

# Moody's Mega Math Challenge 2017

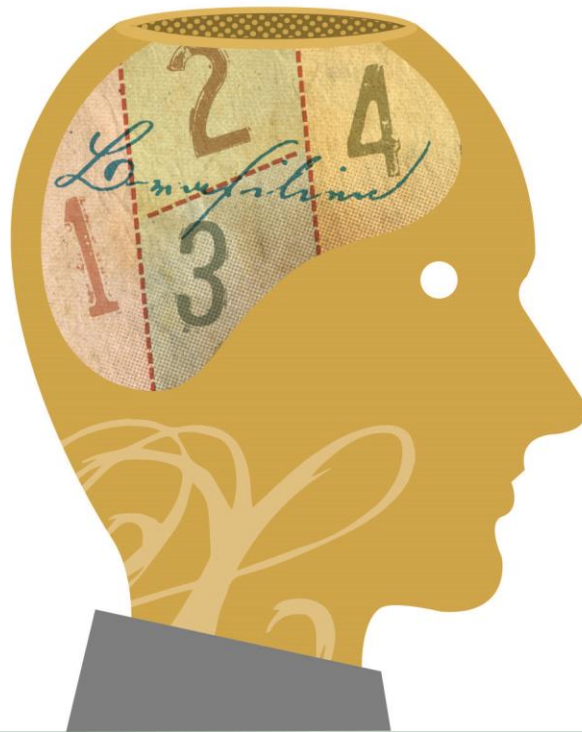
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**Moody's Mega Math Challenge Finalist, \$5,000 Team Prize**



**Moody's  
Mega  
Math  
Challenge**

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## From Sea to Shining Sea: Looking ahead with the National Park Service

### Executive Summary

One of America's greatest blessings is often among its most overlooked: its national parks. America's national parks stand as a hallmark of natural beauty, with unparalleled geographic and biological diversity. However, in light of worrying global warming trends, the well-being and longevity of the parks must be considered. In particular, our coastal parks are directly at risk due to rising sea levels and climate change. Our model seeks to predict coastal parks' vulnerability to these factors.

First, we developed a model to predict future sea level changes for five coastal parks. Global average sea level is affected by two main factors: thermal expansion of water and glacier meltwater. However, local sea levels vary due to location-specific land uplift resulting from isostatic rebound and tectonic plate activity. Taking into account these factors, a function was created to predict the yearly change in local sea level.

We used our model to generate a corresponding sea level change risk rating of high, medium, or low for each of the five coastal parks. Over a time frame ranging from 10 to 50 years, our model predicts a medium sea level change risk rating for Acadia, Cape Hatteras, and Padre Island. This signifies that these parks may endure moderately damaging consequences (infrastructure damage and wildlife loss). On the other hand, Olympic National Park and Kenai Fjords are predicted to be at low risk for sea level change.

In addition to sea level rise, parks are susceptible to a variety of other climate-based factors. Thus, we developed a model that yields a vulnerability score which offers a broader perspective on a park's susceptibility to climate change. This model incorporates individually calculated vulnerability scores of each park to factors such as natural disasters, biodiversity shifts, and temperature change. These values are then weighted and summed to generate the final vulnerability score, which ranges from 0 (least vulnerable) to 10 (most vulnerable). Hatteras has the highest vulnerability score of 4.47, while Padre has the lowest score of 3.12. Holistically, each park has a relatively low score, signifying that none are extremely vulnerable to climate change.

Given all the threats America's national parks face, it is important that the Parks Service apportion its funds to best address its various concerns. However, these are not limited to climate concerns. The Parks Service also works on increasing environmental awareness, promoting a love of nature in younger generations, and is also responsible for simple maintenance and cannot solely address climate change risk. In order to determine how best the Parks Service should use its funds, we created a projection of yearly visitors to each park in order to determine whether more money should be directed to promotion and increasing public interest, or to park maintenance and mitigation of the consequences of climate change. We found that attendance is projected to increase fairly steadily, so funds set aside for upkeep and preservation of parks should not be decreased.

The future of America's national parks is of the utmost importance. Our models attempt to provide answers to pressing questions regarding sea levels, climate change vulnerability, and attendance. These models seek to shed light on how the park system may change in the upcoming years and provide guidance to the NPS, ensuring that future generations will also have to opportunity experience the beauty of America's national parks.

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# 1 Problem Restatement

The problem we are tasked with addressing is as follows:

1. Predict sea level changes over the next 10, 20, and 50 years. Assign a sea level change risk rating of high, medium, or low to each of the five parks.
2. Taking into account the likelihood and severity of climate-related events in each park, develop a model that can yield a climate-vulnerability score for any NPS coastal unit.
3. Predict long-term changes in visitors for each of the five parks. Using this prediction, advise NPS on the allocation of future financial resources.

## 2 Tides of Change

Global sea levels have been rising faster in recent decades than ever before. The acceleration of sea level change is one of many manifestations of increasingly problematic climate change, which threatens the ecological stability of our planet. The rising sea level poses an especially dire concern to coastal environments and threatens several United States national parks located near seashores. In order to assess the level of risk that these parks face as a result of rising oceans, it is first necessary to develop a model for local sea level increases over time.

