MathWorks Math Modeling Challenge 2020

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M3 Challenge THIRD PLACE—$10,000 Team Prize

JUDGE COMMENTS

The summary is well written and clearly outlines the approach used to address each of the three issues for incorporating electric trucks into the national trucking network. Readers would be eager to continue reading the report if the summary included the results of the number of charging stations/chargers and the best corridor to first install charging stations.

Using the Lotka-Volterra competition model was an interesting choice to explore the type of competition between diesel and electric trucks for the delivery of products. The parameters in the L-V model are very well explained and the selection of values for the parameters is well explained and justified by available data. The weaknesses of the model are clearly acknowledged and reasonable estimates are made with explanation and desire to modify as additional information is obtained.

A few questions to ponder:

Why did you use the Lotka-Volterra competition model?

Is there any range of parameters where your Lotka-Volterra model would exhibit periodic oscillations in the electric/diesel fleet sizes as is sometimes observed in animal population modeled by the Lotka-Volterra equations?

How sensitive are your results to changes in values of the parameters used in all three of your models?

Keep on Trucking: U.S. Big Rigs Turnover from Diesel to Electric

Team 13202

Feb 2020

0 Executive Summary

Over the past few decades of human existence, global levels of carbon dioxide gas have risen to alarming levels, catching the attention of scientists around the world and prompting humans as a whole to reduce our carbon footprint. One such effort has been made to reduce greenhouse gas emissions by diesel semi-trucks by replacing them with electric semi-trucks, which have zero emissions on their own. With the advent of a new line of electric semi-trucks by Tesla©, it is now possible to begin an analysis of the impacts of such a decision.

Primarily, we sought to create a prediction of the proportion of electricity powered semi-trucks compared to the total number of semi-trucks over the next two decades, reporting values at 5, 10, and 20 years into the future. By comparing the growth of the populations of diesel and electric semi-trucks to biological populations in nature, we adopted a Lotka-Volterra competition model to develop our prediction model, where semi-trucks were separated by short-haul and long-haul trucks due to their differing competitive advantages between diesel and electric trucks. We solved a system of differential equations to solve for the proportion of electric semi-trucks for 5, 10, and 20 years in the future, which comes out to be 38.6%, 46.5%, and 59.9%, respectively. Our results match the expectation that electric-powered semi-trucks will become more advantageous economically and environmentally in the future.

However, before the trucking industry can switch to a fully electric based model, the infrastructure for heavy duty Class 8 electric vehicles has to be developed. The biggest challenge to this is the charging technology. Because of the limitations of the capacity of electric batteries as well as the relatively slow charging times, placement of charging stations as well as the number of chargers has to be carefully planned out. To plan this, we developed a program that takes into information of exit ramps on highways, as well as the annual average daily truck traffic (AADTT), interpolating from the annual

average daily traffic (AADT) if the statistic from the AADTT was missing, to plan and predict the best location for charging stations, as well as the number of chargers at each station. This came out to be roughly 1 station every 50 miles to maximize the convenience of drivers as well as accounting for the limited efficiency of electric vehicles, with each station ranging from 50-500 chargers, depending on the amount of traffic.

After finalizing plans on the necessary infrastructure, we needed to figure out a quantitative approach to prioritize which corridors need to be built first. By quantifying seemingly qualitative factors, we were able to better analyze and predict community responses to proposals to build such infrastructure. The 4 factors we considered were the anticipated usage of these electric charging stations, the greenhouse gas emissions, the community motivation and enthusiasm, and, last, the cost-budget ratio. By analyzing these 4 factors, we take into account the infrastructure costs, the predicted utilization, the environmental impact, and the overall public support. Through this model, we were able to quantitatively predict which corridors should be prioritized, creating a foundation to begin the process of transforming the trucking industry to a fully electrical industry.

Contents

1 Part I: Shape Up or Ship Out

With the announcement of new electric semi-trucks hitting the market potentially within the next year, it is important to have a model which predicts the impact that such an invention would have on the market of trucks in the near future. Such a model would allow producers of electric semi-trucks to predict the success rate of their product and thus have room to improve upon it to maximize its success in the market. Additionally, it allows for scientists to come up with metrics to estimate the ecological benefits that electric semi-trucks will have. Based upon the current number of semi-trucks in the United States and the current market demand, we developed a model which predicted the proportion of electric-powered semi-trucks in the U.S. 5, 10, and 20 years in the future.

1.1 Assumptions

1. *The relationship between the number of diesel-powered semi-trucks and electric semi-trucks can be modeled as a competition model between two species.* Like two biological species fighting for a common limited resource, diesel and electric semi-truck populations share the limiting factor of consumers within the market of semi-trucks.

2. *The purchase of electric semi-trucks will replace older diesel semi-trucks.* The main benefits of electric semi-trucks are significantly lower greenhouse gas emissions and long-term projected cost efficiency due to the price of electricity, so individuals or companies motivated to buy electric semi-trucks will likely dispense of their diesel trucks to significantly improve these metrics [1].

3. *There will be no new types of semi-trucks that will take a significant portion of the market in the next 20 years.* Given the aforementioned motivations for the creation of electric semi-trucks as less harmful environmental impact and greater cost efficiency, it is unlikely that a new type of truck will be better in these realms [2].

4. *Tesla's electric semis will not encounter major competition in the realm of electric trucks.* Given Tesla's success and de facto monopoly in the electric car market, we believe this will carry over to electric trucks. Additionally, they are one of the largest amongst companies that have released or plan to release electric trucks in the near future [3].

5. *Tesla obtains their electricity from renewable energy sources.* Given Tesla's motivation for the creation of electric vehicles stemming from a desire to reduce the stress on the environment, this is a reasonable statement [4].

6. *The carrying capacity of semi-trucks or correspondingly the number of consumers in the semi-truck market will remain constant.* Production of semi-trucks in general has remained between 120,000 and 220,000 over the past 9 years, with no significant increasing or decreasing trend, indicating that the population of trucks has reached an equilibrium phase [5].

