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The Lion's (Car) Share of the Business: Viable Models of Car Sharing by City

Executive Summary

The car sharing industry has the potential to redefine urban lifestyle in the foreseeable future. The car sharing enterprises are proliferating in urban regions, and there are a significant number of factors leading to a propensity among millennials to not own vehicles of their own. Large automotive corporations are also heavily investing in fully autonomous vehicles, in hopes that revolutionary technological advances will vastly increase the prospects of car sharing. The most pressing issue for car sharing issues is thus to determine optimal business models for particular regions which maximize both the short term revenues and long term sustainability of such models which account for the ineluctable technological advances.

The overarching purpose of our work is to develop a series of mathematical models which can be employed when considering the many facets of implementing car sharing models. We begin by completing a cohort analysis of sorts on the market available car sharing companies. With two primary factors that drivers consider when making decisions about car sharing—amount of time spent in the car and number of miles driven on a daily basis—we grouped American drivers into nine different categories and forecasted how the percentage of the American populace in each category will evolve in forthcoming years. The results that we found were quite interesting; a significant number of people are being deterred from commuting to work in vehicles that they own, which exhibits that there is certainly an increasing market available for car sharing companies to capitalize on.

In addition, we found that out of the cities of Poughkeepsie, NY; Richmond, VA; Riverside, CA; and Knoxville, TN, Richmond is best suited for the round trip car sharing model, while Riverside is best suited for the other three car sharing models analyzed (one-way car sharing floating model, one-way car sharing station model, and fractional ownership). In addition, the one-way car sharing station model is best for all four cities.

When the potential new technologies of autonomous and eco-friendly cars were considered, we found that the best suited cities for each car sharing method were still the same, while the one-way floating model became the best model for Riverside and Knoxville.

Section 1: Introduction

Section 1-1: Background

The growing popularity of car sharing in urban areas points to a gradual revolution in the automobile industry. Car ownership becomes increasingly disadvantageous in areas where street congestion is a regularity and parking is often a dubious proposition at best. In such circumstances, car sharing has found a place for itself, smoothly integrating into an existing, thoroughly developed public transportation system. While public transportation would fulfill the day-to-day mobility requirements of the majority of people, car sharing essentially fills in the gaps, in the individualistic lifestyles of those who value private transportation yet shirk from the prospects of keeping a car. In heavily urbanized locations, where the search for parking often composes half the trip, car sharing is even more useful a system; furthermore, parking spaces are often reserved for vehicles in the car sharing industry. Many cities which have incorporated car sharing have also established bus and cycle lanes, legislature from which a drive towards public transportation and car sharing is evident. Car sharing is even more lucrative to customers due to its eliminating the need to pay for gasoline and regular maintenance, accruing enormous annual savings. And of course, limiting the number of cars on the street is a much needed environmentally benign movement.

Currently, the success of car sharing is largely due to technological advances in databasing and bookkeeping, accurate billing, and dependable car location accountability. However, General Motors, Ford, and other vehicle manufacturers are investing hefty sums of money into autonomous vehicle research, with possible end goals of doorstep pickup and delivery, transporting individuals around the city without the need of a human driver. With forthcoming revolutionary research in the field of artificial intelligence in the context of vehicle autonomy coupled with all of the aforementioned benefits, car sharing has the potential to usurp the archetypal necessity of car ownership for a significant population of the populace.

Section 1-2: Restatement of the Problem

Given the inevitability of the car sharing revolution, we have developed models to accomplish the following:

- 1) We determine how the two factors of amount of time driving a car and number of miles driven on a daily basis partition the population of American drivers.
- 2) For each of the cities of Knoxville, TN; Poughkeepsie, NY; Richmond, VA; and Riverside, CA, we determine the viability of the following four popular business models for car sharing participation in that particular city and then rank their prospects of success.
 - a) Round trip car sharing

- b) One-way car sharing to and from a location of the customer's choice, which requires jockeys to reposition vehicles
 - c) One-way car sharing to and from predefined stations
 - d) Joint ownership of vehicles
- 3) We consider autonomous vehicles in our model for the previous component (2), a development which would mitigate the cost for human jockeys and for predetermined pickup and dropoff locations.

Section 2: Who's Driving?

Section 2-1: Factors Considered in the Model

We aim to reconcile the amount of time spent in the car with the number of miles driven in a day, two factors on which likelihood of car sharing is heavily dependent. Although data are readily available on the amount of miles driven in a year, computing the time spent in a vehicle requires synthesis of more variables. We stratify our analysis into two distinct classifications of regions: those that fall within a metropolitan statistical area and those that do not.

Metropolitan service areas are generally considerably urbanized, with relatively high population and a developed economic internetwork.

