MathWorks Math Modeling Challenge 2019

Nicolet High School –
Team # 12831 Glendale, Wisconsin
Coach: Mike Weidner
Students: Zach Godkin, Gabe Guralnick, Savir Maskara, Ryan Mortonson

MathWorks Math Modeling Challenge Third Place
$10,000 Team Prize

***Note: This cover sheet has been added by SIAM to identify the winning team after judging was completed. Any identifying information other than team # on a MathWorks Math Modeling Challenge submission is a rules violation.

***Note: This paper underwent a light edit by SIAM staff prior to posting.
Executive Summary

After a hard-fought public victory over smoking, the specter of childhood nicotine usage has once again seized the hearts and minds of teens across the world. It is estimated that as many as 37% of 12th graders have tried vaping in the last year. Though many high schoolers perceive it as harmless, vaping has been linked to cancer and respiratory disease. Even worse, teens who vape are far more likely to move on to cigarettes than their peers, inviting a lifetime of coronary heart disease and lung cancer. In order to curb this dangerous trend, it is vital to understand how vaping will spread, what influences the adoption of vaping and other illicit drugs, and how it negatively impacts one’s life.

To determine how e-cigarette use will spread among teens, we first examined the spread of cigarette usage, a historically comparable example. Using trends in cigarette usage as a basis for our analysis, we created a Gaussian bell curve which predicts the percentage of high school students that will use e-cigarettes in any given year. Using our model for the percentage of high school students who vape, we introduced such factors as the number of high school students in a given year and how likely quitting is. Summing the next 10 years, our model predicts that there will be 12,012,432 new users of e-cigarettes between 2019 and 2029. We also used our model to predict how many teens will begin smoking cigarettes as a result of e-cigarette usage. We then applied the knowledge that a teen who vapes is 4.78 times more likely to smoke than a teen who doesn’t vape. Using our previous summation and ignoring teens who will try cigarettes regardless of vaping, our model showed that vaping will create 3,531,655 new cigarette users in the next decade.

To create a system for predicting whether a given person will use a given drug, we created a Java program that takes in information related to various factors that play a significant role in propensity for drug use. These factors include gender, age, race, academic performance, socioeconomic status, and social influence. We created a system of multipliers to judge how likely a person is to use a given drug based on their expression of different variations of each factor. The multipliers were found by comparing the percentage of people who display the desired variation to the percentage who display a “control” variation. We then created a Java program that generated a random high school class of 300 seniors and analyzed the rates of drug adoption. In the average 300-student high school class, with an average distribution of propensity factors, we found that 199 students use nicotine, 16 use marijuana, 31 use alcohol, and 8 use opioids.

To create a robust, singular, and objective metric for the impact of substance use, we measured impact in time. Time represents the amount of time in hours that is lost directly or indirectly due to one lifetime of drug use. To convert monetary losses to time, for example, we assumed a median hourly wage and divided the monetary loss by that median salary to find the hours lost. We used a similar method to evaluate other factors, including drug cost, time spent under the influence, health costs, shortened lifespan, and legal penalties. Once all factors were totaled we found the loss in time for a lifetime of drugs use. Least to greatest: marijuana, 13,134 hours; alcohol, 66,129 hours; nicotine, 203,788 hours; and opioids, 387,895 hours.
Global Assumptions

G.1: All analysis of drug spread, usage, and effects are based on data pertaining to the United States.

Global Definitions

G.1: “Vaping,” “e-cigarette usage,” “vapes”, and “e-cigarettes” will be used interchangeably to refer to smokeless tobacco products which use vaporized nicotine as the ingestion method.

G.2: “Teen” refers to students enrolled in high school (grades 9–12).

1: Darth Vapor

1.1: Restatement

We are asked to create a model to predict how nicotine use due to vaping will spread among high school students over the next 10 years and must compare the predicted spread of vaping to the spread of cigarette usage over a similar timespan.

1.2: Local Assumptions

1) Counts for the number of cigarette and e-cigarette users include only members of the population who are in high school for any amount of time during the 10-year period. 
_Justification:_ Vaping is of most concern for teens since it affects their still-developing brains\(^{[16]}\). Additionally, the problem specifies “a new generation.”

2) The progression curve of vaping use among high schoolers will resemble the progression curve of the usage of cigarettes\(^{[3]}\) (a Gaussian bell curve). 
_Justification:_ In both substances, the addictive chemical is nicotine, which has similar effects on humans regardless of the ingestion medium.

3) 2011 was the meaningful start of the vaping/e-cigarette usage among teens. 
_Justification:_ Data that we have found indicate that in 2011 about 2% of high schoolers reported having vaped. As much of our data only go back to this time, we have decided to count it as the start for the purposes of comparison to cigarette usage.

4) The problem statement asks us to “Analyze how the growth of this new form of nicotine use compares to that of cigarettes.” We interpret “that of cigarettes” to refer to the growth pattern of cigarettes since 1900.

5) The number of high school students is constant for the next 10 years. While the number of high school graduates is expected to fluctuate, it doesn’t significantly increase or decrease\(^{[2]}\). 
_Justification:_ In 2011, the public school student population was 14.789 million. In 2021, this population is only expected to grow to 15.431 million\(^{[38]}\). Private schools will see
even less of an increase in population. We have decided that this fluctuation is not significant enough to influence our data.

