

MathWorks Math Modeling Challenge 2018

Middlebury Union High School–

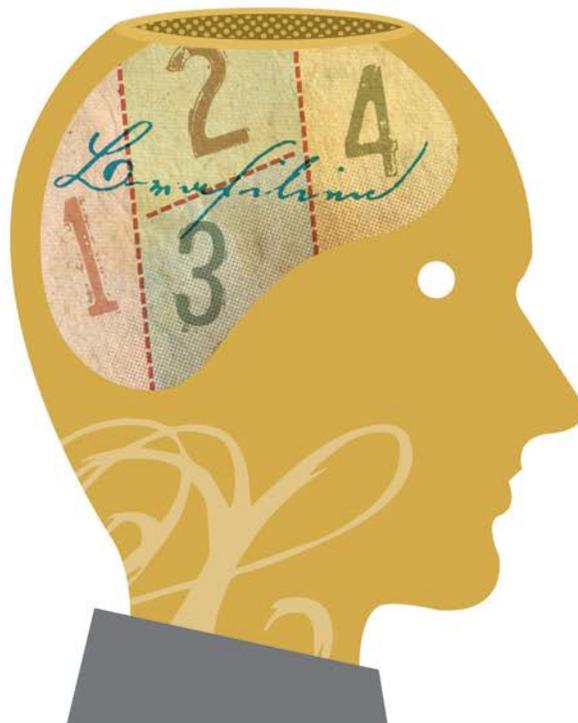
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MathWorks Math Modeling Challenge Finalist

\$5,000 Team Prize



Lettuce Reduce Food Waste

Team #10278

Executive Summary

While undernourishment is one of the world's largest problems, in America this problem has largely been replaced by that of food insecurity. Food insecurity is defined as the state of being without reliable access to nutritious and affordable food. Millions of Americans suffer from food insecurity, which is only compounded by the immense amount of food wasted each year by Americans. This food is thrown away throughout the entire production process and if saved could feed many of the 42 million Americans living in food insecurity today.

Our major task was split into three parts. First, we created a mathematical model to determine whether or not a given state in the United States could feed their food-insecure population using only the food waste generated within that state. For the second part, we designed a model to predict the amount of food waste a given household produces in one year, given the annual household income and the number of individuals in the household. Our final task was to utilize mathematical modeling to consider several options for the repurposing of food waste within a community we are a part of and to provide insight on the strategies and methods the community should pursue to maximize repurposed waste at minimal cost.

For our first task, "Just eat it!" we broke the question down into a comparison between the quantity of food needed to feed the food-insecure population of a given state and the quantity of edible food that can be reused from the food waste generated by that state. We analyzed each of these quantities on their own, with food waste expressed in dollar amounts, and compared two final values: dollars of food needed to feed the state's food-insecure population vs. dollars of edible food available from the state's generated food waste.

For the second part, "Food Foolish?," we compiled a set of data tables to inform a model calculating food waste: the amount of food waste in dollars a given household produces in one year, based on the minimum necessary spending on food required to consume a nutritional diet vs. the average annual food expenditure per income bracket. The first data table provides the cost to nutritiously feed an individual for one year, based on age and gender. The second table provides data on spending on food in one year, based on household income, and can be adjusted to account for the number of individuals in the home.

For the third part, "Hunger Game Plan?," we evaluated several options for repurposing food waste in our school community, including delivering food waste directly to food-insecure families, donating waste to food banks, feeding food scraps to livestock, composting, and putting food waste in the landfill. In evaluating the feasibility and profitability of each of these methods, we determined the costs and benefits associated with implementing each of them in our school community. We then analyzed the data to determine the optimal solution. We conclude that our school community should donate 65.9% of the recoverable food waste to the local food bank, 11.6% should be fed to livestock, and 22.5% should be sent to the landfill.

