

Moody's Mega Math Challenge 2017

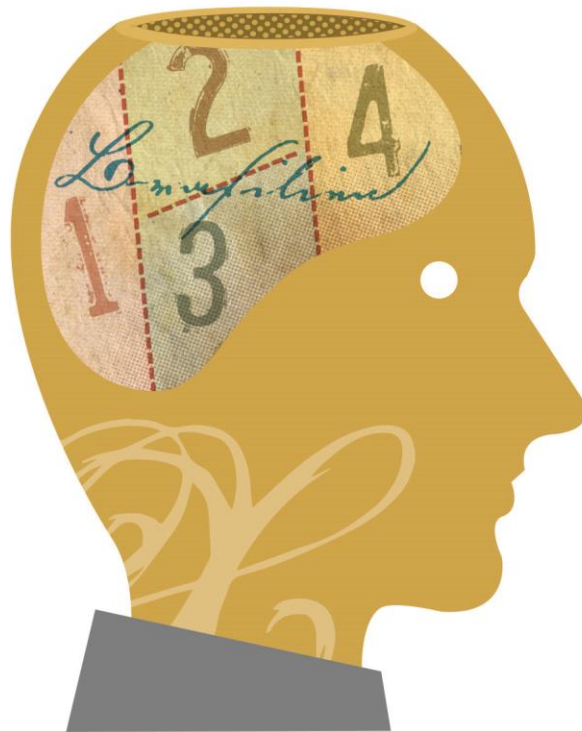
Westford Academy –

Team #9653 Westford, Massachusetts

Coach: Lisa Gartner

Students: Nihar Sheth, Harshal Sheth, Kartik Singh,
Adithya Vellal

Moody's Mega Math Challenge First Runner Up, \$15,000 Team Prize



**Moody's
Mega
Math
Challenge**

***Note: This cover sheet has been added by SIAM to identify the winning team after judging was completed. Any identifying information other than team # on a Moody's Mega Math Challenge submission is a rules violation.

***Note: This paper underwent a light edit by SIAM staff prior to posting.

Executive Summary

To the National Park Service:

The United States National Park System is enormously important to the people of our nation. Unfortunately, the parks, consisting of pristine natural lands, are also some of the regions most vulnerable to the effects of a changing climate throughout the country.

In order to help assess the magnitude of this threat, our team developed models to project climate risk on various coastal units over the next 50 years. First, we developed a model to predict risk due to sea level rise, focusing on five particular national parks (although the model could be applied to any coastal unit). Our model is based upon the realization that a given park's sea level risk depends primarily on two factors: the projected local mean sea level (MSL) rise, and the area of the park that is directly exposed to the coast. We tested the model on the five "focus parks"—Acadia, Cape Hatteras, Padre Island, Olympic, and Kenai Fjords—and the results matched our expectations. Over the next 50 years, Padre Island, Acadia, and Cape Hatteras were all projected to fall into the "high risk" category, given their rapidly rising sea levels and high coastal exposure. On the other hand, Kenai Fjords, which is experiencing decreasing sea levels, was projected to be at "low" risk due to sea level change.

Next, we developed a model to give any coastal unit a climate vulnerability index, based on the likelihood and severity of negative climate-related events occurring there over the next 50 years. The vulnerability index was measured in the dollar value of damage per acre of park land expected over the next 50 years. We determined that the three largest climate-related threats to consider in coastal areas are sea level risk, hurricanes, and wildfires. Although there are other climate factors, such as temperature and air quality, we determined that these did not affect "climate vulnerability" in any significant way compared to the others, and thus chose to leave them out. Similar to the first model, we tested the model on the five focus parks, although this model can be applied to any coastal unit. Overall, the results made sense, and the model is at least reasonably predictive. Padre Island National Seashore was found to have the highest vulnerability index, based on its exposure to rising sea levels, hurricanes, and wildfires. On the other hand, Kenai Fjords National Park in Alaska, which does not have a history of hurricanes or wildfires and is actually experiencing falling sea levels, had a vulnerability index of 0.

Finally, we built upon the first two models we designed to create a model which predicted visitor changes for each park. We used the United States population, the local temperature, and our climate vulnerability index to predict the visitorship for each of our five focus parks. This was done by training a multivariate regression model on all three of the above features, and the resulting visitor level predictions were mostly in line with our expectations based on the results of the previous two models. Based on our model, we recommend that the NPS use its limited resources on the five focus parks in the following order: Cape Hatteras National Seashore, Olympic National Park, Kenai Fjords National Park, Acadia National Park, Padre Island National Seashore. Our model suggested that Kenai Fjords's visitor rates will benefit the most in the future, while its overall rates will still be rather small compared to the other parks. On the other hand, Padre Island National Seashore is projected to have very few visitors 50 years from now and may not be a financially sustainable operation at the time.

Contents

1	Introduction	3
1.1	Restatement of the Problem	3
2	Tides of Change	3
2.1	Restatement	3
2.2	Assumptions	3
2.3	Creating a Model	4
2.4	Validating Our Model	6
2.5	Sensitivity Analysis	6
3	The Coast Is Clear?	6
3.1	Assumptions	6
3.2	Creating a Model	8
3.2.1	Sea Level Rise	8
3.2.2	Hurricanes	9
3.2.3	Wildfires	10
3.2.4	Putting It All Together	12
3.3	Analysis	12
3.4	Limitations	13
4	Let Nature Take Its Course?	14
4.1	Assumptions	14
4.2	Creating a Model	15
4.3	Results	15
4.4	Analysis	16
4.5	Limitations	16
5	Conclusions	17

1 Introduction

Climate change is one of the most pressing problems of our time, and our nation's treasured national parks will likely be among its first victims. Many of our national parks lie along the coast, where they will be affected not only by sea level rise as global temperatures increase, but also by exposure to violent hurricanes. National parks are often located in more extreme locations where violent weather events are more common. Thus, it is important for the National Park Service to assess the risk that climate-related events pose to the national parks, and to plan accordingly.