6) The number of high school seniors who graduate per year is constant at a value of 3,478,027.

*Justification:* In 2013, there were 3,478,027 high school students. Given that the student population is remaining constant per assumption 5, we do not foresee a significant increase in this number over a 10-year period.

7) The percentage of e-cigarette users who will succeed quitting vaping will be similar to that of cigarette users attempting to quit (5%)\(^1\).

*Justification:* Because vaping is a newer phenomenon, there isn’t a satisfactory amount of data to predict this percentage. The addictive ingredient, nicotine, however, is the same regardless of consumption method, so difficulty quitting will likely be similar to cigarettes\(^1\).

7) Student populations are equally distributed among grades 9, 10, 11, and 12.

### 1.3: Variables

#### 1.3.1: Variables for Finding the Number of New People Who Will Be Vaping in the Next 10 Years that Started in High School

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>t</td>
<td>Time in years since 2010</td>
<td>years</td>
</tr>
<tr>
<td>V(t)</td>
<td>Percentage of high school students who vape with respect to time</td>
<td>%</td>
</tr>
<tr>
<td>N(t)</td>
<td>Number of new high school students who vape with respect to time</td>
<td>#</td>
</tr>
<tr>
<td>G</td>
<td>(constant) Number of high school graduates each per year = 3.478 million(^{[37]})</td>
<td>#</td>
</tr>
<tr>
<td>Q</td>
<td>(constant) Percentage of people who will not quit vaping = 95%(^{[1]})</td>
<td>%</td>
</tr>
</tbody>
</table>

#### 1.3.2: Additional Variables for Finding Resultant Smokers Due to the Spread of Vaping

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>R(_S)</td>
<td>Risk level of smoking for a teen who hasn’t vaped (% that will smoke)</td>
<td>%</td>
</tr>
<tr>
<td>R(_SV)</td>
<td>Risk level of smoking if someone has vaped</td>
<td>%</td>
</tr>
<tr>
<td>S(_G(t))</td>
<td>Predicted number of new smokers as a result of vaping with respect to time</td>
<td>#</td>
</tr>
</tbody>
</table>
1.4: Solutions and Results

1.4.1: Finding the Number of New e-cigarette Users over the Next 10 Years Due to Exposure to Vaping in High School

Since the problem asks about the spread of nicotine use among teens, we decided to create a model that predicts the increase in nicotine users using the number of graduating high schoolers who use e-cigarettes, therefore excluding new nicotine users that start as adults.

We used the following data points derived from sources [4] and [5]:

<table>
<thead>
<tr>
<th>Year (where 2010 = 0)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Percentage High Schoolers that Vape</td>
<td>1.5</td>
<td>2.8</td>
<td>4.5</td>
<td>13.4</td>
<td>15.3</td>
<td>12</td>
<td>14.6</td>
<td>24.3</td>
</tr>
</tbody>
</table>

The following Gaussian regression resulted: \( V(t) = 31.9e^{-(t-12.65)/7.598^2}, \ r^2 = .83. \)

The previous function & data are represented graphically as follows:

Our prediction matches the historical rise and fall in cigarette usage\(^{[14]} \) in shape; however, the period of time is smaller by a factor of about 4.8.

In order to determine the number of high schoolers who will graduate each year and who vape, we multiplied \( V(t) \) by \( G \), which equals 3,500,000.
We then multiplied that number by 0.95 since 5% of e-cigarette users will be able to quit vaping\cite{1}, so the remaining 95% (Q) are counted for the 10-year total.

The resultant model for the number of new e-cigarette users in a given year who will continue to use nicotine is

\[ N(t) = \frac{V(t)}{100} \times G \times Q = \frac{31.9e^{(-((t-12.65)/7.598)^2)}/100 \times 3,500,000 \times 0.95}{\{V(t) is divided by 100 to convert the percent to a decimal\}} \]

To find the total number of new nicotine, based on our model for assumed growth in users, we integrated the above equation, giving us

\[ \int (\frac{V(t)}{100} \times G \times Q \times dt) \]

\[ \int (\frac{31.9e^{(-((t-12.65)/7.598)^2)}/100 \times 3,500,000 \times 0.95 \times dt}{\{\text{The indefinite integral shows the growth in the total number of new e-cigarette users who were exposed to vapes in high school, represented graphically in the window } y[0, 12,500,000] \text{ and } t[0, 27] \text{ as}\}}) \]

To find the total new nicotine users that started in their teens, we then added \(3 \times V(t) \times G\) to account for 3 grade levels of nicotine users in year 10 of our model (9, 10, 11) that aren’t accounted for by the model since they do not graduate during the trial period. We also set the limits at 9 and 19 since we start measuring in 2019 and continue until 2029.

\[ 3 \times V(10) \times G + \int_9^{19} (\frac{V(t)}{100} \times G(t) \times Q \times dt) \]

\[ 3 \times 0.283 \times 3,500,000 + \int_9^{19} (\frac{31.9e^{(-((t-12.65)/7.598)^2)}/100 \times 3,500,000 \times 0.95 \times dt}{\{\text{Based on our model, the expected number of new e-cigarette users who start in their teens over the next 10 years is equal to}\}}) \]