Table of Contents

Executive Summary	1
Problem 1: Just eat it!	3
A. Restatement of the Problem	3
B. Assumptions	3
C. Analysis of the Problem	3
D. Design of the Model	4
Part 1	4
Part 2	5
Part 3	7
E. Limitations	7
Problem 2: Food Foolish?	8
A. Restatement of the Problem	8
B. Assumptions	8
C. Design of the Model	8
D. Application of Model in Four Provided Scenarios	9
E. Limitations	11
Problem 3: Hunger Game Plan?	11
A. Restatement of the Problem	12
B. Assumptions	12
C. Design of the Model	12
Resource Cost	12
Resource Limitations	13
D. Conclusions	15
E. Limitations	16
References	17

Problem 1: Just eat it!

A. Restatement of the Problem

We need to create a mathematical model to determine whether or not a given state in the United States could feed its food-insecure population using only the food waste generated within that state. We then applied the model to Texas.

B. Assumptions

During our work, we made the following assumptions:

1. The data table titled “Weight percentages of food losses and waste (in percentage of what enters each step)” that appears on the second tab of the provided Texas_food_data spreadsheet is representative of all states in North America and Oceania as the data are each less than 10% of the total population and we expect usage to be relatively uniform across the region due to similar lifestyles and consumption habits.
2. Reusing food waste in one stage of the *food production-consumption process* does not impact how much food moves onto the next stage in production.
3. In developing our model to provide each individual with a healthy diet, we assumed no dietary restrictions or preferences, i.e., gluten/lactose intolerance, vegetarian/vegan lifestyles.
4. We assumed that the dietary requirements recommended by the USDA for the average person also applies to the food-insecure population as the dietary requirements of food-insecure people and non–food-insecure people are not independent
5. Our model assumed that the ratio of the prices per pound in each food group is represented by the average price of a sample of the more common foods in that category. To then find the actual price per pound of the 5 types of food we scaled our prices so that our prices when applied to dietary breakdown of the recommended daily diet we get the same price as that which is listed for the daily diet ("Meat Price Spreads.").
6. We assumed that food-insecure individuals consume a diet that is proportional to that recommended by the USDA but scaled down by some shortfall (U.S. Department of Health...)

C. Analysis of the Problem

This question can be generalized to a comparison between the amount of food needed to feed the food-insecure population of a state and the amount of edible food

that can be reused from the wasted food in that state. As food insecurity is related to the consumption of a healthy diet we decided to clarify if food waste could provide food insecure people with a healthy diet.

D. Design of the Model

Our model is comprised of three parts. The first is an analysis of the dietary requirements of the food-insecure (FI) population of a given state, the second is an analysis of the consumable nutritional resources available in the food waste of a given state, and finally we compare parts one and two, this comparison determines if a state could adequately support its FI population with repurposed food waste.

Part 1

We defined meeting the food needs of the FI population of a given state as when their diet matched that recommended by the USDA, which sets daily consumption targets for grains, dairy, protein (meat and fish), fruit, and vegetables. We determined the percent shortfall of food required by the average FI individual on an annual basis by dividing the food expenditures of an FI individual minus the minimum food expenditure of a non-FI individual by the minimum food expenditure of a non-FI individual (“How We Got the Map Data.”). This percent shortfall was calculated to be 0.2815, meaning that food waste would need to cover approximately 28% of a given individual's diet. We used this to calculate the yearly shortfall in pounds for each food type. Then using approximations for the per pound cost of each of the five types of food we created the FI Need vector, a vector whose entries denote the yearly cost of the average FI individual's food shortfall.