7. *The birth rate for diesel and electric semis can be modeled using the probability density function of an exponential distribution. Birth rate is defined as the new units of a certain good produced in an year divided by the total operating units in the market at that time.* Because electric semi-trucks are a new product, the birth rate tends to be high following its release, because there are few units in the market at that time. Over time, however, this rate decreases at a decreasing rate approaching a stable birth rate as the total operational units of the market stabilizes. Because electric semi-trucks are a new product, they will disrupt the stability of diesel semi-trucks, causing their birth rate to decrease.

8. *The birth rate for existing populations of semi-trucks will remain constant. An existing population in the context of semi-trucks is a type of semi-truck in the market whose birth rates don't have a significant upward or downward trend.* As the birth rate for existing types of semi-trucks has remained in the same ballpark for the past 9 years,

alluding to assumption 6, we project it to remain in the same range in the foreseeable future [5].

9. *The batteries of Tesla's electric semis will increase in capacitance over time, resulting in a greater maximum range for travel.* Lithium ions have almost tripled in capacitance the past five years and are expected to essentially double by 2025 [6][17].

10. *Tesla's estimates for the growth in the total units of electric semis sold in their first four years will be approximately accurate.* Given the fact that the product is not yet available in the market, Tesla's estimates are the greatest indication of how their trucks will do. Furthermore, we are not using their explicit numbers, only their projected percent growth from the first to second year of operation and from the second to third year of operation.

11. *The maximum range of an electric truck is directly proportional to the capacitance of its batteries.* The discharge rate of energy of a battery remains constant over time, so increasing the capacity of a battery while its power output remains the same will linearly increase the time and distance that the truck can drive [18].

12. *The average annual mileage of a diesel semi-truck will be equivalent to the average annual mileage of an electric semi-truck.* It is reasonable to assume that the demand and usage of truck driving is independent of the type of truck which is doing the work.

1.2 Model Development

The nature of competing goods and services in the market is reminiscent of the competition between two species for a common resource necessary for their survival. As per assumptions 1, 3, and 4, we use a competition model for the relationship between diesel and electric semi-trucks because they both are reliant upon consumers to buy their products in order to establish their "survival" in the market in future years.

In order to model the interaction between diesel and electric semi-trucks, we establish a classic Lotka-Volterra competition model, commonly used amongst ecologists to describe the dynamics between two biological species sharing a similar resource, as entailed by assumptions 3 and 4 [7]. However, unlike traditional Lotka-Volterra models, we implemented time-dependent models of the birth rates and the competitive factors of the species since, under normal circumstances, species change and adapt to other competing species to gain an advantage.

Additionally, because of the difference in functionality of short-haul and long-haul semi-trucks, we separated the two types of trucks when investigating the future proportions of their populations.

1.2.1 Parameters in Lotka-Volterra Model

Population of Semi-Trucks ($N(t)$). The average number of short-haul semi-trucks within the past ten years is $1.92 \cdot 10^6$, and the average number of long-haul trucks within the past ten years is $1.76 \cdot 10^6$ [5], which we took to be the initial values, $N(0)$, of the short- and long-haul semi-trucks.

Birth Rate ($r(t)$). The birth rate represents a percentage of the number of new babies of a species, or in this case, new trucks that are entering the population divided by the existing population. This variable represents the rate of the growth of a species. This statistic varies with time and will be defined as part of the model derivation.

Carrying Capacity (K). The carrying capacities represent the maximum number of semi-trucks that can be supported by the market. As per assumption 6, this number is assumed to be constant for the United States, at $1.92 \cdot 10^6$ for short-haul trucks and $1.76 \cdot 10^6$ for long-haul trucks [5]. We assume that the current market is already at carrying capacity because the market for trucks has already stabilized, so introducing a new "species" would not change the maximum number of trucks, only the proportion of trucks that are gas. Therefore, the carrying capacities of diesel semi-trucks

and electric semi-trucks are the same.
Competitive Effect ($b_{12}(t)$). The competitive effect b_{12} is defined as the effect of competition of population N_2 on N_1, which is characterized by the cost efficiency, environmental impact, and maximum driving range without refueling/recharging of each type of truck. The competitive effect of species 1 on species 2 is the multiplicative inverse of species 2 on species 1, and a value of 1 for the competitive effect of species 1 on species 2 in the Lotka-Volterra model means that individuals from both species have effectively the same chance of obtaining the resources necessary for survival [7]. Section 1.3.1 delineates how the competitive effect will be calculated.

1.3 Model Derivation

The Lotka-Volterra model takes the form of a system of differential equations as follows:

$$\frac{dN_D}{dt} = r_D(t)N_D\left[1 - \frac{N_D}{K_D} - b_{DE}(t)\frac{N_E}{K_D}\right],$$

$$\frac{dN_E}{dt} = r_E(t)N_E\left[1 - \frac{N_E}{K_E} - b_{ED}(t)\frac{N_D}{K_E}\right],$$

where the subscripts D and E represent diesel and electric semi-trucks, respectively [7].

1.3.1: Deriving the Competitive Effect Function

We determined that the competitive effect should take into account different attributes of diesel and electric semi-trucks, just as competing species have different attributes that give them advantages or disadvantages compared to each other. We decided upon the following three attributes for determining this effect: cost, environmental impact, and maximum range of the average semi-truck for each type. We believe these three attributes account for a majority of the factors that consumers consider when choosing between the trucks. We then multiply the ratio of the electric to diesel values for each of the attributes to obtain our competitive effect to obtain the competitive effect of electric semi-trucks on diesel semi-trucks, as shown below. We can then take the multiplicative inverse of this value to obtain the competitive effect of diesel semi-trucks on electric semi-trucks:

$$b_{\text{DE}} = \frac{E_{\text{cost}}}{D_{\text{cost}}} \cdot \frac{E_{\text{env}}}{D_{\text{env}}} \cdot \frac{E_{\text{range}}}{D_{\text{range}}}.$$

