).

FS is thus expressible as

$$FS_n = (1-u) \times \frac{2 \times \alpha_n}{I}, \tag{6}$$

where u is the unemployment rate for those holding a given degree.

To calculate the Financial Benefit (FB), the Financial Stability term was multiplied by the income available to the worker after necessary payments for items such as food and electricity are made. This leaves half of the original income (based on the 50/30/20 plan cited previously). Multiplying the available income and Financial Security, we obtain the Financial Benefit FB_n:

$$\begin{aligned} FB_n &= (1-u) \times \frac{2 \times \alpha}{I} \times \frac{I}{2} \\ &= (1-u) \times \alpha. \end{aligned} \tag{7}$$

It can be seen that the worker's Financial Benefit is based purely on the unemployment rate and assets, which themselves are based upon the net income provided by the job. This makes sense, as from an economic perspective, better jobs pay more and are more secure.

3.4 Testing and Justification

Using (5) and (7), we calculated the Financial Benefits over time for five different career paths: a high school degree (HS), a STEM Associate degree from a community college (STEM CC), a STEM Bachelor's degree from a combination of community college and undergraduate school (STEM CC+UG), a STEM Bachelor's degree from an undergraduate school (STEM UG), and a non-STEM Bachelor's degree from an undergraduate school (non-STEM UG). The FB curve for each of the paths over 40 years was graphed and is shown below (we obtained data for starting salaries and increases in salary over time from the U.S. Census and NACE):