### 2.1 Assumptions and Justifications

In order to generalize our model for local sea level increases, we made the following assumptions:

1. **The surface area of the ocean will not change significantly when the volume increases.** While the surface area does increase slightly, this was found to be insignificant when taking into consideration the volume of the entire ocean.
2. **The total volume of the Earth's glaciers can be represented by rectangular prism with a square base.** This facilitates volume calculations and roughly approximates the physical form of individual glaciers.
3. **The height of the rectangular prism is 2.1 km.** This is the average height of the major ice caps located at the poles [11].
4. **The density of the Earth's crust increases proportionally with depth.** This assumption was necessary in order to simplify calculations for the isostatic equilibrium of land masses, used in Part 2.4.
5. **The rate of tectonic activity is constant over time.** Because tectonic plates move so slowly, the difference in movement from year to year is unlikely to differ significantly. This simplifies calculations for land uplift, also used in Part 2.4.

## 2.2 Model Overview

There are a multitude of factors that lead to changing sea levels, but we chose to focus on a few major ones. While wind patterns, ocean current trends, and groundwater do affect sea level changes, they do not contribute as much as other, more dominating phenomena. We chose to break up regional changes in mean sea level (*SLR*) into changes caused by global factors, changes, and local factors. The main global factors were thermal expansion of water and the melting of glaciers and ice caps due to increasing global temperatures. The predominant local factor was land uplift due to tectonic and glacial activity. Combining these three factors into one equation gives us

$$SLR = TE + MI - U, \quad (1)$$

where  $TE$  is the change due to thermal expansion,  $MI$  is the change due to melting ice, and  $U$  is the land uplift factor.

## 2.3 Global Factors Affecting SLR

### Thermal Expansion

As water heats up, it expands. Since the global temperature is increasing, a major factor that plays into SLR is the simple increase in volume of the warmer oceans [6].

The change in the volume of the mean global sea level due to thermal expansion can be calculated using two distinct equations which, when set equal to each other, can output the change in depth,  $\Delta D$ , of the global sea level due to thermal expansion.

In physics, volumetric thermal expansion can be calculated by

$$\Delta V = \beta V_O \Delta T, \quad (2)$$

where  $\Delta V$  is the change in the ocean's volume,  $\beta$  is the coefficient of volume thermal expansion,  $V_O$  is the initial volume, and  $\Delta T$  is the change in global temperature.

For our model, we substituted  $V_O$  with  $D_i * SA$ , where  $D_i$  is the initial depth of the ocean and  $SA$  is the surface area of the ocean. This is a valid substitution as  $Volume = Depth * Surface Area$ .  $D_i$  is the max depth at which temperature changes occur under the ocean surface. This was determined to be 1000m, the lower limit of the thermocline, which is the transition layer between warmer mixed water at the ocean's surface and the colder, deeper water down below [22]. From research,  $\beta$  was determined to be  $1.5 * 10^{-4} C^{-1}$ , which is based on a mean salinity of 35 parts per thousand and a mean upper ocean temperature of  $10^\circ C$  [23].

Finally,  $\Delta T$  was calculated by performing an exponential regression on historical temperature data. While this regression does not account for the sawtooth shape of the data, it does demonstrate the accelerating increase in temperature that the data show. The regression equation was found to be

$$\Delta T = 0.00106 * e^{0.0014t}, \quad (3)$$

where  $t$  is the current year.

The change in the ocean's volume can also be calculated by

$$\Delta V = SA * \Delta D, \quad (4)$$

where  $\Delta D$  is the change in depth of the ocean.

By setting the two equations equal to each other and substituting the calculated values for  $SA$  and  $\Delta T$ , we get

$$SA * \Delta D = \beta * (D_i * SA)(0.00106 * e^{0.0014t}). \quad (5)$$

Cancelling out  $SA$ , substituting the known values for  $\beta$  and  $D_i$ , and solving for  $\Delta D$ , we get

$$\Delta D = 0.000159 * e^{0.0014t} * 1000, \quad (6)$$

an equation for the yearly change in sea level ( $\Delta D$ ) in mm due to thermal expansion as a function of time  $t$ , the current year.

### Glacier Melting

Glacier melt is another major factor that contributes to *SLR*. When glaciers (including ice caps and sheets at the poles) melt, a large volume of water is added to the ocean water body.

To find the change in volume per year  $V_n - V_{n+1}$ , a 3rd order polynomial regression was run on the change in average thickness (m) of the glaciers with respect to years since 1945. In order to validate the results of the regression, we examined the residuals and found a random scatter. This suggests that the regression model was efficient.