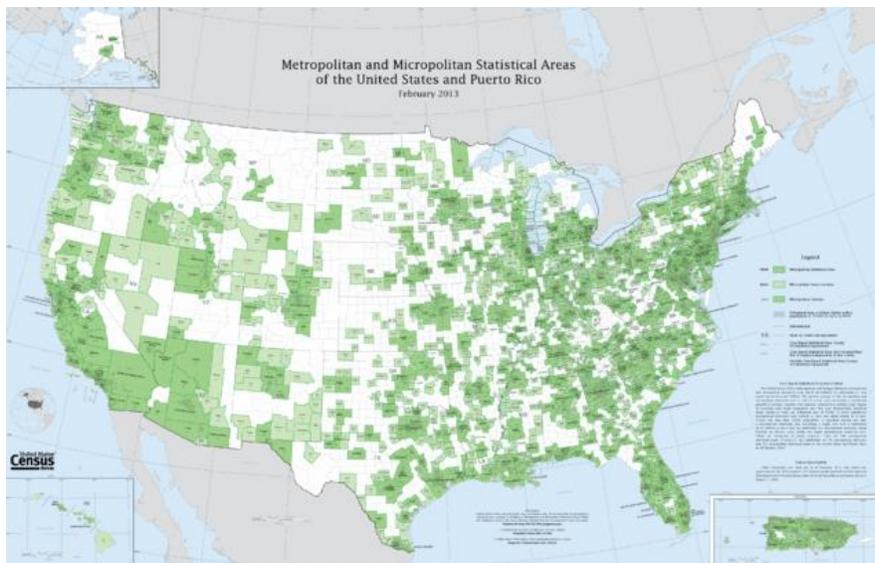


Figure 2-1: Map of the metropolitan statistical areas in the United States and Puerto Rico. The intensity of the color directly corresponds to the population of the metropolitan statistical area.

We then further partition the metropolitan statistical areas into categories contingent on population, as shown in Figure 2-2.

		Unweighted Frequency	Unweighted Percent
Code	Meaning		
-9	Not ascertained	2	0.0
01	In an MSA of Less than 250,000	17,018	11.3
02	In an MSA of 250,000 - 499,999	12,800	8.5
03	In an MSA of 500,000 - 999,999	16,066	10.7
04	In an MSA or CMSA of 1,000,000 - 2,999,999	32,508	21.7
05	In an MSA or CMSA of 3 million or more	40,975	27.3
06	Not in MSA or CMSA	30,778	20.5
Total		150,147	100.0

Figure 2-2: The categories in which metropolitan service areas fall, along with the number of metropolitan areas that fall within the category expressed as a raw frequency and as a percentage of the total number of areas.

Within each of these metropolitan service area partitions, we find the average commute speed within the metropolitan service area, and on synthesizing those average speeds with the population and number of each type of metropolitan service area, we compile a data set composed of the amount of time individuals spend driving a car per day. With this data set, we stratify drivers into low, medium, and high categories in both time spent driving and miles driven, with each category representing a third of the data range. Running a Java program we have written, we determine the percentages of the populace that fall within the nine categories formed from crossing the two aforementioned three category sets (time spent driving and miles driven).

Section 2-2: Assumptions and Simplifications

- Assumption: The population we consider drives privately owned vehicles.
 - Justification: The problem statement is to determine the percentage of drivers that fall into each of nine categories, which are indicators of their propensity to consider car sharing. Thus our analysis focuses on individuals who are considering car sharing, the majority of whom are car owners.
- Simplification: People travel within their metropolitan service area (or lack thereof).
 - Justification: The majority of travel occurs within a metropolitan service area. The commute to and from work, social networks that create day-to-day interactions, and trips to points of interest will all be largely confined by the metropolitan service area due to convenience and time constraints.
- Simplification: People within a metropolitan service area travel at the same speed (the average commute speed) regardless of their end destination.
 - Justification: The majority of the time spent in a car is in the process of commuting to and from work, and the average speed to travel within an area in

noncommuting endeavors does not drastically differ from the average commute speed.

Section 2-3: The Model

2001	Low Time	Medium Time	High Time
Low Distance	31.22%	0.69%	0.00%
Medium Distance	0.91%	28.53%	0.04%
High Distance	0.00%	1.50%	37.11%

Figure 2-3: Table demonstrating the the percentage of the American populace that fell within the intersection of the categories found in the rows and columns in the year 2001.

2009	Low Time	Medium Time	High Time
Low Distance	33.17%	1.44%	0.00%
Medium Distance	1.04%	29.92%	0.06%
High Distance	0.00%	1.04%	33.34%

Figure 2-4: Table demonstrating the the percentage of the American populace that fell within the intersection of the categories found in the rows and columns in the year 2009.

The above two tables display the evolution of the nine categories in which the driving population fall. Considering each row of the table to be a row of a matrix and each column of the table to be a column of a matrix, we model the yearly evolution of the categories with the

$$X_{2009} = \begin{pmatrix} .33167 & .01436 & 0 \\ .0104 & .29923 & 0.00058 \\ 0 & .01042 & .33335 \end{pmatrix}$$

following matrix A. X_{2001} and X_{2009} follow directly from Figures 2-3 and 2-4.

