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M3 Challenge Finalist, $5,000 Team Prize

Moody’s Mega Math Challenge 2016: Share and (Car) Share Alike

Abstract

Recently, the use of car sharing has become more and more popular. With the popularity of Zipcar increasing, owning a car has become less of a milestone and more of a burden. However, with the introduction of this new method of transportation, there are still questions about the staying power of the new transportation sector, and the effectiveness of the different types of car-sharing business models. Zipcar itself has become successful with its hourly rates, but other programs include round trip car sharing, one-way car-sharing floating model, one-way car-sharing station model, and fractional ownership.

We first determined the percent distribution of people within the country in categories based on the amount of time they use cars and the amount of miles they drive. Using the normal model, we were able to model both time and mileage and created a table of nine intervals which are a combination of time and distance on a rating from low to medium to high. Our model gives the percentage of people within each interval. We analyzed the existing information from every zip code in the United States (excluding Alaska and Hawaii), specifically, the commuting time for each zip code and the distance traveled for a stratified random sample of zip codes. We used strata of non-metropolitan and metropolitan regions. Ultimately, we found that the majority of people travel a medium amount of time and that the largest interval was a combination of medium time and long-distance.

Then, using a computer simulation in MATLAB, we were able to model round trip sharing, floating one-way sharing, station one-way car sharing, and fractional sharing. The simulation employs a Markov chain in which 10,000 individuals are followed through 20 timesteps. Their new state depends on a probability which involves sigmoid functions involving the amount of other people already involved in car sharing, and their classification as described in the previous section as it affects the incentive of affordability. That affordability was calculated by finding the average cost of each program for each characteristic possibility determined in the previous section. From the model, we were able to conclude that the one way car-sharing station model is the most cost efficient of the car-sharing programs, regardless of the city characteristics. We recommend implementing this model of car sharing in the four cities studied.

After this was done, we then looked at how technological advances in future cars would affect the results of section 2. We looked at how self-driving cars and power of negligible cost for transportation would affect the success of each of the four car-sharing models. We assumed, perhaps optimistically, that power in the future will be free. Thus, people traveling the least distance would save the least money on gas and that people traveling the farthest would save the most money when changing to a car-sharing model. Furthermore, we considered that individuals could make money working while they were sitting in their self-driven cars. We factored in these considerations into our Markov chain transition matrix. We show that in the future, station one-way car sharing and fractional ownership models of car sharing will increase in popularity.

1 INTRODUCTION

1.1 Background

In recent years, car sharing has become a more prevalent and convenient means of transportation. Through companies like Zipcar and Uber, people are finding that the need to own a car is diminishing. Large corporations are investing in car-sharing systems [1] in hopes that it will bring along new business [2]. Specifically, Ford's vice president of research says they "see this as a business we want to be in." Some companies are even starting their own car-sharing services [3].

Nevertheless, there are still major factors to consider when deciding between car sharing and owning a personal car. People value the convenience of being able to take their own care whenever they would like, yet owning a car creates a lot of hassle. Through car sharing, people would not have to deal with maintenance or repairs for their cars. Through companies like Uber and Lyft, people don't even need to worry about driving or parking. Given the new boom of alternative transportation, people are going to have to make decisions on whether or not owning a car is worth it.

1.2 Restatement of the Problem

1. How should car-sharing companies model their clientele? What percent of the population drives low, medium, and high amount of miles in a day? What percent of the population drives low, medium, and high amounts of hours in a day?
2. There are a few different types of car-sharing programs in today's world. A few are: round trip car sharing where vehicles are rented by the day, hour, or mile, or some combination of the three, and are picked up from and returned to the same point, the one-way car-sharing floating model where cars are rented on demand and are returned to defined areas, the one-way car sharing station model where customers pick up and drop off cars at existing stations, and fractional ownership where multiple owners jointly purchase a private car. Based on the following cities, which car-sharing model works the best? Poughkeepsie, NY; Richmond, VA; Riverside, CA; Knoxville, TN.
3. New advances on self-driving automobiles and fuel efficient cars are predicted to become the new norm. With the possibility of having a car that does not require human participation, how will car sharing be affected, specifically for the aforementioned four cities?

2 WHO'S DRIVING?

2.1 Assumptions and Justifications

In order to further generalize our model for the percentage of drivers by time driven and distance driven, we made the following assumptions and simplifications:

Assumption: People eat locally so that local business travel is negligible.
Justification: We assume that all grocery stores, pharmacies, doctors' offices, etc., are within the immediate vicinity of people's homes and are therefore negligible.
People who eat locally or travel locally are more likely to walk than use vehicular transportation.
Assumption: No members of the population will move from one characterization to another.
Justification: The time it takes to commute to work should remain constant if a person has the same job. The number of jobs moving closer to people's homes and the number of jobs moving farther from people's homes should cancel out and therefore be negligible.
Assumption: There is no population growth.
Justification: The numbers of births and deaths in the short term are roughly equivalent.
Assumptions: Driving children to school or other leisure activities is negligible.
Justification: The time it takes to drive children to school should be accounted for by the commute time to work.
Assumption: People who don't own cars are negligible.
Justification: People who don't own cars are not driving cars and therefore do not contribute to the use of cars in America.
Assumption: We assume that the commute for children to and from school is provided by public school bus systems and therefore irrelevant to our calculations.
Justification: Public schools providing busing for private schools is a common occurrence nationwide.
Assumption: Hawaii and Alaska are excluded from data.
Justification: There is no interstate travel for Hawaii (island hopping) or Alaska (few cities accessible by road), so their commute time is unmeasurable by time and distance in cars.

