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M3 Challenge First Runner Up, $15,000 Team Prize

***Note: This cover sheet has been added by SIAM to identify the winning team after judging was completed. Any identifying information other than team # on an M3 Challenge submission is a rules violation.

***Note: This paper underwent a light edit by SIAM staff prior to posting.

Executive Summary

In 2013, the transportation industry produced over half of all US-generated carbon monoxide, over half of all nitrogen oxides, and nearly one fourth of all hydrocarbons emitted into the atmosphere.Though the consequences of such irresponsible contamination are of long term detriment, the current transportation system is also undoubtedly integral to the economic prosperity of the United States of America and, more broadly, the world. How can we continue to reap the benefits of cheap transport while simultaneously reducing our carbon footprint?

The answer may be car-sharing, the practice of sharing a vehicle for daily commute. Offering the potential for cheap travel, reduced traffic, less air pollution, decreased foreign dependence on oil, and diminished need for land reserved for parking, car-sharing is an alluring possibility. But is it a practical one? Our team has been has been asked to find out by creating a mathematical model that will determine which car-sharing method will generate the greatest participation in four cities: Poughkeepsie, NY; Richmond, VA; Riverside, CA; and Knoxville, TN.

Our job was to first determine our consumer base by categorizing the types of commutes undertaken by drivers in the United States. Trips are categorized into short, medium, and long by both their distance and their duration. We began by examining existing data about the distribution of trips taken in the United States. Next, we made some reasonable assumptions about the duration of these commutes particularly with respect to their dependence on the distance. From this, we were able to estimate the percentage of United States drivers that fell into each combination of short, medium, and long distances and times. We found that most of the drivers fell on the diagonal of the table of trip time vs. trip distance. Most of the drivers were also within or close to the short time and short distance bin.

Next, we were tasked with evaluating the viability of various forms of car-sharing services in four cities. We decided to compare the cost for the consumer of the different forms of transportation, choosing money as our evaluation metric. Using results from behavioral economics and the concept of implicit cost (opportunity costs beyond the stated prices for car-sharing services), we were able to predict the number and location of car-sharing stations we would need for each type of system. Also, we were able to predict ridership under the assumption that people make rational decisions, comparing costs and benefits to make a choice.

Understanding that technological progress is never stagnant, we adjusted our mathematical model for the future by considering which car-sharing method was the most attractive when 100% of vehicles were electric and possessed autonomous driving capability. Interestingly, autonomous vehicles eliminated two of the car-sharing methods, the round trip and one-way floating because, without the need for a station or parking (an autonomous car can pick the customer up directly), they essentially devolved into one-way stationary sharing, which, of course, is no longer stationary. However, despite significant savings from the implicit cost of walking, the one-way car-sharing model costs customers more than twice as much as the fractional ownership model, making the fractional ownership model the clear optimal car-sharing choice, both in the present and in the future.

Share and (Car) Share Alike: Modeling New Approaches to Mobility

Team #7733

April 25, 2016

Contents

1 Introduction

1.1 Background

In the US, car ownership is more than a convenience, it is a right of passage from adolescence into adulthood, it is a vehicle of mobility, independence, convenience and, in an now urbanized America [1], necessity. Whether for transportation to work or conveyance to little league baseball game, the 256 million registered cars in the US [2] serve a purpose. Yet, they also sit largely idle [3]. According to Reinventing Parking, the average car sits parked for 95% of the day, leading many commuters to wonder whether owning a car, despite its numerous benefits, is worth it.

Enter car-sharing, the practice of sharing a vehicle for daily commute. At once, car-sharing appears to appeal to everyone for something. For the financially conscious, car-sharing promises a potentially cheaper form of travel. For the environmentally conscious, shared ownership promises potentially reduced traffic, air pollution, oil dependence, and land reserved for parking. Though largely concentrated in urban and collegiate environments, novel companies like Zipcar, RelayRides, and Enterprise CarShare are ambitious, seeking to expand into new markets and utilize new technology to do it.

Currently, four unique methods of car-sharing exist. The first method, round trip car-sharing, allows customers to rent cars by some combination of day, hour, and mile from a nearby station and to return them to that same point when finished. The second method, one-way car-sharing floating model, allows customers to rent cars on demand and return them to defined areas where a "jockey" will then manually reposition the vehicle. The one-way car-sharing station model, the third model, permits participants to pick up and drop off vehicles at any given station, not necessarily where they picked it up. The last method, fractional ownership, is the most literal version of "car-sharing," as it is a model whereby multiple individuals jointly purchase, maintain, and operate a private car.

