Elk River High School – Team #5560, Elk River, Minnesota  
Coach: Curt Michener  
Students: Joe Evans, Chase Gauthier, Zach Glasgow, Jordan Haack, Peter Jones

M3 Challenge Third Place Team, Cum Laude Team Prize: $10,000

***Note: This cover sheet has been added by SIAM to identify the winning team after judging was completed. Any identifying information other than team # on an M3 Challenge submission is a rules violation.

***Note: This paper underwent a light edit by SIAM staff prior to posting.
“STEM Sells: What is Higher Education Really Worth?”

I SUMMARY

Dear High School Administrators,

Our team has recently developed a model for calculating the total cost a student will need to pay for college, beyond just the “sticker price” that the college advertises. Our model takes into account many factors, such as the costs of tuition and other expenses, as well as how much the family can afford to contribute to a college education, given their circumstances. We consider the fact that the student might need to take out loans and pay interest on those loans in addition to the “sticker price.” We tested our formula for finding the cost for six sample students to attend three typical colleges, and the results were intriguing. We found that, all things considered, middle income families pay the most for college, because they receive fewer grants than lower income families but have to take out larger loans than the upper class. We found that having extra siblings in college simultaneously benefitted lower income students (who received larger Pell Grants) but hurt middle and upper class students by dividing up their family contributions. Our model could easily be adjusted to account for President Obama’s plan to give free two-year college opportunities to qualifying students. Although our model displayed a weakness in determining costs for large, upper class families, we have found a revision that would improve a future version of our model.

We also created a model that accurately represents the benefits and drawbacks for students pursuing higher education. Acknowledging that different levels of education offer different rewards and different costs, we developed a tool to weigh these options against each other. We believe this tool, if you choose to implement it, will prove tremendously useful to the students at your school when the time comes for them to make the important decisions related to higher education. By fully understanding how a college education will affect their lives over time, you can ensure that your students have all the information they need to make such life-changing decisions.

Finally, we created a model to help your students with the difficult task of determining whether they will be satisfied with their chosen careers. We recognize that every student will not take the same path, and we developed a model to help them see this. This model takes into account the student’s affinity for certain activities which pertain to their desired career choice, as well as what type of college they plan to attend and the average income of their desired profession. This model accounts for satisfaction derived from being on the job, as well as weighing the freedom given to you by your available income to pursue other goals. It is important for these students to see the value of choosing a career in which they will not only succeed, but also be satisfied and enjoy themselves. While this model is somewhat subjective, it is paramount that career options and the true worth of higher education be explored for each pupil. This model will allow your students a necessary view into their future. Thank you for your time.

-Team #5560
II INTRODUCTION

A) Background History

According to the Institute of Educational Sciences, 3.1 million high school students graduated on time in 2012 (National Center for Educational Statistics). These graduates have many paths available to them, including attending a four-year university, attending a two-year certificate program, or entering the workforce immediately (with only a high school education). Each path has its benefits and its drawbacks, and each graduate must make a decision that will profoundly impact the rest of his/her life. It is therefore important to assess the true cost of taking each of these paths and the benefits they will give the graduate.

While colleges make tuition costs apparent during the application process, the actual cost paid by students varies greatly. The real cost of attending college takes into account such factors as the amount received in need-based funding, other grants, and the student’s academic profile. Currently, graduates looking to attend college are expected to coordinate their own payment plan, which often includes taking out loans to make up the difference between the amount they can pay and the amount required by the school, minus grants and other aid. These loans come in two forms: public and private. Private loans work just like any other loan would, although the details of the loan such as interest rate and length of the loan are discussed and accepted by both the lender and the graduate. Public loans are controlled by the federal government, with a fixed interest rate for the length of the loan period as well as some protections guaranteed to all applicants. Depending on the situation, a graduate may apply for one, both, or neither type of loan. These loans accrue interest which must be paid off by the applicant after graduation in a similar manner to a mortgage or other long-term loan. In this report we assess the total cost of attending college, including interest accumulated by student debt.

B) Restatement of the Problem

We were asked to create a model to determine the total worth of higher education compared to the true cost of attendance. This model will be applied to a number of faux graduates representing a wide range of circumstances to determine how useful college would be for an individual. We were further asked to create a model that will determine the short- and long-term rewards and liabilities of pursuing various levels of higher education, ranging from a high school diploma only to a four-year bachelor’s degree. Finally, we were asked to develop a ranking system that high school students could use to help them make informed choices regarding their educational future.

C) Relevance

This model is extremely important as it will be a valuable tool to over 3 million graduating high school seniors (National Center for Education Statistics) this year alone, not
to mention the millions more who will be able to use this model in the future. Going to college puts an individual in $24,800 of debt (U.S. News). Some individuals may be better served not going to college, saving themselves from the debt and starting in the workforce four years earlier. It is important for everyone to have a tool to have a complete and unbiased view of the costs of higher education while making their decision after graduation. Other individuals would be well served with a two-year associate’s degree rather than a four-year bachelor’s, and it is important for our model to include data for these individuals as well. Finally, if a four year degree is the best option for an individual, our model will give them a realistic view of their future, including the total amount they will have paid for their education and the effect it will have on their projected quality of life compared to someone with other levels of education.

III Looking Beyond Sticker Prices

A) The Problem

We were asked to create a model to determine the cost of higher education, factoring in the true cost of debt accumulated during and after college. We were asked to determine how this cost would vary from graduate to graduate based on their family’s income, scholarships, and the costs of financing a college education.

B) Assumptions

- Students will contribute 20% of their assets and 50% of their income, while parents will contribute 12% of their assets. (These constants could be changed in our model; however, our data assumes these values.)
- The amount of money for college available to the student from parents depends only on income, taxes, caring for dependents, and a broad category of other expenses (including house and car payments, utilities, etc.) that account for an amount proportional to the total income (which is a viable assumption, according to CNN Money).
- If a parent has multiple children in college, they will contribute to their children’s college expenses equally. A student will only contribute to his/her own college fund.