$$X_{2001} = \begin{pmatrix} .31219 & .00688 & 0 \\ .0091 & .285 & 0.00041 \\ .002 & .01505 & .37114 \end{pmatrix}$$

$$X_{2009} = A^8 X_{2001}$$

$$A^8 = X_{2009} * X_{2001}^{-1}$$

In order to do so, we employ a diagonalization process, decomposing matrix A as follows:

$$A^8 = SJS^{-1}$$

J is a diagonal matrix in which the diagonal entries are the eigenvalues of the matrix A^8 , and the columns of S are composed of the eigenvectors of A^8 . As A^8 is diagonalizable and A^8 furthermore possesses real eigenvalues and eigenvectors which are elements of \mathbb{R}^3 , the matrix A itself can also be diagonalized. It can easily be shown that the eigenvectors of A will be the same as those of A^8 , but the eigenvalues of A will be the eigenvalues of A^8 raised to the power of $\frac{1}{8}$. As such,

$$A = SJ^{1/8}S^{-1}$$

Through diagonalization, we obtain the following:

$$S = \begin{pmatrix} -0.984844 & 0.849593 & 0. \\ -0.173123 & -0.526183 & 0. \\ 0.0105332 & 0.0363787 & 1. \end{pmatrix}$$

$$J = \begin{pmatrix} 1.06436 & 0 & 0 \\ 0 & 1.04464 & 0 \\ 0 & 0 & 0.9 \end{pmatrix}$$

$$S^{-1} = \begin{pmatrix} -0.790905 & -1.27702 & 0. \\ 0.260221 & -1.48032 & 0. \\ -0.00113574 & 0.0673031 & 1. \end{pmatrix}$$

And we deduce that

$$A = \begin{pmatrix} 1.0074 & .00316 & 0 \\ .00027 & 1.0056 & 0 \\ 0.00043 & -.0006 & .98692 \end{pmatrix}$$

Thus, for a particular year, the matrix representing the percentage of the populations in each category is as follows:

$$X_{year} = A^{year-2001} X_{2001}$$

This model is essentially a quasi-Markov chain, given that the matrix A is a quasi-stochastic matrix (with all entries as opposed to each column) summing to 1 and that the model considers intercategory population shifts. Furthermore, this model lends itself to a cohort analysis of sorts of the market available to car sharing companies. From this model we draw Figure 2-5, the evolution of each category over time.