Cost is a combination of upfront and refueling costs for each truck. The upfront cost is the expense to actually buy the truck, and refueling cost is how much fuel/electricity costs for the truck each year, based on average annual mileage. The lifetime of the average diesel semi-truck ranges from 500,000 to 800,000 miles [8], so we used 500,000 for short-haul and 800,000 for long-haul. Based on the annual mileage for short-haul and long-haul diesel trucks, we determined the number of years diesel semi-trucks last. We multiplied this duration by the annual refueling cost and added to the upfront cost to obtain the total cost. This is shown below:

$$E_{\text{Cost}} = E_{\text{Cost Up Front}} + E_{\text{Cost Fuel}} \cdot \frac{E_{\text{Total Distance}}}{E_{\text{Annual Distance}}},$$

$$D_{\text{Cost}} = D_{\text{Cost Up Front}} + D_{\text{Cost Fuel}} \cdot \frac{D_{\text{Total Distance}}}{D_{\text{Annual Distance}}}.$$

We assume $E_{\text{Total Distance}}$ and $D_{\text{Total Distance}}$, which is the lifetime mileage, and $E_{\text{Annual Distance}}$ and $D_{\text{Annual Distance}}$, the annual mileage, to be equal by assumption 12. For short-haul, $E_{\text{Cost Up Front}}$ is \$150,000 and estimated $D_{\text{Cost Up Front}}$ is \$113,636 [1]. $E_{\text{Cost Fuel}}$ or annual cost for electric semi-trucks is \$10,233, and $D_{\text{Cost Fuel}}$ or annual cost for diesel semis is \$25,584 [1]. Total distance is 500,000 miles as aforementioned, and annual distance traveled by a short-haul semi-truck is 42,640 miles [5]. We thus get E_{Cost} as \$269,993 and D_{Cost} as \$413,636 for short-haul total cost. For long-haul, $E_{\text{Cost Up Front}}$ is \$180,000 and estimated $D_{\text{Cost Up Front}}$ is \$136,364 [1]. $E_{\text{Cost Fuel}}$ or annual cost for electric semi-trucks is \$28,517 and $D_{\text{Cost Fuel}}$ or annual cost for diesel semis is \$71,292 from the same source as for short-haul [1]. Total distance is 800,000 miles as aforementioned, and annual distance traveled by a long-haul semi-truck is 118,820 miles. We thus get E_{Cost} as \$347,279 and D_{Cost} as \$554,563 for long-haul total cost.

Environmental impact is measured as total pounds carbon dioxide (CO_2) emitted per year for each truck. This was measured differently for diesel and electric semis due to the lack of data for the latter group. Given the CO_2 emission of trucks per gallon of diesel, we multiplied this value by the number of gallons used per mile, and then multiplied this by the annual mileage for short- and long-haul to obtain the total pounds of CO_2 emitted per year for the average diesel semi-truck. For electric semis, we obtained the average pounds of CO_2 emitted by the production of electricity from renewable sources as measured by kilowatt hours in 2017 as per assumption 5. We multiplied this by the kilowatts used per mile, and then the annual mileage to determine the total pounds of CO_2 emitted per year for the average electric semi-truck. This is shown below:

$$D_{\text{Env}} = \frac{\text{Pounds } CO_2}{\text{Gallon}} \cdot \frac{\text{Gallons}}{\text{Mile}} \cdot \frac{\text{Miles}}{\text{Year}} = \frac{\text{Pounds } CO_2}{\text{Year}}$$

and

$$E_{\text{Env}} = \frac{\text{Emissions}_{\text{total}}}{\text{Electricity}_{\text{total}}} \cdot \frac{\text{Emissions}_{\text{renewable}}}{\text{Emissions}_{\text{total}}} \cdot \frac{\text{Electricity}_{\text{total}}}{\text{Electricity}_{\text{renewable}}} \cdot \frac{\text{Kilowatts}}{\text{Mile}} \cdot \frac{\text{Miles}}{\text{Year}}.$$

The average pounds of CO_2 per gallon of diesel burned is 22.38 pounds for all diesel semi-trucks [9]. We averaged the range of miles per gallon for diesel vehicles given in the prompt to get 1/6.64, yielding 0.151 $\frac{\text{Gallons}}{\text{Mile}}$ for all diesel semi-trucks. For short-haul, we have 42,640 annual mileage, giving 143,717 pounds of CO_2 for D_{env} [5]. For long-haul, using 118,820 annual mileage, giving 400,483 as our value for D_{env} [5]. For electric semi-trucks, we found $\text{Emissions}_{\text{total}}$ and $\text{Emissions}_{\text{renewable}}$ to be 3.967 trillion and 1.228 trillion pounds, respectively [10]. $\text{Electricity}_{\text{total}}$ and $\text{Electricity}_{\text{renewable}}$ we determined to be 3.82 trillion and 2.628 trillion kilowatt hours respectively [11][10]. We found that 2 kilowatt hours are used per mile [16]. For short-haul, the annual mileage is again 42,640 miles, and for long-haul, it is 118,820

[5]. We thus get 41,322 pounds for E_{env} for short-haul, and 115,148 pounds for E_{env} for long-haul.

Finally, range is the distance the truck can travel with full fuel. The maximum range for Tesla's electric semi-trucks is 300 miles for short-haul and 500 miles for long-haul [1]. The average gallon tank size for diesel trucks is between 120 and 300 gallons [12], so we estimated short-haul diesel trucks to have a maximum range of $150 \cdot 6.64 = 996$ miles and long-haul diesel trucks to have a maximum range of $250 \cdot 6.64 = 1660$ miles.