1.1 Restatement of the Problem

Sea level rise is a hot-button issue in today's world. It threatens millions of people who live on coasts around the world and at home. In addition, it threatens our nation's treasured national parks. It is important to build a model to classify the country's different national parks into categories based on their sea level rise risk. More generally, it is important to classify the parks based on risks of any climate-related event hitting them, in order to allow the National Park Service (NPS) to appropriately allocate mitigation and restoration resources. Finally, due to the NPS's limited funding, it is important to prioritize which parks should garner the most financial resources, based on long-term visitation outlook.

2 Tides of Change

2.1 Restatement

This part of the problem asks us to build a model to determine sea level risk ratings for five selected coastal national parks.

2.2 Assumptions

- The mean sea level (MSL) rise for a given area is determined by three factors: ice caps melting, the thermal expansion of water, and vertical elevation changes in the Earth's surface in different areas.

Justification: It is well known through climate science principles that these three factors are the determinants of sea level rise for a given area [6].

- The MSL trend data provided by NOAA takes into account all of the above factors.

Justification: Sea levels in this data are measured relative to a fixed reference point. Therefore, it takes into account not only global sea level change, but also elevation changes [7].

- Rate of MSL change is and will remain constant on both a global and local park level.

Justification: Global MSL change is projected to remain constant at 1.7–1.8 mm/year into the future. Elevation changes in areas are based on geological factors (e.g., tectonic

shifts), which change slowly and constantly on a scale of decades and centuries (barring any major geological shifts, such as earthquakes). Thus, since global MSL change and elevation changes are expected to remain approximately constant, local MSL changes should remain constant as well [7].

- This constant rate of MSL change can be determined based on MSL trends from the past 20 years.

Justification: Although 20 years is not an optimally lengthy timeline, it is the longest timeline for which complete and accurate MSL data could be found for all parks. As explained above, MSL trends do not tend to change drastically, so this data should suffice.

- The sea level change risk is directly proportional to the change in sea level.

Justification: It is reasonable to assume linear proportionality in sea level change risk as it relates directly to sea level.

- A park with a lot of coastline relative to its area is more vulnerable to a change in sea level than a park with less coastline relative to its area.

Justification: Floods caused by a rise in sea level will affect a larger relative area in a park with a high ratio of coastline to area (e.g., Cape Hatteras National Park) than in a park with a low ratio of coastline to area (e.g., Olympic National Park).

- The entire coastline of a park is at sea level.

Justification: This simplifying assumption is reasonable because we can expect the majority of a park's coastline to be at sea level anyway.

- The national park coastlines have a grade of 0.07% upward.

Justification: This is based on the average slope grade of coastline from the datasheet of the Coastal Vulnerability Index from the Woods Hole Institute [9].

2.3 Creating a Model

Before designating the national parks as having low, medium, or high sea level change risks, we must quantify the sea level change risk for any given coastal national park.

To begin, we define $X(t)$ as the projected change in sea level in millimeters at a coastal national park t years after 2016.

Using the data provided [7], we determined $X(t)$ for each park:

National Park	$X(t)$
Acadia National Park	2.18t
Cape Hatteras National Seashore	3.84t
Kenai Fjords National Park	-2.62t
Olympic National Park	0.14t
Padre Island National Seashore	3.48t

Table 1: Percent of Park Flooded by Sea Level Change

National Park	$R(10)$	$R(20)$	$R(50)$
Acadia National Park	0.94%	1.89%	4.72%
Cape Hatteras National Seashore	4.71%	9.42%	23.55%
Kenai Fjords National Park	0.00%	0.00%	0.00%
Olympic National Park	0.01%	0.01%	0.03%
Padre Island National Seashore	1.90%	3.79%	9.49%

Obviously, the most direct danger posed by rising sea levels is permanent flooding of coastal areas. We quantify the sea level change risk of a park, R , at a time t years from 2016 as

$$R(t) = \frac{F(t)}{A},$$

where $F(t)$ is the area of the park in square miles that is projected to be submerged by the ocean t years from 2016, and A is the total area of the park, also in square miles.

To find the area of the park that we expect to be flooded by a given sea level change of $X(t)$, we multiply the length of the park's coastline in miles, C , by $X(t)$ and by a scalar quantity D . D is a constant such that $D \cdot X(t)$ gives the distance penetrated inland by the ocean in miles as a result of the sea level increasing by $X(t)$ millimeters. So,

$$F(t) = C \cdot D \cdot X(t).$$

To find the value of this constant D , we assume that the coastlines for all the parks are sloped with a grade of 0.07% [9]. Given this, we can determine how far inland the ocean will penetrate given a sea level increase of 1 millimeter using a simple proportion:

$$\frac{1}{D} = \frac{0.07}{100},$$

$$D \approx 1400.$$

So, for every 1 millimeter increase in sea level, the ocean can be expected to penetrate about 1.4 meters, or 8.70×10^{-4} miles.

Hence,

$$F(t) = 0.00087CX(t).$$

So, the final sea level change risk model is

$$R(t) = \frac{000087CX(t)}{A}.$$

We use this risk model to determine the sea level change risk for each of the five given national parks for the given time spans; the resulting risks are shown in Table 2. Coastline measures were approximated using Google Maps, and park areas are sourced from a list of national parks in the U.S. [4].