2.2 Model for Commute Time

We used our assumptions to create two different models. The model of time uses the average commute time for each zip code using figures from the 2010 Census. The model for distance compares urban settings, which have lower mean distances, to rural settings.
Since the model of commute time for zip codes is unimodal and symmetric, we can assume that the model is normally distributed. Therefore, we were able to set the low, medium, and high ranges using z-scores:

$$z = \frac{y-\mu}{\sigma} \rightarrow y = z * \sigma + \mu$$

where:
z = z-score for the commute
y = commute time of a sample
μ = mean commute time (this will be used to represent the mean of models)
σ = standard deviation of the commute time (this will be used to represent the standard deviation of models)

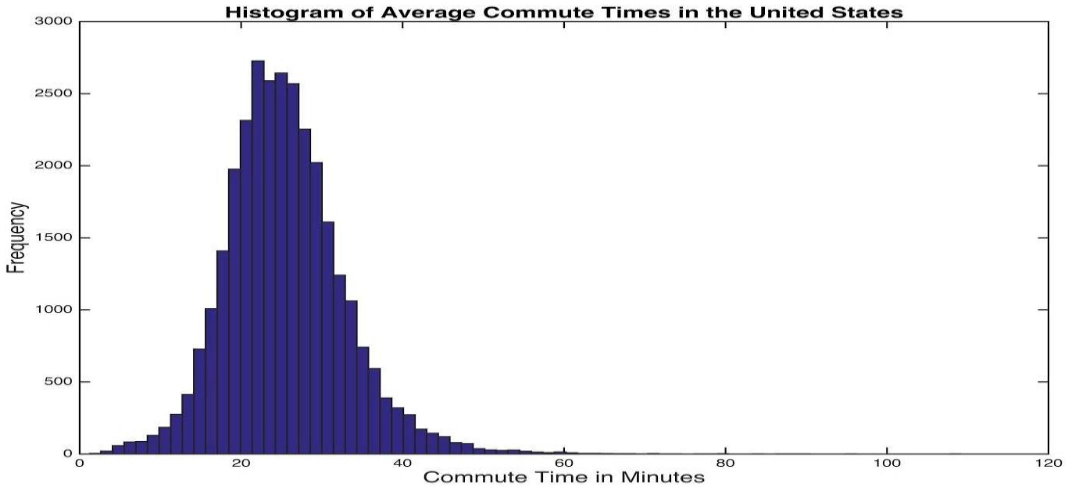


Figure 1: The frequency histogram of commuting times for 98,929 U.S. zip codes [12]. The mean commute time μ = 25.48 minutes and the standard deviation $\sigma = 7.4$ minutes.

To set the low, medium, and high ranges, we used the 68-95-99.7 rule and set the low commute time as having a z-score $[-\infty, -1]$, the medium commute time as having a z-score $(-1, 1)$, and the high commute time as having a z-score $[1, \infty]$. Using the z-score equation, we determined that the low, medium, and high ranges were [0, 18.08],

(18.08, 32.88), and [32.88, ∞], respectively (the z-score of negative infinity becomes zero because zero is the lowest possible commute time).

2.3 Model for Distance

For our distance model, we took into account urban areas, which contain about 85% of the population, and rural areas, which contain about 15% of the population. To do this, we used a stratified random sample or urban and rural zip codes so that we would have a representative sample of approximately 15 rural and 15 urban zip codes. The following images depict the distributions of distance for rural and urban areas.

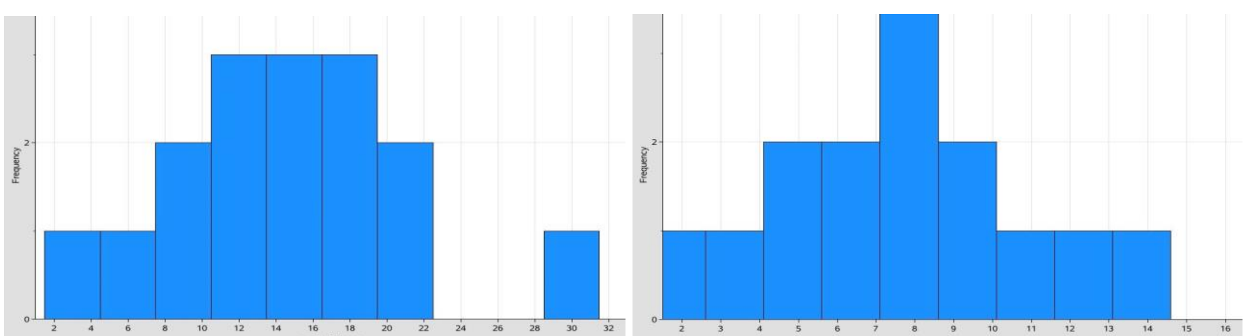


Figure 2: The (left) histogram is the frequency distribution of rural travel distances. The (right) histogram is the frequency distribution of travel urban.

Since both sets of data are both unimodal and symmetric, we can assume the data is normally distributed. Therefore, we can retrieve the mean and standard deviation for both graphs. For rural areas $\mu_{rural} = 14.48$ and $\sigma_{rural} = 7.00$. For urban areas $\mu_{urban} = 7.60$ and $\sigma_{urban} = 3.23$. Since rural areas represent 15% of the population and urban areas represent 85% of the population we can create a model for the combination of these populations:

$$\mu_{total} = 0.15 * \mu_{rural} + 0.85 * \mu_{urban} = 8.63$$

$$\sigma_{total} = \sqrt{0.15^2 * \sigma_{rural}{}^2 + 0.85^2 * \sigma_{urban}{}^2} = \sqrt{0.15^2 * 7.00^2 + 0.85^2 * 3.23^2} = 2.94$$

This mean and standard deviation are the sample statistics for the model of distance for both the rural and urban populations (the total population). Therefore, we can set low, medium, and high ranges for the distance traveled (in miles) using z-score intervals of $[-\infty, -1]$, $[-1, 1]$, and $[1, \infty]$. These intervals become $[0, 5.69]$ (low), $(5.69, 11.57)$ (medium), and $[11.57\ \infty]$ (high) in the context of the model (all numbers are in terms of miles for the intervals).