1.2 Restatement of Problem

With the increased ubiquity of car-sharing, we have been asked to develop a model that encompasses the following:

1. Determine the percentage of current U.S. drivers in every combination of three categories—low, medium, high—of driving time and driving distance.
2. Determine which car-sharing model will generate the greatest participation in Poughkeepsie, NY; Richmond, VA; Riverside, CA; Knoxville, TN, based on our mathematical model, taking into account the unique characteristics of each city.
3. Determine which car-sharing model will generate the greatest participation in Poughkeepsie, NY; Richmond, VA; Riverside, CA; Knoxville, TN, when considering emerging automobile technologies such as autonomous and electric vehicles.

2 Who's Driving?

The major factors affecting consumer decisions about car-sharing are distance traveled and time spent traveling. Each factor can be classified into short, medium, and long, and a grid of nine possible combinations can be created. To properly evaluate the viability of car-sharing programs, we first determine the percentage of Americans whose typical commute falls in each category.

2.1 Assumptions and Simplifications

- **Assumption:** The typical commute for a majority of the population is short.

  Justification: Most people take live close to the destinations they want to visit, such as work, or are near a grocery store, as most people live in an urban or suburban area.

- **Assumption:** For a given distance, time is distributed normally.

  Justification: For a given distance, there are a myriad number of random factors that affect commute time such as weather, traffic, and construction, leading to a normal distribution of driving time within a given distance range.

- **Assumption:** The maximum drive length is 2860 miles

  Justification: 2680 miles is the width of the United States and few reasonable people would venture, in a car-sharing service, beyond that.

- **Simplification:** Each driver will fall into only one distance-time category.

  Justification: Because car-sharing will be predominately used for day-to-day activities, we will examine only the typical transit for each driver. Thus each driver will only fall into only a single category based on his or her typical transit.

- **Assumption:** The short/medium and medium/long distance divisions are 8.25 miles and 30 miles respectively.

  Justification: The short-medium distance division of 8.25 miles is the average length of the four cities we were asked to analyze. Traveling across the city would constitute a "medium" distance trip for most commuters. From personal experience, a 30-mile trip would be categorized as "long."

- **Assumption:** The short-medium and medium-long time divisions are 10 minutes and 30 minutes.

  Justification: With little grounds for establishing time divisions, we deferred to personal experience, categorizing a trip under 10 minutes as ”short” and a trip over 30 minutes as “long.”

2.2 Looking at Distance

Using our assumptions above, we looked at data from the 2009 National Household Survey and determined that the average distance traveled in a car was quite short and fell sharply with increasing distance [4]. We approximated this distribution to be exponentially decreasing and involve distance inversely proportional to some constant, which represents the increase in distance of a trip that would result in half the population deciding to make the trip. We approximated this value to be 4 based on anecdotal evidence. The result was then scaled to make the area under the curve equal to 1. This allowed us to take the integral of our equation to get the percentage of drivers in each driving range interval. Our distribution equation was

$$distance\_distr(x) = \frac{1}{5.77078} \times \left(\frac{1}{2}\right)^{\frac{x}{4}}.$$

We integrated our distribution to get the percentages of drivers in each category:

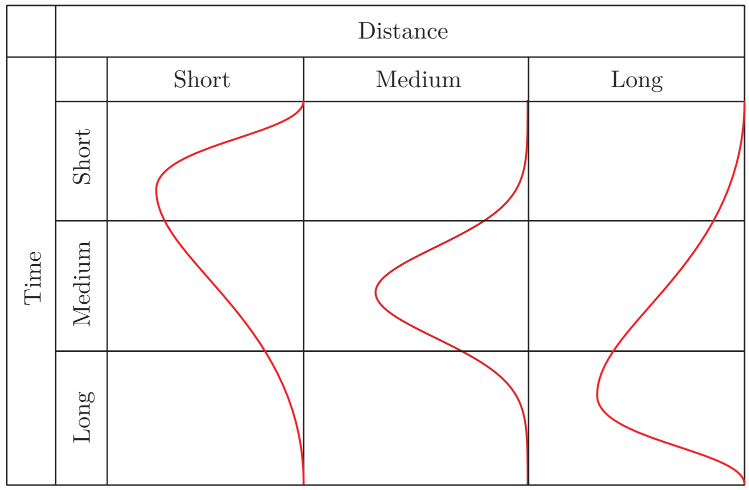
$$76.06\% = \int_0^{8.25} distance\_distr(x)dx.$$

This is repeated for each of the three divisions. The results are as follows:

Category	Range	Percentage
Short	$0 < x < 8.25$	76.06%
Medium	$8.25 < x < 30$	23.39%
Long	$30 < x < 2560$	.55%

2.3 Looking at Time

Because the distance and time are not independent (since longer trips will, of course, tend to take longer time), we found the mean and variance for the transit time within each distance category. We then used this to generate a distribution which we used to find what percentage of drivers from each distance category fell into each time category. The following schematic explains this approach:



The mean and variance for each category are as follows:

Category	Range	μ	σ
Short	$0 < x < 10$	10.5	53.1
Medium	$10 < x < 30$	26.6	137.8
Long	$30 < x$	81.0	5267

For each of these time ranges, we then determined the percentage of the normal curve each interval occupied within the distance category through the normal cumulative distribution function. The results are below:

		Distance		
		Short	Medium	Long
Time	Short	39.78%	6.694%	3.1771%
	Medium	52.36%	53.528%	7.715%
	Long	0.3725%	38.6047%	75.889%

Because the normal curve extends below zero and the duration of a trip cannot be less than zero, we truncated the curve at $t = 0$. Because of this, our percentages do not yet add up to 100%. To fix this, we scaled the percentages in the above table so that they equal 100%. This results in the following data:

		Distance		
		Short	Medium	Long
Time	Short	43.00%	6.773%	3.661%
	Medium	56.60%	54.164%	8.890%
	Long	0.4026%	39.063%	87.449%

2.4 Combining Distance and Time

The percentages obtained from our analysis of time and data were combined by multiplying the percentage in each distance category into the corresponding percentage in each time category. The calculation is shown in the table below.

		Distance		
		Short	Medium	Long
Time	Short	43.00% × 76.06%	6.773% × 23.39%	3.661% × 0.55%
	Medium	56.60% × 76.06%	54.164% × 23.39%	8.890% × 0.55%
	Long	0.4026% × 76.06%	39.063% × 23.39%	87.449% × 0.55%

This produced the following final results for our estimated percentages of commute characteristic combinations.

		Distance		
		Short	Medium	Long
Time	Short	32.71%	1.584%	0.0199%
	Medium	43.05%	12.67%	0.0489%
	Long	0.306%	9.14%	0.4810%

2.5 Critical Analysis

Drivers cannot be reasonably categorized into having typical driving any category; all drivers take trips of many different lengths.

2.5.1 Sensitivity Analysis

The upper bound on the distance function has no effect as the function's value decreases to near zero values beyond a distance of 20 miles. The constants in the distance function, however, were very sensitive to change.

2.5.2 Comparison to Actual Data

We looked at an actual dataset and found the number of drivers in each time distance bin.

		Distance		
		Short	Medium	Long
Time	Short	37.80%	.48%	0.0027%
	Medium	20.17%	14.07%	0.32%
	Long	0.46%	3.78%	4.19%

Our model had a weaker correlation between time and distance than the dataset.

3 Zippity Do or Don't

car-sharing takes advantage of the fact that cars are not in use continuously and thus can service multiple individuals. This defrays the cost for the individual and spreads it across all the users in the group. There are multiple ways of accomplishing this goal.

3.1 Assumptions and Simplifications

- **Assumption:** People make judgments based on cost.

  Justification: As a measure of value, money provides an effective metric for an item or service's perceived worth. While there are additional subjective factors at , such as comfort or privacy, these are difficult to accurately quantify and are also reflected in the cost.

- **Assumption:** People will walk until the perceived value of walking is equal to the cheapest possible taxi service to bring them from their current position to their desired location.

  Justification: To determine the amount of distance that people will walk (and subsequently analyze the placement of car rental stations), it is necessary to determine the willingness of people to walk. Under the previous assumption, people will select the cheaper means of transportation. This suggests that individuals will walk until the cost of walking exceeds the cost of riding a taxi service to their desired location.

- **Assumption:** Commuter preferences have remained relatively constant since 1967. **Justification:** Although it is likely that there has been some change in commuter preference, there has not been a recent study on the perceived utility of walking, a value with is necessary for our analysis. More modern surveys and research can provide the answer to this question and should not significantly affect the outcome of our analysis.

- **Assumption:** There are no discounts (e.g., student discounts, promotions, etc.).

  Justification: Promotions are not distibuted evenly across the population, and their usage is difficult to measure.

3.2 The Utility Function

To determine the placement of car rental stations for those car-sharing methods that require it, we referred to a 1967 behavioral economics study done to analyze the placement of bus stations to maximize usage by the public. In this study, the primary factors considered were the total walking time to the station (TW), total time of the trip (TT), and the total cost of the trip (C) [5]. These factors were used to define a utility function (U), a measure of the perceived worth of taking public transportation:

$$U(TW, TT, C) = -0.147TW - 0.0411TT - 2.24C$$

The coefficients on this model indicate the importance that the average household places on each factor (i.e., its marginal utility). By taking a ratio of these numbers, we can identify the rate at which one factor will substitute for another.