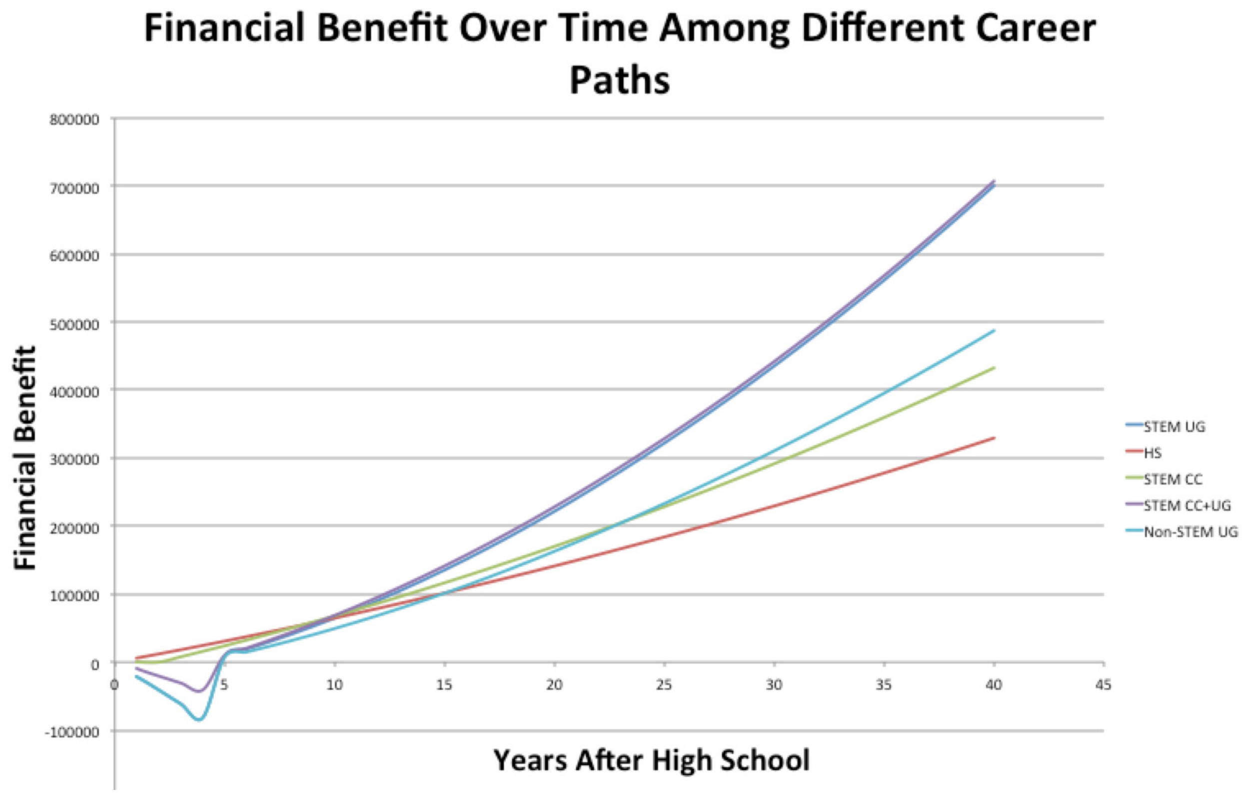

Figure 5: *Financial Benefit Curves for Various Career Paths.*

As expected, the three paths that require an undergraduate education experience an initial dip, as student loans/debt accumulates. However, because people with degrees are often employed in higher-paying jobs, the financial benefit from these academic paths outpaces the other academic paths that require less school, like the high school diploma and Associate's degree. Using the HS path as a comparative baseline, the financial benefit of obtaining any 4-year degree (STEM UG, non-STEM UG, or STEM CC + UG) surpasses the benefit of working straight out of high school in 10 years after high school graduation. Meanwhile, the financial benefit of obtaining a 2-year degree from a community college surpasses the benefit of the HS path in 17 years (note that the financial benefit of community college is not negative because we assume President Obama's initiative pays for two years of community college).

Incidentally, we can compare these results to the research report "Education Pays" published by CollegeBoard. They looked at the cumulative net earnings of a high school diploma, Bachelor's degree, and Associate's degree. According to their model, the Associate degree academic path surpasses a high school graduate's net earnings at age 34, and the Bachelor's degree academic path surpasses a high school graduate's net earnings at age 32 if they take advantage of federal grants, as we have assumed in our model. Age 34 is 16 years after high school graduation, which is almost identical to our estimate of surpassed financial benefit after 17 years with a 2-year degree. Age 32 is 14 years after high school graduation, which is still close to our estimate of surpassing the financial benefit of just a high school diploma in 10 years with a 4-year degree.

At the end of a 40-year career, we can compare the differences in financial benefits obtained from getting a college degree. The financial benefit of graduating with just a high school diploma has an index value of 330,000. Meanwhile, the benefit of obtaining a 2-year Associates degree from a community college is indexed at 430,000. The benefit of obtaining a 4-year non-STEM Bachelor's degree is 480,000. Finally, the financial benefit of both a 4-year STEM university degree, or 2-year community college plus 2-year university STEM degree, is indexed at a whopping 710,000, almost twice the benefit of just a high school diploma. Note that Financial Benefit does not correlate with net income. We would also like to disclaim that these index values are relative; chances are economic conditions will change drastically over the course of a 40-year career, so these FB values are just projections. However, one conclusion is evident—obtaining a 4-year STEM degree is greatly beneficial.

3.5 Sensitivity Analysis

Though we did not have time to actually calculate sensitivity, there are ways in which we could have done it. We could have varied the 50/30/20 rule, as this was a major assumption, and determined how much the results depended on that assumption. We could also have varied our results in Part 1, which affects the debt to be paid. Varying any other factors (like unemployment rate) and determining the effect on financial benefit would be suitable too.

4 Part 3: Life is Short...A Game Theory Approach

4.1 Analysis of Problem

While it is easy to measure the technical, quantitative factors that affect the quality of life, it is a much more difficult task to measure the subjective, qualitative factors. To do so requires a measure of an individual's values. A model that adequately covers both fronts will quantify many subjective factors, such as desire to remain close to family, ambition, and desire to leave an impact upon the community.

4.2 Assumptions and Justifications

- **Assumption 1:** The six traits we use are comprehensive enough give a good educational fit for the student.

  Justification: This is a simplifying assumption. With more traits, the model is more comprehensive but also more complex and computationally intensive.

- **Assumption 2:** The student is not aiming for any education beyond the undergraduate level.

Justification: It is difficult to find data on the salaries and careers of graduate students, since there is a smaller proportion of them when compared to the population of undergraduate students; thus, this is a simplifying assumption.

- **Assumption 3:** We only need to use two traits to create the dominant strategy.

  Justification: Given that we need to create a 2-by-n matrix, it is necessary to truncate the matrix so that one side may have only two "strategies," as is required for this solution. This is explained in greater detail in the solution.

4.3 Design of the Model

In order to account for the effect of qualitative factors on overall quality of life, a personal preference survey can be developed to determine what a student's values are. Voting theory can then be applied to quantify the strength of their values. A second survey can be given the general public to see how individuals that followed various career paths ranked their values to see what values are associated with which academic paths. Finally, game theory can be applied to compare the student's self-determined values against those determined by the population at large to suggest possible career paths.