As seen in the graph, glacier thickness decreases each year. Using the change in **thickness** generated by the regression,  $\Delta \text{Glacier Volume}$  (the total volume (km<sup>3</sup>) of melted ice) can be calculated.

$$\Delta \text{Glacier Volume} = V_{n+1} - V_n, \quad (7)$$

where  $V_n$  is the total volume (km<sup>3</sup>) of the Earth's glaciers in the  $n$ th year since 2016 and  $V_{n+1}$  is the total volume (km<sup>3</sup>) of the Earth's glaciers in the following year.

To calculate the  $V_O$ , or the total glacier volume in 2017, the total estimated lateral surface area, 5.1 million km<sup>2</sup> [13], is multiplied by the average height, 2.1 km:

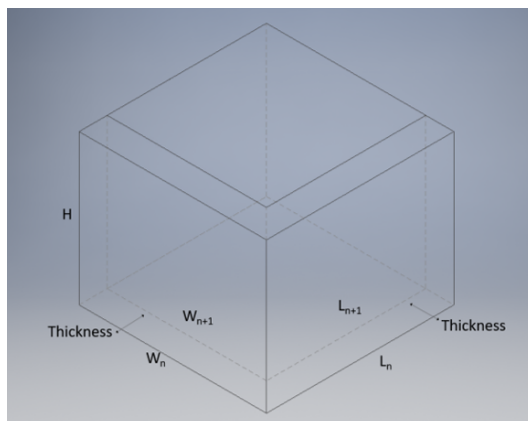
$$V_O = 5.1 * 10^6 \text{km}^2 * 2.1 \text{km} = 1.071 * 10^7 \text{km}^3. \quad (8)$$

Because the base of the glacier is assumed to be square, we calculated  $L_O$  and  $W_O$  by finding the square root of the surface area, yielding 2,258.32km. Then, using the baseline values of  $V_O$ ,  $L_O$ ,  $W_O$ , the volume in the following year,  $V_{n+1}$  (in this case,  $V_1$ ), can be calculated.

To find  $V_{n+1}$ , the length ( $L_{n+1}$ ) and width ( $W_{n+1}$ ) of the future glacier must be calculated by subtracting the thickness from both the current year's length and width. Assuming the height of the glacier remains constant, the future volume of the glacier is

$$V_{n+1} = L_{n+1} * W_{n+1} * H, \quad (9)$$

where  $L_{n+1} = L_n - \text{Thickness}$  and  $W_{n+1} = W_n - \text{Thickness}$ .



**Figure 1:** Graphical Depiction of Change in Glacier Volume in 1 Year

Finally, once  $V_n$  and  $V_{n+1}$  are calculated, the recursive function calculates MI, the *SLR* contributed by glacier melt per year.

$$MI = \frac{(\Delta \text{Glacier Volume})}{K_{glacier}} \quad (10)$$

where  $\Delta \text{Glacier Volume}$  is the total volume ( $\text{km}^3$ ) of melted ice and  $K_{glacier} = 392.277$ .  $K_{glacier}$  signifies that for every  $392.277 \text{km}^3$  of glacier melt, there will be 1 mm of *SLR*.

## 2.4 Local Factors Affecting SLR

### Isostatic Rebound and Tectonic Plate Activity

Land uplift is an increase in absolute land levels due to various geological phenomena. The two major causes of land uplift we considered were isostatic rebound and tectonic plate activity. Isostatic rebound is an increase in the level of land masses that were weighed down by ice sheets and glaciers. As the ice melts, the weight is lifted, and the level of the land increases. Tectonic plate activity can lead to an increase in land levels at subduction zones, which occur predominantly at coastal areas. At subduction zones, a continental tectonic plate is pushed upwards by an oceanic plate sinking into the Earth's mantle.

These two types of land uplift affect the five parks under consideration differently. Acadia National Park is not located at a subduction zone, so isostatic rebound is the main cause of land uplift. Isostatic rebound essentially entails the return of land masses to equilibrium after a deformation. We modeled isostatic rebound linearly using historical data, estimating that the rebound can be approximated by Hooke's Law. The yearly rebound value was estimated to be 0.15 mm a year.

Cape Hatteras National Seashore and Padre Island National Seashore are both affected similarly because they are both located on barrier islands. Neither is located near a tectonic plate boundary or near glacial activity, so we determined that both experience negligible land uplift.

Kenai Fjords National Park is located in southern Alaska, which has been experiencing significant uplift as a result of glacial melting and tectonic activity. However, copious data

exist documenting the effects of these changes. A study conducted at the University of Alaska at Fairbanks found that the rate of vertical uplift in Homer, a town located on the same peninsula as Kenai Fjords National Park, was 9.4 mm upwards per year. This is much more uplift than has been observed in other locations around the world, and has actually led to a decline in the relative sea level in southern Alaska. The rate of uplift has been fairly constant over the past twenty years, and so we expect it to remain the same going forwards.

Olympic National Park is located on Washington's Olympic Peninsula, which is on an accretionary wedge, a special kind of subduction zone. Therefore the sort of land uplift it experiences is nearly entirely tectonic. The rate of uplift was measured in 2001 by the Yale Geology Department to be 0.5 mm per year. Using the same assumption as for Kenai Fjords, we expect this number to be the same in the future.