C) Approach

Our model stems from the idea that the college will be charging the student a certain cost to attend the school each year, simply calculated by costs incurred minus aid granted. So we begin with the simple equation,

\[ P = C - A, \]

where \( P \) represents the total price the student will need to pay directly to the school (though the student will likely pay more than this, in the form of student loans, unless they can
immediately cover their total costs), \( C \) is the “sticker price” of the school (including travel, supplies, and living expenses), and \( A \) is financial aid from all sources. So when we consider the price the student will pay throughout the course of their education, which we will let be \( X \), we get another simple equation,

\[
X = P + I,
\]

where \( I \) is the interest the student will incur if they weren’t able to pay their costs of attendance right away, which, statistically, is the majority of students. 69% of graduating seniors had student loan debt, with an average debt of $28,400 (U.S. News)

At the time of their enrollment, a student and his family will only be able to contribute a certain amount of money to their education, and will only win a certain amount of scholarships (if any), while the rest must be covered by a loan. First, we will calculate how much money a student and his/her family can afford to pay during their enrollment based on their individual family circumstances. This number, which we will call \( FC \) (the family contribution, which includes both the student and parent), depends on many factors. Namely, the parents’ and student’s income and assets, as well as the number of dependents the parents will otherwise need to support.

The total family contribution to all colleges \((FC_{total})\) for all of their dependents in college comes from the parents’ and student’s income and assets. We get the equation

\[
FC_{total} = \min(\alpha(Al_{parents}) + \alpha(Al_{children}) + \alpha(AA_{parents}) + \alpha(AA_{children}), P),
\]

where \( Al \) stands for Available Income (income that can be contributed to the college costs), \( AA \) stands for Available Assets (assets that can be contributed for college costs), and the function \( \alpha(n) \) represents the amount of money that can be contributed to the college of the total money available. We include the minimum function because the family cannot contribute more money than \( P \), the total price to attend.

The expected student contribution, adhering to the standard set forth by the Department of Education in the FAFSA form, is 20% of the student’s total available assets (bank account balances, investment worth, property owned, etc.) and around 50% of their total available income (total tax allowances subtracted from total income). Similarly, according to the standards of the Department of Education, the parents are expected to contribute about 12% of their assets (ifap.ed.gov).

However, the most important factor is the amount of money that is available for the college fund from the parents’ income. A large portion of the parents’ income is used, though, and cannot be reserved for a college fund. These expenses primarily include taxes, house/car payments, living expenses, and caring for dependents.

First, we will consider taxes paid versus total income. Note that total taxes include federal, state, and local taxes. We found some statistics from the IRS’s website:
<table>
<thead>
<tr>
<th>Income (Dollars)*</th>
<th>Total Taxes Paid (%)*</th>
</tr>
</thead>
<tbody>
<tr>
<td>$13,000</td>
<td>17.4</td>
</tr>
<tr>
<td>$26,100</td>
<td>21.2</td>
</tr>
<tr>
<td>$42,000</td>
<td>25.2</td>
</tr>
<tr>
<td>$68,700</td>
<td>28.3</td>
</tr>
<tr>
<td>$105,000</td>
<td>29.5</td>
</tr>
<tr>
<td>$147,000</td>
<td>30.3</td>
</tr>
<tr>
<td>$254,000</td>
<td>30.4</td>
</tr>
</tbody>
</table>

*Data taken from irs.gov

After performing our calculations, we found that the best regression for this data set was a logarithmic regression ($r^2 = 0.91$). This model retains accuracy for all but abnormally high incomes, and these high incomes ($110,000+ per year) will be analyzed later in our report. Our model is as follows:

$$\text{parents income after taxes} \approx \text{income} - 4.699\ln(\text{income}) + 25.867$$

Second, we need to consider how much money the parents spend per year on house and car payments, as well as utilities. The average American spends 43% of their money on these costs, regardless of income levels (CNN Money). We will assume that this value is accurate enough for our purposes. A proportional approximation is sensible because people with larger incomes generally buy larger houses and more expensive cars.

Finally, we need to consider the cost of raising children. We found the following data for how much money the average American family spends on raising a child aged 15-17 over different income levels (Daily Mail).
By plotting midpoints and performing a linear regression, we found the following regression ($r^2 = 0.98$) for the data:

\[
\text{cost per child per year} \approx 6027 + 0.127(\text{income}).
\]

This is logical, because a more wealthy family spends more money on each of their individual children than the average family does. However, in a single parent household, this cost is on average $400 more expensive per month, due in general to daycare costs (Care), so

\[
\text{cost per child per year} \approx 6027 + 0.127(\text{income}) + 4800(2 - \#\text{Parents})
\]

Lastly, it is essential to note that it *isn’t possible* to calculate $\alpha(AI_{parents})$. The amount of money the parents will contribute will vary significantly based on the parents. Some parents may contribute no money, while others are willing to pay their child’s entire college fund. Therefore we will define $\rho$ to be the proportion of the available income the parents are willing to contribute, so that (let $\text{income} = i$)

\[
\alpha(AI_{parents}) = \rho \ast (\text{available income})
\]

\[
= \rho \ast (\text{income} - \text{taxes} - \text{home & living & other expenses} - \text{child expenses})
\]

\[
= \rho \ast (i - 4.699ln(i) + 25.867 - 0.43i - (6027 + 0.127(i) + 4800(2 - \#\text{Parents}))\#\text{Kids}).
\]

So now we know that

\[
FC_{\text{total}} = \min(\alpha(AI_{parents}) + AI_{students} \ast 0.5 + AA_{parents} \ast 0.12 + AA_{students} \ast 0.2, P),
\]

where $\alpha(AI_{parents})$ is defined as above.