To better demonstrate our process we've included a detailed description of how we calculated the cost of the yearly shortfall for grains. We found that an individual needs 9 ounces of grain per day, which translates to 205 pounds of grain annually (U.S. Department of Health...). To calculate the cost of the average yearly FI individual's shortfall in grains we multiplied the yearly grain requirement by the percent shortfall and our approximation for the per pound cost of grain:

$$\begin{aligned} \text{Yearly Requirement}(lbs) \times \% \text{Shortfall} \times \text{Cost of Food}(\$/lbs) &= \text{Cost of Yearly Shortfall} \\ 205(lbs) \times .2815 \times 2.91(\$/lbs) &= 168.22(\$) \end{aligned}$$

This process was repeated for all 5 food groups, yielding the following FI Need vector:

$$\begin{aligned} \mathbf{v}_{\text{FI Need}} &= \{ \text{Grain}, \text{Dairy}, \text{Protein}, \text{Fruit}, \text{Vegetables} \} = \\ &= \{ 168.22, 77.33, 141.33, 236.97, 281.36 \}, \end{aligned}$$

where the entries are the dollar costs of the amount of food needed to cover a FI individual's yearly shortfall for that type of food.

Part 2

In making our determinations about the food waste generated at each stage of the food production-consumption process (FPCP)¹, we decided to use the percentages from the Food and Agriculture Organization of the United Nations (FAO), Global Food Losses and Food Waste, 2011 provided in the Texas_food_data source. The percentages of food waste are listed by food category² for each step in the FPCP.

We selected to use the monetary value of cash receipts by food category by state from the USDA's Economic Research Service provided to us in the Texas_food_data as the input for our determinations about the food waste generated in a given state in a given year. We elected to use cash receipt data to represent initial quantity of produced food in a given state because similar cash receipt data is available for all fifty states. Using the food category groupings provided to us in the Texas_food_data set, we summed cash receipt values for the crops in each of the seven food categories to determine a total cash receipt value for each food category. We let this value represent the quantity of food at the end of the Agricultural Production step, assuming that food loss and waste generated at the agricultural production stage does not make up part of the income represented in the receipts.

We considered the FAO's descriptions of food wastes and losses at the other four steps of the FPCP and determined which steps result in food waste that is suitable for human consumption. For example, food losses and wastes in the Post-harvest Handling and Storage and Processing and Packaging steps of the FPCP include losses due to spillage, degradation, discards during sorting, and death during transport (for livestock) (FAO. 2011. Pages 2&3). As these food losses and wastes directly impact food quality, safety and consumability, we assumed that no food waste generated in these steps is suitable for repurpose as human food. We assumed that food losses and wastes in the Distribution: Supermarket Retail and Consumption steps of the FPCP are suitable for repurpose as human food because these wastes happen within food-retail and household locations where the food has been cleared for human consumption. To account for food waste resulting from spoilage and degradation in supermarkets, retail locations, and households, we assumed that the time until expiration for each of the food categories is a representation of how much of the waste of that food category is salvageable given the inevitable time-lag due to transportation/transfer of food from a retail/household location to FI individuals, and the chance that a given food will meet its expiration or best buy date within that transportation time. We chose one month as our redistribution time, and estimated the average time in which foods of each category expire when stored at optimal conditions by comparing the listed food items from page 24 of the 2011 FAO study (FAO. 2011.) to the storage periods in the table

¹ The *food production-consumption process* (FPCP) is our title for a five-step sequence consisting of the five steps for which there exist food waste percentage data from the FAO data set. In order, the five steps are Agricultural Production; Post-harvest Handling and Storage; Processing and Packaging; Distribution: Supermarket Retail; and Consumption.

² The seven *food categories* are Cereals, Roots and tubers, Oilseeds and pulses, Fruits and vegetables, Meat, Fish and seafood, and Milk.

titled “Storage Periods for Retaining Food Quality” in the paper on Food Storage (Albrecht, Julia. Pages 4-9) Converting the estimations for expiration times to percentages of 30 days, we then used these percentages to scale how much of the food would remain usable during the time between transport from retail/household to FI individuals.