**Table 1:** Yearly Land Uplift Rate ( $U$ ) for National Parks

National Park	Yearly Land Uplift Rate (mm/year)
Acadia	0.15
Cape Hatteras	0.0
Kenai Fjords	9.4
Olympic	0.5
Padre Island	0.0

## 2.5 SLR Model

By combining the equations from the aforementioned factors, we get the final SLR model for yearly sea level increases in years:

$$SLR = (0.000159 * e^{0.0014t}) + \left( \frac{\Delta \text{Glacier Volume}}{392.277} \right) - U. \quad (11)$$

### Projections

The net change in sea level over  $n$  years can be determined using a sum of the yearly change in each of those years:

$$\sum_{i=2016}^{Final\ Year} TE_i + MI_i - U, \quad (12)$$

where *Final Year* is the current year. We summed  $TE$  and  $U$  using two separate summations and programmatically summed  $MI$  by writing a recursive Java program. Running the model on the 5 given National Parks using Table 1 for the  $U$  values gives us the net change in sea level over the 10, 20, and 50 year periods. These outputs are shown below.

**Table 2:** Net Change in Sea Level (mm) for National Parks

National Park	10 Years	20 Years	50 Years
Acadia	38.0611	78.679	250.40
Cape Hatteras	39.711	81.829	258.054
Kenai Fjords	-63.689	-115.571	-221.346
Olympic	34.211	71.329	232.554
Padre Island	39.711	81.829	258.054



### 2.5.1 Sensitivity Analysis

In order to determine which variables have the greatest impact on the net change in sea level, we took one factor and shifted its value by 5% to see the resulting net change in sea level.

**Table 3:** Sensitivity Analysis Values

Factor	Shift %	10 Years	20 Years	50 Years
$\beta$	$\pm 5.00$	$\pm 0.00806$	$\pm 0.00818$	$\pm 0.008526$
$k_{Glacier}$	$\pm 5.00$	$\pm 0.02499$	$\pm 0.027078$	$\pm 0.06041$

As the shifts in net change in sea level are so small, this supports our model since small changes in factors do not have a profound impact on the overall output of the model.

## 2.6 Assessing Sea Level Change Risk

### Defining Risk Thresholds

We used the net sea level rise projections over 10, 20, 50 years to determine the risk to each park. Four of the five parks (all except Olympic National Park) abut a coastline. The National Oceanic and Atmospheric Administration (NOAA) has estimated risk levels for coastal cities based on the amount of sea level increase by 2100. We used these brackets to assess the risk to the four coastal national parks. In order to evaluate the risks over the required time frames, we recalibrated the NOAA brackets as shown:

**Table 4:** Coastal Risk Brackets by Sea Level Increase (mm)

Risk Level	10 Years	20 Years	50 Years
<b>Low</b>	< 35	< 40	< 100
<b>Medium</b>	35–120	40–240	100–600
<b>High</b>	< 120	< 240	< 600

### Risk Ratings for Each National Park

Comparing the net changes shown in Table 2 to the brackets shown in Table 3, we see that of the coastal parks, Acadia, Cape Hatteras, and Padre Island have a medium risk rating, while Kenai Fjords has a low risk rating because it experiences a net decrease in sea level.

The situation is slightly different for Olympic National Park because it is further inland than the other parks. We first wanted to see if the sea level increases would even lead to the park's boundaries being affected. Using the average elevation of the park (about 0.75 miles) and its average distance from the coast (about 6 miles), we found the angle of elevation of the park to be  $1.87^\circ$ . This leads to the following projections for inland ocean intrusion:

**Table 5:** Inland Intrusion for Olympic National Park (m)

Time Frame	Intrusion (m)
<b>10 Years</b>	1.044
<b>20 Years</b>	2.176
<b>50 Years</b>	7.094

In 50 years, our model predicts that sea levels will have reduced the Washington coastline by 7 meters. However, since Olympic National Park is over 9000 meters inland, the model does not predict that the park will be affected by the sea level at all. So we can safely say that Olympic National Park is at a low risk level.

The risk rankings are summarized below:

**Table 6:** Summary of Risk Ratings

<b>National Park</b>	<b>10 Years</b>	<b>20 Years</b>	<b>50 Years</b>
<b>Acadia</b>	Medium	Medium	Medium
<b>Cape Hatteras</b>	Medium	Medium	Medium
<b>Kenai Fjords</b>	Low	Low	Low
<b>Olympic</b>	Low	Low	Low
<b>Padre Island</b>	Medium	Medium	Medium

However, it must be said that the margin of error on these predictions only increases with time due to the uncertainty involved with extrapolation. So some of these parks, particularly Cape Hatteras and Padre Island, which are both narrow barrier islands, may actually be at high risk in the future depending on how climate trends change.