Now if a family has more than one child in college, we will assume that the parents will contribute equally to each child’s education (and that the students will only contribute to their own funds). Therefore for each individual child’s expected family contribution, the contributions from the parents will be divided by the number of kids in college at that point. We then get

\[
FC_{\text{yearly}} = \min\left(\frac{\alpha(AI_{parents}) + AA_{parents} \ast 0.12}{\# \text{kids in college}} + 0.5AI_{students} + 0.2AA_{students} \ast \frac{P}{\text{years}}\right).
\]

Now we need to calculate how much money the student will need to take out in the form of loans, and how much more money will need to be paid in interest throughout the student’s life. The size of the loan can simply be found by subtracting the amount the student’s family can pay during enrollment from price of attending the school. If we let the size of the loan be $L$, we see that

\[
L = P - FC_{\text{yearly}} \ast \text{years},
\]

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so now we will need to calculate the amount of money the student will need to pay in interest, by taking into account inflation and average interest rates.

We want to consider the fact that inflation will make it slightly easier for the students to pay off their debt, and we want our value for \( X \) to be represented in today’s (2015) dollars. Using the Fisher Equation, we know that the real interest rate is the best way to estimate the interest the student will incur in terms of the worth of money today. According to Fisher (Wikipedia)

\[
\text{real interest rate} = \text{nominal interest rate} - \text{inflation rate}.
\]

For our purposes, we will assume that this is an equality. Furthermore, we know that the average nominal interest rate for college loans (a Stafford Loan, which is generally compounded daily) this year is 4.66% annual interest for undergraduate students (Dept. of Education). Also, because we lack the ability to predict future inflation rates, we will assume a constant inflation rate of 1.38% (Mitchell). This gives us the value

\[
\text{real interest rate} = 4.66\% - 1.38\% = 3.28\%,
\]

which is the value we will assume to be the interest rate, accounting for inflation, for the average loan taken out by a college student for his/her education. And since we got our value for a yearly interest rate, we can now calculate our value of \( I \) in terms of \( L \). Assuming a \( t \) year repayment period, we get that (through the interest formula)

\[
\text{amount paid on loan} = L \left(1 + \frac{r}{n}\right)^{nt} = L(1.0328)^t = L + I_t.
\]

(Note that \( n = 1 \), since we have an annual interest rate and we compounded yearly.) Therefore the amount of interest paid, \( I_t \), more than the given price, is approximately given by

\[
I_t = L(1.0328)^t - L.
\]

For the typical 10-year loan, this is \( I_{10} \approx 0.38L \). For a 15-year loan, \( I_{15} \approx 0.62L \). The amount of time a student will spend repaying their loan is dependent on their income after graduation, so we won’t assume a particular value for \( t \).

Now we will discuss calculating some values of \( C \), our costs incurred for the college. This is in effect our “sticker price,” but modified for inflation and other factors. We will need to consider many important factors, such as tuition, travel, food, and living expenses. We will need to separate schools into different categories and analyze their costs separately. We will use the categories that the College Board uses, namely, Two-Year (including Community College), Public Four-Year (in-state and out-of-state), and Private Four-Year. We will describe typical costs for each category, as well as give examples.

Tuition will include the cost spent on classes (later will will consider money saved by taking AP, IB, CLEP, SAT II’s, or PSEO classes) and supplies needed to attend. Living expenses will cover dorms, food, entertainment, and other costs such as travel. We will assume that these costs will not change based on the family’s income (although, for example, a lower income student may not fly to and from college over breaks, and may incur less travel costs).
<table>
<thead>
<tr>
<th>Type</th>
<th>Avg Tuition &amp; Supplies*</th>
<th>Avg Living Expenses and other Amenities*</th>
<th>Total Cost per Year</th>
</tr>
</thead>
<tbody>
<tr>
<td>Two-Year (or Community College)</td>
<td>$3,347</td>
<td>$7,705</td>
<td>$11,052</td>
</tr>
<tr>
<td>Public Four-Year (In-state)</td>
<td>$9,139</td>
<td>$9,804</td>
<td>$18,943</td>
</tr>
<tr>
<td>Public Four-Year (Out-of-state)</td>
<td>$22,958</td>
<td>$9,804</td>
<td>$32,762</td>
</tr>
<tr>
<td>Private Four-Year</td>
<td>$33,231</td>
<td>$11,188</td>
<td>$44,419</td>
</tr>
</tbody>
</table>

*Data taken from The College Board for 2014-2015.

For calculating the total cost of college for a given family (C), factors that need to be addressed are Advanced Placement credits, Tuition (including tuition inflation), and Living Expenses. For calculating the financial aid (A), we need to consider Federal Grants, State/Local Grants, and Institutional Grants. We have selected eight colleges to act as examples, using data we found on College Calc. The eight colleges were selected according to region, with two chosen from each of the Southeast, Midwest, Northeast, and Western Coast regions of the United States. Two colleges were then selected from one state in a given region, one a private university and the other a public university. After the colleges were selected, we examined the total cost for each of the colleges for one year without any benefits being used, such as AP credits or grants. Because public institutions have different costs depending upon whether or not the student is a resident of the state the school is in, there are different values for in-state and out-of-state costs.