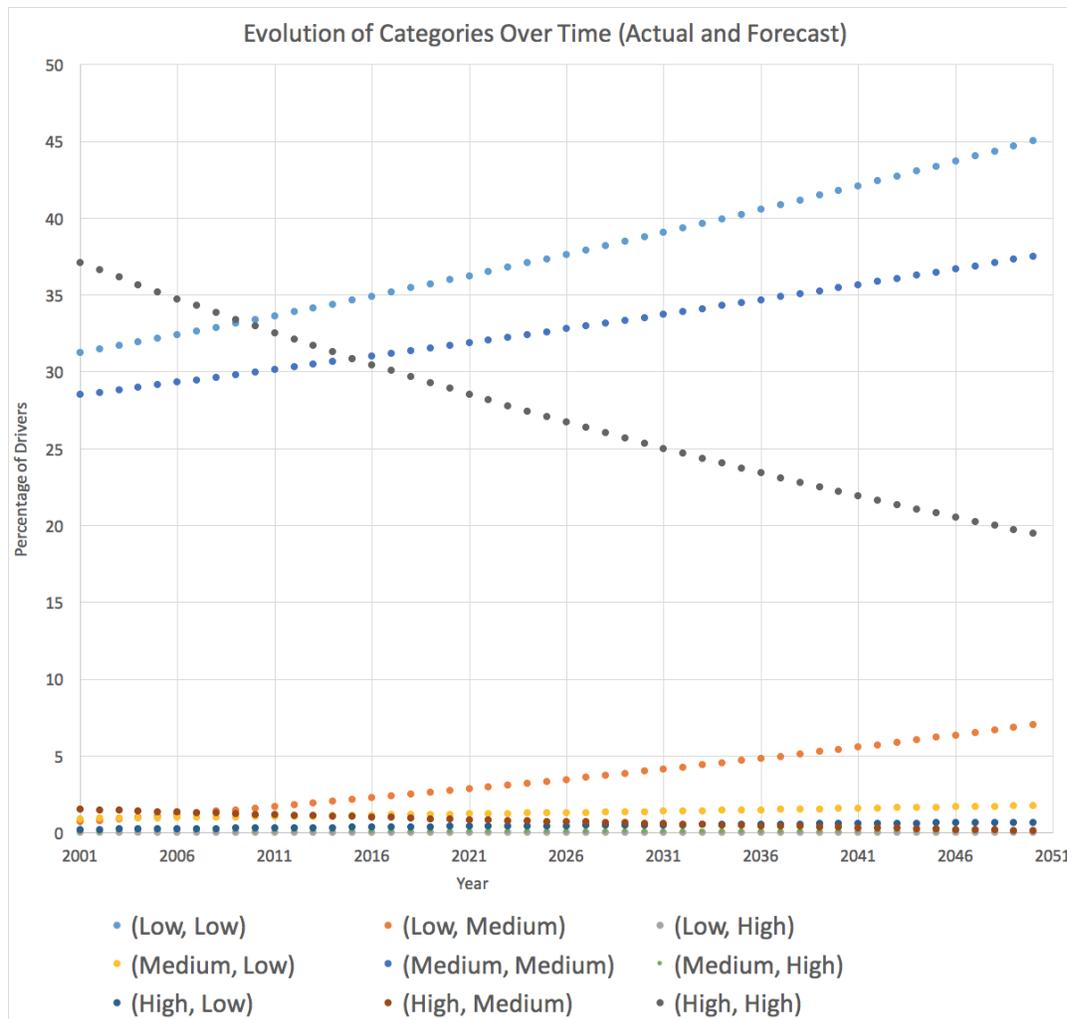


Figure 2-5: Scatterplot which shows the evolution of each of the nine categories over time. In the legend, categories are specified by two parameters, the first of which represents the driving distance category and the second of which represents the driving time category. (Low, Medium), for instance, would represent the percentage of drivers that drive for a low amount of distance and spend a medium amount of time driving on a daily basis.

Section 2-4: Results

There are three primary categories of note in the above graph (Figure 2-5): People who spend a low amount of time driving and drive a low number of miles, people who spend a medium amount of time driving and drive a medium number of miles, and people who spend a high amount of time driving and drive a high number of miles. Evidently, the number of drivers who spend a low amount of time driving and drive a low number of miles is increasing, a trend which may reflect the growing prevalence of teleworking; however, the trend may also indicate many drivers' being deterred from commuting to work in vehicles of their own. Similarly, the number of drivers who spend a medium amount of time driving and drive a medium number of hours is increasing. However, the number of drivers who spend a high amount of time driving to drive a high number of miles is steadily and more rapidly decreasing; from these three trends we conclude that a significant number of people are being deterred from commuting to work in their own vehicles, which demonstrates the increasing available market on which car sharing companies may capitalize.

Section 2-5: Strengths and Weaknesses

The amount of available data points were perhaps both the largest strength and the largest weakness of the model. We are able to access the raw data from the National Household Traffic Survey rather than just summary statistics. This allowed for more accurate percentiles and the ability to group people in more descriptive ways. However, we are only able to access this data for the 2001 and 2009 surveys. While it is available for earlier years, the survey asked different questions, and some values necessary for our calculations were not available. Having data sets for more years would have made it possible to more accurately determine the coefficients in our quasi-Markov chain matrix.

Section 3: Zippity do or don't?

In recent years, as car sharing has gained popularity, various car sharing companies have come up with their own unique business models. The four major models are as follows:

- Round Trip Car Sharing: Vehicles must be rented from and returned to the same station. One example of round trip car renting is Zipcar.
- One-Way Car Sharing (Floating): Vehicles are rented from and returned to general areas rather than specific stations. The starting and ending area do not have to be the same. Currently, the floating model requires "jockeys" to reposition vehicles between trips.
- One-Way Car Sharing (Station): Vehicles are rented from and returned to existing stations. However, the start station and final station do not have to be the same.
- Fractional Ownership: Several owners pitch in to jointly buy a private car.

Different car sharing models may be most preferable for a new car sharing company in different cities, as the viability of a business model is dependent on various city-specific factors such as convenience, cost, and demographics. In particular, car sharing is known to be trending amongst millennials, who generally possess the income and desire for private transportation as well as a certain aversion to the cumbersome responsibility of owning and maintaining a car.

We create success models for the four car sharing business models based on various city-dependent factors and run them on data compiled from the vastly different cities of Knoxville, TN; Poughkeepsie, NY; Richmond, VA; and Riverside, CA, all of which already host Zipcar stations.

Section 3-1: Factors Considered in the Models

3-1-1: Walkability

The walkability metric is based on walk score, which is defined to be the feasibility of walking to amenities in a particular location. The proprietors of Walk Score define it in accordance with the following description:

Walk Score®	Description
90–100	Walker's Paradise Daily errands do not require a car.
70–89	Very Walkable Most errands can be accomplished on foot.
50–69	Somewhat Walkable Some errands can be accomplished on foot.
25–49	Car-Dependent Most errands require a car.
0–24	Car-Dependent Almost all errands require a car.

Figure 3-1: A scale of walkability according to WalkScore.com

Calculation of walk score occurs through a patented system which “analyzes hundreds of walking routes to nearby amenities,” with points awarded through a decay function based on distance to amenities in various categories. Amenities within a 5 minute walk are given maximum points, and those over a 30 minute walk are given no points. Additionally, the walk score takes into account pedestrian friendliness based on population density and road metrics.

The greater the walkability of an area, the less the necessity for cars and, consequently, the fewer the users of car sharing. As a result, demand for car sharing will be greater when the walk score is lower. In order for our walkability metric to increase with demand for car sharing, we set it to $(1 - \text{walk score})$, where the walk score is a decimal percentage between 0 and 1 rather than an integer between 0 and 100. Naturally, in large cities the walk score varies by neighborhood, resulting in a walk score gradient often concentrated at college campuses or downtown, as shown in the figure below. Because all walk scores from a city are not easily available, we use the average walk score of each city to compute our final walkability metric.