2.4 Final Analysis and Results

Using the z-score ranges for both the distance and time models, we conducted a simple random sample of all U.S. zip codes for 30 sample zip codes [12]. We then used a website that takes zip codes and gives the average commuting distance for that zip code as an output [4]. Since we took a simple random sample (SRS), the resulting sample is statistically representative of the United States. As a result, we can use the distance model and the time model to create a 9-combination table and compare each combination to its corresponding area for the 30 zip code model.

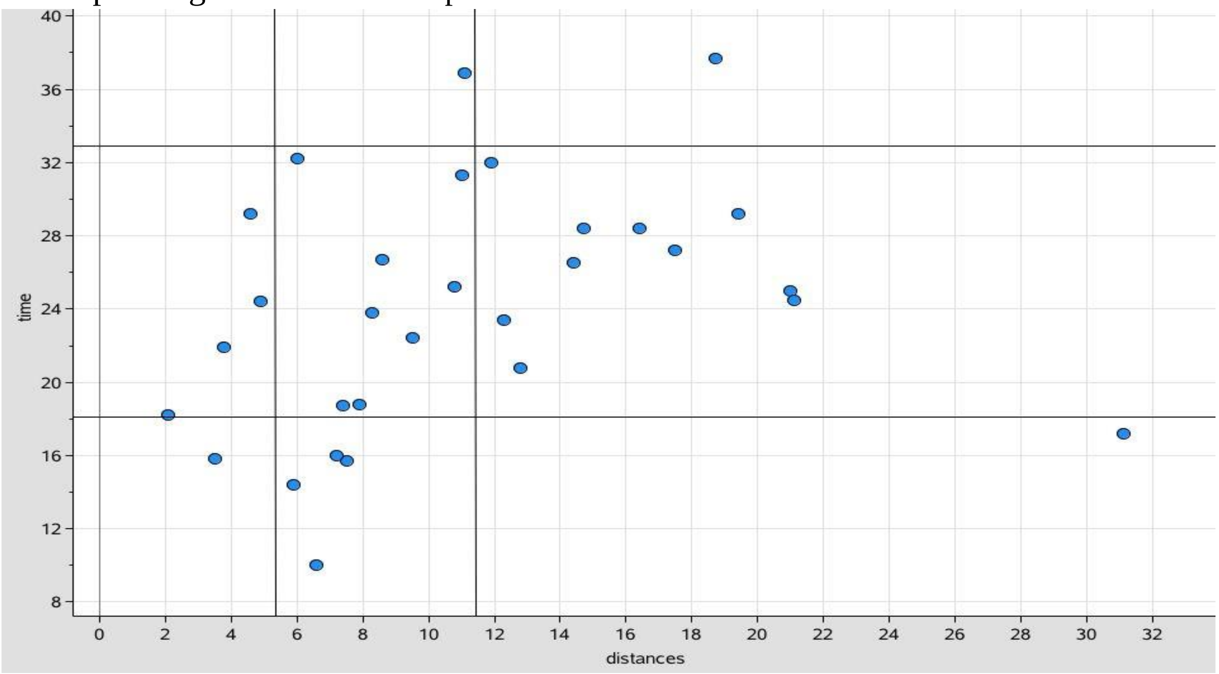


Figure 3: This image contains the 30 zip codes in their place. The lines represent the low, medium, and high margins for distance and time. The point on the right has high influence and is highly improbable since it would result in a speed of nearly 110 mph.

Figure 3 leads us to the probabilities of time and distance, based on the number of zip codes within each "interval box." These probabilities represent the percentage of drivers who drive for a "low, medium, or high" time and a "low, medium, or high" distance.

	Low (miles)	Medium (miles)	High (miles)
Low (time)	1/30, 3.33%	4/30, 13.33%	1/30, 3.33%
Medium (time)	4/30, 13.33%	8/30, 26.67%	10/30, 33.33%
High (time)	0/30, 0.00%	1/30, 3.33%	1/30, 3.33%

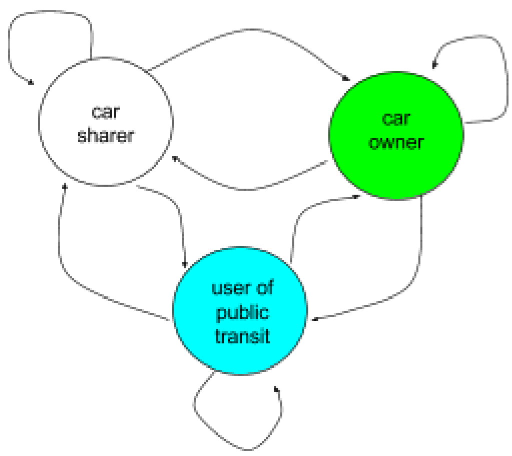
According to our model, those who drive long (high) distances in a medium amount of time represent one third of the total number of drivers nationwide. It is not surprising that those who drive long distances in short (low) periods of time or those who drive short distances in a large amount of (high) time represent a small percentage of the total population (3.33% and 0%, respectively) of the United States.

3 ZIPPITY DO OR DON'T?

3.1 Assumptions and Justifications

Assumption: Each person is in exactly one of three states: a car sharer, a car owner, or public transit user or pedestrian.
Justification: For simplification purposes, we assumed that a person does not use more than one mode of transportation.