However, other factors need to be considered in order to find the real cost of each one of these colleges in particular. In the area of transfer credits, we have taken the average number of AP classes taken by each person throughout the United States. The average number of AP classes taken by each individual person is 5 (The College Board). From this information, we picked the top 5 most taken AP tests, which are, in order, English Language and Composition, United States History, English Literature and Composition, Calculus AB, and United States Government and Politics (The College Board). It was noted that Calculus AB has the highest number of persons at or above a score of 3 and persons with a 5, with 57.7% and 22.6% (AP Credit Policy), respectively. Therefore, we decided that the student would receive a 4 on this exam and a 3 on the other four. From these assumptions, we determined that the average student would receive a different amount of credits based on each college's policies, which a range of AP credits being from 0 to 25 credits given. We then assumed, because of the nature of the four-year programs, that we take these credits out of the average of 120 credits needed for graduation. We then determined the average Federal Grants,
State/Local Grants, and Institutional Grants. The data are displayed on the table below to find the total cost to the family.

<table>
<thead>
<tr>
<th>Universities</th>
<th>Original Cost (4 years)</th>
<th>Transfer Credits</th>
<th>Federal Grants (per year)</th>
<th>State/Local Grants (per year)</th>
<th>Institutional Grants (per year)</th>
<th>Cost to Family</th>
</tr>
</thead>
<tbody>
<tr>
<td>Florida State University - In-State</td>
<td>$69,640</td>
<td>16</td>
<td>$4,462</td>
<td>$2,413</td>
<td>$2,304</td>
<td>$30,484</td>
</tr>
<tr>
<td>Florida State University - Out-of-State</td>
<td>$127,452</td>
<td>16</td>
<td>$4,462</td>
<td>$2,413</td>
<td>$2,304</td>
<td>$80,556</td>
</tr>
<tr>
<td>Harvey Mudd College</td>
<td>$249,840</td>
<td>0</td>
<td>$6,845</td>
<td>$9,223</td>
<td>$27,193</td>
<td>$76,796</td>
</tr>
<tr>
<td>Pennsylvania State University - In-State</td>
<td>$115,216</td>
<td>4</td>
<td>$4,385</td>
<td>$3,533</td>
<td>$6,301</td>
<td>$56,192</td>
</tr>
<tr>
<td>Pennsylvania State University - Out-of-State</td>
<td>$165,512</td>
<td>4</td>
<td>$4,385</td>
<td>$3,533</td>
<td>$6,301</td>
<td>$104,812</td>
</tr>
<tr>
<td>University of California - Berkeley - In-State</td>
<td>$118,288</td>
<td>13</td>
<td>$5,080</td>
<td>$12,130</td>
<td>$9,393</td>
<td>$7,012</td>
</tr>
<tr>
<td>University of California - Berkeley - Out-of-State</td>
<td>$209,800</td>
<td>13</td>
<td>$5,080</td>
<td>$12,130</td>
<td>$9,393</td>
<td>$55,276</td>
</tr>
<tr>
<td>University of Miami</td>
<td>$230,304</td>
<td>0</td>
<td>$5,374</td>
<td>$5,163</td>
<td>$23,765</td>
<td>$93,096</td>
</tr>
<tr>
<td>University of Minnesota - Twin Cities - In-State</td>
<td>$92,348</td>
<td>25</td>
<td>$4,611</td>
<td>$3,636</td>
<td>$4,867</td>
<td>$29,840</td>
</tr>
<tr>
<td>University of Minnesota - Twin Cities - Out-of-State</td>
<td>$117,348</td>
<td>25</td>
<td>$4,611</td>
<td>$3,636</td>
<td>$4,867</td>
<td>$49,632</td>
</tr>
<tr>
<td>University of Pennsylvania</td>
<td>$240,008</td>
<td>0</td>
<td>$6,777</td>
<td>$2,853</td>
<td>$35,371</td>
<td>$60,004</td>
</tr>
<tr>
<td>University of St. Thomas</td>
<td>$182,376</td>
<td>8</td>
<td>$4,567</td>
<td>$3,503</td>
<td>$14,517</td>
<td>$82,820</td>
</tr>
</tbody>
</table>

Ultimately, the best way to calculate the value of $P = C - A$ for a specific college is to look at the website of the college itself.
We now need to calculate the cost of the college for the whole duration spent there. College fees increase every year in a fairly linear manner (The College Board). We will therefore assume that the college price inflation rate stays constant for the next four years. We have an inflation rate of 0.025 for a two-year college, 0.035 for a public four-year college, and 0.022 for a private four-year college (The College Board).

We can now calculate our average values of \( C \), the total cost incurred for the entire duration of attendance, for these types of colleges, with inflation taken into consideration, for the school year starting in 2015 (so we apply inflation for the fact that our data came from 2014).

\[
C_{\text{two-year}} = (1.025^1 + 1.025^2)(11052) = $23,874 \\
C_{\text{four-year public in state}} = (1.035^1 + 1.035^2 + 1.035^3 + 1.035^4)(18943) = $82,638 \\
C_{\text{four-year public out of state}} = (1.035^1 + 1.035^2 + 1.035^3 + 1.035^4)(32762) = $142,923 \\
C_{\text{four-year private}} = (1.022^1 + 1.022^2 + 1.022^3 + 1.022^4)(44419) = $187,666
\]

We now must consider the amount of aid the student will receive in the form of grants and scholarships (we have already considered loans). We will calculate the amount of aid the student will get from the government; however, we cannot determine a formula to calculating how much scholarship money a student will receive from outside sources or from the school for merit-based accomplishments. We will let this incalculable value be \( S \), and we will not attempt to model the student’s merits. Students can plug in the value themselves and see how that affects their costs. If we let \( Pell \) represent the value of the Pell Grant awarded to a student, we get that

\[ A = Pell + S. \]

It would be redundant for us to include the formula for calculating a Pell Grant because it can be found on the FAFSA.