3.2.1 Limitations of Utility Function

While the Domencich–McFadden utility function is useful as a reference and for rough modeling, it only accounts for 93% of households, it does not consider several factors such as average community income, environment, weather, and rate of crime that may impact how willing an individual is to walk relative to drive.

3.3 The True Cost of Transportation

When we compare the coefficients of travel time and cost,

$$\frac{-0.0411\ min^{-1}}{-2.24\ dollars^{-1}} = 0.0183\ dollars \cdot min^{-1} \times \frac{60\ min}{1\ hr} = 1.10\ dollars \cdot hr^{-1}.$$

This implies that the the average family in 1967 would be willing to substitute \$1.10 for an hour of transit time. Similarly, comparing the coefficients of walking time to total time,

$$\frac{-0.147\ min^{-1}}{-0.0411\ min^{-1}} = 3.58.$$

This implies that the average family perceives walking to be 3.58 times more costly than being in transit. Thus the average family will be willing to increase their transit time by up to 3.58 minutes to sacrifice 1 minute of time spent walking to the station.

Combining these two results, we concluded that the average person felt that walking costs about 1.10 $dollars \cdot hr^{-1} \times 3.58 = 3.94\ dollars \cdot hr^{-1}$ in 1967 and waiting in transit costs 1.10 $dollars$. After adjusting for 618.92% inflation since then, people will be willing to pay \$24.32 to not walk for 1 hour and \$6.81 to decrease transit time by 1 hour.

3.4 How Far Will I Walk?

To determine how far people are willing to walk to a car-sharing station, we used the assumption that people will make judgments based on cost. People will walk until the perceived cost of walking equals the cost of hiring the cheapest taxi service to transport them to the station. UberX appears to provide the least expensive service ($\$1.05 + \$0.15\ min^{-1} + \$0.70\ mile^{-1}$), so it was used as a benchmark.

3.4.1 Cost of UberX

The explicit cost of UberX is given by the stated prices where t_{drive} is the time spent driving to the car-sharing station:

$$Cost_{uberX,explicit} = \$1.05 + \$9.00\ hour^{-1} \times t_{drive} + \$0.70\ mile^{-1} \times d.$$

The implicit cost of UberX can be described by the perceived cost of waiting for the car (t_{wait}) and time spent driving to the station (t_{drive}):

$$Cost_{UberX,implicit} = \$6.81\ hour^{-1} \times (t_{drive} + t_{wait}).$$

According to Forbes, the average wait time for UberX is 8 minutes, or 0.133 hour, so $t_{waiting} = 0.133\ hr$.

Combining the explicit and implicit costs, the total cost of hiring an UberX is given by

$$\begin{aligned} Cost_{uberX,total} &= Cost_{UberX,explicit} + Cost_{UberX,implicit} \\ &= \$1.05 + \$9.00\ hr^{-1} \times t_{drive} + \$0.70\ mile^{-1} \times d + \$6.81\ hr^{-1} \times (t_{drive} + 0.133). \end{aligned}$$

3.4.2 Cost of Walking

The explicit cost of walking is free. The implicit cost, on the other hand, is given by

$$Cost_{walking,implicit} = \$24.32\ hr^{-1} \times t_{walk},$$

where t_{walk} is the time spent walking.

The total cost of walking, then, is simply

$$Cost_{walking,total} = Cost_{walking,implicit} = \$24.32\ hr^{-1} \times t_{walk}.$$

3.4.3 Putting It Together

To determine the distance people are willing to walk, we set the cost of walking equal to the cost of driving:

$$Cost_{uberX,total} = Cost_{walking,total},$$

$$\begin{aligned} \$1.05 + \$9.00\ hr^{-1} \times t_{drive} + \$0.70\ mile^{-1} \times d + \$6.81\ hr^{-1} \times (t_{drive} + 0.133) \\ = \$24.32\ hr^{-1} \times t_{walk}. \end{aligned}$$

Solving for the total walking time,

$$t_{walk} = \frac{1\ hr}{\$24.32} \times [\$1.05 + \$9.00\ hr^{-1} \times t_{driving} + \$0.70\ mile^{-1} \times d + \$6.81\ hr^{-1} \times (t_{drive} + 0.133)].$$

According to our model in Section 1, the average time for a car to drive a short distance is 10.5 minutes, or 0.175 hours. Furthermore driving from one's house to the station is the same distance as walking from one's house to the station. Since distance is a product of rate and time and the average person walks at a rate of 3.1 miles per hour,

$$d = t_{walk} \times 3\ mph.$$

Using these new values in the original equation for t_{walk}, we find that

$$t_{walk} = 0.208\ hours.$$

This tells us that 93% of households would be willing to walk for 0.208 hours to the nearest station, which is an implicit cost of $5.05. Assuming each person walks at the average pace, 3.1 mph, each person is willing to walk 0.645 miles per trip.