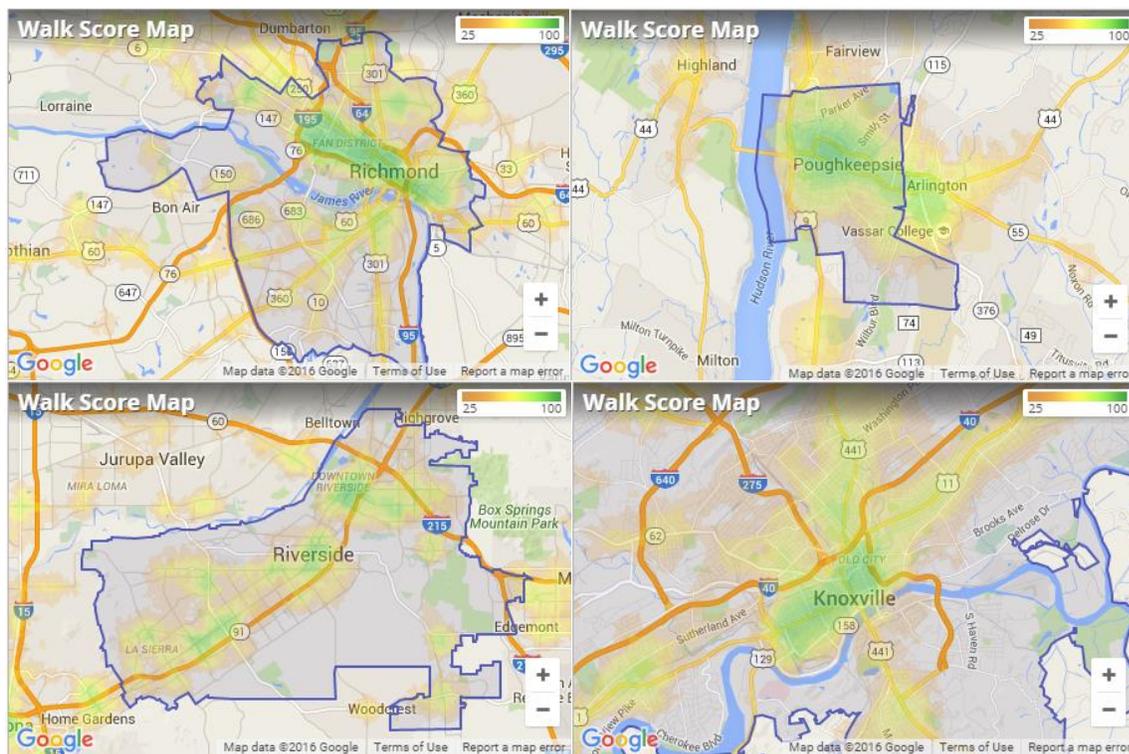


Figure 3-2: Maps extracted from Walk Score, representing the walk score index of each of the four cities: Richmond in the top left, Riverside in the bottom left, Poughkeepsie in the top right, and Knoxville in the bottom right. The greener a region, the higher its walk score.

3-1-2: Population Density

The density metric simply measures the average population density of a given city by taking the population of the city and dividing by the geographic area of the city. The denser a city, the fewer parking spaces per person, and the lower the likelihood that an arbitrary person will own a car. At the same time, an extremely high density may completely eliminate the need for any cars, owned or car shared, and as a result car sharing popularity may in fact decrease with very high densities. Although the effect of population density is unclear, car sharing popularity is undoubtedly expected to change with city population density.

3-1-3: Millennial Percentage

Naively speaking, as millennials are the target demographic for car sharing, the greater the percentage of millennials, the greater the chances a car sharing company will succeed in a city. At the same time, more lucrative car sharing businesses may deter millennials, who do not have the spending money for such models of car sharing, so it is possible that more millennials may result in more costly business models being less viable in certain cities. However, being the target demographic, millennials' tendencies and preferences will greatly affect the success of a particular car sharing model. The percentage of millennials is taken from the number of millennials in the city divided by the total city population from the 2010 U.S. Census.

3-1-4: Median Income

Different median income values in different cities might rightly affect a car sharing business model's chances of successes, since lower-income residents of a city would rather rely on public transport, while extremely high-income residents might prefer owning a car. A medium-to-high income value resident would be the most likely to participate in car sharing, so a city with more of those, on average, would thus more easily allow a car sharing business to prosper.

Section 3-2: Assumptions and Simplifications

- Assumption: The results from modeling car sharing in large cities can be generalized to smaller cities.
 - Justification: We believe that the factors that affect success of car sharing in large cities generalize readily to small cities.
- Assumption: The Cost of Living Index is proportional to the cost of gas and maintenance.
 - Justification: The average cost of living index is set to 1. As cost of living increases, we expect costs such as gas prices to increase as well. Therefore, multiplying such values by the cost of living will adjust for the fact that gas prices and car maintenance costs are not independent of city.
- Assumption: A car can be driven for 150,000 miles before it must be replaced.
 - Justification: This value is taken from Consumer Reports.
- Assumption: At best 4 people can share a car in fractional ownership.
 - A sedan can usually fit at most 4 adults comfortably, and decreasing the number of participants increases the up-front cost for each person.
- Simplification: Zipcar is representative of round trip car sharing.
 - Justification: Zipcar's car sharing system uses round trip car sharing, and it is currently one of the most popular such companies.
- Simplification: Turo is representative of a one-way car sharing floating model.