We classify the risks into these categories by simply taking the 33rd and 67th percentiles of the fifteen risk ratings that were produced. Areas that are expected to be less than 0.02%

Table 2: Sensitivity Analysis of $R(t)$ for Acadia National Park

Rate of Change of Sea Level (mm/year)	$R(10)$	$R(20)$	$R(50)$
1.96	0.89	1.78	4.45
2.18	0.94	1.89	4.72
2.40	1.09	2.18	5.45

flooded are classified as low (green). Areas expected to have less than 4.72% flooded but above 0.02% risk are classified as medium risk (yellow). Areas expected to be flooded 4.72% or more are classified as high risk (red).

2.4 Validating Our Model

To check our model, we look at the extremes that were generated.

Our model gives Cape Hatteras National Shore the highest sea level risk rating out of the five national parks. Physically speaking, this makes sense. Cape Hatteras is a small, narrow strip of land, making it extremely susceptible to permanent flooding as a result of rising sea levels. Its high risk rating reflects this.

On the opposite end of the spectrum, our model gives Kenai Fjords National Park the lowest risk ratings. Again, physically speaking, this makes sense. Kenai Fjords National Park is large, so its coastal areas make up a smaller proportion of its total area. Hence, a smaller proportion of the park's area will be affected by permanent flooding caused by rising sea levels.

2.5 Sensitivity Analysis

A final step in analyzing our risk model is measuring how sensitive it is to changes in the function $X(t)$, which yields the projected change in sea level for a park t years after 2016. Ultimately, the value produced by $X(t)$ for a given park depends on the predicted rate of change of sea level at that given park. By altering this value and observing the resulting change in risk rating, we can understand how sensitive our model is to change.

For this analysis, we alter the rate of change of sea level for Acadia National Park:

The two rates above and below the mean are the 5th and 95th percentiles for Acadia National Park [7]. The results for these values do not stray too far from those for the mean value, which illustrates that our model responds reasonably to small changes in the rate of change of sea level.

3 The Coast Is Clear?

3.1 Assumptions

- A park's vulnerability to climate-related events is determined by the expected value total park asset loss as a result of the event, normalized by the number of acres in the park.

Justification: Measuring expected value of asset loss (in dollars) is a standardized method for comparing vulnerability. This must be normalized by size of the park, because a park with much more in assets, such as Olympic National Park, will naturally have a higher asset loss expected value, but this does not necessarily make it more vulnerable to climate-related events.

- Park asset losses will primarily be derived from sea level rise, hurricanes, and wildfires for coastal parts (of all climate related events). The vulnerability index model will focus on these factors.

Justification: These climate-related events will lead to asset damage. Other climate events, such as heat index, temperature, and air quality, are all very transient in nature and would not incur a comparable level of asset damage. Other natural disasters, such as earthquakes, are not climate related.

- Inflation and time value of money are irrelevant to this issue.

Justification: These deal with the specifics of valuing money over time, which is not relevant. In this case, money is merely used as a standard measure for severity, and changes in inflation and interest rates do not concern us.

- The value of a park is uniformly distributed across all of its land.

Justification: One patch of forest or coast is typically not significantly more valuable than another patch within the same park; overall, value is fairly uniformly distributed.

- One acre of national parkland is worth \$1195.

Justification: The total value of all national park land across the country is \$62 billion [10], and there are 51.9 million acres of park land [4]. Thus, each acre of national park land is, on average, worth \$1195 dollars.

- Storms that are ranked below Category 1 on the Saffir–Simpson scale are not powerful enough to cause any appreciable damage to national parks.

Justification: Based on the description of storms provided with the Saffir–Simpson scale, it is reasonable to assume that no serious damage is caused by tropical storms, tropical depressions, or extratropical storms.

- No hurricanes will occur in the states of Alaska and Washington in the next 50 years.

Justification: Historical records suggest that Alaska and Washington rarely, if ever, have hurricanes [2]. It is reasonable to assume that no hurricanes will occur in these areas in the next 50 years.

- Hurricane rates will not change over the next 50 years.

Justification: It is still too early to determine if human activity has an effect on hurricane rates [1].

- There will be no wildfires in Alaska over the next 50 years.

Justification: There is no record of forest fires in Kenai Fjord National Park [5]. Given this and the fact that Alaska's climate is generally quite cold, we can reasonably assume that wildfires will not occur in Alaska in the next 50 years.

3.2 Creating a Model

A model for a park's vulnerability must take into account both the severity and likelihood of climate events.

Different climate events cause different types of damages, but the severity must be comparable across all of the events. As such, we define the severity of a climate event as the dollar amount of damage that it causes to the park in a given year.

Quantifying the likelihood of a climate event is a bit less straightforward. For some events, likelihood can be expressed as the probability of the event occurring at a given time. For other events, it makes more sense to express the likelihood of an event as an expected value of that event's effects.

With severity and likelihood defined, we must decide which climate events to consider in our model. There are only certain events that can significantly damage the asset value of a park. Given that these are coastal parks, we define these events to be: hurricanes, wildfires, and permanent flooding caused by rising sea levels. Other conditions, such as high heat indices or low air quality, while likely to drive away visitors in the short term, will not cost the park significantly in the long run. From this perspective, the three events we consider will likely prove much costlier. We place our focus on measuring the impacts of those events.

Ultimately, we define the vulnerability score for a park as the dollar amount of damages the park can expect in a given year due to hurricanes, wildfires, and permanent flooding due to rising sea levels. We determine this value for a park by modeling the dollar amount of damages the park can expect from each event individually and then summing these values.