Thus we obtained the initial competitive effect of Tesla electric semi-trucks on diesel semi-trucks to be 1.654 for short-haul and 1.673 for long-haul. However, due to our assumption 9, the competitive effect is not a constant value, as the overall competitiveness of Tesla electric semi-trucks will increase as new cycles of the product are introduced. We reasoned that the competitive effect of Tesla electric on diesel will increase significantly initially and over time slow down and approach an asymptote as many improvements are made immediately for new products and decrease in quantity over time. Initially, the cost and environment attributes are an advantage for Tesla, but the range factor is a disadvantage. Thus, to estimate the asymptote value we projected that eventually the range of Tesla's electric semis will rival that of diesel semis, or numerically that $\frac{E_{\text{range}}}{D_{\text{range}}} = 1$ for both short- and long-haul. That translates to an asymptote value of 5.491 for short-haul and 5.554 for long-haul. We decided to use the right half of the logistic curve to model the competitive effect as a function of time. Recalling that the competitive effects of each of the two types of trucks on each other are multiplicative inverse, we will just determine the competitive effect of electric trucks on diesel trucks. The general equation for a logistic curve is

$$x(t) = \frac{a}{1 + \left(\dfrac{1}{x_0} - 1\right) e^{-rt}}.$$

For short-haul, the curve should have a value of 1.654 at $t = 0$, and 5.491 as t approaches infinity. We can thus solve for variables a and x_0 of the logistic curve. We determine $a = 5.491$ and $x_0 = 2.32$. Now we must determine r. Recognizing that we are modifying the range of electric semi-trucks and that the range of a truck is dependent on the capacitance of the truck's batteries as per assumption 11, we need an estimate for the growth in efficiency of the capacitance of batteries at some timestamp in the future. Capacity for lithium-ion batteries, which are used for electric vehicles, stands at 302.2 GWh as of 2019, and plants with another 603.8 GWh are planned to open in 5 years [13]. We thus determined that the range of electric trucks will multiply by a factor of $\frac{603.8}{302.2} = 1.998$ in 5 years, giving 3.3047 as the competitive effect at $t = 5$. Plugging this in, we obtain that $r = .2509$. For long-haul, the curve should have a value of 1.673 at $t = 0$, and 5.554 as t approaches infinity. Using the same method as we implemented for short-haul, we obtain $r = .274$. This gives us the following two equations for the competitive effect of Tesla electric semi-trucks on diesel semi-trucks as a function of time for short-haul and long-haul, respectively:

$$b_{\text{DE}} = \frac{5.491}{1 + 2.32e^{-0.2509t}},$$

$$b_{\text{DE}} = \frac{5.554}{1 + 2.32e^{-0.2514t}}.$$

1.3.2 Deriving the Birth Rate Function

The next and last coefficient we determined was the birth rate. Alluding to assumption 7, we will use the probability density function, or effectively the derivative, of an exponential distribution for the birth rate of both diesel and electric semi-trucks. As per assumptions 2 and 12 the birth rate of electric semi-trucks will eventually reach the current birth rate of the existing population of trucks as determined by assumption 9 because their population will stabilize, we want our probability density function of the exponential distribution to approach a nonzero value. As per assumption 8, value can be as the average of new semi-trucks produced over the past 9 years, which has no significant trend, divided by the total number of semi-trucks in operation. The average is 165,488 [5], and the total number of semi-trucks used in 2017 was 3.68 million [14], yielding 0.0450 as the current birth rate. We can write the general form of our exponential distribution as $f(x) = c + \lambda e^{-\lambda t/k}$, where c is the current birth rate, and λ and k are constants to be solved for. For diesel semi-trucks , since the birth rate will eventually reach 0, since electric semi-trucks will replace diesel semi-trucks, we can write the general form of our exponential distribution as $f(x) = \lambda_2 e^{-\lambda_2 t/k}$, where λ_2 is a constant unrelated to λ and k has the same value as for the electric semi-truck.

Let's first determine the birth rate of Tesla electric semi-trucks. The eventual birth rate c is the birth rate of the existing population of diesel semi-trucks. As explained by assumption 10, we used Tesla's projections for the birth rate of electric semis sold in the first three years to obtain the coefficients for our birth rate as a function of time. More specifically, we obtained a birth rate of 5 trucks/year in the first year, and 2 trucks/year in the second year [15]. Plugging these ordered pairs of $(\mathbf{1}, \mathbf{5})$ and $(\mathbf{2}, \mathbf{2})$ into our equation, we determined that $\lambda = 12.5$ and $k = 13.638$. Adding this to our added constant yields a final equation of $f(x) = .045 + 12.5e^{-12.5t/13.638}$ for the birth rate of Tesla electric semi-trucks. Because k has the same value for both equations we now only need to find λ_2. At t = 0, or in 2020, we estimate the birth rate of diesel semi-trucks to be the same as that of the average for the past 9 years, or .045 [5]. Thus we get $\lambda_2 = .045$, yielding a final equation of $f(x) = .045e^{-.045t/13.638}$ for the birth rate of diesel semi-trucks.

These two equations are the same for short- and long-haul, as we project the two groups to grow at the same rate due to lack of data for Tesla electric semi-trucks. Thus we have the following equations for the birth rates for diesel and Tesla electric semis for both short- and long-haul:

$$r_E(t) = 12.5e^{-0.9163t} + 0.0450,$$

$$r_D(t) = 0.0450e^{-0.07332t}.$$

1.4 Results

After deriving our time-dependent functions for the competitive effects and birth rates of the semi-trucks, we substituted them into our Lotka-Volterra differential equations and coded a numerical differential equation solver in Matlab to produce the plots of the projected populations of diesel and electric semi-trucks over a 20-year period. Figures 1.4.1 and 1.4.2 depict the comparative populations of short-haul and long-haul semi-trucks, respectively.