- Justification: Turo’s car sharing system uses a one-way car sharing floating model, and it is currently one of the most popular companies using that model.
- Simplification: Hertz is representative of a one-way sharing station model.
 - Justification: A one-way sharing station model is fairly similar to how car rental works, so Hertz, a popular car rental company, must represent this model well.

Section 3-3: The Model

For each model of car sharing other than fraction ownership, we determine a metric K (also known as “desirability”) representing the success of and participation in a representative of a given car sharing model, and find that metric for a given set of 16 large American cities. For round trip car sharing, we use Zipcar as our representative company, taking the total count of Zipcar cars in each of our cities based on information on the Zipcar website. To obtain K, our success metric for Zipcar, we then determine the count of Zipcar cars per million people in the population of the city. For the one-way car sharing floating model we use Turo, taking the number of Turo cars available for use on a specific day in each city by searching for available cars in each city on the Turo website. We then determine the number of available Turo cars in the given city per one million people in that city to obtain our metric K for Turo. We use Hertz to represent a one-way car sharing model, determining the number of Hertz rental stations available in a city by searching for rental stations in each city through the Hertz website. Then we set the number of rental stations per million people in the city, our success metric for Hertz.

We deduce that walkability, density, millennials, and income are the most significant factors leading to fluctuations in demand between models and cities. For each model, we determine the set of predictors that would result in the best adjusted R-squared value after generating multiple linear regression models predicting the metric divided by population in millions for that model. In order to make the results from each regression model comparable, we normalize the models based on a large American city, which we choose to be Chicago. The final coefficients for each predictor variable in each model are provided in Figure 3-3 below.

Factors	Model 1	Model 2	Model 3
Walkability	0.01811	0.07035	0
Density	0	0	1.280E-4
Millennials	13.30	-11.49	0
Income	4.392E-5	0	-7.457E-5

Constant	-6.212	0.2107	5.148
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Figure 3-3: The coefficients for each of our predictor variables in models 1-3.

The adjusted R-squared from models 1-3 are 64.87%, 20.14%, and 16.20%, respectively. Because the adjusted R-squared values are analogous to the amount of variation explained by the predictors, it follows that Model 1, wherein the predictor variables accounted for 64.87% of the variation, is fairly accurate, while Models 2 and 3, with much lower adjusted R-squared values, are mediocre in terms of describing the car sharing participation based on the predictor variables. Although it is possible that models other than the multiple linear regression model may fit the data, the residuals from each model have fairly linear normal probability plots, fairly normal histograms, and seemingly randomly distributed residuals with respect to predicted values, so the multiple linear regression model appears to be an appropriate fit to the data sets.

In Model 1, the p-values for the walkability, millennials, and income coefficients were 0.105, 0.003, and 0.021, respectively, signifying that if we assume that there was no correlation between our response variable and each of our predictors, there is a less than 11% chance we would have seen results as extreme or more extreme than ours due to variation. In the cases of millennials and income, the likelihood is less than 1% and 3%, respectively, so low that we must conclude that there is, in fact, a correlation between the predictor variables and our response variable, the success metric.

In contrast, the p-values for the walkability and millennials coefficients in Model 2, the Turo model, were 0.038 and 0.261, respectively, meaning that for all coefficients, there is a less than 27% chance (and less than a 4% chance for the walkability coefficient) that we would have gotten our results from variation if we had assumed no correlation between our response variable and each of our predictors. In the cases of walkability and vicinity, there is most likely a correlation between our predictors and the response variable due to such low probabilities. Millennials is more ambiguous of a correlation, because a 26% chance is more than 1 in 4 and is not low at all.

In Model 3, the p-values for the walkability, millennials, and income coefficients were 0.163, 0.283, and 0.131, respectively. This means that there is a less than 29% chance for all coefficients (and less than a 17% chance for the walkability and income coefficients) that we would have gotten our results from variation if we had assumed that there was no correlation between our response variable and each of our predictors, so we have evidence that there is correlation between our predictors and the response variable.

Car sharing method 4 is markedly different than the other car sharing methods as the car is privately owned and all costs fall directly on the owners. Assuming, as mentioned earlier, that, at best, 4 people could share fractional ownership, we conclude that each person must only pay 1/4 of the cost to purchase the vehicle upfront, as well as 1/4 of the gas cost. However, while the cost is more reasonable, fractional ownership is much less desirable. Owning a private car is nice and allows for added comforts such as the placing of useful items in the glove compartment and customizing the preset radio stations. While any given person is driving the car, they will have an extremely high satisfaction value. However, it is inevitable that there will be times that multiple people will want to drive the car to different locations. In these situations, not everyone can be happy and some people may incur additional costs if they decide to use some other form of transportation while other participants use the shared car. We decide that for times of conflict, satisfaction will be just below that of the other methods, so for a 4-person shared car, each given person will be unable to use the car about half of the times they would like to.