3.2.1 Sea Level Rise

The likelihood of sea level rise is assumed to be 100%, since we can expect sea level rise to occur constantly over the next 50 years. The important realization with sea level rise is that it causes permanent damage to the affected part of the park. Once the sea moves over an area and submerges it, the value of that area is completely eliminated.

We use the outputs from the sea level rise risk model in part 1 to determine the percent of each park that we predict will be flooded 50 years from now. The total cost of sea level rise can thus be determined by

$$C_{\text{sea level rise}} = 1195 \cdot A \cdot P,$$

where C is the total cost of sea level rise, A is the total number of acres in the park, P is the percent of the park that is flooded, and \$1195 is the value of an acre of park land as stated in our assumptions.

Normalization must be done on these values to make them more easily comparable to each other. This is done by divided by size of the park, since larger parks will naturally have

Table 3: Sea Level Rise Damage

National Park	A (acres)	P (%)	C (\$)	N
Acadia National Park	49052	4.72	2766729	56.4
Cape Hatteras National Seashore	30351	23.55	8541454	281.4
Kenai Fjords National Park	669984	0.00	0	0.0
Olympic National Park	922650	0.03	330770	0.3585
Padre Island National Seashore	68288	9.49	7744235	113.4

higher expected damage figures. This is done as follows:

$$N = \frac{C}{A},$$

where N is the normalized vulnerability index for sea level rise.

Intuitively, this makes sense. Cape Hatteras and Padre Island are the most exposed to the ocean and stand to be most affected by sea level rise. This is illustrated by their high damage values of approximately \$281.4 and \$113.4 per acre, respectively, which lie well above the values for the other parks.

3.2.2 Hurricanes

The expected damages caused by hurricanes for any particular park over a 50 year period is defined as follows:

$$\sum_{i=1}^{50} C_{\text{hurricane severity}} \cdot C_{\text{hurricane likelihood}}.$$

The severity of a hurricane is calculated by estimating the asset loss in dollars that would result from hurricanes of various strengths hitting each particular park. This is determined by using the formula below:

$$C_{\text{hurricane severity}} = \text{number of acres per park} \cdot \text{money lost per acre}.$$

This metric is then normalized, for the sake of easing comparison, by dividing by the park size, so we end up with a final hurricane severity of dollars per acre for each park.

We use the acreage of each national park [4], while the money lost per acre is set to an arbitrary dollar value. This approach was chosen due to the lack of existing data on the effects of hurricanes on coastal national parks. We assign a damage per acre value that is proportional to hurricane category. Specifically, prior calculations estimate that the average value of an acre of national park land is \$1195. Based on this value we estimate that a category 5 hurricane will destroy roughly 1/30 of the value of park land, so our category 5 damage per acre value is \$40. We then reduce the damage per acre by \$8 for every decrease of 1 in hurricane category, giving us values of \$32, \$24, \$16, and \$8.

The likelihood of a hurricane hitting each national park is determined using a state landfall probability calculator that utilizes a Poisson distribution [8]. The calculator uses hurricane data which specifies the number of each category hurricane that made landfall in the last 150 years. This data is used to determine the average rate at which each category

hurricane makes landfall in each coastal state, which enables a likelihood calculation for each category of hurricane that can be performed individually for each coastal state. These likelihood calculations make use of the assumption that hurricane rates will stay constant over the next 50 years. It is useful to note that some states, such as Alaska and Washington, have likelihood equal to zero. The multiplication of the severity and likelihood is then performed for each particular combination of state and hurricane category and summed over the whole 50-year period.

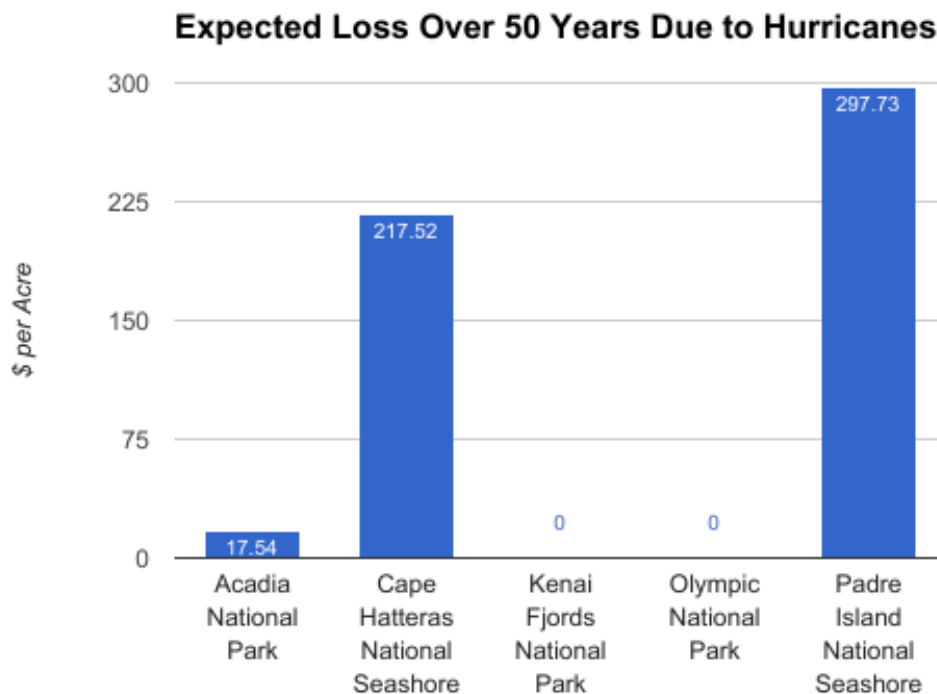


Figure 1: Total Loss over 50 Years Due to Hurricanes

3.2.3 Wildfires

The damages from wildfires in a given year is given by

$$C_{\text{wildfire}} = \text{expected number of acres burned} \cdot \text{cost per acre burned.}$$

We use a value of \$322.48 [3] as the cost per acre burned. This value is assumed to be constant over time and applicable for any given park. It should be noted that this cost per acre value was derived from data concerning wildfires in California, and therefore there is a possibility that it is not as applicable to other national parks as we assume.