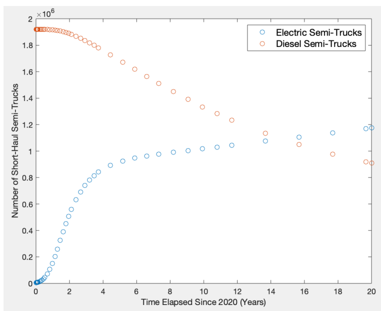

Fig 1.4.1: Projection of Populations of Short-Haul Semi-Trucks

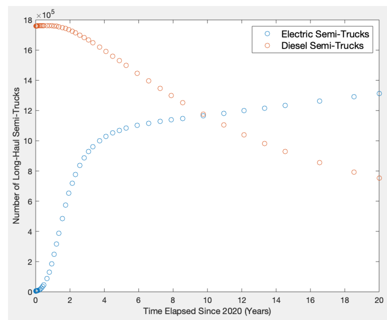

Fig 1.4.2: Projection of Long-Haul Semi-Trucks

Using these plots, we found the proportion of diesel and electric semi-trucks for their respective haul types by dividing each specific population by the sum of the two populations. Figures 1.4.3 and 1.4.4 depict the comparative proportions of short-haul and long-haul semi-trucks, respectively.

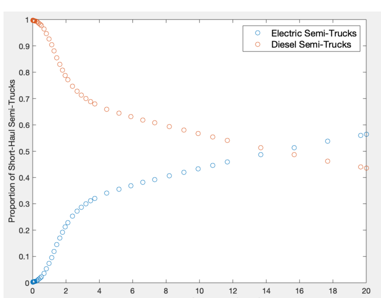

Fig 1.4.3: Projection of Proportions of Short-Haul Semi-Trucks

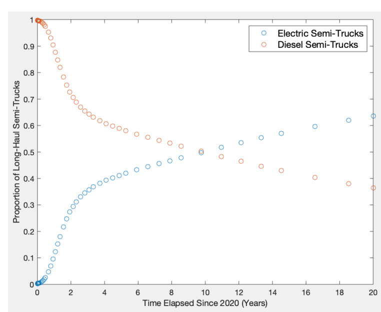

Fig 1.4.4: Projection of Proportions of Long-Haul Semi-Trucks

Proportion of Electric Semi-Trucks (Table 1.4.1)			
	5 Years	10 Years	20 Years
Electric Short-Haul Semi-Trucks	35.5%	43.3%	56.4%
Electric Long-Haul Semi-Trucks	41.5%	50.0%	63.5%
Total	38.6%	46.5%	59.9%

Our model concludes that the proportion of electric semi-trucks will be 38.6%, 46.5%, and 59.9% of the total population of semi-trucks at years 2025, 2030, and 2040, respectively.

1.5 Strengths and Weaknesses

Our model is adaptive, taking into account the changing birth rates of the different semi-trucks and the changing competitive effects which each type of semi-truck will have on each other as technology advances, and producers will then adapt newer models of semi-trucks to better compete in the market. This then ameliorates the drawbacks of adopting a Lotka-Volterra model, which assumes a linear birth rate and constant competitive factor for both species, an unrealistic assumption in practice. Furthermore, the Lotka-Volterra competition model is a valid modeling technique amongst ecologists, which helps give credibility to our approach.

On the flip side, our model does have weaknesses as well. We have many assumptions, which push at the applicability of the model. However, given the lack of data for Tesla electric semi-trucks, as they have yet to be released, we don't believe this to be a fatal weakness. Additionally, there are other factors we didn't consider when determining the attributes and formulas for these attributes of the competitive effect. One such example is maintenance costs for the two types of trucks. Again, however, given the lack of data we don't believe this hampers the legitimacy of the model we have construed.

2 Part II: In It for the Long Haul

As electric trucking becomes more prevalent to reflect environmental concerns, major trucking routes would undergo significant installations of charging infrastructure to support the changing vehicle types. One of the most significant challenges to switching to an all-electric trucking model is the process of charging vehicles. With current technology having slow and inefficient charge times, planning and creating a model to predict locations where charging stations are necessary will greatly expedite the process of changing to an all electric trucking industry. Based on the traffic flow in each trucking route, we developed a model to determine the number of stations and the number of chargers at each station necessary to support single-driver, long-haul traffic in the event where all trucking becomes electric. We then applied our model to the specified corridors to find their respective amount of stations and chargers needed. In the following model, we define a charging station as a cluster of chargers that are near exit ramps of highways, rather than having multiple stations per exit ramp in order to provide more organized and comprehensive results.

2.1 Assumptions

1. *All electric trucks are long-haul trucks.* This assumption will result in a slight over-approximation on charges required, given that some electric trucks in the Annual Average Daily Truck Traffic statistic do not fall into Class 8 vehicles, so they will have a slightly higher efficiency [19].

2. *The truck flow at a certain point in the route is uniform across the different time intervals.* Due to trucking not being bound by usual work day schedules, many truckers actually drive equally both at night and at day [23].

3. *Drivers will try to fuel up at 20% battery remaining.* By refueling at this percentage, drivers leave roughly 50 miles to find a fuel station, while also avoiding refueling excessively. Also, based on Tesla's own website, battery levels should ideally never hit zero and should be

lowest at around 20%. Therefore, when battery levels hit 20%, drivers will try to recharge as soon as possible [24].

4. *We are able to ignore effects like traffic when accounting for the spacing between adjacent charging stations.* Because of the construction of electric vehicles, traffic and other idling scenarios will not result in a significant loss of charge or battery [24].

2.2 Model Development

In order to model and determine the number of charging stations and chargers required, our model has to be able to support the current level of long-haul traffic. To determine the necessary resources to fuel existing long-haul traffic, we used the Annual Average Daily Truck Traffic (AADTT) statistic to predict and estimate the number of trucks present on a daily basis.[19] Data that was missing was interpolated using Annual Average Daily Traffic and values from similar states. This, combined with data from the North American Council For Freight Efficiency (NACFE) on the battery efficiency of modern Class 8 electric vehicles allows us to estimate the time required to charge and the distance traveled per full charge [21]. Our model decided to use DC Fast Charging as the primary charging tool for Class 8 electric vehicles. Utilizing any other charger would result in exorbitantly long waiting times and limit the productivity of the trucking industry [22].

2.2.1 Data and Variable Breakdown

Distance Per Full Charge (D). According to assumption 3, drivers will drive from 100% charge to 20% charge. This, combined with the amount of miles a full Class 8 electric vehicles can drive results in an average distance of 200 miles following a recharge [21].