## 2.7 Forecasting over 100 Years

Models forecasting something as volatile as the climate are likely to have a great deal of error, especially over time frames as long as 100 years. Human behavior is likely to affect climate patterns more and more, leading perhaps to more and more extreme trends. Or perhaps global climate trends will become steadier as more and more countries commit to environmentally friendly practices. The variability in expected climate patterns is tremendous, and so it is impossible that one simplified model will be able to accurately account for sea level changes over a period as long as a century.

## 2.8 Model Revisions

Many of the assumptions we made in creating this model dealt with the guess that climate conditions would influence all parts of the globe equally. This results in significant error between our model's output and reality, because global climate patterns vary immensely and cannot be simplified to this extent. In addition, we chose to ignore cyclical trends in climate patterns, such as the effects of the El Niño and La Niña phenomena. These occur every few years and often have tremendous impact on sea levels. Finally, coastline erosion also leads to changes in measured sea level and varies regionally. Accounting for these various factors would help streamline our model and fit it better with real-world results. Finally, the effects of human activity (such as wetland drainage, deforestation, dam construction, and more) on sea level have not been well modeled by climate scientists and vary as well. This usually leads to a small discrepancy between sea level predictions and reality as well. Managing to account for more of these factors would probably lead to a better model for sea level change.

### 3 The Coast Is Clear?

#### 3.1 Model Overview

Six major factors were identified to contribute to the climate vulnerability score, or CVS, based upon the National Park Service Climate Change Response Strategy and ecological principles: biodiversity, hurricanes, wildfires, earthquakes, temperature, and air quality [25]. The rationale for including each factor is expanded upon in section 3.3.

##### Calculating Total CVS

The total CVS ranges from 0 to 10, where 0 is the lowest vulnerability and 10 is the highest vulnerability. It is calculated by summing the individual CVS values:

$$CVS = 0.3 * B + 0.25 * H + 0.2 * WF + 0.15 * EQ + 0.05 * T + 0.05 * AQ, \quad (13)$$

where the coefficients represent each contributing factor's weighting factor,  $B$  represents biodiversity CVS,  $H$  represents hurricane CVS,  $WF$  represents wildfire CVS,  $EQ$  represents earthquake CVS,  $T$  represents temperature CVS, and  $AQ$  represents air quality CVS. Calculated CVS values per the 5 coastal units provided are shown below:

**Table 7:** Severity Values of Natural Disasters on Each National Park

National Park	Hatteras	Acadia	Kenai	Olympic	Padre
CVS Value	4.47	3.20	4.14	4.076	3.12

The formulas to obtain specific scores for each factor are explained in 3.3.

#### 3.2 Assumptions and Justifications

In order to generalize our model of the Climate Vulnerability Score, we made the following assumptions:

- The biodiversity score score only needs to include the species richness.**  
While the biodiversity does include other factors, the temperature score will account for the other two major factors (optimal conditions of keystone species and dominant producer species).
- All hurricanes of the same storm category have the same wind speed.**  
While the wind speed of each hurricane of the same storm category may vary, the average would be the average of the wind speed thresholds between boundaries.
- The distance from the national park to the nearest fault line is inversely proportional to the severity and frequency of earthquakes on the park.**  
The closer a park is to a fault line, the park will be on average closer to the epicenter of an earthquake, increasing the severity. Since earthquakes originate near faultlines, parks near faultlines will encounter more of these natural disasters.

### 3.3 Contributing Factors to CVS

#### Biodiversity Score (BS)

A leading principle in the theory of genetics and ecological stability is that the greater diversity of a system, the more resilient it proves to be. A greater variation in genetic diversity, and therefore species diversity, leads to an increased ability of the ecosystem as a whole to weather climate disturbances. Therefore, biodiversity is the most weighted factor at 30%, as it is the greatest contributor to ecological stability and recovery time.

The pure concept of biodiversity is mostly dependent on species richness and may also be affected by the optimal conditions of keystone species and dominant producer species. However, for the purposes of determining the CVS score for NPS coastal units, the biodiversity score will only include the species richness, and the temperature score will account for the other two factors.

The biodiversity score can be calculated by

$$B = 10 \left( 1 - \frac{R}{R_m} \right), \quad (14)$$

where  $R$  is the species richness of the biome in question, and  $R_m$  is the maximum value of species richness of a biome in the 8 biomes [24].  $1 - \frac{R}{R_m}$  is necessary to maintain the CVS scale trend in the biodiversity score, so climate vulnerability increases with the BS. By including a coefficient of 10, the biodiversity score has an effective possible range of 0–10.

#### Temperature Score (TS)

As temperature ranges widely from area to area, the temperature score for each coastal unit will be evaluated based on the optimal temperatures for each unit's keystone species and dominant producer species. Keystone species are generally defined as predators that significantly impact biodiversity, physical features, and climate by their presence in an ecosystem. If the average temperature of an ecosystem strays from a keystone species' optimal temperature, ecosystem stability may be significantly affected. Furthermore, the optimal temperature of dominant producer species will also be considered, as they are most responsible for providing the energy in the form of biomass required to sustain organisms higher on the food chain.