The expected number of acres burned, on the other hand, is expected to change over time, and thus a more sophisticated model is required. The expected number of acres burned, $A(t)$, can be predicted using past trends in the number of acres burned, provided by the NPS [5]. There were no data for Kenai Fjords National Park, but based on its location (Alaska), it

was assumed that no forest fires took place. We sum the number of acres burned in each location in each year, and take a four-year moving average of this to smooth these data and reduce the effect of spikes and drops in forest fire severity. We plotted these data, as shown in Figure 2.

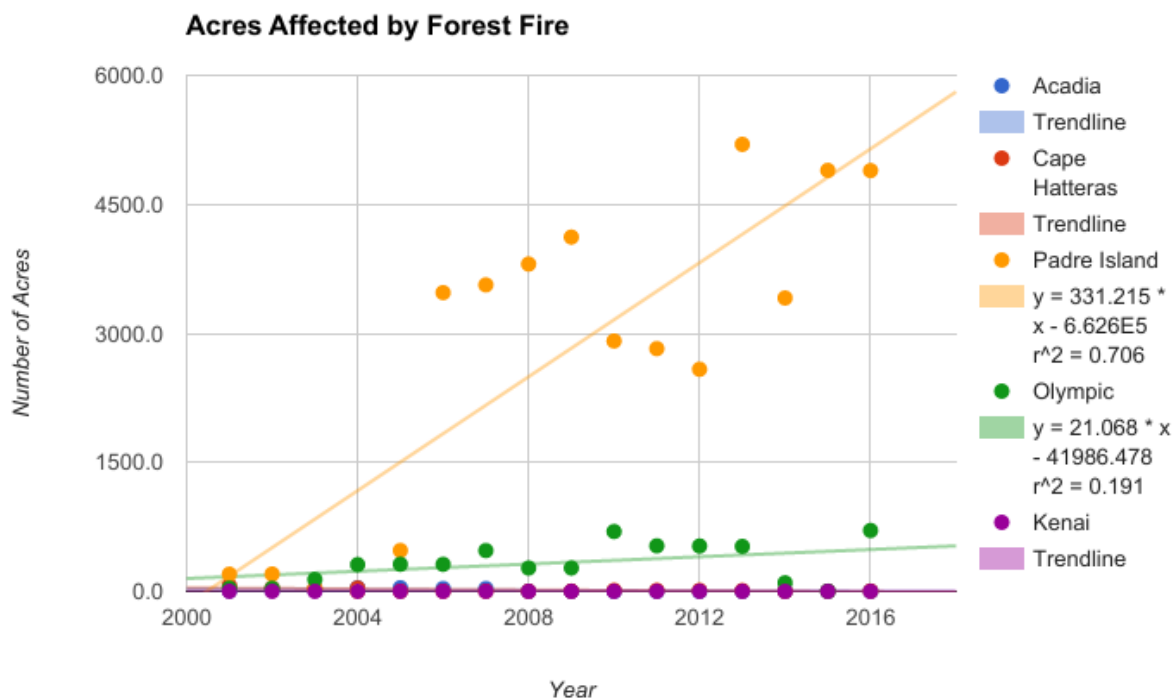


Figure 2: Approx. Acres Burned in Five NPS Units

Trendlines were also added to Figure 2. However, the number of acres burned in Acadia National Park and Cape Hatteras National Seashore, and the numbers outputted by the trendline equations, were extremely small relative to the number of acres burned in the other parks, and thus these two parks were also negligible. In addition, the R^2 value for Olympic National Park was a low value, 0.191, because of two outliers in the data. We removed these outliers and found new trendlines, as shown in Figure 3.

A linear model is roughly appropriate for both data sets, as shown by the lack of a discernible pattern in the residuals plots. As such, it does make sense to run a linear regression on these data.

The value of R^2 for Olympic National Park indicates that about 77.4% of the variation in acres burned can be explained by the change in time. The value of R^2 for Padre Island indicates that about 70.6% of the variation in acres burned can be explained by the change in time. As such, these trendlines can be trusted with a reasonable amount of confidence.

The vulnerability in each year was calculated by extrapolating into the future using the calculated trendlines. Using a summation similar to that used in the section on hurricane