12,012,432 users

1.4.2: Finding the Number of New Smokers as a Result of the Spread in Vaping

We discovered through our research that students who vape are more likely (4.78 times more) to become cigarette smokers\cite{15}, meaning \(R_{SV} = 4.78 \times R_S\). We created a model, therefore, to predict the number of new cigarette users, resultant from the spread of vaping.

\[ R_S = 9.8\% \cite{5} \]

\[ R_{SV} = 4.78 \times R_S = 39.2\% \]

If a teen vapes, they have a 39.2\% chance of using cigarettes. The number of e-cigarette users can therefore be multiplied by 0.392 to predict how many teens will use cigarettes. We subtract
9.8% of that answer to remove teens that would have smoked anyway to find how many teens smoke due the spread of vaping.

\[ N(t) \times R_{SV} - N(t) \times R_S \]

\{N(t) is defined in part 1.4.1\}

The total number of new smokers who smoke only due to vaping is found through integration:

\[ \int N(t) \times R_{SV} - N(t) \times R_S \times dt \]

\[ S_G(t) = (R_{SV} - R_S) \int N(t) \times dt \]

Extrapolating the model from 2019 to 2029, we get:

\[ 0.294 \times (3 \times 0.283 \times 3,500,000 + \int_0^{19} (31.9e^{-(t-12.65)/7.598})/100 \times 3,500,000 \times 0.95 \times dt) \]

\[ S_G(t) = 3,531,655 \text{ new cigarette users due only to the spread of vaping} \]

1.5: Validation

Because we cannot know the future, we validate our model through comparison to past models, how strongly our predictions correlate to current data, and if our predictions make sense in the context of society.

As already demonstrated, our prediction model for e-cigarette usage matches the shape of cigarette usage (a Gaussian bell curve). This is to be expected since both employ the same addictive ingredient, so their growth should be similar. Our data differs in that it takes about \( \frac{1}{5} \) of the time to rise and fall when compared to rates of cigarette usage. At first the fact that it happens in a much shorter time frame seems to invalidate our prediction, but given that efforts are already being made to reduce vaping and that technology allows a much faster dispersion of information, it makes sense that success in curbing vaping would happen in a much shorter time frame.

Our model also predicts that e-cigarette usage will peak at around 32%. This matches closely with historic cigarette usage, which peaked at 29% of high school students 45 years ago\(^{21}\). The data points validate our model, too. The \( r^2 \) number was .83. While imperfect, we are very happy with this correlation coefficient given how few data points are available to us and that vaping is a relatively new phenomenon.

The logarithmic trend for total number of new e-cigarette users also makes sense because the rate of adoption will decrease over time just like it did for cigarettes\(^{14}\), but users will find it hard to quit\(^{1}\). This pattern will result in the population of e-cigarette users increasing at a decreasing rate over time.

1.6: Strengths & Weaknesses

1.6.1: Strengths

- Our model focuses on adoption of e-cigarettes by high school students, who are the population most at risk to permanently change their neurochemistry and become addicted\(^{16}\).
- Our model also accounts for the increase in smokers of standard cigarettes due to vaping, which poses additional health hazards, including increased blood pressure, coronary heart disease, stroke, and lung cancer.
- Our model takes into account the graduation of high school students and the resultant buildup in the general population that uses e-cigarettes. The model also considers that a certain number of users will be able to quit vaping.
- Our model accounts for the (hopefully) likely decline in e-cigarette usage due to prevention efforts.

1.6.2: Weaknesses
- We had to work with a very low sample size since vaping only affected 2% of teens as recently as 2010. We had only 8 years of data to work with, so our model will likely be proven somewhat inaccurate.
- Our model doesn’t account for the adoption of e-cigarettes among adults or other nicotine consumption methods beyond vaping and smoking. The brain continues developing until at least age 25, so we miss a vulnerable constituent of the population.
- Our Gaussian regression assumes that the rate of adoption of vaping will cease as quickly as it exploded. The data that represent usage of cigarettes, on the other hand, suggest that the actual model should skew left.

Part 2: Above or Under the Influence?

2.1: Restatement

We are asked to create a model that predicts the likelihood that a given person will use a given substance, with the substances being nicotine, marijuana, alcohol, and un prescribed opioids. This likelihood is based on social influence, characteristic traits (of the user), and characteristics of the drug itself. We must demonstrate this model by predicting the number (out of a class of 300) of high school seniors who will use these substances.

2.2: Local Definitions

1) **Socioeconomic status**: A measure of an individual’s economic and social position in relation to others based on wealth, income, and parental education level.
2) **Family Income per Person**: Total family income in USD divided by number of persons in the family.
3) **Social Influence**: The total number of peers who use any sort of drug.
2.2: Local Assumptions

1) The effect of a factor in determining the likelihood for drug use is independent of other possible factors (e.g., the effect of a high schooler’s household income level on drug use does not change due to their gender).

2) We have interpreted “social influence” in the problem statement to refer to the number of peers that a given person knows who use drugs.

3) The factors that influence a person’s propensity to use drugs are genetics, socioeconomic status, social (see 2) influence, school performance (average letter grades), gender, age, and race.

   Justification: The factors defined here which were not included in the problem statement come from a study by the CDC[6]. We decided to limit the scope of our analysis to high school students, as that is what our end test will be on.

4) We have interpreted “nicotine” in the problem statement to refer to both traditional smoking methods and modern vaporizers. We have used whichever data are available for each factor.