To calculate the cost of one trip for the company, we sum the cost of the car per trip, the cost of maintenance, and the cost of gas. This is then multiplied by a cost of living coefficient for the city to take into account the higher costs of maintenance, gas, and cars in more expensive cities. The cost of living is a number which over the United States has an average value of 1 and is a good indicator of how prices will change based on location. The cost of employees is then added to get a final cost.

The cost of the car + cost of maintenance + the cost of gas is found to be a constant based off of national averages for cost of maintenance per mile, cost of gas per mile, average cost of a car, and average lifespan of a car:

$$\frac{30000 \text{ dollars}}{\text{car}} * \frac{30 \text{ miles}}{150000 \frac{\text{miles}}{\text{car}}} + \frac{.608 \text{ dollars of maintenance}}{\text{mile}} * 30 \text{ miles} + \frac{.15 \text{ dollars of gas}}{\text{mile}}$$

* 30 miles = 28.74 dollars

In order to obtain the final results, the metric K derived from the regression for that particular model (also known as the “desirability”) and city combination is divided by the total cost for that model and city.

Section 3-4: Results

The final results are included in the table below.

Town and Business Model	City Dependent Spending	Employee Spending	Cost of Living Index	Cost	Desire	Desire/Cost
Riverside 3	28.74	0	1.17	50.4387	1020197.508	20226.48301
Riverside 2	28.74	12	1.17	45.6258	885140.2796	19399.99473
Knoxville 3	28.74	0	0.84	36.2124	666549.1072	18406.65372
Knoxville 2	28.74	12	0.84	36.1416	576225.9084	15943.56388
Richmond 3	28.74	0	1.01	43.5411	662940.947	15225.63617
Richmond 1	28.74	0	1.01	29.0274	347826.4284	11982.69319
Knoxville 1	28.74	0	0.84	24.1416	281181.1096	11647.16132
Riverside 1	28.74	0	1.17	33.6258	329408.2637	9796.295218
Riverside 4	28.74	0	1.17	33.6258	277041.625	8238.960114
Richmond 2	28.74	12	1.01	41.0274	313686.7425	7645.786536
Knoxville 4	28.74	0	0.84	24.1416	160361.25	6642.527836
Richmond 3	28.74	0	1.01	29.0274	187349.75	6454.238065
Poughkeepsie 3	28.74	0	1.22	52.5942	125058.7172	2377.804344
Poughkeepsie 4	28.74	0	1.22	35.0628	26809.125	764.6030836
Poughkeepsie 1	28.74	0	1.22	35.0628	17764.87435	506.658748
Poughkeepsie 2	28.74	12	1.22	47.0628	20683.30975	439.4831959

Figure 3-4:

It can be seen that Riverside was consistently a good area regardless of the business model utilized. This is because Riverside had a higher population than anywhere else as well as a low cost of living, both of factor in heavily into the final metric. The order as seen from this table is Riverside, Knoxville, Richmond, then Poughkeepsie for all business models except the first model. In the first model Richmond was found to be the best. This is because Richmond has a high millennial population which was heavily weighted in the regression of the first model.

Section 3-5: Strengths and Weaknesses

The regression models for car sharing models 1 and 2 are fairly good, accounting for a significant amount of the variation in the desirability metric K. Unfortunately, the models for car sharing models 3 and 4, while serviceable, are not as strong. In addition, the models do not take into account parking costs, which might affect the costs incurred car sharing companies. However, a great strength in the model lies in the fact that it can be fairly easily applied to any

city in America, as the predictor statistics needed to run each model are easily accessible for nearly every city in America.

Section 4: Road Map to the Future

After analyzing car sharing companies in the present, a look ahead into what the future holds in store yields interesting questions. In recent years, alternative energy fueling for cars, as well as self-driving cars, have been proposed. Therefore, we now modify our model from the previous section to examine the possible effect of the introduction of these emerging technologies on car sharing companies once vehicles incorporating such technologies have entered the arena.

Section 4-1: Factors Considered in the Model

The model is based on the one from section 3, and in addition to the factors from section 3, we consider the two technological advances that we are adding in this section: self-driving and environmentally friendly cars.

4-1-1: Self-Driving Cars

Autonomous cars decrease costs associated with paying employees. This only affects the one-way car sharing floating model as the service would no longer have to pay for “jockeys,” decreasing costs significantly. However, autonomous cars are difficult to make and will, at least initially, have a higher cost than human-driven cars.

4-1-2: Eco-friendly Cars

Eco friendly cars decrease the total gas cost per trip. It is assumed that by the year of the breakthrough, cars will be efficient enough that the gas cost for the average trip will be negligible. The effect of eco-friendliness is neither cost dependent nor model dependent.

Section 4-2: Assumptions and Simplifications

- Assumption: The initial cost of an autonomous vehicle will be \$150,000.
 - Justification: This is the estimate given by Google for the initial cost of their autonomous car.
- Assumption: The cost will remain at this value until some major technological breakthrough, at which point the buying price will decrease exponentially.
 - Justification: Before a technological breakthrough, it makes sense that the cost of a technology would not decrease significantly. After a breakthrough, the manufacturing costs would decrease significantly, so the buying price would do so as well.
- Assumption: The cost of an autonomous car will never go below the 2015 average car cost on the time frame that we will be examining.