Time to Recharge (T). This variable can be derived from assumption 3. Statistics on the average charging time of a Class 8 electric vehicle yields an average charging time of 30 minutes [20].

Daily Average Volume of Trucks (V). This variable yields the average number of trucks that pass through any given section of highway per day.

Assumption 2 helps simplify the processing of this data as the number of truck flow is assumed to remain roughly constant throughout all times of the day.

Distance between Stations (S). This variable represents how far apart adjacent charging stations are in a highway. The smaller S is, the more convenient it is for truck drivers, but also the more costly it is for construction.

2.2.2 Model Derivation

In order to first determine the optimal spacing between adjacent stations, we can use assumption 3 to calculate the most convenient positioning for drivers to refuel. Since 20% charge can travel roughly 50 miles, we try to position each charging station as close to miles apart from each other as possible. By calculating the positioning between adjacent charging stations, we are now able to predict the required number of chargers per station by determining the number of trucks that pass through the station on average.

Since refueling takes 30 minute intervals, the most logical way to record the number of trucks that need refueling is to determine the number of trucks that pass through a refueling station every 30 minutes. This can be determined with the following equation:

$$V_{\text{30 Minutes}} = V \cdot \frac{T}{24}.$$

However, this function only outputs the total number of trucks that pass by, and not all trucks actually have to refuel every time. After every recharging, a Class 8 electric vehicle should be able to travel a minimum distance of D; therefore, on average, each truck would pass through $\frac{D}{S}$ charging stations before refueling. Computing this value yields an average of refueling every 4 charging stations. From here, we can deduce that 25% of trucks actually need to refuel at each charging station. Thus, the total number of trucks that need to be refueled during any given 30 minutes is

$$V_{\text{Refuel}} = V \cdot \frac{T}{24} \cdot \frac{1}{4}.$$

In order to keep up with this demand, the amount of chargers per charging station should be the same as the number of vehicles needed to be charged.

From here, we were able to create a program that takes in an array representing different corridors. The arrays contain information on the locations of mile markers and exit ramps as well as the Annual Average Daily Truck Traffic, with the first variable determining which mile markers should have charging stations, and the second variable determining how many chargers are present at respective charging stations [21].

2.3 Results and Example Corridors

Using data from 5 different popular trucking corridors and routes, the number of charging stations and chargers required were determined. A detailed analysis of the route from Minneapolis, MN, to Chicago, IL, is provided below as well for reference.

Charging Stations and Chargers Required for Sample Corridors (Table 2.3.1)					
	San Antonio to New Orleans	Minneapolis to Chicago	Boston to Harrisburg	Jacksonville to Washington DC	Los Angeles to San Francisco
Stations Required	10	8	7	13	6
Chargers Required	1559	1720	1708	1640	1243

Locations and Quantity of Charging Stations on I94 (Table 2.3.2)	
Location of Charging Station in Miles from Minnesota	Number of Chargers Required
52	346
104	115
160	59
231	99
257	84
306	121
356	350
404	550

Although the number of chargers are significantly large, these numbers are reasonable because of the short mileage of Class 8 electric vehicles as well as the long charging time. [21] Regular gas powered commercial heavy-duty trucks can travel roughly 750 miles before refueling, and pumping takes as few as 10-15 minutes. [22] This means that gas powered trucks are about 9 times more efficient when it comes to pumping, which justifies the large number of chargers required.

2.4 Strengths and Weaknesses

A quantitative sensitivity analysis is not applicable due to the nature of the model, so we will proceed with a qualitative breakdown of our program's strength and weaknesses. Our model is capable of predicting the necessary resources in order to sustain current trucking practices and models, and by basing our criteria on existing habits and conveniences for truck drivers, a potential switch to electric vehicles following our models plan will not bring about any drastic changes for drivers and the business. For example, by maintaining a similar frequency of charging stations as gas stations, the route planning for truck drivers remains roughly the same. Another strength of this model is actually its over-approximation. Because the model slightly over-approximates by assuming all trucks have the efficiency of a Class 8 semi truck, there will be slightly more chargers than necessary, helping account for daily fluctuations or random perturbations.

However, the weakness of this model also lies in its over-approximation. In doing so, the predicted costs of this model are higher than they would be normally, creating a higher economic strain and demand for chargers. Furthermore, our model also depends heavily on the reliability of truck drivers to have predictable behavior. Because of the small distance an electric powered truck covers, there is limited flexibility for truckers to deviate from planned routes. Furthermore, our model does not account for the possibility for improved technology, both in charging equipment and in the efficiency of electric vehicles. Despite this, our model is a reasonable prediction for a plan of action to implement electric vehicles into the trucking industry.

3 Part III: I Like to Move it, Move it

As part of the switch to eco-friendly trucking, certain areas will transition first as some areas are more readily equipped to deal with the change better. Also, some areas have more resources and are more willing to undergo the change. Knowing this, we were tasked with creating a model which would allow us to determine a ranking of the routes we used in Part II to undergo the changes first. We first determined various factors that a government would consider when determining whether they would want to undergo a change. Using these factors, we were able to create a point system for each of the factors out of 1000. We then used the sum of these points to determine our ranking (first being the highest points, and last being the smallest points).

3.1 Assumptions

1. *A state's environmental budget accurately reflects the ability of a state to fund area-specific environmental projects.* According to the .S. Gov-

ernment's Office of Management and Budget, the budget reflects support for high priority projects [44]. Thus, it is reasonable to assume that the budget is a measurable indicator of what sections the government and people prioritize and will fund.

2. *All four factors—Anticipated Usage (U), Greenhouse Gas Emissions (GHG), Community Motivation (M), and Cost (C)—are equally weighted in importance in determining the ranking of the five corridors.* There is no concrete evidence that any factor is substantially greater in importance than another; assuming that each factor is weighted equally allows for a more quantitatively-determined ranking system.