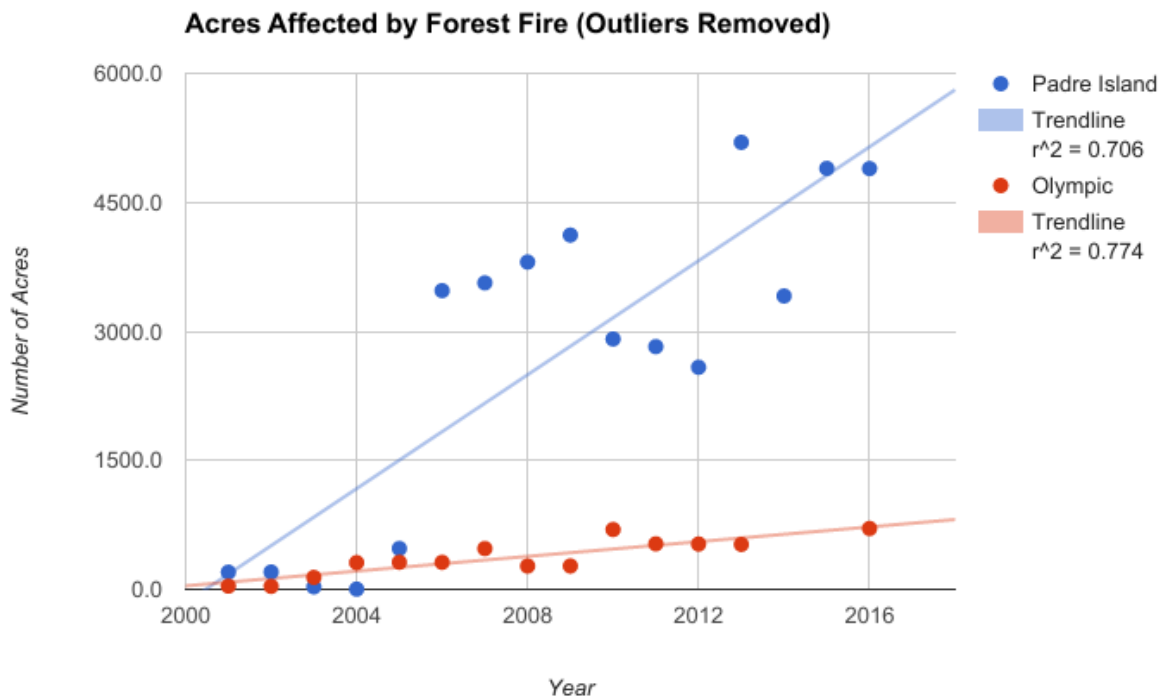


Figure 3: Approx. Acres Burned in Two NPS Units, with Outliers Removed

vulnerability, the total vulnerability to forest fires in the next 50 years was calculated. These sums were then normalized by dividing the dollar cost by the size of the park, in acres; these results are shown in Figure 4.

3.2.4 Putting It All Together

To calculate the total vulnerability index for a given park, we sum together the vulnerability index values for the three events.

$$N_{total} = N_{sea} + N_{hurricane} + N_{fire}.$$

The results of this calculation are shown in Table 4 and are visualized in Figure 5.

3.3 Analysis

On the whole, the results make sense. Padre Island, which is devastated by frequent wildfires, derives a high component of its vulnerability index from wildfires. Cape Hatteras, which is surrounded by ocean and is right in the middle of “Hurricane Alley,” derives most of its vulnerability from sea level rise and hurricanes, and not much from wildfires. Kenai Fjords National Park, which has very little history of climate-related catastrophes, has a predictably low vulnerability index. Olympic National Park, known for its vast forests, derives the vast

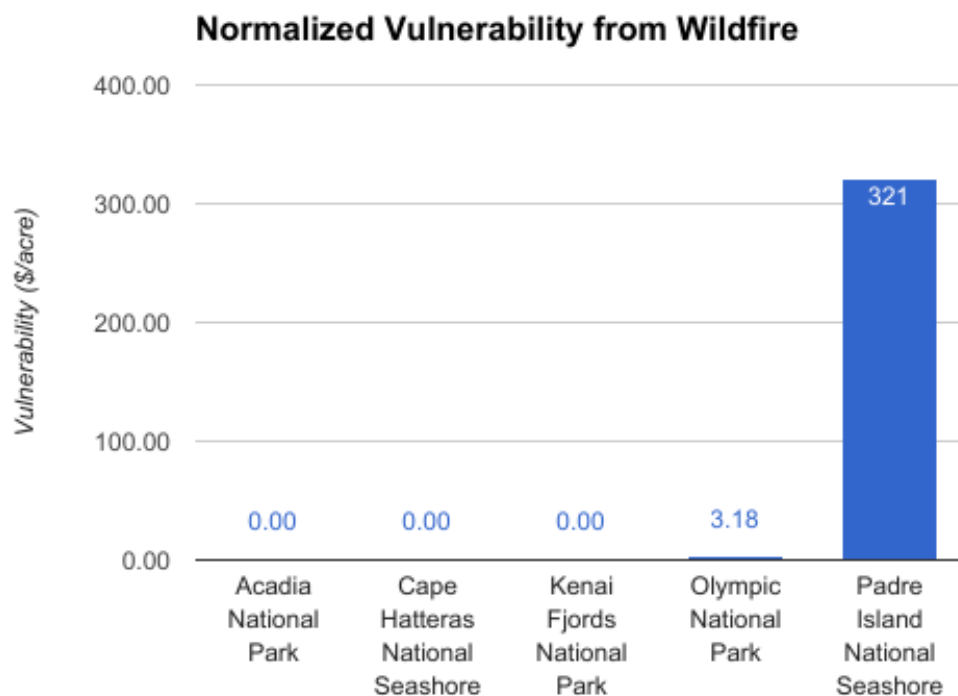


Figure 4: Normalized Total Loss over 50 Years Due to Wildfires

Table 4: Vulnerability Index

National Park	Sea level	Hurricane	Wildfire	Total
Acadia National Park	56.4	17.5	0.0	73.9
Cape Hatteras National Seashore	281.4	217.5	0.0	498.9
Kenai Fjords National Park	0.0	0.0	0.0	0.0
Olympic National Park	0.3585	0.0	3.18	3.5
Padre Island National Seashore	113.4	297.7	321.0	732.1

majority of its (overall low) vulnerability from wildfires, which spread quickly through forests. Based on total vulnerability, Padre Island is the highest. This makes sense since it is close to the ocean, so it is impacted by sea level rise; it is on the Gulf Coast, so hurricanes are an issue; and it is hit by frequent wildfires. It is strongly affected by all three, so the high vulnerability index makes sense. On a high level, the model appears reasonable.