We can now calculate \( X \) for our six scenarios.

\[
X = P + I = C - A + I = P + ((1.0328)^t - 1)L \\
X = P + ((1.0328)^t - 1)(\max(0, P - \frac{(Al_{\text{parents}})^{\frac{\alpha P_{\text{parents}}}{\text{# kids in college}}} + 0.12}{0.5 AI_{\text{student}} + 0.2 AA_{\text{student}}}) \times \text{years})
\]

where

\[
P = C_{\text{total}} - (S_{\text{yearly}} + Pell_{\text{yearly}}) \times \text{(years attending)} \\
C = \text{total "sticker price"} \\
S_{\text{yearly}} = \text{yearly scholarships received} \\
t = \text{number of years needed to pay back loan} \text{ (generally } t = 10 \text{ or } 15 \text{)} \\
i = \text{parents' untaxed income} \\
Al_{\text{parents}} = \rho \times (0.57i - 4.699ln(i)) + 25.867 - (6027 + 0.127i + 4800(2 - \# P_{\text{parents}})(\# K_{\text{ids}}) \\
\rho = \text{the proportion of their available income the parents will contribute.}
\]
D) Sample Cases

For our six cases, we will first assume that $t = 10$ (the time it takes the average graduate to pay off his/her debt) and that $\rho = 0.5$. Note that the effect of the single parent family mainly affects the cost of raising the child. We will also assume that the student isn’t receiving any merit-based aid ($S = 0$), and that the student isn’t bringing any college credits with them. For the family with a $35,000$ annual income, we will assume that the assets are negligible. For the family with a $75,000$ annual income, we will assume $10000$ in assets, and $25,000$ in assets for the family with a $125,000$ annual income. We will assume that our child is the middle child, so that there are always two dependents attending college at a time. We will assume that the student has no income or assets. These assumptions are reasonable and necessary to complete the Pell Grant form and for our model.

We will consider the cost paid for 3 different “sticker prices” and compare them.

<table>
<thead>
<tr>
<th>(income, #kids, #parents)</th>
<th>Pell Grant</th>
<th>EFC by calculator*</th>
<th>FC by model</th>
<th>total paid ($X$) when $C_{2 \text{yr}} = $50,000</th>
<th>total paid ($X$) when $C_{4 \text{yr}} = $125,000</th>
<th>total paid ($X$) when $C_{4 \text{yr}} = $200,000</th>
</tr>
</thead>
<tbody>
<tr>
<td>(35000,1,1)</td>
<td>3780</td>
<td>1775</td>
<td>2327</td>
<td>55060</td>
<td>148188</td>
<td>251755</td>
</tr>
<tr>
<td>(75000,1,1)</td>
<td>0</td>
<td>11297</td>
<td>12385</td>
<td>59610</td>
<td>153743</td>
<td>257310</td>
</tr>
<tr>
<td>(125000,1,1)</td>
<td>0</td>
<td>27005</td>
<td>25259</td>
<td>50000</td>
<td>134128</td>
<td>237695</td>
</tr>
<tr>
<td>(35000,3,2)</td>
<td>5730</td>
<td>0</td>
<td>0</td>
<td>53219</td>
<td>140962</td>
<td>244530</td>
</tr>
<tr>
<td>(75000,3,2)</td>
<td>1880</td>
<td>3580</td>
<td>2536</td>
<td>61920</td>
<td>158364</td>
<td>261932</td>
</tr>
<tr>
<td>(125000,3,2)</td>
<td>0</td>
<td>11065</td>
<td>6069</td>
<td>64421</td>
<td>163366</td>
<td>266933</td>
</tr>
</tbody>
</table>

*finaid.org

E) Analysis

First, our calculated FC by the model is fairly close to the values given by the FEC calculator on finaid.org. The only category with a significant difference is the case with a five-person, high income family. However, this deviation can be simply explained by the fact that our logarithmic regression fails at high income values. If we had more time, we could adjust our model by changing the equation

$$\text{parents income after taxes} \approx \text{income} - 4.699 \ln(\text{income}) + 25.867$$

to account for the fact that total taxes level off at about 31%, to say perhaps

$$\text{parents income after taxes} \approx \max(\text{income} - 4.699 \ln(\text{income}) + 25.867, 0.69 \times \text{income})$$
to account for this fact. This would fix the fact that we overestimated taxes for the $125,000 student, which caused an underestimated value for the family contribution. Unfortunately we lack the time to adjust our model, so for now our model has the limitation that it underestimates family contributions for families above $110,000 income and therefore overestimates final cost for those students.

As it turns out, the people whose family’s will pay the most are people whose family is middle class. The lower class students will pay less due to Pell Grants lowering their total costs, while the upper class students will pay less because they will not need to take out a large loan. However, the system also favors the students whose parents pay a large portion of their available income (high values of $\rho$) who are generally more rich, because then the student doesn’t have to pay them by him/herself.

Finally, lower class students are clearly at an advantage when they have siblings in college, because they get a larger Pell Grant without losing any of their generally negligible parent contribution to their sibling. However, middle and upper class students are greatly disadvantaged by having a sibling because it hurts their family’s financial contribution more than it helps their pell grant, which leads to a larger loan.

Though not included in this chart, the students at the greatest disadvantage are students whose parents chose not to contribute to their college fund. For example, if the student described by $(125000,3,2)$ had parents who chose not to contribute to his college fund, he would need to pay $276,180, according to our model (and all by himself, too), which is considerably higher than the other cases.

If President Obama’s recent suggestion to make two years of community college free for qualifying students (2.5 GPA minimum and making progress towards a degree) went into effect, it would affect our model in a trivial way. We will simply not have to consider tuition in our process of calculating $C$ when the conditions for his plan are met, and then plug-and-chug from there.

IV Show Me the Money!