3. *Fully charging a battery is charging the truck 80% of its original battery (for example, a truck that will be fully charged can be charged from 20% to 100%).* Evidence suggests that optimally a battery's charge should never go below 20% to ensure maximum efficiency [48]. Also, in our assumptions in Part 2, we assumed that drivers will try to refuel once their fuel hits 20%.

4. *The states that encompass the corridor will help pay for another state's lack of funding for the development of charging stations if needed.* We assume states will have a natural disposition to support other states if they lack the funding in order to expedite the construction process so all parties can benefit.

3.2 Developing the Ranking

To develop the ranking system used for the five corridors, we considered four factors: Anticipated Usage, Greenhouse Gas Emissions, Community Motivation, and Cost-Budget Ratio. Each of these factors is ranked on a scale of 1-1000, with a higher value indicating that the corridor should be targeted for development the most.

3.2.1 Variables and Parameters

Distance (D). We define D as the distance, in miles, traveled by all trucks along a certain section in a corridor in a day. D can be calculated by the summation of the amount of distance traveled by all trucks along each partitioned strip in a corridor.[43] Thus, we have the equation

$$D = \sum_{i=1}^{n} d_i t_i$$

where n represents the amount of partitioned strips along the corridor, d_i represents the distance in miles in the ith partition, and t_i is the amount of trucks in the ith partition. The compiled data for the total distance traveled by all vehicles in a day is documented below.

Total Distance Traveled by Class 8 Vehicles in a Day (Table 3.2.1)	
Corridor	Distance Traveled (10^7 Miles)
San Antonio - New Orleans	7.01
Los Angeles - San Francisco	3.67
Jacksonville - Washington DC	6.28
Minneapolis - Chicago	4.067
Boston - Harrisburg	4.453

Anticipated Usage (U). We define anticipated usage as the expected amount of charges that occur in a given day. It is important to take into account the amount of charges that occurred, because the outflow of money from the truck corporations using these charging stations becomes an inflow of money towards the local governments running the stations. This revenue can then be used in other areas (i.e., infrastructure, education, etc.) within the community. Thus, a corridor with a greater number of charges occurred and revenue generated would receive greater benefits to its community, meaning that it would be more useful to develop that corridor first.

To calculate how many charges occur, we can first divide the total distance traveled daily, D, by the amount of distance traveled per full charge, which happens to be 200 miles per full charge [43], to obtain the total number of full charges required. Thus, $\frac{D}{200}$ yields our Anticipated Usage index for a corridor.

Greenhouse Gas Emissions (E). One of the benefits to electric trucking is that it emits no greenhouse gases [45]. With greenhouse gases being the primary factor for climate change, it is imperative to reduce greenhouse gas emissions as a way to combat climate change [46]. Consequently, if the implementation of electric trucking in a corridor greatly reduces the amount of greenhouse gas emissions relative to its current emission level, it is more useful to develop in that corridor first.

The percent change in greenhouse gas emissions due to electric trucking can be determined through the equation

$$GHG_{\text{change}} = \frac{GHG_{\text{trucks}}}{GHG_{\text{i}}},$$

where GHG_{change} is the absolute value of the percent change in greenhouse gases, GHG_{trucks} is the amount of greenhouse gases released by the trucks along the corridor, and GHG_{i} is the total amount of greenhouse gas emissions in a state surrounding a corridor.

The expression $\frac{D}{\text{mileage}}$, where mileage $= 7.5$ and represents the average number of miles per gallon for a diesel truck, calculates the amount of gallons of diesel used among all trucks along a corridor [47]. Thus $GHG_{\text{trucks}} = GHG_{\text{gallon}} \cdot \frac{D}{\text{mileage}}$, where $GHG_{\text{gallon}} = .0101$ represents the amount of greenhouse gas emissions for each gallon of diesel burned, would yield the total amount of greenhouse gas emissions in metric tons for all trucks [42]. Then the equation for GHG_{change} can be rewritten as

$$GHG_{\text{change}} = 0.00134\frac{D}{GHG_i}.$$

Community Motivation (M). To determine community motivation, we used several different variables.

S_i represents the percent of chargers in a state that surrounds a corridor. A community as a whole will be more motivated to do a project if they have a big role in the project. Also, the more of the infrastructure there is in a state, the more revenue the local government can attain from it. Therefore, we concluded that the greater the percent of chargers/infrastructure in the state, the more invested the community will be in the project.

C_i represents how much a state cares about the environment. This is is a component of community motivation as it is a way to show how much the local government does already to combat climate change. This variable is calculated through the formula

$$C_i = \frac{B_i}{G_i}.$$

B_i represents the budget each state allots to combat climate change and protect the environment. This is used to determine how much a community cares about the climate as it is the sheer amount of money they spend on the environment [25][26][27][28][29][30][31][32][33][34][35][36][37][38][39].

G_i represents the state's GDP. We use this value to calculate how much a community cares as it takes into account the relative wealth of the states [40].

Thus, we divide B_i by G_i because it gives us an idea of how much of a state's wealth the local government is willing to spend on improving the environment. This gives us a much more accurate and quantitative value for determining how much a community cares.

Finally, to determine the value M on each of the routes, we calculate the sum of the products of C_i and S_i for each of the states on the route:

$$M = \sum_{i=1}^{n} C_i S_i,$$

where n is the number of states the corridor passes through. Computing yields the following values for different states.

Community Motivation Values (Table 3.2.2)			
States	B_i (Million \$)	G_i (Billions \$)	S_i (Percent of chargers in state)
TX	854	1896	58.3
LA	134	264	41.7
CA	12400	3155	100
FL	1535	1100	1.9
GA	30	620	22.9
SC	647	248	22.04
NC	90	590	22.2
VA	733	557	30.8
MN	324	383	6
WI	261	349	72
IL	550	901	22
MA	61.5	599	13
CT	17	287	31
NY	5388	1740	13
PA	550	817	42

Cost-Budget Ratio (R) The implementation of a large-scale infrastructure project can be expensive; however, to understand the potential economic impact of this development, the cost must be interpreted relative to each state's financial status. Having a low Cost-Budget Ratio would indicate that the corridor should be targeted for development because the economic impact of the implementation would be low. Thus, we defined R_i to be $\frac{\text{cost}_i}{B_i}$, where cost_i is the cost in dollars of building chargers in each state and B_i is each state's budget, as defined in Community Motivation. Cost can be calculated through $cost_i = chargers \cdot \$51000$, where chargers is the number of chargers in each state and \$51000 is the cost to build 1 charger [41].