Therefore, the keystone species and dominant producer species as well as optimal temperature must be identified for each coastal unit. These temperatures, along with the average temperature for each coastal unit, will be used to calculate the % difference between each pair

$$T_k = \frac{|O_k - A|}{O_k} \quad (15)$$

and

$$T_p = \frac{|O_p - A|}{O_p}, \quad (16)$$

where  $T_k$  is % difference in optimal temperature of keystone species  $O_k$  and average temperature  $A$ , and  $T_p$  is % difference in optimal temperatures of keystone species  $O_p$  and average temperature  $A$ . To scale these resultant values from 1–10 for the overall temperature score, the resultant % differences  $T_k$  and  $T_p$  will be rated from 1–5 based on general threshold values as shown in the following table.

**Table 8:** Threshold Values for % Temperature Difference and Corresponding Ratings (1–5)

Rating (1-5)	Threshold
1	0%–5.00%
2	5.01%–15.00%
3	15.01%–30.00%
4	30.01%–50.00%
5	> 50.01%

For the five coastal units provided, the choices of keystone and dominant producer species as well as their optimal temperature are as shown:

**Table 9:** Optimal Temperature of Keystone and Dominant Producer Species per Coastal Unit

Name	Keystone Species, Temperature(°F)	Dominant Producer, Temperature(°F)
AC	Beavers, 32.0–82.4	White Birch, 20–25
CH	Oysters, 68.0	Sea Oats, 72.0–95.0
KF	Black Oystercatcher, 62.0–73.0	Phytoplankton, 33.8–42.8
OL	Salmon, 44.6–60.8	Sitka Spruce, 50–75
PA	Blue Crab, 59.0–70.0	Widgeon Grass, 65.3–86.0

As the temperature score may contribute to the biodiversity score, it is weighted more lightly at 5%.

### Air Quality Score (AQS)

The air quality score is dependent upon the air quality index (AQI), which represents overall air quality in terms of amount of pollutants within a given area. As AQI, unlike occurrence of severe natural disasters, stays stable over time and also may contribute to the biodiversity score, it is weighted more lightly at 5%. Currently, there are accepted rating thresholds for AQI. To scale the AQ from 1–10, ratings have been assigned to each defined threshold.

Air quality can be rated as shown:

**Table 10:** Threshold Values for AQI and Corresponding Ratings (0–10)

Rating (0–10)	Threshold
0	0.00
2	0.01–50.00
4	51.00–100.00
6	101.00–150.00
8	151.00–200.00
10	>201

### Natural Disasters

Each of the three major natural disasters (hurricane, wildfire, earthquake) is given an individual CVS which is calculated by

$$CVS = \text{Weighting Factor} * \text{Contributing Factor Score}, \quad (17)$$

and

$$\text{Contributing Factor Score} = \text{Frequency Score} * \text{Severity Score}. \quad (18)$$

The product of the frequency and severity scores are divided by the maximum possible value and multiplied by 10 to put the contributing factor score onto a 0–10 point scale.

### Weighting Factors

Of the 3 natural disasters in the CVS function, hurricanes were determined to be the most costly to park wildlife and geography. They generate strong winds and extremely high precipitation causing widespread defoliation and animal death (due to drowning and food shortage). In addition, recovering from the loss of land and biodiversity is difficult. Thus, hurricanes are given a weighting factor of 25%, the highest of the natural disasters.

Wildfires were determined to be moderately damaging to parks. Fewer animals die due to wildfires than hurricanes; most animals flee, while small insects die most often. While trees are burnt down, wildfires are often integral to the health of the ecosystem. Thus, wildfires are given a weighting factor of 20%.

Earthquakes are the least damaging to the parks. There are the least amounts of deaths. Normally, animals are merely scared, while trees topple. Thus, earthquakes are given a weighting factor of 15%.

### Frequency Score

The frequency score is calculated by finding the average number of natural disasters per year. The following frequency scores were calculated using the provided data. Frequencies for earthquakes were not calculated because the earthquake severities were evaluated based on the distance from the national park to the nearest fault line (which encompasses both the severity and frequency, since frequency and severity are directly proportional).

**Table 11:** Frequency Values (occurrences per year) of Natural Disasters at Each Park

Natural Disaster	Hurricane	Wildfire
<b>Hatteras</b>	1.60	4.30
<b>Acadia</b>	0.25	3.65
<b>Kenai</b>	0.20	0.00
<b>Olympic</b>	0.05	20.15
<b>Padre</b>	0.50	2.05

### Severity Score

The severity score is unique to each natural disaster and park. To calculate the severity score for hurricanes, the average wind speed for each park is divided by the maximum possible wind speed. To calculate the severity score for wildfires, the average acreage lost per wildfire is divided by the national average acreage lost per wildfire, 48 acres (Cabbert). Finally, the severity score for earthquakes is defined as the distance from the park to the nearest fault line.