Figure 1: Table of Storage Periods for Food Categories (at optimal storage conditions)

Food Type	Time (days)	Percent of 30 days
Cereal	30	100%
Roots and tubers	30	100%
Oilseeds and pulses	30	100%
Fruits and vegetables	7	23%
Meat	3.5	12%
Fish and seafood	5	17%
Milk	7	23%

Accumulating each of our assumptions concerning which steps of the FPCP generate food waste suitable for repurposing and what percentage of that food would not go bad before it could be delivered to FI individuals, we applied the percentages of food wasted at each step in the FPCP for each food category to the initial cash receipt value for the respective food category. For each food category, we subtracted the waste generated in Post-harvest Handling and Storage and Processing and Packaging steps (as we deemed food waste from these steps repurposable) and then found the waste generated by only the Distribution: Supermarket Retail and Consumption steps. Finally, we applied the percentages for expiration periods to determine the final amount of usable food that can be recovered from food waste.

Figure 2: Table of Edible Food by Food Category from Waste Generated in Texas

Food Category	Salvageable Food (in thousands of dollars)
Cereals	229,796
Roots and tubers	35,213
Oilseeds and pulses	30,937
Fruits and vegetables	127,984
Meat	179,965
Fish and seafood	1,078

Milk	65,391
Total	670,364

Justification for how to clump “food categories” into food groups/type: Looking at the listed food items in each food group (FAO. 2001. page 24), we determined which one of the five types (Grains, Dairy, Protein, Fruit, Vegetables from the USDA food types for a healthy diet) the food was. We then used these groupings to calculate the value of the each of the five types of food, giving the Texas Supply Vector:

$$\mathbf{v}_{\text{supply vector}} = \{\sum \text{value of Grain foods}, \dots, \sum \text{value of Vegetable foods}\}$$

$$\mathbf{v}_{\text{supply vector}} = \{2.30E + 08, 6.45E + 07, 1.86E + 08, 1.50E + 07, 1.77E + 08\}$$

Part 3

After the analysis of both the amount of food required for an average FI individual, we could then multiply by a scalar to find the needs of an entire population. In the case of Texas, there are 4,320,050 individual considered to be FI. Thus we can multiply the number of individuals, in any state, by the needs vector to determine the state needs vector for that state. In the case of Texas,

$$\mathbf{v}_{\text{Texas Need}} = \mathbf{v}_{\text{FI Need}} \times P_{\text{FI population of Texas}} =$$

$$\{7.27E + 08, 3.34E + 08, 6.11E + 08, 1.02E + 09, 1.22E + 09\}$$

From Part 2 we were able to determine that the total amount of food waste that could be repurposed for human consumption. This is represented by the Texas Supply Vector

$\mathbf{v}_{\text{Texas supply}} = \{229795618.2, 64456675.32, 186207131, 14963733.27, 177342036.2\}$
Thus in order to feed the FI population, assuming a balanced diet, the following must be true:

$$\mathbf{v}_{\text{Supply}} - \mathbf{v}_{\text{Need}} = \{\geq 0, \geq 0, \geq 0, \geq 0, \geq 0\}.$$

However,

$\mathbf{v}_{\text{Texas supply}} - \mathbf{v}_{\text{Texas need}} =$
 $\{-4.97E + 08, -2.70E + 08, -4.24E + 08, -1.01E + 09, -1.04E + 09, -3.24E + 09\},$
meaning Texas is unable to entirely make up for the food shortfall of FI Texans with potentially repurposable food waste.

E. Limitations

While we believe that dollar amount is a feasible thing to use for our comparison, food prices do vary based on location, season, and market fluctuation.

Our scaling of the amount of food that is salvageable in Figure #2 is somewhat arbitrary, based on the assumptions made in Figure #1; however, we still feel that this is a necessary method to quantify data that would otherwise remain unknown, thus making our model even more inaccurate. In addition, the data we drew from for the Texas example only encapsulated 78.1% of the food produced in Texas. However, given the magnitude of the difference between supply and need, this missing 21.9% wouldn't have skewed the outcome significantly.

Problem 2: Food Foolish?