3.5 Round Trip Car-Sharing

Using Zipcar's price model, vehicles are rented by 30 minute increments for X dollars, including gas and insurance, and are picked up from and returned to the same point. Participation in this service requires a $7 membership fee per month. Additional taxes, T dollars, are levied by state.

3.5.1 Assumptions

- **Assumption:** Commuters only return their rideshare once per day.

  Justification: Consumers must return the rideshare for every time they take it out. Most consumers do not keep rideshares overnight. Therefore, consumers can be assumed to return their cars once per day.

- **Assumption:** The average commuter has the car for 9 hours and 45 and Y minutes on weekdays, where Y is the time required to walk back to the station where his or her car is parked.

  Justification: This takes into account the standard 8 hour work day, the standard 30 minute lunch break, the 75 minutes the average commuter travels, and the Y time it takes for the average commuter to walk back to the station where his or her car is parked.

- **Assumption:** The average commuter takes the car out twice on weekends for a duration of four hours and Y minutes.

  Justification: Based on anecdotal evidence, this is enough to do errands or have an evening out and then walk back to the station.

- **Assumption:** The time spent inside the station is negligible.

  Justification: Opening the door and turning the car on can be reasonably assumed to take only a few seconds

- **Assumption:** The cars follow an L-shaped snake geometry to travel between points.

  Justification: The grid pattern is a very common city layout pattern. This assumption prevents paths from traveling through buildings and infrastructure.

- **Assumption:** No pickups and drop-offs will occur at airports.

  Justification: It is illegal in some areas, and to simplify the model, we decided to ignore this possibility.

- **Assumption:** No taxes.

  Justification: No places we considered had additional taxes on car-sharing.

3.5.2 Finding Y

Using the snake geometry approach, to find the average distance traveled by a walker, we assume there is an arbitrary walker who must walk in an L-shaped pattern at a distance R away from the station. The set of all locations where the walker could be forms a rhombus of side length $R\sqrt{2}$, and perimeter $4R\sqrt{2}$. Integrating for all distances R, the distance R traveled for each walker along the rhombus, from a minimum distance of 0 to the maximum distance of .645 miles for each trip,

$$\int_0^{0.645} 4\sqrt{2}R^2 \, dR = 0.506.$$

This gives the sum of the distances traveled by all arbitrary walkers. The mean can be computed by taking the total distance of all possibilities divided by the number of all possible arbitrary walkers, which in this case is simply the area of the largest rhombus (of side $.645\sqrt{2}$).

Thus, dividing 0.506 by $(.645\sqrt{2})^2$ (because the rhombus is a square) yields .608 miles as Y (time required for the commuter to walk back to the station where his or her car is parked).

3.5.3 Implicit Cost

The average person makes approximately 2 trips per day or 730 trips per year and drives, on average, for approximately 25 minutes per trip. This is justified by the average commuter driving 75 miles.

In one year there are 12 months, 52 weekends, and 260 weekdays. The implicit cost for the average commuter is

$$\begin{aligned} \textit{implicit cost} = {} & Y \textit{ mile/walking trip} \times 1 \textit{ hour}/3.1 \textit{ mile} \times \$24.32/\textit{hour} \\ & \times 2 \textit{ walking trips/day} \times 365 \textit{ days/year}. \end{aligned}$$

Since Y = 0.608 mile,

$$\begin{aligned} \textit{implicit cost} = {} & 0.608 \textit{ mile/walking trip} \times 1 \textit{ hour}/3.1 \textit{ mile} \times \$24.32/\textit{hour} \\ & \times 2 \textit{ walking trips/day} \times 365 \textit{ days/year}. \end{aligned}$$

Thus

$$\textit{implicit cost} = \$3482.00.$$

3.5.4 Explicit Cost

Based on membership fees and rates, we arrive at the following for explicit cost:

$$\begin{aligned} explicit\ cost = \$7\ membership\ fee/month \times 12\ months/year \\ + 52\ weekends/year \times (X/0.5\ hr \times (4+Y)\ hr)/weekend \\ + 260\ weekdays \times [(9.78333+Y)\ hr \times X/.5\ hr]/weekdays. \end{aligned}$$

Thus the to total explicit cost is given by

$$5625.54\ hours/year \times \$X/hour + \$84.00/year.$$

3.5.5 Total Cost

Summing the two costs, for each city,

City	x	Total
Poughkeepsie, NY	\$4.25/hour	\$27474.545/year
Richmond, VA	\$4.5/hour	\$28880.93/year
Riverside, CA	\$4.25/hour	\$27474.545/year
Knoxville, TN	\$4.25/hour	\$27474.545/year

3.6 One-Way Car-Sharing Station Model

3.6.1 Assumptions

- **Assumption:** All stations are operational (cars are present when they are needed and space is available when someone needs to park).