3.2 Design of the Model



To model the behavior of each person in the city, we used discrete Markov chains (figure on the right). We model the probabilities that each person will change states or stay in the same state below.

The following letters are assigned as follows to identify the states: S - shared-car users, C - car owners, and P - public transit

In the following state transition matrix, row and column 1 correspond to shared-car users, row and column 2 correspond to car owners, and row and column 3 correspond to public transit users. We modelled the transition matrix as follows:

$$\begin{bmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{1+e^{-s(t)*k+cp}} & 1-\frac{1}{1+e^{-s(t)*k+cp}} & 0 \\ \frac{1}{1+e^{-s(t)*k+cp}} & 0.4*(1-\frac{1}{1+e^{-s(t)*k+cp}}) & 0.6*(1-\frac{1}{1+e^{-s(t)*k+cp}}) \end{bmatrix}$$

Let X_i refer to the state of an individual at time step i. Furthermore, let $s(t)$ equal the number of individuals currently sharing cars.

We then we let

$$P(X_{n+1}=S|X_n=C)=\frac{1}{1+e^{-s(t)^*k+cp}}$$
$$P(X_{n+1}=S|X_n=P)=\frac{1}{1+e^{-s(t)^*k+cp}}$$

where p is the price of the car-sharing service and k and c are constants. We use a sigmoid function to model this probability because the probability must be between 0 and 1. Furthermore, the sigmoid function models the behavior of this probability well because when $s(t)*k - cp$ is negative, the probability that a person would convert to a car-sharing plan is close to zero because the price would be worth more than the popularity of the car-sharing plans.

We assume that a car owner would not begin using public transportation for simplification purposes. This is reasonable because we are assuming that no one is moving out of the city they live in. If they found it necessary to own a car, it would be unlikely that they would rid themselves of a car completely. Thus, as

$$P(X_{n+1}=S|X_n=C)+P(X_{n+1}=C|X_n=C)+P(X_{n+1}=P|X_n=C)=1$$

we can solve to find

$$P(X_{n+1}=C|X_n=C)=1-\frac{1}{1+e^{-s(t)^*k+cp}}$$

To represent the probabilities of changing from a public transit user to a car owner, we used values of .4 and .6 to represent the intuitive greater likelihood of someone using public transportation to continue using public transportation, and the lesser likelihood of them buying a car and becoming a car owner. Because the three probabilities must add to 1, we have both of these equations:

$$P(X_{n+1}=P|X_n=P)=.6*(1-\frac{1}{1+e^{-s(t)*k+cp}})$$
$$P(X_{n+1}=C|X_n=P)=.4*(1-\frac{1}{1+e^{-s(t)*k+cp}})$$

We could find no available data on transitioning from shared cars to other forms of transportation. As a result, we assumed that the probabilities were all 1/3 because it is reasonable to assume that some people stop using shared cars because they are too expensive and, instead use public transportation. Some will save enough money to prefer to afford their own cars, and some will enjoy using shared cars. We consider that a person joining a shared car plan will be likely to drop the plan and purchase a vehicle, especially considering that many individuals who currently enter shared-car plans are college students and will likely buy a car in the future [13].

Thus, we let

$$P(X_{n+1}=S|X_n=S)=P(X_{n+1}=C|X_n=S)=P(X_{n+1}=P|X_n=S)=\frac{1}{3}$$

3.2.1 Cost to User

In order to determine the cost differences among each class of drivers, we calculated the cost of each car-sharing program. We based these calculations on the time and mileage data for each of the nine types of people and current prices of each of the four models in today's market. For the model, we first found the mean of each commute time bracket, t, found in section 2 and then multiplied it by 2 to indicate both ways. This number will vary based on the categorization of the driver. This total is multiplied by some constant m which we chose to be 1.5, representing the amount of driving that one might do in one day relative to commute time for other activities. We feel that this constant of proportionality is reasonable because job opportunities are more abundant as one moves away from their neighborhood, leading to a longer commute. This is then multiplied by the cost of the car-sharing service's rate per minute, p. Finally, we add the yearly membership fee of divided by the number of days in a year, y. Therefore, we obtain the equation

$$(m(2t+w))p+y$$

for the cost of round trip car sharing, where w is what our assumption is that the average work day, between part time and full time jobs, is roughly 6 hours long, or 360 minutes, where y is a 70 dollar membership fee for the year, and where p is 0.15 dollars.
For the one-way car sharing, we use the equation

$$(m\times 2t\times p)+y$$

where p varies depending on the floating model of business, 0.41 dollars, and the station model of business, 1.67 dollars, and where the floating model has a yearly fee of 35 dollars and the station of 70 dollars.
Thusly, we attain these costs in dollars per day for each car usage characterization per each car-sharing method:

Table 1	Low t = 15.08 mins	Medium t = 25.48 mins	High t = 35.87 mins
Round Trip	$60.98 per day	$65.66 per day	$70.33 per day
One Way Floating	$18.65 per day	$31.44 per day	$44.22 per day
One Way Station	$7.74 per day	$12.96 per day	$18.16 per day

In order to quantify a price for fractional ownership, we added the cost of owning and maintaining a car to the cost of gas for that car based on the mean mileage of each car mileage bracket from section 2. This way, we could calculate an average cost for each type of person. First the mean mileage of each car mileage bracket was divided by the average miles per gallon of a car, 25.5 mpg [9]. This quotient was multiplied by the average cost of gas per gallon, which was $2.3 [8]. After this, we calculated the cost of owning a car without considering gas and added them. This was done by considering that the average cost of buying a car is $33,560, *h*, and that the average life of a car is 8ight years, *m* [6]. The average cost of buying the car was then divided by the years and added to $593, *o*, which is the average cost of owning a car per year [7]. Then we multiplied this sum by 1/365 and 1/(4.5) to get the average cost of owning a car without gas per person considering that there are 365 days in a year and 4.5 people who own one car [10]. To get the cost of driving per day we then added the average cost of gas per day and the average cost of owner a car per day.