- Justification: Since autonomous cars require more advanced, and therefore more expensive, technology to run, we find it reasonable that in an 11-year time span following a breakthrough, the cost of the self-driving technology will not decrease significantly enough to make the cost of an autonomous car less than that of an average car in 2015.
- Simplification: The cost of maintenance will not go down over time and can be considered as the constant 18.24 dollars per trip.
 - Justification: While the technological advanced parts will cause an increase in car costs, the cost of maintenance on the engine, tires, etc. will not go up.
- Assumption: The desire to use the technology will be constant over time.
 - Justification: The new technology will not make the customer experience any better but instead will lower costs over time.

Section 4-3: The Model

We consider two technological advances, autonomous cars and eco-friendly cars, as mentioned.

Google estimates that the initial cost of their first autonomous car will be \$150,000 (Consumer Reports). As previously mentioned, we assume the price of a self-driving car to remain at approximately this value until a technological breakthrough occurs. While we do not know when this breakthrough will occur, we call this year b and examine trends in the ten years after this breakthrough occurs. It is assumed that the cost of an autonomous car will never go below the 2015 average car cost: \$30,000. It is worth noting that therefore all values are in dollars considering 2015 inflation. It is assumed that at year b there would be the point $(0, 150000)$. By considering how prices trended over time with other technologies (such as the iPhone, computers, and the early invention of the car) it is decided that within 5 years of a wide release of the technology the price would go down 50% of the total amount it would ever go down. Using these two points to run regression the following formula is found as:

$$y = 18.24 \text{ dollars} + (120 * .863^x) \text{ dollars per car} * \frac{30 \text{ miles}}{150000 \text{ miles per car}}$$

y = the cost of the car,

x = years after the technological breakthrough.

Section 4-4: Results

From the exponential analysis the following data were found for the cost and could then be used to find Desire to Cost ratios for each city. At this point, the second business model is overall the best because the extra cost of labor is no longer required due to self-driving cars. Thus, the original convenience that came with this business model is weighted more heavily.

The new order, as before, should be Riverside, Knoxville, Richmond, then Poughkeepsie. There is no change in the order because the main difference as the technology increases is that the second business model improves. There are little differences in how the different cities would react to the new technology.

Town and Business Model	Cost	Desire	Desire/Cost
Riverside 2	33.04310692	885140.2796	26787.44107
Knoxville 2	23.72325625	576225.9084	24289.49476
Riverside 3	49.56466039	1020197.508	20583.1635
Knoxville 3	35.58488438	666549.1072	18731.24274
Richmond 3	42.78658717	662940.947	15494.13007
Richmond 1	28.52439145	347826.4284	12193.99997
Knoxville 1	23.72325625	281181.1096	11852.55121
Richmond 2	28.52439145	313686.7425	10997.14057
Riverside 1	33.04310692	329408.2637	9969.046328
Riverside 4	33.04310692	277041.625	8384.248661
Knoxville 4	23.72325625	160361.25	6759.664369
Richmode 4	28.52439145	187349.75	6568.054233
Poughkeepsie 3	51.68280827	125058.7172	2419.735332
Poughkeepsie 4	34.45520551	26809.125	778.0863473
Poughkeepsie 2	34.45520551	20683.30975	600.2956432
Poughkeepsie 1	34.45520551	17764.87435	515.593336

Figure 4-1: Desire-to-cost ratio for autonomous car technology 10 years after breakthrough.

Section 4-5: Strengths and Weaknesses

One strength of the model for autonomous and eco friendly cars is that it is time dependent, thus allowing projection of results for a significantly long amount of time. However, it is not dependent on the year but rather on the year after some breakthrough. More research could have been done to estimate when this breakthrough would occur. Additionally, we could incorporate our results from section 2 to account for the fact that people are driving less now than they used to.

Section 5: Conclusion

Our work models the market for car sharing enterprises, the relative prospects of four business models in four unique locations, and the adjusted prospects of the same four business models in the same four locations considering imminent technological advances.

Our market analysis demonstrates that an increasing portion of the American populace is being and will be deterred from spending a considerable amount of time and driving for considerable distances in personally owned vehicles, thus revealing entirely new population segments for the car sharing market to target. We arrive at such a conclusion using a quasi-Markov chain model in order to conduct nine different cohort analyses.

Furthermore, our multiple regression and comparison of car sharing methods determines that the one-way station method has the most potential for success in all four cities and furthermore, that of the four cities, Riverside is the most promising location for all but the round trip method.

We would suggest that car sharing companies take the following tables into account when considering which city to expand to or deciding upon business method.

	Best City
Round trip	Richmond
One-way floating	Riverside
One-way station	Riverside
Fractional Ownership	Riverside

Table 5-1: The best cities for each car sharing model.

City	Best Method
Poughkeepsie	one-way, station
Richmond	one-way, station
Riverside	one-way, station
Knoxville	one-way, station

Table 5-2: The best car sharing model for each city.

Finally, our model, when taking into account autonomous and eco friendly cars, indicates that the ride sharing business will be even more profitable in the future. We would suggest that companies invest in the car sharing business, as several already have.

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