A. Restatement of the Problem

The problem prompts us to create a model to examine how different demographic factors and income affect a household's waste. The problem prompts us to also consider different lifestyle choices such as where the food is purchased, such as grocery stores, the school cafeteria, and restaurants. The problem also asks us to demonstrate how our model works when applied to several scenarios, including a single parent and toddler with an income of \$20,500, a family of four composed of two parents and two teenage children who have an annual income of \$135,000, an elderly couple with an annual income of \$55,000, and a single 23-year-old who has an annual income of \$45,000.

B. Assumptions

1. This model assumes that food waste can be represented as the difference between the actual food expenditure and the necessary food expenditure based off USDA dietary requirements (U.S. Department of Health...).
2. The model also uses the assumptions made about food prices and utilized the same price adjustment factor as was used in **Part 1 of Just Eat It!**. We used the assumptions and adjustment factor to convert recommended food requirements into dollar amounts as a way to quantify food waste.

C. Design of the Model

In order to calculate a monetary amount required for a healthy diet we considered the effects of both age and gender on an individual's nutritional requirements. We decided to use data from the US Department of Health and Human Services and the US Department of Agriculture to determine the amount of food from each basic food group that an individual of a given age and gender would need in a day. We used the midpoint of each recommendation range (in cups per day) to calculate pounds per food group needed each year, and converted this to dollars with the same prices and adjustment factor used in the above Just Eat It! section.

Figure 3: Table of Necessary Food Expenditure based off Dietary Requirements for Age/Gender Groups

Men		Women	
Age Range	Yearly Cost (dollars)	Age Range	Yearly Cost (dollars)
1-3	1527.96	1-3	1527.96
4-8	2044.39	4-8	1931.54
9-13	2519.04	9-13	2356.54
14-18	3058.50	14-18	2623.63
19-30	3234.91	19-30	2623.63
31-50	3215.59	31-50	2571.12
51-70	2987.11	51-70	2466.53
71+	2987.11	71+	2466.53

After calculating the amount of money required to feed an individual for one year based on their age and gender, we gathered data on probabilistic spending based on income bracket. The Figure 3 table is a measurement of average food expenditure based on income range and family size.

Figure 4: Table of Annual Food Expenditure based on Income (Consumer Expenditure Survey, 2016)

Income Range	Average Household Size (people)	Average Food Expenditure (dollars)
<\$15,000	1.6	3,768
\$15,000 - \$29,000	1.9	4,437
\$30,000 - \$39,999	2.3	5,221
\$40,000 - \$49,999	2.5	6,028
\$50,000 - \$69,999	2.6	6,739
\$70,000 - \$99,999	2.9	8,436
\$100,000 - \$149,999	3.1	10,351
\$150,000 - \$199,999	3.1	13,550
\$200,000 and more	3.2	16,054

D. Application of Model in Four Provided Scenarios

Single parent with a toddler, annual income of \$20,500:

According to our model, the necessary food expenditure required for a toddler of either gender (ages 1-3) is \$1,527.96 per year (**Figure 3 Table**). Since the gender and age of the single parent is unknown, we assume they are between 19 and 30 years old and average the annual cost of food for the genders in that age range: $(\$3,234.91 + \$2,623.63)/2 = \$2,929.27$. Adding the necessary food expenditure required for the toddler to that required for the single parent, we determine that the household needs to spend a minimum of \$4,457.23 on food per year to meet their basic dietary and nutritional needs.

The data in the **Figure 4 Table** shows that an average household with an annual income of \$20,500 spends an average of \$4,437.00 per year on food. We accept this value as is, based on the fact that a toddler does not eat as much as an adult, and thus, one toddler and one adult are reasonably approximated by 1.9 individuals. This household is most likely in danger of being FI because the necessary spending for food for one year is greater than what a household of this size typically spends. This suggests that food waste is negligible.

$$\begin{aligned} \text{Necessary food expenditure} &> \text{actual food expenditure} \\ \$4,457.23 &> \$4,437.00 \end{aligned}$$

Family of four (two parents, two teenage children), annual income of \$135,000:

Since we do not know the genders or exact ages of any of the four individuals in this household, we compute the necessary food expenditure by averaging between the genders and assuming an age range for each individual. We assume the parents both fall within in the 31 – 50 year-old age range and the adolescents within the 14 – 18 range. Averaging the appropriate data from **Figure 3 Table**, yields a necessary yearly food cost of \$11,645.25.