   Justification: Sources used in part 1 indicate that cigarette use in the United States is going down and that e-cigarette use is much more prevalent. Both are still present, though, so we have included both in our model as was possible.

5) Gender and grades are distributed equally among a given population (e.g., 50% male, 50% female; 25% for A/B/C/D).

6) For all other factors, each person (when “created”) has a random chance of presenting one of the factors (e.g., 25% of being each race, calculated on a per-person basis).

   Justification: Determining chances of various factors being present in a population of 300 high school seniors would require complex and difficult variables that we decided not to include in our study due to time constraints.

2.3: Variables

2.3.1: Actual Variables

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_x$</td>
<td>The multiplier of a person’s likelihood to use a given drug</td>
<td>-</td>
</tr>
<tr>
<td>$%_{\text{desired}}$</td>
<td>The percentage of people exhibiting the desired variation of a factor</td>
<td>%</td>
</tr>
<tr>
<td>$%_{\text{control}}$</td>
<td>The percentage of people exhibiting the base variation of a factor</td>
<td>%</td>
</tr>
<tr>
<td>$P_{\text{drug}}$</td>
<td>The probability of a given person to use a given drug</td>
<td>-</td>
</tr>
</tbody>
</table>
2.3.2: Multipliers

In order to accurately represent the effects of factors independent from one another, levels are measured in comparison as a multiplier compared to a chosen standard (per factor). In the table below, the multiplier of the chosen standard level is assigned a value of 1 (e.g., with gender, a male is 1.763 times more likely to have used drugs than a female). These values are extrapolated from various sources.

<table>
<thead>
<tr>
<th>Factor</th>
<th>Level</th>
<th>Multiplier (Nicotine)</th>
<th>Multiplier (Marijuana)</th>
<th>Multiplier (Alcohol)</th>
<th>Multiplier (Opioids)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(F_g): Gender</td>
<td>Female</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Male</td>
<td>1.763(^{[6]})</td>
<td>1.03(^{[11]})</td>
<td>0.961(^{[7]})</td>
<td>1.010(^{[11]})</td>
</tr>
<tr>
<td>(F_r): Race</td>
<td>White</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>African American</td>
<td>0.750(^{[6]})</td>
<td>0.655(^{[11]})</td>
<td>0.88(^{[9]})</td>
<td>0.677(^{[11]})</td>
</tr>
<tr>
<td></td>
<td>Hispanic</td>
<td>1.132(^{[6]})</td>
<td>1.073(^{[11]})</td>
<td>1.02(^{[9]})</td>
<td>1.108(^{[11]})</td>
</tr>
<tr>
<td></td>
<td>Other</td>
<td>0.984(^{[6]})</td>
<td>0.644(^{[11]})</td>
<td>0.991(^{[9]})</td>
<td>0.634(^{[11]})</td>
</tr>
<tr>
<td>(F_a): Age</td>
<td>9th Grade*</td>
<td>0.527</td>
<td>0.502</td>
<td>0.561</td>
<td>0.718</td>
</tr>
<tr>
<td></td>
<td>10th Grade</td>
<td>0.672</td>
<td>0.752</td>
<td>0.722</td>
<td>0.750</td>
</tr>
<tr>
<td></td>
<td>11th Grade*</td>
<td>0.836</td>
<td>0.876</td>
<td>0.861</td>
<td>0.875</td>
</tr>
<tr>
<td></td>
<td>12th Grade</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>(F_s): School Performance</td>
<td>A</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>1.484(^{[12]})</td>
<td>1.762(^{[10]})</td>
<td>1.175(^{[10]})</td>
<td>1(^{[10]})*</td>
</tr>
<tr>
<td></td>
<td>C</td>
<td>1.935</td>
<td>2.381</td>
<td>1.270</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>D/F</td>
<td>2.387</td>
<td>3.143</td>
<td>1.381</td>
<td>5</td>
</tr>
</tbody>
</table>
### F<sub>E</sub>: Parental Level of Education

<table>
<thead>
<tr>
<th>High School</th>
<th>1</th>
<th>1</th>
<th>1</th>
<th>1</th>
<th>1</th>
</tr>
</thead>
</table>

### F<sub>I</sub>: Family Income Per Person

<table>
<thead>
<tr>
<th>Income Range</th>
<th>1</th>
<th>1</th>
<th>1</th>
<th>1</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>≤ $4,872</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

### F<sub>P</sub>: Peer Influence

<table>
<thead>
<tr>
<th>Influence</th>
<th>1</th>
<th>1</th>
<th>1</th>
<th>1</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Less than 4 Alcohol- or Drug-using friends</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>4 or more AOD using friends</td>
<td>1.686&lt;sup&gt;[17]&lt;/sup&gt;</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* the only data found provided values for heroin but not all opioids
** no data found for these values

### 2.4: Solution

For this task, we needed to create a model that would determine whether a given person would have used a given drug and then use this model on a sample population of 300 high school seniors. We decided that the first necessary step for this task would be to find a quantitative system to judge the likelihood of a given person to use one of four drugs. We limited our research and model to the factors outlined above.