**Table 12:** Severity Values of Natural Disasters on Each National Park

Natural Disaster	Hurricane	Wildfire	Earthquake
Hatteras	69.91	0.31	4.81
Acadia	75.17	2.27	2.04
Kenai	0.00	0.00	9.34
Olympic	0.00	0.69	9.58
Padre	68.45	19.24	6.33

### 3.4 Model Revisions

Many averages we made on the severities and frequencies of the contributing factors for the Climate Vulnerability Scores were based on limited data. Not all of the data was provided, and hence, our values may not be fully representative of the parks for all years. Another feature that could have been considered would be modeling overall resilience of an ecosystem based on past performance which could utilize the climate vulnerability score calculated for this part of the problem. One way to accurately model the resilience would be to utilize the Ramberg–Osgood relationship generally used to model the nonlinear graph near the yield point of stress vs. strain of materials. Ecological stability is similar to a stable equilibrium; that is, when an ecosystem is perturbed, it tends to return to equilibrium up to a certain level, where it no longer demonstrates elastic behavior. Instead, it begins to show plastic behavior, where the ecosystem is still surviving but no longer can return to its original equilibrium. Furthermore, in this region, equilibrium is no longer constant, which would accurately model extreme events that permanently alter ecosystem dynamics.

## 4 Let Nature Take Its Course?

The National Parks Service, like any federal agency, does not have bottomless coffers to reach into for funds and must decide how best to apportion the money available to it. It also has a number of tasks to address, including facility repair, facility maintenance, new construction, employee wages, educational programs, and park promotion. As with any organization, it is important to apportion these funds to ensure they do the maximum good. Two major factors that the National Parks Service must investigate are climate vulnerability and projected visitors. Climate vulnerability will affect the amount of money that the National Parks Service must put into repair and maintenance, while changes in projected visitors could drive adjustments in funding towards educational programs, new construction, and marketing. We created a projection for yearly visitors to each park and compared it with the results from the previous part.

### 4.1 Assumptions and Justifications

1. **The United States' population can be modeled logistically.** This assumption carries over from standard population dynamics analyses.
2. **The average age in the United States does not vary in the long term.** While the baby boomer generation is aging, we assume that in the long term these differences even out and the average age remains nearly constant.
3. **The gender split in the United States is nearly 50-50.** Statistically speaking, this assumption should hold.

4. **Nominal incomes increase steadily with inflation, and yearly inflation occurs at 2.0%.** In the long term, we expect that economic conditions will remain fairly stable, and that wage increases will not outpace inflation. We assume that U.S. inflation will, in the long term, match expectations and remain at the historic average of about 2.0% [20].
5. **The urban population of the United States is growing at a rate of about 0.1% yearly.** Demographic data show that the urban population of the United States is on an upwards trend and is increasing on average at this rate. We expect this trend to continue in the future [21].
6. **The area of preserved national parkland will stay the same.** Accounting for increases in preserved parkland would overcomplicate the model because the government's attitude towards conservation often varies from administration to administration, making it too hard to accurately predict.

## 4.2 Visitor Model Overview

A 2007 report published by the United States Department of Agriculture (USDA) provided a model of total yearly national park visitors based on socioeconomic factors [19]. We simplified the model to make it more applicable for the long term and added time evolution to some of the variables. We took this socioeconomic model for total visitors and scaled it down so that the numbers were consistent with the provided historical data for each park. This gave us our final projections for each park, up to the year 2050.

## 4.3 Socioeconomic Model

### 4.3.1 Probability of Visiting a Park

The USDA report used the following equation, which describes the “wilderness recreation participation probability,” or the probability that an American person visits a national park site:

$$P(\text{visit}) = \frac{1}{1 + e^{-XB}}, \quad (19)$$

where  $X$  is a row matrix containing the means of explanatory variables, and  $B$  is a vector containing parameters for each variable. We multiplied the probabilities output by this function by the total U.S. population to get total projected yearly park visitors.

### Variables and Their Parameters

The USDA report included a number of variables which affected annual park visitors, but some of these were far less important than others and less likely to matter in the long term. We isolated age, gender, U.S. citizenship, membership of an environmental society, average income, distance from nearest national park, and education level as demographic variables affecting the likelihood of a park visit. The row matrix  $X$  contains mean values for these explanatory variables. For example, the average age in the U.S., the proportion of males, the proportion of U.S. citizens, etc. We used census data and values calculated by the USDA report authors to populate matrix  $X$ . Some of the variables, such as income and urban population, are more variable over time than others, so we used historical data



to calculate the average yearly percentage increase in both and model their time evolution. Matrix  $X$  was thus populated with expected values for all the variables. The report authors had calculated the parameters associated with each variable, so we used those coefficients in the parameter vector  $B$ . The table below shows the values for each variable in matrix  $X$  and vector  $B$ , where  $t$  is measured from 1997.

**Table 13: Explanatory Variable Values**

Factor	Means ( $X$ )	Parameters ( $B$ )
Age	37.7	-0.19
Gender	0.5	0.634
USA-Born	0.882	1.31
Member	0.229	0.768
Income	$5.96 * 1.015^t$	0.088
Miles	75.7	-0.002
Education	0.208	0.101
Urban Pop.	$0.785 * 1.001^t$	-0.139

The dot product  $X \cdot B$  was evaluated for every time  $t$  from 0 to 53, allowing us to plug a number into equation (17). This gave us the probability that a resident of the United States visits a national park in the  $t$ -th year after 1997.