Figure 4 Table estimates that a household with an annual income of \$135,000 spends \$1,0351.00 on food per year. However, a household with this income only averages 3.1 people, so adjusting for the fact that four people depend on this income, the yearly household food expenditure would more reasonably be $(10,351.00/3.1) \times 4 \text{ people} = \$13,356.13$. This household most likely generates around \$1,710.88 of food waste per year because the necessary spending for food for one year is less than what a household of this size typically spends.

$$\begin{aligned} \text{Necessary food expenditure} &< \text{actual food expenditure} \\ \$11,645.25 &< \$13,356.13 \end{aligned}$$

$$\begin{aligned} \text{Actual food expenditure} - \text{Necessary food expenditure} &= \text{Annual food waste} \\ \$13,356.13 - \$11,645.25 &= \$1,710.88 \end{aligned}$$

Elderly couple, living on retirement, annual income of \$55,000:

We assume that individuals living on retirement are within the age range of 71+ and we average between the genders for food costs from the **Figure 3 Table** to be representative of the individuals in the relationship because we cannot assume a

heterosexual couple. This indicates a necessary yearly food expenditure of \$5,453.64.

The **Figure 4 Table** estimates that an annual income of \$55,000 predicts yearly food expenditures of \$6,739.00, but if we account for the average number of people in a household of this income size (2.6), a more reasonable estimate for yearly food expenditure for two people is: $(\$6,739.00/2.6) \times 2 = \$5,183.85$. Since the average yearly food expenditure for a household of this size is less than the necessary food expenditure predicted for this elderly couple, this household is most likely in an FI state. Thus we predict that food waste is negligible.

$$\begin{aligned} \text{Necessary food expenditure} &> \text{actual food expenditure} \\ \$5,453.64 &> \$5,183.85 \end{aligned}$$

Single 23-year-old, annual income of \$45,000:

Our model predicts, using data from **Figure 3 Table**, that a 23-year-old individual will require \$2,929.27 in food expenses per year (averaging the expenses for each gender in the 19-30 year-old age range).

According to **Figure 4 Table**, the average household with an income of \$45,000 spends approximately \$6,028.00 on food per year. However, a household in this income bracket supports an average of 1.9 adults, so this number must be adjusted to accurately predict the food expenditures for a single person. Accounting for the average number of people per household in this income bracket, a more accurate prediction for food expenditure for this scenario is $\$6,028.00/2 = \$3,014.00$ per year. Therefore this household most likely produces around \$84.73 of food waste per year.

This household most likely generates around \$84.73 of food waste per year because the necessary spending for food for one year is less than what a household of this size typically spends.

$$\begin{aligned} \text{Necessary food expenditure} &< \text{actual food expenditure} \\ \$2,929.27 &< \$3,014.00 \end{aligned}$$

$$\begin{aligned} \text{Actual food expenditure} - \text{Necessary food expenditure} &= \text{Annual food waste} \\ \$3,014.00 - \$2,929.27 &= \$84.73 \end{aligned}$$

E. Limitations

While we believe food wastage is most likely more common than seen in the above scenarios, our calculation of necessary food expenditures is idealistic because it assumes individuals actually follow recommended dietary requirements. In reality, Americans tend to purchase more cheaper grains and bypass the expensive (but highly recommended) fruits and vegetables, causing the average cost spent on food to be lower than what individuals should be spending in order to maintain a healthy diet.

Problem 3: Hunger Game Plan?

A. Restatement of the Problem

The problem asks us to develop a mathematical model to determine the optimal strategy of repurposing food waste in our community. Focusing on our school, we must repurpose a maximal amount of food while keeping cost to a minimum.