We made use of a system of multipliers in order to calculate the amount that each variation of a given factor influences the likelihood of a person having used a drug. These numbers occur when all other factors are held constant. These multipliers were found using percentages of the United States teen population that make use of each drug when the factor is not present versus when it is:

\[ F_{factor} = \frac{\%_{desired\ multiplier}}{\%_{control\ multiplier}} \]
These percentages were found through research of various sources. $\%_{control}$ is generally just the variation present in the first row of our data table for each factor. For example, according to the CDC, 63% of “A” average students have drunk alcohol compared to the 80% of “C” students who have used the same substance\(^{10}\). This means that a C student is 1.27 times more likely to drink alcohol, so the $F_S$ value for C grades is 1.27.

Using this process, we calculated the multiplier values for a set of factors including gender, race, age, school performance, parental education, family income, and peer influence. We then decided that, as our data are represented in multipliers, the overall multiplier for a given person’s usage of any one drug can be represented as

$$F_{overall} = F_G * F_R * F_A * F_S * F_E * F_I * F_P$$

In this model, $F_{overall}$ represents the multiplier for a given drug.

We then moved to testing our system of multipliers on a sample population of 300 high school seniors. We decided to make use of technical computing in the form of a Java program to accomplish this task due to the necessity of a large number of calculations and the need to store a large amount of data. This would have been extremely difficult to compute without the use of looping systems in a program. The code (.java files) for this simulation is available in Appendix A.

Our program makes use of object-oriented concepts inherent in the Java language to split the task into a “Driver” class (ProbGenerator.java) and a “Person” class (Person.java). The Person class takes in a constructor which includes information relevant to the various factors outlined in 2.3 (e.g., gender). It then includes four methods, one for each drug we were tasked to analyze. Each method, through conditional statements, determines each $F$ multiplier for the person based on the provided information about the factors. It then determines $F_{overall}$ for the drug using the above formula. The Driver class, then, generates an Array of 300 Persons. We simplified the generation of a population of high school seniors through assumptions 5 and 6 outlined in sections 2.2. This generation was accomplished through the use of random values that were compared to probabilities that we defined for each of the factors. The Driver then constructs a two-dimensional Boolean array to store the drug use values for each Person (true if the Person has used the drug, and false if it hasn’t). In order to convert from a multiplier to a probability, we divided the national percentage of high schoolers who have used each drug by the drug’s multiplier for a given person:

$$P_{drug} = \frac{\%_{national \ (avg)}}{F_{overall}}$$

The following table outlines the national percentages used:

<table>
<thead>
<tr>
<th>Drug</th>
<th>Percentage of High Schoolers (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nicotine</td>
<td>42.5%(^{[5]})</td>
</tr>
<tr>
<td>Marijuana</td>
<td>43.6%(^{[5]})</td>
</tr>
<tr>
<td>Alcohol</td>
<td>58.5%(^{[5]})</td>
</tr>
</tbody>
</table>
The Driver then generates more random values. If the generated value (from 0–100) is less than \( P_{\text{drug}} \), the value for that drug for that person is set to \textbf{true}. We then iterate through the generated array, counting the occurrences of each drug being used. This information is then outputted.

### 2.5: Results

In 100 tests of 300-student high school senior classes, our model outputted the following data:

<table>
<thead>
<tr>
<th>Drug</th>
<th>Average Number of Students</th>
<th>Percentage of Students (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nicotine</td>
<td>199</td>
<td>66.33</td>
</tr>
<tr>
<td>Marijuana</td>
<td>16</td>
<td>5.33</td>
</tr>
<tr>
<td>Alcohol</td>
<td>31</td>
<td>10.33</td>
</tr>
<tr>
<td>Opioids</td>
<td>8</td>
<td>2.67</td>
</tr>
</tbody>
</table>

### 2.6: Validation

When compared to the national average percentages for high school teens’ use of each of the given drugs, it is clear that our model is not accurate. The percent error values for each drug are shown below:

<table>
<thead>
<tr>
<th>Drug</th>
<th>Percent Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nicotine</td>
<td>56.07</td>
</tr>
<tr>
<td>Marijuana</td>
<td>-87.78</td>
</tr>
<tr>
<td>Alcohol</td>
<td>-82.34</td>
</tr>
<tr>
<td>Opioids</td>
<td>-25.83</td>
</tr>
</tbody>
</table>

These data do not indicate that our model is ineffective, however. In fact, the data generated are very consistent with the constraints placed on our model by assumptions 2.2.5 and 2.2.6. For example, family incomes per person of less than $4,872 per year are associated with an \( F_i \) value of 1 for all drugs. For nicotine specifically, this multiplier is much greater than those of the other income groups present in the data. In the real world, however, this lowest income group makes up much less of the average student population, while our model creates a mostly
even distribution of income groups among the students generated. This results in the inflated data seen above and the large percent error values calculated. Creation of a more accurate model would require extensive research into the demographic makeup of an average school and would require much more generalization on our part to represent the entire United States, as this makeup can vary greatly due to location.

With these constraints in mind, the data generated do validate the capabilities and accuracy of our model. Given even distributions of the various factors, our model correctly identifies the fact that nicotine use would be present among the largest number of students. Our model also accurately shows that alcohol use is more common than marijuana use and that marijuana use is then more common than opioid use. These data are consistent, though with much smaller values, with the researched national data.

2.7: Strengths & Weaknesses

2.7.1: Strengths
- Our model formats the generated data in an easily readable way.
- Our model accounts for the differing addictive nature of each of the given drugs through the calculation of a unique multiplier for each factor for each drug.
- Our model generates the average value for each drug in a sample of 100 schools of 300 seniors.
- Our model could easily be modified to more accurately generate the student population.