$$(d/mpg) * g + (1/4.5)(1/365)((h/m) + o) = cost/day$$

Table 2	Low	Medium	High
Fractional Ownership	13.15 dollars per day	21.20 dollars per day	29.84 dollars per day

3.3 Categorizing the Cities

We researched all of the zip codes belonging to each city. We looked up the average commute time and distance from each zip code and used the parameters described in section 2 to assign categories to each zip code. The amount of each category was divided by all of the zip codes in each city to make a percentage of each category. The categories are labelled as follows: LL- low distance low time, LM- low distance medium time, LH- low distance high time, ML- medium distance low time, MM- medium distance medium time, MH- medium distance high time, HL- high distance low time, HM- high distance medium time, HH- high distance high time.

	LL	LM	LH	ML	MM	MH	HL	HM	HH
Knoxville , TN	2/16	1/16	0/16	3/16	8/16	0/16	0/16	2/16	0/16
Richmond, VA	2/20	2/20	0/20	1/20	15/20	0/20	0/20	0/20	0/20
Riverside, CA	1/9	3/9	0/9	0/9	0/9	0/9	0/9	5/9	0/9
Poughkeepsie, NY	0/2	0/2	0/2	0/2	2/2	0/2	0/2	0/2	0/2

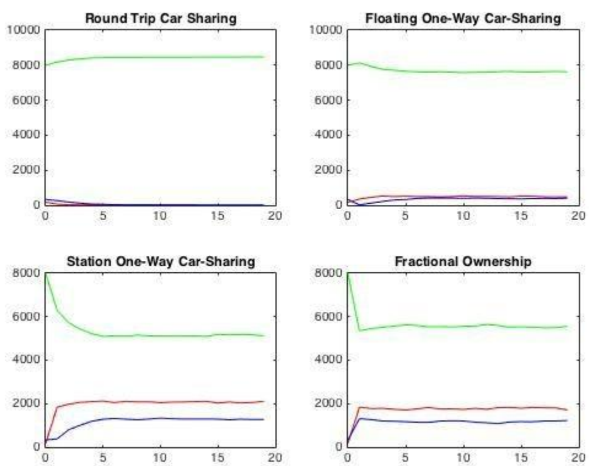
3.4 Computer Simulations

Using MATLAB, we were able to simulate the transition of states using Markov chains. The prices we found for the different car-sharing companies were dependent on time and distance, so we decided to give time a value of high, medium, or low and distance a value of high, medium, or low. The simulation consisted of 10,000 people who were each assigned a value based on researching their zip code and finding their place on the "interval" diagram used in Figure 3. The simulations were assigned a high, medium, or low value based on in which of the nine sections they were located. Then, using Markov chains, the persons underwent a transition into a new or the same state over many timesteps, using the transition matrix.

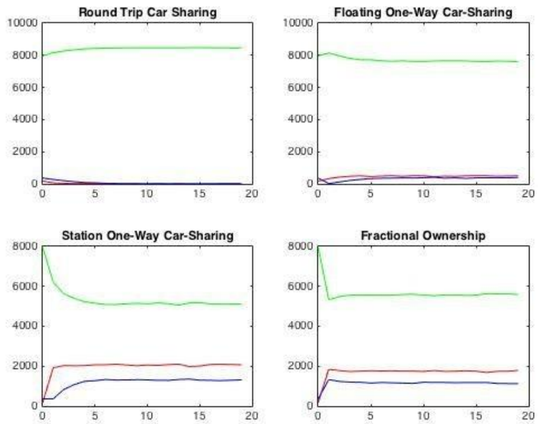
We set $k = 0.001$ and $c = .1$ so that price and popularity had roughly equivalent weights on the decision of an individual of the city. We will discuss the sensitivity of our model to these parameters in section 5.
The colors are represented as follows:

- Green: car owner
- Blue: public transit
- Red: car sharing

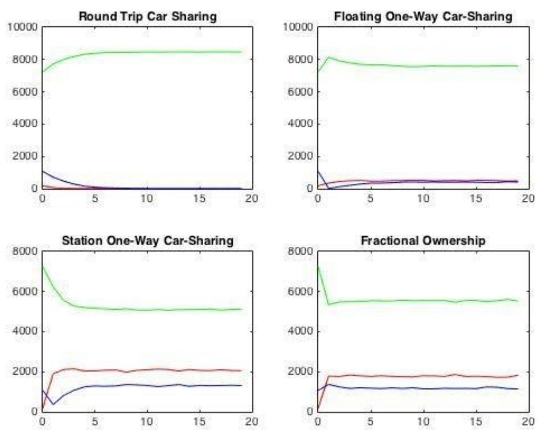
Poughkeepsie:



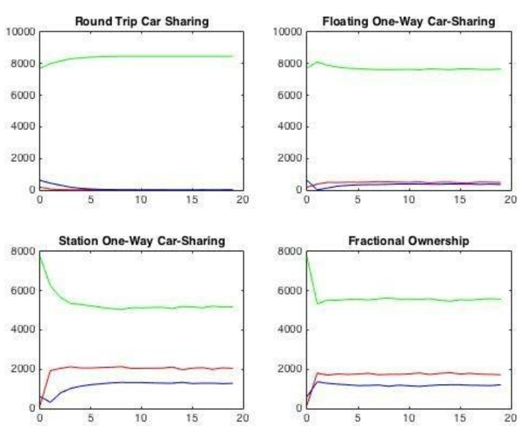
Richmond:



Riverside:



Knoxville:



The above images represent the results of the computer simulations. It is clear that the cities, though they had slightly different population types, all behaved in a similar way. This can be attributed to the fact that the probabilities that a citizen of a city changed states or remained in the same state was independent of the type of city he or she lived in. We based the probabilities only on the popularity of the shared cars and the prices we calculated for each of the nine types of drivers.