To calculate the Cost-Budget Ratio score for each corridor, R_c, we used the following equation:

$$R_c = 1000 - \frac{1000}{n} \sum_{i=1}^{n} \frac{\text{chargers} \times 51000}{B_i},$$

where n is the number of states for each corridor. We summed the Cost-Budget Ratios of all states in a corridor and then averaged the value to find the average Cost-Budget Ratio among the corridor. In order to standardize the score from 1 to 1000, we multiplied the average Cost-Budget Ratio by 1000 and subtracted it from 1000. These operations maintain the idea that a low Cost-Budget Ratio would yield a higher score overall in determining the corridor targeted for development. We were able to determine the number of chargers required to be built in each state based off our model from Part II.

3.3 Compiling the Factors and Results

We scaled all the factors to a range of 0 to 1000 to ensure each factor is equally weighted. We are able to come up with the following data tables for each of the 4 factors.

Overall Combined Factors for Each Corridor (Table 3.2.3)				
States	U_c	E_c	M_c	R_c
San Antonio - New Orleans	1000	807	120	102
Minneapolis - Chicago	92	764	153	618
Boston - Harrisburg	212	734	187	995
Jacksonville - Washington DC	771	828	267	1000
Los Angeles - San Francisco	100	1000	1000	159

Summing up each of these factors yields our determined importance for each different corridor based on environmental factors, economic opportunities, community motivation, and existing budgets. From this we can determine the rankings of importance, which are as follows:

1. Jacksonville, FL - Washington, DC
2. Los Angeles, CA - San Francisco, CA
3. Boston, MA - Harrisburg, PA
4. San Antonio, TX - New Orleans, LA
5. Minneapolis, MN - Chicago, IL

3.4 Strengths and Weaknesses

Our model takes into account various factors that state and government legislatures will consider when determining whether to implement new infrastructure or not. Our model was able to quantify factors like Community Motivation in order to take into account various pivotal factors. Another strength of our model is that it breaks down factors by state boundaries rather than just corridors. This allows us to analyze the influence of state budgets as well as account for the differing values of communities, to better predict the public response to constructing such electric vehicle corridors.

A weakness of this model is assuming that all the factors will be equally weighted. In practice, Community Motivation, while important, will never have the same impact as the cost of infrastructure. In fact, cost will be by far the most heavily weighted of the factors; however, without a way to quantify how to weight different factors, doing so would result in inaccurate results. Despite this, our model still provides a comprehensive approach to prioritizing which corridors to be built first.

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5 Code Appendix in Matlab

```
function dydt = osc2(t,y)
    dydt = zeros(2,1);
    dydt(2) = (12.5*exp(-0.9163*t)+0.0450)*(y(2) - 0.0000004208*y(2)^2-
        0.0000004208/(5.491/(1+2.32*exp(-0.2509*t)))*y(1)*y(2));
    dydt(1) = (0.0450*exp(-0.07332*t))*(y(1) - 0.0000004208*y(1)^2 -
        0.0000004208*(5.491/(1+2.32*exp(-0.2509*t)))*y(1)*y(2));
end
```

```
function [T,Y] = call_osc2()
    tspan = [0 20];
    y1_0 = 1920000;
    y2_0 = 5220;
    [T,Y] = ode15s(@osc2,tspan,[y1_0 y2_0]);
    plot(T,Y(:,1),'o')

end
```

```
function [T,Y] = call_osc3()
    tspan = [0 20];
    y1_0 = 1920000;
    y2_0 = 5220;
    [T,Y] = ode15s(@osc2,tspan,[y1_0 y2_0]);
    %plot(T,Y(:,2),'o')
    %hold on;
    %plot(T,Y(:,1),'o')
    %hold off;
    X = Y(:,2)./(Y(:,1)+Y(:,2));
    plot(T,X,'o')
    hold on
    Z = Y(:,1)./(Y(:,1)+Y(:,2));
    plot(T,Z,'o')
    hold off
    %legend({'Electric Semi-Trucks','Diesel Semi-Trucks'},'FontSize',12)
end
```

```
function dydt = longHaul(t,y)
    dydt = zeros(2,1);
    dydt(2) = (12.5*exp(-0.9163*t)+0.0450)*(y(2) - 0.0000005208*y(2)^2-
        0.0000005208/(5.554/(1+2.32*exp(-0.2514*t)))*y(1)*y(2));
    dydt(1) = (0.0450*exp(-0.07332*t))*(y(1) - 0.0000005208*y(1)^2 -
        0.0000005208*(5.554/(1+2.32*exp(-0.2514*t)))*y(1)*y(2));
end
```

```
function [T,Y] = call_osc4()
    tspan = [0 20];
    y1_0 = 1760000;
    y2_0 = 4780;
    [T,Y] = ode15s(@longHaul,tspan,[y1_0 y2_0]);
    plot(T,Y(:,1),'o')

end
```

```
function [T,Y] = call_osc5()
    tspan = [0 20];
    y1_0 = 1760000;
    y2_0 = 4780;
    [T,Y] = ode15s(@longHaul,tspan,[y1_0 y2_0]);
    plot(T,Y(:,2),'o')
    hold on;
    plot(T,Y(:,1),'o')
    hold off;
    %X = Y(:,2)./(Y(:,1)+Y(:,2));
    %Z = Y(:,1)./(Y(:,1)+Y(:,2));
    %plot(T,X,'o')
    %hold on
    %plot(T,Z,'o')
    %hold off
    legend({'Electric Semi-Trucks','Diesel Semi-Trucks'},'FontSize',12)
end
```