2.7.2: Weaknesses
- Our model fails to account for the fact that some variations of each factor are more likely than others.
- Our model does not allow for any change over time - this issue is further compounded by the fact that the data gathered for multipliers came from different sources from different time periods.
- Our model generalizes the percentage of high schoolers who use each drug due to a lack of specific information about the sample high schoolers.

Part 3: Ripples

3.1: Restatement

We are asked to create a model for the impact of substance use that includes both financial and non-financial factors. We must use this model to rank the impacts of alcohol, marijuana, nicotine, and opioids
3.2: Local Assumptions

1) Any one-time purchases related to drug use (e.g., a marijuana pipe or vape pen) will not be counted in overall cost.
2) Lifetime users of a substance are addicted to their substance.
3) There are 365 days in a year.

3.3: Variables

3.3.1: Constants:

$32500^{[19]}$ - median American income. Equates to roughly $16/hour
50 years - average American span of drug use for nicotine, marijuana, and alcohol
24 hours - average hospital stay for a drug user

3.3.2: Establishing the Hierarchies of the Effects of Drug Use

Not only does drug usage affect the user, but it has a significant impact on the people and relationships around the user. Because of this distinction, the effects of drug use have been separated into two categories: Primary Impacts and Tertiary Impacts. Primary Impacts are impacts that come at direct cost to someone, whether that someone is the user or a victim of the user’s actions. These can be impacts such as the fiscal cost of the drug, time lost while under the influence, or property damage. Tertiary Impacts are impacts that indirectly affect multiple people. These impacts can range from the lost productivity of a workplace after an addict is fired to the vast trauma surrounding a suicide.

3.3.3: All Considered Variables and Their Respective Categories

<table>
<thead>
<tr>
<th>Primary</th>
<th>Tertiary</th>
</tr>
</thead>
<tbody>
<tr>
<td>● Cost of the drug</td>
<td>● Emotional distress of an addicted family member committing suicide</td>
</tr>
<tr>
<td>● Time lost under the influence</td>
<td>● Lost productivity in the workplace</td>
</tr>
<tr>
<td>● Medical expenses due to drug use</td>
<td>● Stress of a family member</td>
</tr>
<tr>
<td>● Loss of employment</td>
<td>● Cost to taxpayers</td>
</tr>
<tr>
<td>● Emotional distress</td>
<td></td>
</tr>
<tr>
<td>● Legal penalties</td>
<td></td>
</tr>
<tr>
<td>● Property damage</td>
<td></td>
</tr>
<tr>
<td>● Domestic abuse</td>
<td></td>
</tr>
<tr>
<td>● Shortened lifespan</td>
<td></td>
</tr>
</tbody>
</table>

*Not all variables mentioned were included in the modeling due to time and data constraints.*
3.4: Solution & Results

3.4.1: Establishing a Time as a Common Unit for Comparison

Many important impacts of drug use are purely subjective, and therefore can have varying degrees of importance on a person’s life. Because of this, all measurements will be done in reference to averages. Also, due to the subjectivity of the impacts, all measurements will be converted into time. Time is the measurement of the hours that a victim has lost due to drug usage. Time loss can be literal, such as in the case of lost time while under the influence. However, time can also be indirectly lost through the opportunity cost or fiscal cost of taking drugs. With regards to the fiscal cost, the time lost will be calculated by taking the fraction of cost over the average American income per hour. For example, spending $20 on drugs equates to 1.25 hours using the average American income\(^{[19]}\):

\[
Adjusted \ Time \ Lost = \frac{Cost}{Average \ Income \ Per \ Hour} + \frac{Actual \ Time \ Lost}{Average \ Income \ Per \ Hour}
\]

3.4.2 Differentiating between Primary and Tertiary Impacts

A Primary Impact directly impacts one person, so the adjusted time lost is calculated in reference to that one victim. However, because Tertiary Impacts affect a large number of people, these impacts are divided among the people they affect. For example, the cost of addiction to the U.S. Government would be equally divided among all American taxpayers.

3.4.3 Standardizing Time Lost for All Impacts

Because some impacts of drug use are presented every day, while others are more rare, all impacts will be measured in reference to lifetimes. To quantify lifetime impact, the adjusted time lost will be multiplied by the occurrences of the impact over a lifetime of a drug user. For more common impacts, this number will be very large. However, for more rare impacts, like divorce, the number of occurrences per lifetime could be less than one. Given this, the final equation is

\[
Adjusted \ Time \ Lost \ per \ Lifetime = Adjusted \ Time \ Lost \ (per \ Impact) * Occurrences
\]

3.4.4 Recording Time Lost for Each Drug

<table>
<thead>
<tr>
<th>Impact</th>
<th>Time Lost in a Lifetime</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nicotine</td>
<td>115625 hours(^{[23]})</td>
</tr>
<tr>
<td>Marijuana</td>
<td>2459 hours(^{[24]}[27][28])</td>
</tr>
<tr>
<td>Alcohol</td>
<td>281 hours(^{[281]})</td>
</tr>
<tr>
<td>Opioids</td>
<td>168750 hours(^{[36]})</td>
</tr>
</tbody>
</table>

\[Cost \ of \ the \ Drug\]
### 3.5: Validation

Our model found that, in terms of hours lost, the negative impact metric of the four drugs we measured ordered opioids, nicotine, alcohol, marijuana (from most lost hours of productivity to fewest lost hours of productivity). Opioids are, in fact, the most harmful drug on an individual basis\(^42\) according to the Independent Scientific Committee on Drugs. Alcohol is overall the most harmful drug\(^42\), but due to its cost to others. Since our metric focused much more on personal expenses, we should compare our ranking to individual rankings of drug impacts. Considering individual impact, cigarettes are more harmful than alcohol, and marijuana causes the least individual harm\(^41\). Our model is validated, therefore, because its conclusions are supported by external, independent sources.