3.4 Limitations

An issue in the model is the damage estimates for hurricane damage, which, if incorrect, can lead to skewed results. We assume that a category 5 hurricane causes \$40 of damage per acre. However, this was essentially an educated guess due to a lack of available data on hurricane damage in national parks. If this value is actually \$80, for example, all of the hurricane vulnerability values would double. When it comes to hurricane values, the

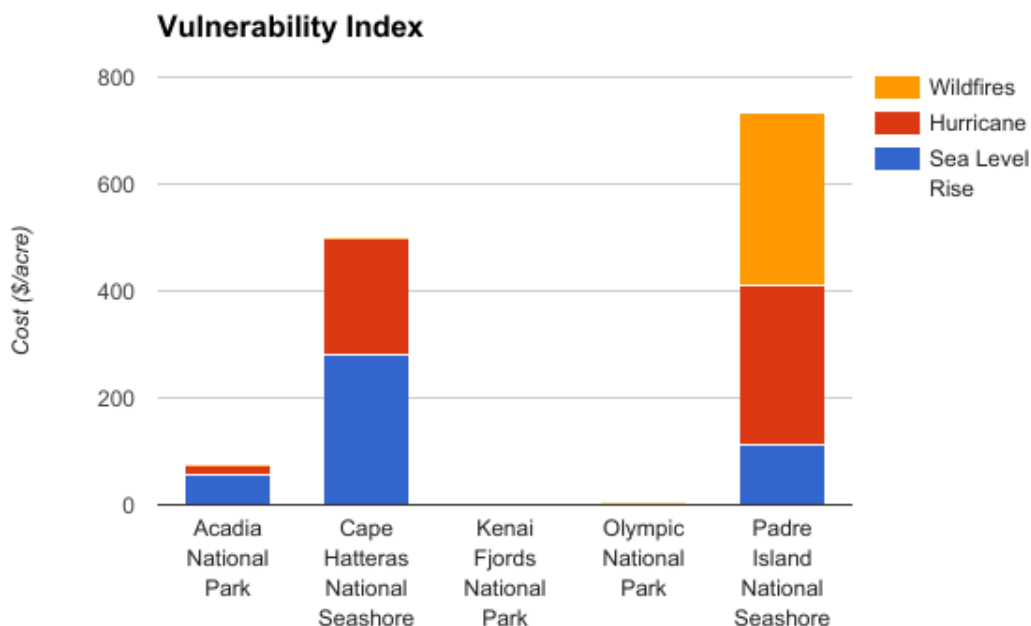


Figure 5: Vulnerability Index

accuracy of the estimates can swing the results.

Other climate related events, such as temperatures and air quality, were not included in the model. Although we believe that these factors pale in comparison to the ones we did include, they could turn out to play an unforeseen role which the model misses.

4 Let Nature Take Its Course?

4.1 Assumptions

- Park entrance fees will remain constant (relative to inflation) over the next 50 years.
Justification: Park manager decisions to change entrance fees are almost impossible to predict and are not the focus of our model.
- The primary factors that determine park attendance are the U.S. population, the vulnerability index for the park, and the temperatures for the year at the park.
Justification: As the population increases, there are more people available to visit national parks, and thus visitor count would likely increase. As parks experience more climate-related events, given by the vulnerability index, visitorship would likely decrease. Temperatures have been shown to have a significant positive correlation with park attendance [11].
- The vulnerability index for each park is held constant over the past 25 years and will remain constant over the next 50 years.

Table 5: Predicted Visitor Counts in 2067

National Park	2016 Visitor Count	2066 Projected Visitor Count
Acadia National Park	3303393	1391408
Cape Hatteras National Seashore	2411711	3081905
Kenai Fjords National Park	346593	1635067
Olympic National Park	3390221	3076239
Padre Island National Seashore	634012	333356

Justification: For the purposes of this regression, we do not calculate vulnerability indices for past years. We do not believe that our vulnerability index holds up in an extrapolation into the past. This approach also still accounts for the relative differences in vulnerability index between the different national parks.

4.2 Creating a Model

We train a multivariate linear regression model in order to take into account multiple factors (U.S. population, vulnerability index, etc.) to predict a single value (visitor count in a given year). The data used for training include the U.S. population, vulnerability index, local temperature, and visitor counts for the past 20 years. We then use this model to predict the visitor count in each of the five national parks 50 years from now.

The regression model is based on two essential equations: a hypothesis function and a cost function [12]. The hypothesis function defines the visitor count for a given year in terms of the above three features as follows:

$$h_{\theta}(x) = m_0 + m_1x_1 + m_2x_2 + m_3x_3.$$

In the above equation, m_0 is an intercept term, while m_1 , m_2 , m_3 are coefficients for the three features (x_1 , x_2 , x_3) that we use. We then adjust the values of m_0 , m_1 , m_2 , and m_3 to minimize mean squared error (MSE) between the hypothesis function and the visitor count. The MSE function, which is our cost function, is defined as

$$J(\theta) = \frac{1}{2m} \sum_{i=1}^n (y_i - h_{\theta}(x_i))^2.$$

To acquire predictions from the model, we must input the local temperature, U.S. population, and vulnerability index in 2066. The data used for local temperature and U.S. population are obtained by performing simple linear regression on the respective factors over time. The predictions for the local temperature and U.S. population in 2066 are then obtained from these models and used as the inputs for the model.