  Justification: The station and fleet should be appropriately sized for the number of users.

- **Assumption:** Time spent waiting in stations is 0 minutes.

  Justification: Building off of the previous assumption, cars should be available when needed. The time spent opening and starting the car can be considered to be negligible.

- **Assumption:** Stations are within walking distance of the destination.

  Justification: People will not use station based ride sharing unless they are within walking distance.

- **Assumption:** Average person commutes 2 trips per day.

  Justification: People must return home and therefore take two trips.

3.6.2 Implicit Cost

This is similar to the round trip cost, but just for one leg. This calculation resuses distance Y from the above part:

$$\frac{0.608\ miles}{trips} \times \frac{\$24.32}{hour} \times \frac{4\ trips}{day} \times \frac{365\ days}{year} = \$6964/year.$$

3.6.3 Explicit and Total Cost

The explicit costs for a one-way trip are determined by the following fees. The variable X is used to represent the specific fees per city:

$$Explicit\ Cost = \left(\frac{\$7\ membership\ fee}{month} \times \frac{12\ months}{year}\right)$$

$$+ \left(\frac{365\ days}{year} \times \frac{\$X}{0.5\ hr} \times \left(1.25 + \frac{0.608\ miles}{\frac{3.1\ miles}{hr}}\right)\right)/day$$

$$= (\$84 + 2344.24\ hours \times \$X/hour)/year,$$

$$Total\ Cost = Implicit\ Cost + Explicit\ Cost = \frac{\$6964 + 2344.24\ hours \times \$X/hour}{year}.$$

City	x	Total
Poughkeepsie, NY	\$4.25/hour	\$17011.02/year
Richmond, VA	\$4.5/hour	\$17597.08/year
Riverside, CA	\$4.25/hour	\$17011.02/year
Knoxville, TN	\$4.25/hour	\$17011.02/year

3.6.4 Critical Analysis

Many bikesharing programs, which are one-way station systems, have had issues keeping stations from becoming full or empty, which create the undesirable scenarios of not being able to park or not having a bike to ride. With bikes, trucks are employed to shift bikes from high bike density, popular destination stations to low density, empty stations. With cars, however, this is significantly less possible because a unique driver is required for each car [6]. This greatly increases the cost of implementation.

3.7 One-Way Car-Sharing Free-Floating Model

3.7.1 Assumptions

- **Assumption:** Cars receive "superpermits" to park in any residential, metered, or general spot for free.

  Justification: A free-floating model is defined by users being able to park anywhere and not only in designated station locations [7].

- **Assumption:** Users can find parking spots available near their destination.

  Justification: A "superpermit" system would make most spots available to the consumer.

- **Assumption:** Users do not need to walk to get from their car do the destination.

  Justification: If parking spots are near the destination, walking from the car to the destination is negligible.

- **Assumption:** Free-floating car users take short trips.

  Justification: Since free-floating systems are concentrated in downtown areas, trips can be assumed to be short.

- **Assumption:** Consumers will walk to the nearest available car.

  Justification: Apps provided by the rideshare service show users where the cars are. Consumers are assumed to be logical and value their time and therefore seek the closest car in order to minimize their utility cost.

- **Assumption:** Although it is a one-way system, most consumers will still take two trips on a daily basis

  Justification: Consumers generally return home for the night, and if they took the car-sharing service to get there, they will probably take it back. This requires two trips. The difference versus a round trip service is that the consumer does not have to pay for car-sharing for the time the car is sitting in the parking lot.

3.7.2 Approach

In a free-floating model, a designated area, referred to as a home area, contains many cars that people drive. The distribution, or car density, or cars in this area is equal to

$$\frac{number\ of\ cars}{area\ of\ service}$$

and can be modified by the ridesharing company by providing more or less cars. In Seattle, Car2Go has 660 cars over a service area of approximately the whole city, 83.78 square miles [8]. This gives a car density value of 7.88 cars/square mile as an approximate car density for a viable one-way free-floating model.

If the consumer walks a distance of R miles, they can walk to a circular amount of the city with area $\pi \times R^2$. The expected value of cars in this circular area is

$$Expected\ Cars = \pi \times R^2 \times 7.88/mile^2$$

In order to expect a car, this value must be greater than 1. This first occurs at a radius of .20 miles.