### 3.6: Strengths & Weaknesses

#### 3.6.1: Strengths
- Standardizes and objectifies different units of measurement without inventing a false or made-up reference point.
- Uses an easily understandable unit of measurement, as opposed to a convoluted or made-up index.
- Accurately scales up the importance of some impacts while accurately lessening the importance of others.

#### 3.6.2: Weaknesses
- Does not factor in abstract emotional costs such as stress or trauma.
- Due to time and data constraints, only factors in a limited number of impacts.
- The usage of multiple averages instead of more in-depth modeling leads to a more simplified representation.
- Does not factor in quality of life variables, nor the societal impact of a certain drug.
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Appendix A: Java Code

```java
public class Person {

    private String gender; // biological sex - male or female
    private String race; // white, african-american (black), hispanic, other
    private int schoolGrade; // school grade, from 9 to 12
    private String avgGrade; // academic grade, from A-F, though D and F are the same
    private String parentEd; // high school or college
    private int income; // family income per person
    private int friends; // number of friends who do drugs

    public Person(String gender, String race, int schoolGrade, String avgGrade, String parentEd, int income, int friends) {
        this.gender = gender;
        this.race = race;
        this.schoolGrade = schoolGrade;
        this.avgGrade = avgGrade;
        this.parentEd = parentEd;
        this.income = income;
        this.friends = friends;
    }

    // multiplies the multipliers of all factors that a person exhibits - values taken from 2.3
    public double getProbMarijuana() {
        double genderMult, raceMult, schoolGradeMult, schoolMult, parentMult, incomeMult, friendsMult;
        genderMult = raceMult = schoolGradeMult = schoolMult = parentMult = incomeMult = friendsMult = 1;

        if (gender.equalsIgnoreCase("Male")) genderMult = 1.03;
        if (race.equalsIgnoreCase("black")) raceMult = .655;
        else if (race.equalsIgnoreCase("hispanic")) raceMult = 1.073;
        else if (!race.equalsIgnoreCase("white")) raceMult = .644;

        if (schoolGrade == 10) schoolGradeMult = 1.498;
        else if (schoolGrade == 11) schoolGradeMult = 1.744;
        else if (schoolGrade == 12) schoolGradeMult = 1.991;
    }
}
```
if (avgGrade.equalsIgnoreCase("B")) schoolMult = 1.762;
else if (avgGrade.equalsIgnoreCase("C")) schoolMult = 2.381;
else if (!avgGrade.equalsIgnoreCase("A"))
schoolMult = 3.143;
if (parentEd.equalsIgnoreCase("college"))
parentMult = 1.265;
if (!income <= 4872) {
    if (income <= 18561) incomeMult = 1.22;
    else if (income <= 32250) incomeMult = 1.76;
    else incomeMult = 2.34;
}
if (friends > 4) friendsMult = 1.686;
return genderMult * raceMult * schoolGradeMult * 
schoolMult * parentMult * incomeMult * friendsMult;
}

public double getProbNicotine() {
    double genderMult, raceMult, schoolGradeMult,
    schoolMult, parentMult, incomeMult, friendsMult;
    genderMult = raceMult = schoolGradeMult =
schoolMult = parentMult = incomeMult = friendsMult = 1;
    if (gender.equalsIgnoreCase("Male")) genderMult = 1 .763;
    if (race.equalsIgnoreCase("black")) raceMult = .75;
    else if (race.equalsIgnoreCase("hispanic"))
        raceMult = 1.132;
    else if (!race.equalsIgnoreCase("white")) raceMult
        = .984;
    if (schoolGrade == 10) schoolGradeMult = 1.275;
    else if (schoolGrade == 11) schoolGradeMult = 1.586
    ;
    else if (schoolGrade == 12) schoolGradeMult = 1.896
    ;
    if (avgGrade.equalsIgnoreCase("B")) schoolMult = 1.
484;
    else if (avgGrade.equalsIgnoreCase("C")) schoolMult
    = 1.935;
    else if (!avgGrade.equalsIgnoreCase("A"))
schoolMult = 2.387;
if (parentEd.equalsIgnoreCase("college"))
    parentMult = .2;

if (!(income <= 4872)) {
    if (income <= 18561) incomeMult = .63;
    else if (income <= 32250) incomeMult = .43;
    else incomeMult = .26;
}

if (friends > 4) friendsMult = 1.686;

return genderMult * raceMult * schoolGradeMult *
    schoolMult * parentMult * incomeMult * friendsMult;

public double getProbAlcohol() {
    double genderMult, raceMult, schoolGradeMult,
        schoolMult, parentMult, incomeMult, friendsMult;
    genderMult = raceMult = schoolGradeMult =
        schoolMult = parentMult = incomeMult = friendsMult = 1;