A) The Problem

We were asked to create a model to determine the short- and long-term liabilities and rewards of entering the workforce with different career preparations, including a STEM (Science, Technology, Engineering, and Mathematics) degree, another four-year degree, a two-year degree, and a high school diploma. The factors to be considered include earning potential and financial stability.

B) Assumptions
The jobs chosen to be averaged for all STEM fields, non-STEM fields, Associate’s Degree, and High School Diploma are representative of jobs held by all individuals with their respective education level, ignoring outliers.

- Income is race and gender neutral.
- Any major life events, such as getting married, having children, or suffering an unexpected illness, will affect all educational paths approximately equally. We can safely assume this because the cost of all these situations would affect an individual in the same way. The cost of getting married would simply double the cost of living for any scenario. The cost of raising a child would be nearly constant for any scenario: food, clothing, a room, etc. The cost of medical procedures and medication would not differ too much person to person either, assuming similar insurance coverage in any case.
- The individual will get an average job with an average income immediately upon graduation, and will not change jobs.
- The individual will not have a job or draw any form of income while receiving their education.
- Real income remains relatively the same during the individual’s entire life. In other words, the relationship between cost of living and income will remain the same, and changes to this relationship are negligible.

**C) Approach**

Our model accounts for an economically unstable period. In an economic boom, job security will play less of a role in our model, and it should be changed accordingly. Due to the unpredictability of the future, we have developed our model to be accurate during the most recent period of economic instability: the Recession of 2008.

In order to properly contrast the short-term vs. long-term liabilities and rewards of an undergraduate STEM degree as compared to other career preparation paths, we considered short term as 10 years after graduating high school and long term as 25 years after graduating high school. The reasoning behind this is to account for years lost by those who attend college vs. those who enter the workforce immediately out of high school. Other factors that would need to be addressed in creating the model include the debt incurred by our subject who chose to attend college and debt accumulated after entering the workforce, compared to someone who entered the workforce right out of high school and incurred no debt. Also, salary is a main factor in considering short term vs. long term. To address this, we selected the top three career paths for a STEM bachelor’s degree, non-STEM bachelor's degree, two-year degree/certificate, and high school diploma only. We then consider each career path case by case. For STEM undergraduate, the careers we chose are computer support specialists, medical scientists, and petroleum engineers. For a non-STEM bachelor's degree, the jobs chosen were supply chain manager, sales manager, and training manager. The jobs chosen for
those with an associates degree are community service specialist, greenhouse manager, and chef. Finally, for those with a high school degree, the jobs chosen were transportation manager, real estate broker, and mechanic.

\[ Economic\ Standing = Net\ Worth \ast Employment\ Rate \]

Economic standing is defined as a measure of two factors—the total value of all the individual’s assets, and the frequency of employment in a given field. We chose these values as the basis for our model because they are measurements of the expected earnings of an individual multiplied by the likelihood that they will earning that amount. Using this method, we define the interval \( \{a, b\} \) as follows: \( a \) is the number of years after high school an individual would enter the workforce, while \( b \) is the year at which the model is being assessed (\( b=10 \) for the short-term assessment, \( b=25 \) for the long-term assessment):

\[
S = N \ast E_r
\]

\[
N = \int_a^b (salary - cost\ of\ living)dt - (Initial\ Debt + Interest\ Accrued) - \int_a^b (yearly\ debt\ payment)dt
\]

\[
E_r = \frac{\text{Total employed in field}}{\text{Total people in field}}
\]

\[
S = \frac{\int_a^b (salary - cost\ of\ living)dt - (Initial\ Debt + Interest\ Accrued) - \int_a^b (yearly\ debt\ payment)dt}{\text{Total people in field}} \ast (Total\ employed\ in\ field)
\]

\( cost\ of\ living\ is\ based\ on\ values\ taken\ from\ MIT’s\ living\ cost\ statistics \)

\( yearly\ debt\ payment = salary \ast \beta \)

\( \beta = Payment\ to\ income\ ratio \)

<table>
<thead>
<tr>
<th>Values of ( \beta )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Debt Payment to Income Ratio (( \beta ))</td>
</tr>
<tr>
<td>$71,681</td>
</tr>
</tbody>
</table>

**D) Analysis**

For the purpose of evaluating the model on a general level, the cost of living was estimated based on statistics from MIT’s living wage data (livingwage.MIT.edu). According to data from the Bureau of Business and Economic Research, Wisconsin accurately represents the cost of living across the United States, assuming cost of living is correlated with average income. Using this, we found that the average cost of living for the average single adult to be
$18,455 per year. This value was applied to all cases considered in the analysis. Student loan debt was considered with a standard student, the case in which the household income was $75,000 with two parents and three children. This standard case was applied to the differing levels of education, using the total debt costs for two-year institutions and the lower end four-year institutions. The values for each of these cases were input into the model for the individual occupations and the averages by field. The values for $S_{10}$ and $S_{25}$ represent the economic standing for each case 10 and 25 years, respectively, after graduating high school (the short- and long-term measurements).