According to the model in part 2, ride sharing, the cost of walking is $24.31 per hour and the average speed is 3.1 mph:

$$cost/mile\ of\ walking = \frac{24.31\frac{dollars}{hour}}{3.1\frac{miles}{hour}} = 7.84\frac{dollars}{mile}.$$

This means that it costs the consumer $7.84 for each mile to walk to the nearest car. Combining,

$$Cost\ To\ Find\ Car = Expected\ Miles\ To\ Find\ A\ Car \times \frac{Cost}{Mile\ Of\ Walking}.$$

This evaluates to $1.58. Finally, the cost for the trip is similar to that of the other models, according to our model in part 1, the average time for a car to drive a short distance is 10.5 minutes, or 0.175 hours. Car2Go costs $0.41/ minute. Assuming two trips a day, and 365 days a year,

$$Cost\ per\ year = \frac{2\ trips}{day} \times (\$1.58 + \frac{.175\ hours}{trip} \times \frac{\$0.41}{minute} \times \frac{60\ minutes}{hour}) \times \frac{365\ days}{year}.$$

The total cost is therefore: $\$4293.09/year$.

3.8 Fractional Ownership

3.8.1 Assumptions

- **Assumption:** The insurance cost per car stays the same even if multiple people are sharing that car.

  Justification: Insurance rates for car-sharing vary but obtaining an insurance rate that matches the cost of a single person insurance is possible.

- **Assumption:** Three people share each car.

  Justification: Three was the most common number of people mentioned for the fractional ownership of normal automobiles [9].

- **Assumption:** Gasoline prices stay constant.

  Justification: The prices of gasoline will increase relatively uniformly with inflation even over the United States, allowing the increase in price of gasoline to be ignored.

- **Assumption:** People drive as much when they car share compared to not car-sharing.

  Justification: People still need to get to places.

3.8.2 Approach

When the cost of ownership of each car is spread over a number of people,

$$cost/year = \frac{repairs + insurance + payment}{number\ of\ people} + 365\frac{days}{year} \times 38.4\frac{miles}{day} \times \frac{gallons}{22.4 miles} \times gas\ price.$$

Where repairs, insurance, and payment are specified in dollars per year and gas price is specified in dollars per gallon, for each city,

State	Repairs[10]	Insurance[10]	Gas[11]	Car Cost[12]	Cost/Year
NY	365	1196	1.929	5600	3594
VA	382	758	1.556	5600	3220
CA	390	906	2.390	5600	3794
TN	353	762	1.528	5600	3194

3.9 Results

Comparing all four car-sharing methods, we found that the cheapest car-sharing method in every city, when taking into account explicit and implicit costs, is the fractional ownership model, followed by the one-way floating model, the one-way car-sharing station model, and, in last, the round trip car-sharing. Because we assumed that the overall cheapest model would garner the greatest participation, the ordering of expense is also the ordering of greatest participation.

Among our four representative cities, the cheapest location to participate in the fractional model is Knoxville, TN; followed by Richmond, VA; Poughkeepsie, NY; and, in last, Riverside, CA.

4 Road Map to the Future

4.1 Assumptions and Simplifications

- **Assumption:** In the future, 100% of vehicles are electric and autonomous.

  Justification: Advances in technology will make entirely electronic and autonomous vehicles possible and economically feasible. For simplicity, we will assume that in this hypothetical future, all cars possess the modern technology.

- **Assumption:** The electricity infrastructure does not significantly change over the next few decades.

  Justification: The electricity generation and distribution systems change slowly and it appears likely that, in the future, electricity will continue to be produced through coal plants.

- **Assumption:** Completely switching from conventional to electric cars reduces greenhouse gases by 33%.

- **Assumption:** Autonomous vehicles eliminate the need to walk to a station.

  Justification: Autonomous vehicles will be able to drive to whatever location needed to pick up users.

- **Assumption:** Electric cars cost $0.05 to drive per mile.

  Justification: This is derived from the cost of electricity and the energy efficiency of electric vehicles.

- **Assumption:** The cost of maintaining electric vehicles is $\frac{1}{3}$ that of conventional cars.

  Justification: The electric vehicle has less mechanical parts thus requires less maintenance.

- **Assumption:** The total costs associated with air pollutants in the US is estimated at anywhere from US$71 billion to $277 billion per year. For simplicity, we will take the lower bound of that estimate, US$71 billion.

  Justification: The amount of pollutants changes relatively slowly, allowing us to use the current pollution costs for the near future.

- **Assumption:** The number of licensed US drivers remains constant at 210M.

  Justification: The number of licensed US drivers has remained constant at 210M since 2009.

- **Assumption:** The cost of ride sharing does not change.

  Justification: While it is reasonable to presume that the cost of car-sharing will decrease as it scales, new legal barriers, taxes, and other unforseen obstacles could just as easily increase the cost. For simplicity, we will assume that the hourly rate of car-sharing does not chage.

- **Assumption:** Insurance premiums for autonomous cars will decrease by 66%. Because autonomous cars are safer and because electric cars cost less to maintain, insurance costs should also decrease. As there is no real world data for insurance premiums for autonomous cars, we presume that insurance premiums will fall at the same rate (by 66%) at maintenance for electric vehicles, an assumption not wholly inconceivable if car accidents are significantly reduced.