Thus, a potential car-sharing service should be most interested in the cities with the highest population densities, in order to accrue the most participants with the least amount of maintenance costs. We rank the cities in the following manner by population density:

1. Poughkeepsie, NY
2. Riverside, CA
3. Richmond, VA
4. Knoxville, TN

For each of the four cities, we found that station one-way car sharing would gain the most popularity. This is attributed to the fact that the cost per day of station one-way car sharing was most cost effective. Furthermore, it was most convenient for individuals to have the flexibility to only travel one way, minimizing the need to pay for time that the car is parked and not in motion. However, we have shown that the behavior of each of the four cities studied is very similar.

4 Road Map to the Future

4.1 Assumptions and Justifications

To make our model, we used generalizations by relying upon the following assumptions and simplifications:

Assumption: Gasoline prices are constant.
Justification: Although the price of gas is not constant, there is simply no way to predict the price of it because of the numerous unpredictable measures and circumstances. Thus, for simplification purposes, we set the price of gasoline equal to its average price of the past 10 years, which is $2.30 [8].
Assumption: People are more willing to buy something that will save them money.
Justification: Most people will buy something if it will save them money. People value their money.
Assumption: People do work during their newfound time on their commutes.
Justification: People will have a lot more free time with self-driving cars. Now, they will have more time to work. We are assuming that with this time, people will work.
Assumption: Future cars will run on a source of negligible cost.
Justification: Cars and energy sources are making great advances every day. Because of this, we are assuming that by the time self-driving cars are available, cars will run on a free power source.

Assumption: People will earn the same amount of money during their extra work time in their cars.
Justification: In the workplace, some jobs pay twice the normal amount for overtime work and some will not pay for work in the car. However, this is different in every place of employment and cannot be predicted.
Assumption: MPG is constant for the short term.
Justification: We needed to simplify this because we needed to find the amount of money that the people would save. This assumption emphasizes how eco-friendly transportation affects the cities.

4.2 Calculating the Money Saved or Earned

To calculate the money saved or earned, we first found the money saved per day through the future's advancements. To do this, we first calculated the average amount of money people spend on gas for the low, medium, and high distance classes. This was done by multiplying the average MPG from the past 10 years, f, which is 25 by one over the miles they traveled which are the mean values from the characterization in 2.4, q. This value was then multiplied by 2.8, l, which is the average gas price from the past 10 years. This gave us the daily average money spent on gas for each distance class [2].

$$f*(1/q)*l = money\ on\ gas$$

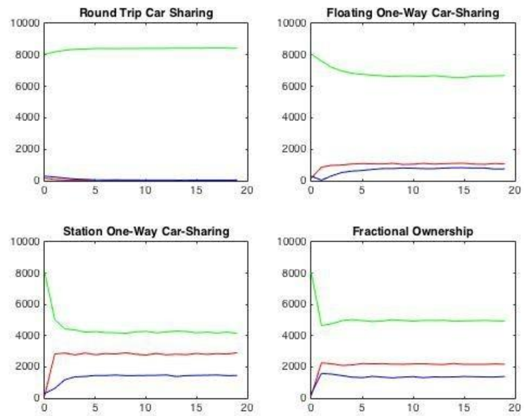
After this, we found that the median U.S. salary is $51,939, m, and that the average hours that people in the United States work is 2,080 hours, s [14]. We then divided $51,939 by 2,080 and then divided this quotient by 60 minutes to get the money earned per minute at work for each distance class. This gave us $0.416/min.

$$(m/s)*(1/60) = money\ earned\ /\ minute$$

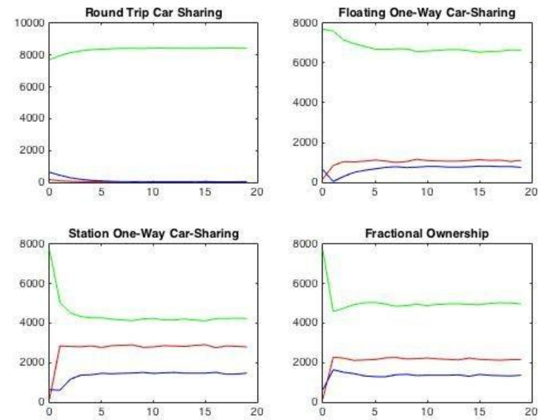
Once this was done, we added the values found within each distance class for the money earned per minute in the self-driven car and the amount of money spent on gasoline. With this information, we subtracted these values from Table 1 in section 3.2.1 based upon its distance class. For example, if the data that we found was for the low distance class, we subtracted the data from all of the low distance classes in Table 2 in section 3.2.1. This was then done for both the medium and high distance classes.
*note that the negative numbers denote that the person is making money

Table	Low t = 15.08 mins	Medium t = 25.48 mins	High t = 35.87 mins
Round Trip	55.87 dollars per day	56.59 dollars per day	60.81 dollars per day
One Way (Floating)	5.45 dollars per day	22.37 dollars per day	34.7 dollars per day
One Way (Station)	−5.46 dollars per day	3.89 dollars per day	8.64 dollars per day
Fractional Ownership	−0.06 dollars per day	12.13 dollars per day	20.32 dollars per day