$$E_r = Employment\ Rate$$

$$MAI = Median\ Annual\ Income$$

<table>
<thead>
<tr>
<th>Non-STEM Bachelor’s</th>
<th>MAI</th>
<th>$E_r$</th>
<th>$S_{10}, S_{25}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Farm Manager</td>
<td>$64,760</td>
<td>.969</td>
<td>$240,638, $867,215</td>
</tr>
<tr>
<td>Funeral Director</td>
<td>$60,390</td>
<td>.966</td>
<td>$199,997, $765,612</td>
</tr>
<tr>
<td>Supply Chain Manager</td>
<td>$79,497</td>
<td>.935</td>
<td>$377,692, $1,209,850</td>
</tr>
<tr>
<td>Average</td>
<td>$68,215</td>
<td>.957</td>
<td>$272,769, $947,543</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>STEM Bachelor’s</th>
<th>MAI</th>
<th>$E_r$</th>
<th>$S_{10}, S_{25}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Computer Scientist</td>
<td>$49,930</td>
<td>.949</td>
<td>$102,719, $522,417</td>
</tr>
<tr>
<td>Medical Scientist</td>
<td>$74,590</td>
<td>.955</td>
<td>$332,057, $1,095,762</td>
</tr>
<tr>
<td>Petroleum Engineers</td>
<td>$108,910</td>
<td>.931</td>
<td>$651,233, $1,893,702</td>
</tr>
<tr>
<td>Average</td>
<td>$77,810</td>
<td>.945</td>
<td>$362,003, $1,170,627</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Associate’s</th>
<th>MAI</th>
<th>$E_r$</th>
<th>$S_{10}, S_{25}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Community Service Specialist</td>
<td>$40,050</td>
<td>.97</td>
<td>$10,835, $101,299</td>
</tr>
<tr>
<td>Greenhouse Manager</td>
<td>$35,000</td>
<td>.89</td>
<td>$-36,130, $57,708</td>
</tr>
<tr>
<td>Chef</td>
<td>$44,240</td>
<td>.913</td>
<td>$49,802, $125,338</td>
</tr>
<tr>
<td>Average</td>
<td>$43,850</td>
<td>.925</td>
<td>$46,175, $124,157</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>No Degree</th>
<th>MAI</th>
<th>$E_r$</th>
<th>$S_{10}, S_{25}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transportation Manager</td>
<td>$49,320</td>
<td>.98</td>
<td>$302,477, $756,192</td>
</tr>
<tr>
<td>Real Estate Broker</td>
<td>$39,800</td>
<td>.99</td>
<td>$211,315, $528,288</td>
</tr>
<tr>
<td>Mechanic</td>
<td>$36,610</td>
<td>.957</td>
<td>$173,743, $434,358</td>
</tr>
<tr>
<td>Average</td>
<td>$41,910</td>
<td>.976</td>
<td>$228,920, $572,302</td>
</tr>
</tbody>
</table>

Based upon the average model values for each level of education, there is a direct, positive correlation between higher education levels and higher economic standing. As we expected, college graduates in a short-term time span suffered compared to those who went directly into the workforce. However, in the long-term, once the debt of the college graduates was paid off, those with a high school degree were surpassed by those who obtained a
bachelor’s degree and went into a STEM field, earning nearly twice as much annually as those with only a high school degree, and the benefits of higher education with respect to a graduate’s economic standing over time significantly outweigh the short-term hardships. Although in the short term, having lower levels of education was more beneficial, it was far better in the long term to go to college and get at least an associates degree, with larger payouts for bachelor degrees in any fields.

V Life Is Short…

A) The Problem

We were asked to determine a ranking system using qualitative and quantitative measures to determine the effect of higher education on overall quality of life. This model will be able to be used by high school students to determine the value of different paths of career preparation.

B) Assumptions

- An unskilled worker will be able to find and work a job which pays $10.00/hour directly out of high school, and will work 40 hours/week for 50 weeks/year
- An individual will sleep on average 8 hours per night every night

C) Approach

Our goal is to define a function, \( Q(C_{im}) \), which outputs a measure of the quality of life a graduate would experience if he were to take a career path \( C_{im} \) out of college \( i \) with major in field \( m \). We will define the quality of life to be a function that has range \([0\%, 100\%]\). A higher output corresponds to a lifestyle with a higher standard of living, job security, lower stress caused by incurred debt, enjoyment derived from the workplace, and job skill. This model only takes into account job satisfaction directly related to career choice. Major life events such as getting married, having children, or buying a house are not modeled. We let the yearly salary for career path \( C_{im} \) be \( S_{im} \).

A student who comes out of college \( i \) with a major in field \( m \) will incur a debt of \( L_{im} \) (which would be found using our model from part one). Instead of attending college (and grad school, if necessary), the student could have landed a low-paying job and been working full time already. Assuming a constant unskilled hourly wage of $10, the student could have made \( \frac{\$10 \times 40 \text{ hours/week} \times 50 \text{ weeks/year}}{\text{year}} = \$20,000/\text{year} \).

So not only will the student have incurred a debt, which will be \( L_{im} \) overall (calculated from our part one model), but they will be $20,000 behind a student who didn’t attend college for every year the student wasn’t working. Therefore the value for \( L_{\text{no college}} = -20,000 \times \text{years of education skipped} \).
We let the happiness level (0-100%) for career path $C_{im}$ during work be $W_{im}$ and the happiness level for career path $C_{im}$ outside of work be $O_{im}$, so we can calculate a person’s satisfaction from a career path based on both of these factors. Assuming a 40 hour work week, and eight hours of sleep each night, a person spends 

$$\frac{40 \text{ hours/week}}{7 \text{ days/week}(24 - 8 \text{ hours/day})} = \frac{40}{7 \times 16} \times 100\% = 35.71\%$$

of their waking hours at work.

If a person spends about a third of their waking hours miserable at work, it is going to decrease their quality of life, even if the higher income leads to a happier life outside of work, unless the amount of income increased is significantly greater than the satisfaction lost during work. Unfortunately, we high school students lack the life experience to comment on the greater significance of life, but we assume that we should strive to find a balance between happiness at work and outside of work, and that this should be done proportionally to the time spent making income and spending it. Therefore, we determine that time will be split into two categories and combined to extract an overall quality of life. The effect of the satisfaction rank will be directly proportional to the amount of time spent in each category: at work $W_{im}$ and time outside work $O_{im}$. We thus have the simple expected value function

$$Q(C_{im}) = 0.3571 W_{im} + 0.6429 O_{im}.$$  

A student should therefore select the career path $C_{im}$, college $i$, and major $m$ that maximizes $Q(C_{im})$.