4.1.1 Pollution

Because US transportation contributes $\frac{1}{3}$ of all air pollution and total air pollution costs $71B, switching to 100% electric vehicles will save

$$\begin{aligned} total\ savings =& \frac{\$71B/year}{3} \\ =& \$26.333B/year. \end{aligned}$$

A savings of $26.333B among the 210M licensed drivers in the US represents a savings of

$$\begin{aligned} total\ savings\ per\ driver =& \frac{\$26.333B/year}{210M\ drivers} \\ =& \$125.38B\frac{1}{driver \times year}. \end{aligned}$$

Since each driver benefits equally from less air pollution, no commuter behavior is changed among the four car-sharing options.

However, the cost of energy (replacing gasoline) changes significantly, to \$0.05 a mile. Also significant is the reduction of implicit costs of walking. Instead of commuters needing to walk to stations in the round trip car-sharing model and one-way car-sharing station model, the autonomous car can pick them up. Similarly, in the one-way car-sharing floating model, the variable cost of labor goes to zero as cars can now "jockey" themselves.

4.1.2 One-Way Car-Sharing Model

Because autonomous cars can pick people up directly from their homes, the need to walk to car stations is eliminated, which creates a implicit cost savings of \$125.38 per driver per year.

Using the current rates for car-sharing, we can compute the explicit costs for the one-way car-sharing model:

$$\begin{aligned} explicit\ cost &= \$7\frac{membership\ fee}{month} \times 12\frac{months}{year} + 2\frac{trips}{day} \times 365\frac{days}{year} \times \frac{\$x}{.5hour} \times 1.25\frac{hours}{day} \\ &= \frac{\$84.00 + 1825hours \times \frac{\$X}{hour}}{year}. \end{aligned}$$

Where x is the half hour car-sharing rate,

$$\begin{aligned} total\ cost &= implicit\ cost + explicit\ cost \\ &= -\frac{\$125.38}{year} + \frac{\$84.00 + 1825hours \times \frac{\$X}{hour}}{year}. \end{aligned}$$

This formula is then used with each of the cities to derive a new cost:

City	x	Total Cost
Poughkeepsie, NY	\$4.25	\$7714.87/year
Richmond, VA	\$4.50	\$8171.12/year
Riverside, CA	\$4.25	\$7714.87/year
Knoxville, TN	\$4.25	\$7714.87/year

4.1.3 Round Trip Car-Sharing Model

With autonomous vehicles, there is no need for a round trip car-sharing model because every benefit of such (not having to walk to a station, having a car immediately at your disposal, etc.) can be achieved through the one-way car-sharing model. Thus, with autonomous vehicles, the round trip model devolves into the one-way, and we no longer consider it a compelling option.

4.1.4 Fractional Ownership Model

This model is modified to use electricity rather than gasoline giving the new equation

$$cost/year = \frac{repairs + insurance + payment}{number\ of\ people} + 365\frac{days}{year} \times 38.4\frac{miles}{day} \times \frac{\$.05}{mile}.$$

The inputs for this model are updated to reflect the assumptions and changes that electric autonomous vehicles will cause:

State	Repairs[10]	Insurance[10]	Car Cost[12]	Cost/Year
NY	122	399	5600	2741
VA	127	253	5600	2694
CA	130	302	5600	2711
TN	118	254	5600	2691

5 Model Assessment

Our model for the first section assumed the most people drove mostly short distances and that the time to travel was normally distributed around each distance bin. This allowed our model to account for the semidependence of time on distance. Weaknesses of our model include the fact that many variables are not explicitly present in our equation. This made it harder to extend this model to cover different cases. The normal distribution that we used for the time also made it harder to analyze the model for sensitivity to different variables.

For the second section, we assigned a cost to everything and compared the car-sharing methods in each city to each other. This allowed us to easily modify the parameters for the third section. While assigning costs to everything required a bit of interpretation, it allowed an easy way to compare each method.

6 Conclusion

In our assessment of car-sharing programs, we first created a model to categorize commutes in the United States by distance and duration based off of existing data and some assumptions about the characteristics of typical car trips. With these results, we began evaluating various car-sharing systems to assist potential companies in their plan of expansion.

Ultimately, through an analysis of implicit and explicit costs, we determined that fractional ownership was the ideal method of car-sharing. Furthermore, even with projected technology advancements, the result held constant. With advances in self-driving cars, we determined that some current systems such as round trip sharing and both one-way models for sharing devolved into a one-way free-floating model. With technological advancements and growing popularity, we can expect great opportunities and benefits from fractional ownership in the near future.

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