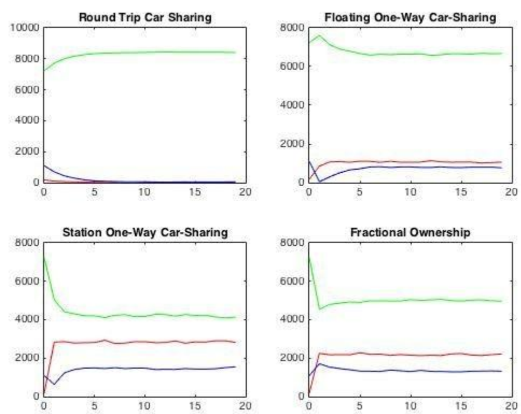
Poughkeepsie :



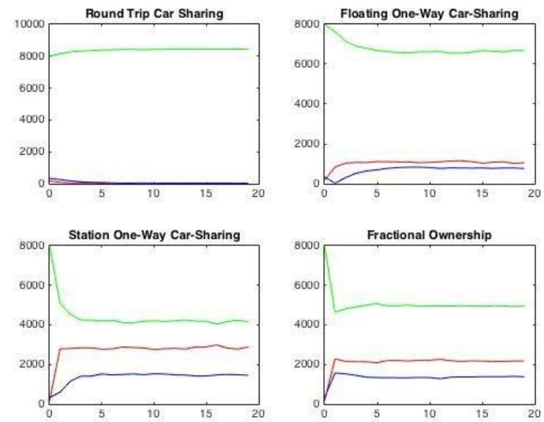
Richmond:



Riverside:



Knoxville:

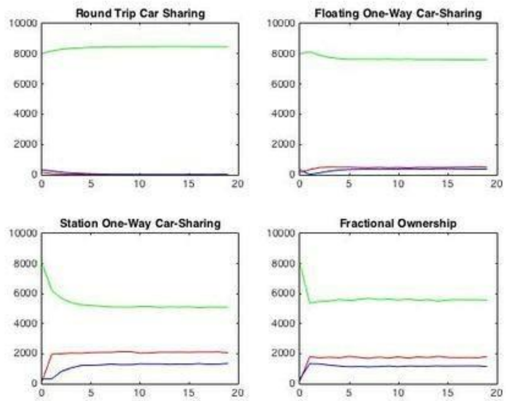


According to the results of the computer simulations, we can see that environmental considerations and efficiency are both significant factors in choosing a transportation service. When factoring into the model that time is money, and a person's time spent driving could be spent working or doing other productive tasks, we see that the incentives to using services with driverless cars would be very attractive. Furthermore, when fuel efficiency and new technology reach new levels, such cars may be able to have negligible fuel use. It is clear that both of these factors would positively impact the participation in certain programs, including station one-way car sharing, if these cars are driverless and run on minimal amounts of fuel. However, we see that participation in round trip car sharing does not improve, as it is still less efficient to commute to destinations and pay for the waiting time of the car.

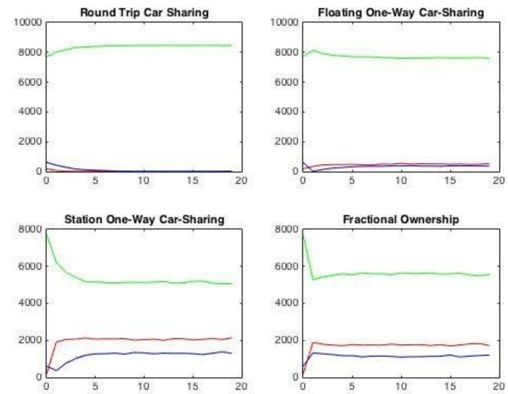
5 SENSITIVITY ANALYSIS

We evaluate the sensitivity of our model to the parameters chosen for k and c in the probabilities of changing states or staying in the same state [15].
When the k value was multiplied by 5, there was very little change in the resulting graphs. The graphs are displayed below.

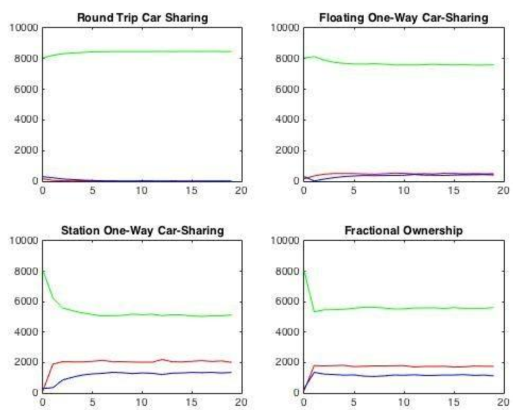
Poughkeepsie:



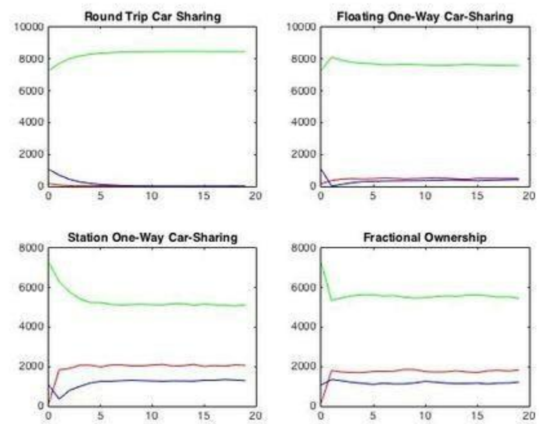
Richmond:



Knoxville:



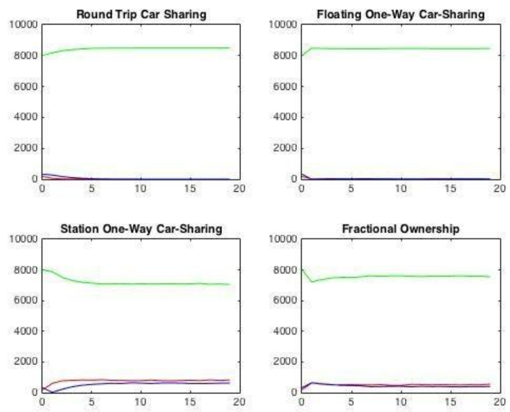
Riverside:



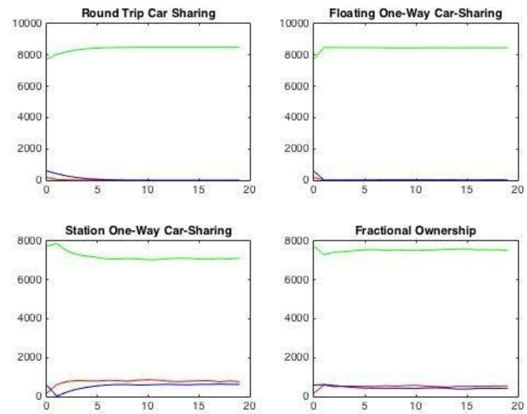
It is clear from the similarity of these graphs with the original graphs that changing the value of k by a significant amount does not affect the results of our model.

In addition, we analyzed the sensitivity of our model to the parameter c. After multiplying c by a factor of 2, we ran the computer simulation again. The graphs are displayed below. There is a higher sensitivity to this parameter, but this change did not affect the recommendations we will make as a result of our model.

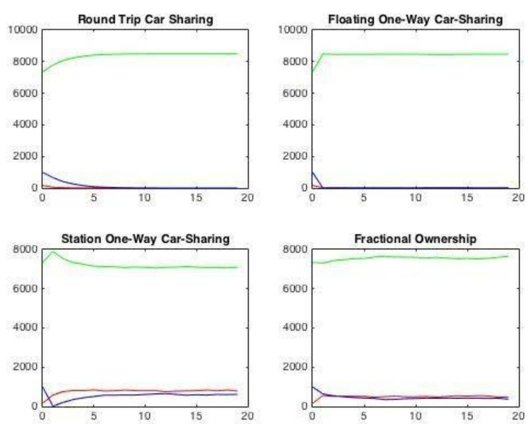
Poughkeepsie:



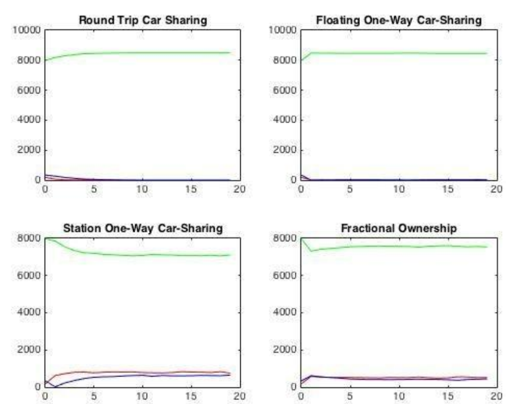
Richmond:



Riverside:



Knoxville:



6 STRENGTHS AND WEAKNESSES OF THE MODEL

Strengths

- We took into consideration the difference between rural and urban areas for distance in a commute which allowed us to ensure that both strata were normal models.
- Our model uses all of the U.S. zip code data (98,929 data points) on commute time and population in order to determine the distribution of the population across the high to low categories for driving time and driving miles.
- The Markov chain model takes into account the individual preferences of different types of people in the cities and accounts for the effects of individual decisions.
- Since our Markov chain model is discrete, we can take into account the number of car sharers into the probabilities of changing states or staying in the same state. The more popular it is to share a car, the more likely another person will hear about the car-sharing programs and join as well.
- Our model is flexible and can be applied to any city in the United States.
- Our model is fairly robust in relation to the parameters *k* and *c*.

Weaknesses

- We assumed the total amount of driving done is proportionate to the commute of each person. We assumed this is reasonable because the commute time is likely related to the distance of opportunities to a person's home.
- Our sample size for distance of commutes was only 30 zip codes. However, this is large enough for a reasonably accurate representative sample of the United States, and time did not allow for a more in depth study. We could not find a census for the data for each zip code like we did for the time.
- We did not consider the fact that a person could use more than one mode of transportation. It is likely that a person would use both public transit and car sharing.
- The transition matrix probabilities did not depend on city-related characteristics. Thus, the four cities behaved in similar ways. However, the cities were not so diverse that this behavior is unlikely. If we were to improve our model, we would choose to consider more factors in our transition matrix relating to city-specific data.

7 CONCLUSION

In this study, we analyzed the types of individuals who live in the United States and categorized them based on their driving patterns. Using these distributions, we were able to use Markov chains to represent the states of individual citizens in the four cities of Knoxville, TN, Riverside, CA, Poughkeepsie, NY, and Richmond, VA.

We tracked these citizens of each city across 20 time states, each representing a 6-month period. We used these data to evaluate the potential for four different models of car sharing: round trip car sharing, the one-way car-sharing floating model, the one-way car-sharing station model, and fractional ownership. We found that four cities had very similar results, so we estimated that a car-sharing business would profit most in the cities with the densest populations, for they would have the most participation in the least area.

From this model, the one-way car-sharing station model is the most cost effective. We attribute this to the high level of flexibility of this model, as well as the fact that it has the lowest cost in today's market. The implementation of this service in the cities studied would garner the highest levels of participation. In the future, we predict that this one-way station model will be even popular, due to fuel efficiency and potential for productivity during commutes.

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