Personal preference surveys have long been used to determine one's interests and desires, such as Edward's Personal Preference Survey, which was developed by Allen L. Edwards. Our model marries a version of this type of preference survey with a voting theory approach to quantify the preferences of the individual. Specifically we focus on six traits: desire to remain close to family, desire to earn money, desire to have free time, desire to make an impact on the world, value of education, and ambition. It is known that voting theory can determine a "best" choice if each member in a group ranks their selections. A common method is to use a Borda count method, which awards points based on the position that the selections received from each individual. This form of count can be simulated even if only one individual is voting, by creating multiple questions and asking the survey taker to rank various values against each other. If the questions ask the participant to rank three traits at a time out of six total traits, there are a total of $\binom{6}{3}$ or 20 questions. Now that we have successfully simulated 20 votes, fractions can be created for $\frac{PointsEarned}{PointsPossible}$ for each trait, thus quantifying them.

The second survey would ask the general public to rate, using a 1–5 system, what they see as their values in their current academic path. That is, a high school graduate with no further education would be asked to rate the six traits that we have chosen, and their response would be averaged with all of the other high school graduates surveyed. A possible result of a survey such as this is shown in the so-called key-matrix below.

$$\begin{array}{l} \\ HighSchool \\ CC_{STEM} \\ CC+UG_{STEM} \\ UG_{STEM} \\ UG_{Liberal} \end{array}
\begin{array}{c} \begin{array}{cccccc} Family & Money & Free\,Time & Impact & Education & Ambition \end{array} \\
\begin{bmatrix} 5 & 1 & 5 & 1 & 1 & 1 \\ 4 & 3 & 3 & 3 & 2 & 3 \\ 3 & 5 & 2 & 3 & 3 & 3 \\ 2 & 5 & 1 & 5 & 5 & 5 \\ 2 & 2 & 3 & 3 & 4 & 2 \end{bmatrix} \end{array}$$

To combine the two surveys together in a meaningful way, game theory can be applied. Given the score of an individual, such as the example score below,

$$\begin{array}{c} \\ Values \end{array} \begin{array}{c} \begin{matrix} Family & Money & FreeTime & Impact & Education & Ambition \end{matrix} \\ \begin{bmatrix} 20 & 30 & 15 & 10 & 18 & 9 \end{bmatrix}, \end{array}$$

a zero-sum game can be created which will yield the academic paths that align most closely with the student's preferences. Using the so-called key-matrix and the student-generated responses, a 5 × 2 payoff matrix can be created to describe the game. For the purposes of this analysis, the matrix must be of the dimensions $2 \times n$ or $n \times 2$, or else the mathematics becomes too cumbersome to work with. Thus we must select the two values that were determined to be the most important to the student, i.e., the two traits with the highest point totals. For the sample case we are working, the adjusted payoff matrix is

$$\begin{array}{l} \\ High\ School \\ CC_{STEM} \\ CC + UG_{STEM} \\ UG_{STEM} \\ UG_{Liberal} \end{array} \begin{array}{c} \begin{matrix} Family & Money \end{matrix} \\ \begin{bmatrix} \frac{10}{3} & 1 \\ \frac{8}{3} & 3 \\ 2 & 5 \\ \frac{4}{3} & 5 \\ \frac{4}{3} & 2 \end{bmatrix} \end{array}$$

In this game, five equations can be created, one for each academic path. The highest intersection between two of the lines that allows for the creation of a horizontal line (equilibrium) can be determined to discover which two options are the best for the student. The equations have the following form:

$$V_{HS} = 5q + (1 - q),$$

$$V_{HS} = 4q + 1, \tag{8}$$

where q is the probability of the first column, and thus $1 - q$ is the probability of the second column.

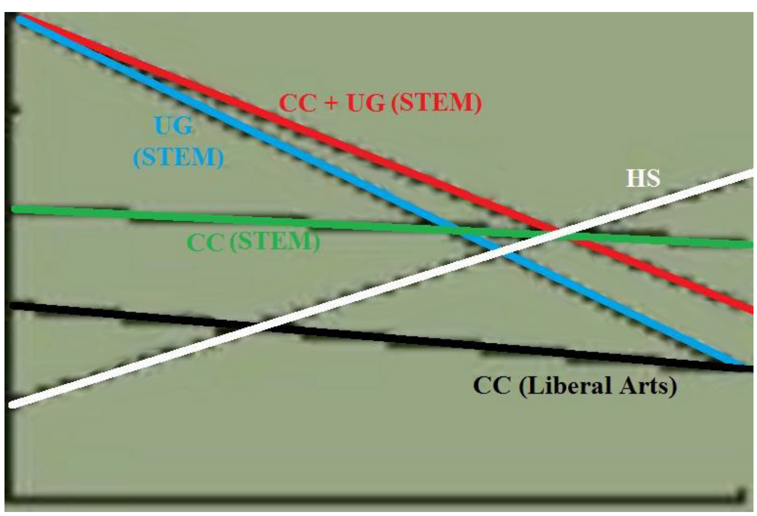

Figure 6: *The five payoff matrix equations (generated on a TI-84 Plus).*

The two lines that created the best intersection were the High School and the combination of community college and a Bachelor's degree in a STEM field. The final discriminator applied was the value of education in the original student survey. If the survey value was high (> 15), the academic path with a higher education was chosen. In this case, the value for education was 18, so the Undergraduate degree path was selected as the best path for the student.