We will assume that a person’s happiness, outside of work, is wholly dependent on their net flow of money, to a certain degree. However, according to a Princeton University study, income had less of an effect as the total income earned increased, to the point where at $65,000 there was no longer a statistically significant correlation between a higher income and more happiness (Quiñones, Princeton University). At this point, there was no correlation between people with an income over $65,000 and their happiness.

![Graph showing income and happiness](image)

This data is clearly best modeled with a logarithmic function, and we found the regression line

\[ Satisfaction = -91.785 + 16.562 \times \ln(\text{income}) \]

had an \( r^2 = 0.71 \). And, since income greater than $75,000 has a negligible effect on happiness, a logarithmic function is a solid way to model happiness, since it grows very slowly. However, it must be noted that this happiness function is extrapolating for incomes greater than $75,000. Now a student who has graduated and enters the workforce with salary \( S_{im} \), but has incurred a debt of \( L_{im} \) and has to pay \( Loan_{\text{yearly}} \) for his/her education a year, and is paying off his/her costs over the next \( t \) years, has a satisfaction level outside of work due to income earned modeled by

\[ O_{im} = 16.562 \times \ln(S_{im} - Loan_{\text{yearly}}) - 91.785. \]

Obviously income isn’t the sole predictor of happiness in a person’s life, but it is the best measurement of a result that directly comes from a student’s college decision.

It is difficult to determine how satisfied a person will be at any given job. What can be done, however, is determining how enjoyable an individual finds similar activities to a specific career path and extrapolate that data. For example, in a STEM Field, the activities will be based on their enjoyment of mathematics and science, based on a scale ranging from 0-100%. \( W_{im} \) will therefore be a self-assessment of how much the individual feels he/she would be satisfied by a career in their field of choice.

\[ W_{im} = \text{self assessment of satisfaction from } 0\% \text{ to } 100\%. \]

Finally, the model for projected satisfaction in a particular field is

\[ Q(C_{im}) = 0.3571 W_{im} + 0.6429(16.562 \times \ln(S_{im} - Loan_{\text{yearly}}) - 91.785). \]

**D) Analysis**

In one example, a student who is trying to decide whether or not to attend a two-year technical college (to become a welder, making $36,000 a year) compared to attending zero college and starting work immediately. The student assigns a score of 60 to working with his hands, similar to a welder, and 60 to the unskilled labor he would be doing with no college education. Because his family’s income is approximately $75,000 annually and he is the only child of a single father, the true cost of attending a two-year college is $59,610. He has no assets, and his father isn’t willing to contribute to his college fund. We calculate

\[ Q(C_{\text{college}}) = 0.3571(60\%) + 0.6429(16.562 \ln(36000 - \frac{1}{10}59610) - 91.785) = 72.1\% \]

\[ Q(C_{\text{no college}}) = 0.3571(60\%) + 0.6429(16.562 \ln(20000 - \frac{1}{10}(-40000)) - 91.785) = 69.8\%. \]

In this case, college doesn’t really have a huge impact on the man’s expected satisfaction with his career path over the next 10 years, college or not. However, if Obama’s plan goes into effect, then the value of \( Q(C_{\text{college}}) \) increases significantly (to 74.1%) and clearly makes college the better option for this man. This is a situation where Obama’s policy would make a big difference in an individual’s decision on whether to obtain a college degree.
Another example case is a student who firmly believes they are destined for college. Her family makes $125,000 annually, and neither of her two sisters is in college yet. Her parents are still together, and her true cost of attending the four-year private university she hoping to be accepted to is $266,933, with a monthly payment of $787.71, which is a yearly payment of $9452.52. Her major is going to be engineering (mechanical, with a starting average salary of $65,000 from this particular university), but she assigns a very small value of 30 to \( W_{im} \) in the subjects of mathematics and science. We see 
\[
Q(C_{\text{college}}) = 0.3571(30\%) + 0.6429(16.562\ln(65000 - 9452.52) - 91.785) = 68.1\%.
\]

This student’s projected satisfaction is relatively low, and she may want to either consider a less expensive university or a different field of study that she would enjoy more. For example, if she pursues a career that she likes enough to assign a value of 80 to \( W_{im} \), even if she makes $15,000 less than before, we see that 
\[
Q(C_{\text{college}}) = 0.3571(80\%) + 0.6429(16.562\ln(50000 - 9452.52) - 91.785) = 82.5\%,
\]
which is clearly the better option for her.

A student needs to balance work enjoyment, student loans incurred, and salary. For some people, choosing \( C_{\text{no college}} \) may give the highest value for \( Q \). Due to the subjectivity of satisfaction, someone who is not very good at a particular activity but still enjoys participating in it will ultimately feel satisfied regardless of the outcome. However, for others, money may play a large factor in their self-worth, and they therefore will assign higher values of \( W_{im} \) to activities with greater applicability to high salary careers. However, the model may also tell a student to sacrifice some of their work enjoyment to pursue a job with a higher salary.

**VI Conclusion**

Overall, we were able to accurately determine the cost of college for students in varying financial circumstances throughout the United States. With our model, we can accurately predict the cost of colleges for different families for any higher educational institute in the United States. For people who received our estimates and were undecided about whether or not to attend to college, they also have the ability to use our model in order to tell them how this will affect them in both the short-term and the long-term areas of their lives. Also, we are able to use another model to calculate the satisfaction level for different career paths they are considering, so they can be ranked. College is a major decision: what college should you go to? what should you major in? what job do you eventually want to train for? These decisions can and will affect the rest of a person’s life. However, with the model we have created, students will be able to make more informed and therefore better decisions for their college plans, helping them achieve greater satisfaction and overall happiness with their lives.
VII Bibliography