4.3 Results

Based on these results, Kenai Fjords National Park will experience a huge surge in visitorship over the next 50 years, and thus it is recommended that more resources be dedicated to this park. On the other hand, both Acadia National Park and Padre Island National Seashore

Table 6: Model Coefficients

Factor	Coefficient
Population (in millions)	-1505.8
Temperature	76558.4
Vulnerability Index	-988.7

will experience significant drops in visitor counts. Padre Island in particular, with its very small projected visitorship, is a candidate for removal from the National Park System if this situation becomes financially necessary. Essentially, financial resources should be allocated based on expected number of visitors in 50 years.

4.4 Analysis

The coefficient table shows that population does not act in the direction that we expected. This is an issue with the model, since it seems unlikely that increased population would actually decrease the number of visitors at national parks. The reason for this unexpected coefficient needs to be further analyzed in future iterations. Vulnerability index clearly negatively correlates with projected population, as expected, while temperature positively correlates. The magnitudes of the correlations should be taken with a grain of salt, since temperature values are much smaller (average around 40) than vulnerability index scores, which are in the hundreds.

The majority of our models seemed fairly reasonable and in line with our previous results and general expectations. For example, the visitorship of Padre Island National Seashore, which was extremely vulnerable to climate change events, was predicted to decrease significantly. The only result that was unexpected was that of Acadia National Park. Our model predicted that its visitor count would decrease more than twofold in 50 years; we predicted that there would not be as much of an impact because Acadia National Park is currently a very popular park and its vulnerability was significantly impacted only by sea level rise.

4.5 Limitations

We understand the dangers of using multivariate regression, and the unknowns that it brings. When using multivariate regression, one risks introducing variables that do not actually influence the output value in expected ways, and one risks having a limited understanding of how the model arrives at the value it actually arrives at. Unfortunately, in this scenario, a multivariate regression model proved to be the most viable option, since a single value (visitor count) needed to be calculated from multiple input parameters. In this case, this risk is limited by careful selection of input parameters and careful analysis of outputs.

Given more time, there are a number of improvements that could be made to this linear regression model. First, we could use a more reliable method to arrive at the model parameters for the year 2066. Our current method, which used linear regression to determine these parameters' values, is not necessarily reliable, and would introduce a second opportunity for modeling inaccuracies (such as the questionable population coefficient). It should be noted,

however, that any value we find, no matter the source, would not be completely accurate, and thus some of this error would nevertheless be inevitable.

5 Conclusions

We first created a model to categorize different coastal national parks into low, medium, and high risk categories for sea level rise risk. In the year 2050, three of the five given national parks fall into our high risk category, one of the five falls into the medium risk category, and the last falls into the low risk category. The results of our model aligned quite well with our expectations. The model also responded favorably to a sensitivity analysis. Our model predicts that rising sea levels pose a real and formidable threat to national parks in all of the coastal regions of the United States.

We then created a model to calculate the climate vulnerability index of any coastal national park, based on sea level rise, hurricane, and wildfire risks. Through application of this model, we found that Padre Island National Seashore is most vulnerable to climate-related events, while Kenai Fjords National Park has essentially no vulnerability.

For our final model, we designed and trained a multivariate regression model which used the projected future U.S. population, temperature of the local area around the park, and the vulnerability index to predict the visitor counts of each coastal park. We tested our model on the five focus parks and concluded that Kenai Fjords National Park would have the highest future visitorship increase, while Padre Island National Seashore will most likely become unsustainable in the future.

References

- [1] *Global Warming and Hurricanes*. <https://www.gfdl.noaa.gov/global-warming-and-hurricanes/>, 2016.
- [2] *HURDAT Re-analysis*. http://www.aoml.noaa.gov/hrd/hurdat/All_U.S._Hurricanes.html, 2016.
- [3] *2009 California Wildfires*. https://en.wikipedia.org/wiki/2009_California_wildfires, 2017.
- [4] *List of national parks of the united states*. https://en.wikipedia.org/wiki/List_of_national_parks_of_the_United_States, 2017.
- [5] *Wildland Fire: Wildland Fire — U.S. National Park Service*. <https://www.nps.gov/fire/wildland-fire/>, 2017.
- [6] Delaware Coastal Programs, *What Causes Sea Levels to Change?* <http://www.dnrec.delaware.gov/coastal/Documents/SLR%20Advisory%20Committee/AdaptEngage/3WhatCausesSeastoRise.pdf>.

- [7] Department of Commerce (DOC), National Oceanic and Atmospheric Administration (NOAA), National Ocean Service (NOS), Center for Operational Oceanographic Products and Services (CO-OPS), *Sea Levels Online: Sea Level Variations of the United States Derived from National Water Level Observation Network Stations*. <https://tidesandcurrents.noaa.gov/sltrends/sltrends.html>.
- [8] D. Fitch, U. Shama, and L. Harman, *Landfalling Hurricane Probability Project*. <http://www.e-transit.org/hurricane/welcome.html>, 2015.
- [9] E.S. Hammar-Klose and E.R. Thieler, *Coastal Vulnerability to Sea Level Rise: A Preliminary Database for the U.S. Atlantic, Pacific, and Gulf of Mexico Coasts*. <https://woodshole.er.usgs.gov/project-pages/cvi/digitaldata.html>, 2001.
- [10] B. Lam, *How Much Are America's National Parks Worth?* <https://www.theatlantic.com/business/archive/2016/07/us-national-parks-worth/492044/>, 2016.
- [11] K. McKinney, *Warm with a Chance of Crowds*. <https://www.npca.org/articles/976-warm-with-a-chance-of-crowds#sm.0001gi6lfty32d0cqj81g841kzzue>, 2016.
- [12] A. Ng, *Multivariate Linear Regression*. <http://openclassroom.stanford.edu/MainFolder/DocumentPage.php?course=MachineLearning&doc=exercises/ex3/ex3.html>, 2012.