### 4.3.2 Population Model

The next step in our modeling process was to determine the value of the U.S. population as a function of time, to be multiplied by the probability calculated in the previous part. To do this, we figured that this value, in millions of people, was given by a logistic equation of the form

$$P = \frac{\alpha}{1 + ke^{c*t}},$$

where  $\alpha$ ,  $k$ , and  $c$  are constants that serve to provide horizontal shrinks and translations that make the equation more suitable for modeling the U.S. population. In particular, we let  $\alpha = 696.27$ ,  $k = 7.908$ , and  $c = -0.0167$ , which were found by performing a logistic regression on the historical U.S. Population Data. Again, we examined the residuals and found they were randomly scattered, resulting in a suitable regression.

From this, our U.S. Population equation is given by

$$P = \frac{696.27}{1 + 7.908e^{-0.0167*t}}, \quad (20)$$

where  $t$  is the current year (since 1900).

### 4.3.3 Scaling the Socioeconomic Model

The socioeconomic model gave us the general trend of all national park visits until 2050. However, these numbers had to be rescaled to match the overall trends exhibited by each park’s visitor data. A proportionality constant was calculated for each park, and the projection was adjusted until it best fit the historic data. The projected results for each park, calculated using a Python script, are shown in Figure 2. Furthermore, Table 14 shows our final projections for the number of visitors to each park in the year 2050.

**Table 14: Final Projected Visitor Values for 2050**

National Park	Projected Number of Visitors
Acadia	3,645,604
Cape Hatteras	3,358,483
Kenai Fjords	415,869
Olympic	4,835,233.015
Padre Island	31,4015



**Figure 2: Visitor Projections for Each National Park**

## 4.4 National Park Service Recommendations

Based on our output from the Visitor Projection model, we witness a general upward trend in visitors for each of the national parks. This is substantiated because the National Park Service has been making efforts to increase the attraction of visitors to National Parks [4]. Additionally, in the Budget Justification report released by the National Park Service for Fiscal Year 2017, we see about 50% of this year's funds being devoted to promotional work, including the Every Kid in a Park program, new construction, and the recently adopted Centennial Challenge, which allows parks to fund and complete projects left on the back burner. Given the fairly steady increase in projected yearly visitors, as compared to the dire situation some parks are in, we believe this 50-50 split between funding to new projects and funding for upkeep and maintenance is sufficient, if not too heavily skewed towards new work. The National Parks Service, we believe, should ensure that the natural beauty of the United States is preserved for future generations to enjoy and appreciate.

## 4.5 Model Revisions

The visitor projection model is based entirely on demographic and socioeconomic factors, which does not incorporate all of the major relevant factors. Visitor numbers are also driven in part by climate change—it has been shown that they vary fairly drastically during the year with seasonal variations in temperature. These trends are likely to hold up as global temperatures increase. Incorporating temperature as a factor would deepen the projection model. In addition, overall climate change and the resultant ecological shifts might change people's motivation to visit a park. If the likelihood of seeing characteristic flora or fauna at a park has been diminished due to climate change, the total number of visitors is likely to be driven down. Incorporating these climate variables into the visitor projections is likely to give a more accurate picture of the future of the park system.

## 5 Conclusion

Our models attempt to provide insight into the future of national parks. First, we sought to find sea level change risks (high, medium, low) for five coastal parks in the following 10, 20, and 50 years. To do so, we developed a model that incorporated factors that contribute to global sea level rise (thermal expansion and glacier melt) and local sea level rise (isostatic rebound and tectonic activity). Ultimately, we determined that Acadia, Cape Hatteras, and Padre Island were at medium risk for sea level change in the given time frame, while Kenai Fjords and Olympic National Park were at low risk. This model responds well to sensitivity analysis, and the sea level change risk predictions hold true within the 50-year time frame. Beyond 50 years, it is nearly impossible to accurately predict the temperature and ensuing sea level.

Next, a model that yields any NPS coastal park's vulnerability score to climate change (on a scale of 0 to 10) was developed. Multiple climate-related factors were weighted and summed to yield a final climate vulnerability score (CVS). Hatteras has the highest score, 4.7, while Padre has the lowest score of 3.12. None of the five coastal parks were determined to be especially vulnerable to climate-related changes, indicating that these parks will not be terribly affected in the event of climate change. The general model can be used

by the NPS to calculate the CVS of any coastal park.

Finally, a model of projected annual visitors to each park was developed based on socio-economic factors and rescaled to match historic trends for each park. We used the projections to determine that funds should be directed towards preservation and counteracting the effects of climate change, more so than towards promotion and increasing interest in the parks. We hope that the National Parks Service can use our findings to plan for the futures of all national parks, ensuring that America's unparalleled natural beauty is preserved for posterity.

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