4.4 Justification

The voting theory and game theory models are both valid approaches in their own right; however, using them in conjunction allows for a good indicator of interest and can allow a student to explore options they may have not considered before. By applying a personal preference survey with a voting theory approach, we were able to effectively quantify the subjective factors that affect quality of life. However, this approach did not lose sight of economic realities, as the economic truths exposed in the first two sections of this paper were applied to create accurate values for the key-matrix. The graph theory approach also allows us to rank the various academic paths in order of potential interest to the student.

5 Strengths and Weaknesses of Model

Our model has several strengths and several weaknesses. Strengths include:

1. Our methods are highly customizable and adaptable. We consider a wide variety of factors (e.g., family income, household size, cost of attendance, etc.) to give a result tailored to the individual or household's preferences and characteristics. Our aggregate statistics, such as total cost, are also customizable and specific to different income brackets and types of colleges.
2. Our model is both original and realistic. Our computations (except for federal aid, which is not calculable using any publicly available formula) are developed from our assumptions and fundamental mathematical/economic principles but are still relatively close to values from other studies.
3. Our model is robust. Moderate changes in initial parameters produce relatively small (at most moderate) changes in our results.

However, our model also has a few drawbacks. These include the following:

1. The flexibility of our model is double-sided—though it is adaptable and well tailored to individual circumstances, a large amount of input is required to obtain the best results. This could be potentially tedious for the user.
2. We made the simplifying assumption that several factors, such as tuition, loan rates, and unemployment rate, do not change. In reality, these factors are not constant. However, in Part 1, a student can account for tuition changes and other price changes, because the costs and aid are summed over four years, so each year can be individually modified.
3. Due to computational limitations, we could not analytically find the optimal strategies

for payoff matrices with more than two columns. Thus, we were restricted to the top two preferences for the user.

4. We failed to consider any graduate or professional school. Doing so would have made our model more complete, but would also have added several layers of complexity.

6 Further Work

Building on our weaknesses, we feel that given the time and resources, we could have improved our model. In Problem 1, large variables such as tuition could have been broken up into components for a more accurate representation of the cost of attendance. In Problem 2, a more detailed study into the nature of unemployment and income rates would allow for Financial Benefits to be more accurate. In Problem 3, more traits could be added to add more options for students participating in the survey. Also, using Brown's Algorithm, an analytical approach to solving games, all six qualities in the student survey can be used to determine an optimized path in a more holistic view.

An intriguing piece of the project we came across was a statistic from the U.S. Census stating that 75% of workers with STEM majors do not work in a STEM field. The implications this has on these industries are great, and it would be beneficial to these industries to study the effect that the 75% would have on the STEM field if they were to begin work a field suited to their major.

7 Conclusion

The first part of our model determines the total amount a household is expected to pay to send a child through college. Given the household factors of annual income, the number of earners in the family, and the number of dependents in the family, we calculated the expected financial aid, in the forms of federal grants, work-study options, and scholarships. We then subtracted this aid from the total cost of attendance of a university. The remaining costs, if any, are to be paid by the household in the forms of out-of-pocket spending or loans. Accounting for interest, we calculated the expected payment amount for six different household scenarios. Our maximum error from values from the U.S. Census was 29%, which is okay because we do not account for all economic fluctuations, and our averages are not entirely representative of American households.

Whereas the first part of our model addresses the costs of college, the second part of our model addresses the overall monetary benefits, and incorporates a graphical cost-benefit analysis of education over time. Using a set of recurrence relations, we compared financial stability and net financial benefit as a function of time for individuals with the following levels of education: high school, STEM at a community college, STEM at a 4-year college, and non-STEM at a 4-year college. We found that students who receive education past high school are initially less financially stable because they incur debt and cannot work as much as high school grads can, but their net financial benefit eventually eclipses that of the high school graduates, about 10–15 years after high school.

Lastly, we used a combination of voting theory and game theory to construct a student preference survey that guides them to their best option in their academic career. The survey determines the best values of the six available traits to run through a zero-sum game to optimize the value of the game for the student. The values generated by the student, especially the Education value, shows the student their ideal path for moving on into the workforce. Through the use of this model, we hope that students can make more informed decisions on their career and academic pursuits and that the needed workforce for the rapidly expanding fields in STEM can be created.

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