    if (gender.equalsIgnoreCase("Male")) genderMult =
        .961;

    if (race.equalsIgnoreCase("black")) raceMult = .88
    ;
    else if (race.equalsIgnoreCase("hispanic"))
        raceMult = 1.02;
    else if (!race.equalsIgnoreCase("white")) raceMult
        = .991;

    if (schoolGrade == 10) schoolGradeMult = 1.285;
    else if (schoolGrade == 11) schoolGradeMult = 1.
        533;
    else if (schoolGrade == 12) schoolGradeMult = 1.
        781;

    if (avgGrade.equalsIgnoreCase("B")) schoolMult = 1
        .175;
    else if (avgGrade.equalsIgnoreCase("C"))
        schoolMult = 1.270;
    else if (!avgGrade.equalsIgnoreCase("A"))
        schoolMult = 1.381;

    if (parentEd.equalsIgnoreCase("college"))
        parentMult = 1.458;

    if (!(income <= 4872)) {
        if (income <= 18561) incomeMult = 1.18;
```java
import java.util.Random;

public class ProbGenerator {
    static Random r = new Random();
    public static void main(String[] args) {
        int avgNicotine, avgMarijuana, avgAlcohol,
            avgOpioid;
        avgNicotine = avgMarijuana = avgAlcohol = avgOpioid = 0;

        for (int k = 0; k < 100; k++) {
            Person[] testPeople = generatePop(300);

            boolean[][] probTest = sampleTest(testPeople);

            int nicotineCount, marijuanaCount, alcoholCount,
                opioidCount;
            nicotineCount = marijuanaCount = alcoholCount =
                opioidCount = 0;

            for (int i = 0; i < 4; i++) {
                for (int j = 0; j < 300; j++) {
                    if (i == 0) {
                        if (probTest[0][j]) nicotineCount++;
                    } else if (i == 1) {
                        if (probTest[1][j]) marijuanaCount++;
                    } else if (i == 2) {
                        if (probTest[2][j]) alcoholCount++;
                    } else {
                        if (probTest[3][j]) opioidCount++;
                    }
                }
            }
        }
    
        avgNicotine += nicotineCount;
        avgMarijuana += marijuanaCount;
        avgAlcohol += alcoholCount;
        avgOpioid += opioidCount;
    }

    avgNicotine = Math.round(avgNicotine / 100);
    avgAlcohol = Math.round(avgAlcohol / 100);
    avgMarijuana = Math.round(avgMarijuana / 100);
    avgOpioid = Math.round(avgOpioid / 100);

    System.out.println("In " + 100 + " tests:\nSample
Size: " + 300 + "\nNicotine: " + avgNicotine + "\nAlcohol: " + avgAlcohol + "\nMarijuana: " + avgMarijuana + "\nOpioid: " + avgOpioid + "\n");
```
44    "\nMarijuana: " + avgMarijuana +
45    "\nAlcohol: " + avgAlcohol +
46    "\nOpioid: " + avgOpioid);
47
48    // generate array of booleans for each person of
49    // whether they have used various drugs based on factors
50    // outlined in 2.3
51    public static boolean[][] sampleTest(Person[] people) {
52        boolean[][] results = new boolean[4][300];
53        for (int i = 0; i < people.length; i++) {
54            int probChecker = r.nextInt(101);
55            if (probChecker <= (42.5 / people[i].
56                getProbNicotine())) results[0][i] = true;
57            if (probChecker <= (43.6 / people[i].
58                getProbMarijuana())) results[1][i] = true;
59            if (probChecker <= (58.5 / people[i].
60                getProbAlcohol())) results[2][i] = true;
61            if (probChecker <= (3.6 / people[i].
62                getProbOpioids())) results[3][i] = true;
63        }
64        return results;
65    }
66
67    // generate a random population of people based on
68    // assumptions outlined in 2.2
69    public static Person[] generatePop(int size) {
70        Person[] result = new Person[size];
71        for (int i = 0; i < size; i++) {
72            String gender;
73            if (i < size / 2) gender = "female";
74            else gender = "male";
75            String race = "ERROR";
76            int raceChk = r.nextInt(4);
77            switch (raceChk) {
78                case 0:
79                    race = "white";
80                    break;
81                case 1:
82                    race = "black";
83                    break;
84                case 2:
85                    race = "hispanic";
86                    break;
87                case 3:
88                    race = "other";
89                    break;
90            }
91        }
92
String grades;
if (i < size / 4) grades = "A";
else if (i < size / 2) grades = "B";
else if (i < 3 * size / 4) grades = "C";
else grades = "F";

String parentEd = "ERROR";
boolean parentChk = r.nextBoolean();
if (parentChk) parentEd = "high school";
else parentEd = "college";

int income = -1;
int incomeChk = r.nextInt(4);
switch (incomeChk) {
    case 0:
        income = 4872;
        break;
    case 1:
        income = 18561;
        break;
    case 2:
        income = 32250;
        break;
    case 3:
        income = 32251;
        break;
}

int peers;
boolean friendsChk = r.nextBoolean();
if (friendsChk) peers = 3;
else peers = 5;

if (income == -1 || peers == -1 || race.equals("ERROR") || parentEd.equals("ERROR")) System.exit(1);

result[i] = new Person(gender, race, 12,
    grades, parentEd, income, peers);
}
return result;