B. Assumptions

1. In order to perform the simplex method, we have to make sure that the required assumptions are not violated. These assumptions include proportionality, additivity, divisibility, and certainty. Proportionality is upheld because we believe that increasing each option will proportionally increase the benefit. For example, we expect the cost-benefit to increase by fifty percent when there is a fifty percent increase in food that is delivered directly to FI individuals. Additivity is upheld because the equations relating the variables are all linear models. To allow for divisibility to be upheld we assume that we can implement any non-zero combination of solutions. While we are not certain of the exact values of our constants, we feel that the choices we have made are representative of the situation.

2. To allow for cost-benefit analysis, we assume that monetary value is an adequate measure of the importance of each use of food. In other words, we consider food waste that is used for human nutrition to provide a cost benefit that is equivalent to the market value of said food.

C. Design of the Model

We chose to consider how best to repurpose food waste in our school community, determining that there were five main options for dealing with the waste: delivering food directly to the homes of food-insecure individuals (which will be referred to as “EAT1”), delivering food to our local food shelf (“EAT2”), delivering food directly to local farms for use as livestock feed (“LIVESTOCK”), repurposing the food at our school with an on-site compost (“COMPOST”), and delivering the food to our local landfill (“LANDFILL”). We further decided that each of these options require varying amounts of money, time (volunteer hours), promotional and educational effort, and environmental impact. In realizing that we have limited amounts of these resources, we determined approximate associated “costs” in each of these areas.

Resource Cost

For each method of dealing with food waste, we factored in vehicular transportation to determine the monetary cost, time expenditure, and environmental impact. We also factored in additional manual labor to determine the

time expenditure, as well as the environmental impact (in pounds of CO₂) of putting the food in a landfill. Additionally, each method of food disposal requires varying levels of care and effort on behalf of the faculty, student body, and kitchen staff. We are measuring this resource cost in the form of “promotional and educational effort,” which is measured as the percentage of the local student government’s available time that must be spent on educating the school’s population.

Resource Limitations

Monetary Cost: We assume that our school would provide \$1,000 to food-repurposing efforts, because we estimate \$1,000 to be our school’s current per-year spending on delivering food scraps to the landfill.

Volunteer hours: We will assign this project to our school’s chapter of National Honor Society, which is a volunteer organization that is required to do community service. Each of its approximately 30 members will each be required to perform 6 hours of service in support of this food waste problem.

Promotion and Education: Absolute quantities are not needed to analyze the promotional effort expended. Instead, we measure the relative promotional effort needed as a percentage out of 100%, the total promotional ability of our student government.

Environmental cost: In an effort to be environmentally conscious, we hope to reduce CO₂ emissions to half of that which our school is currently generating using only the landfill (18,000 lbs CO₂) so 9,000 lbs is our goal and limit.

This setup provided us with the following numerical benefits, costs, and constraints, summarized in Figure 5.

Figure 5: Table of Food Waste Disposal: Costs and Benefits

<i>Food Waste Disposal Methods</i>	<i>Cost: \$ / Year (USD)</i>	<i>Cost: Volunteer Person-Hours / Year (hrs)</i>	<i>Cost: Promotion & Education / Year (% of total available)</i>	<i>Cost: Environmental Impact / Year (lbs CO₂)</i>	<i>Benefit: \$ / Year (USD)</i>
<i>EAT1: Food delivery direct to food-insecure homes</i>	\$1,350	540	150%	10,620	\$32,940
<i>EAT2: Delivery to food shelf</i>	\$450	180	150%	3,600	\$16,470

<i>LIVESTOCK: Deliver to farms for livestock consumption</i>	\$600	296	10%	4,720	\$18,360
<i>COMPOST: Compost at the school</i>	\$0	200	20%	0	\$0
<i>LANDFILL: Deliver to local landfill</i>	\$1,000	120	0%	18,000	\$0
<i>Resource Limitation</i>	\$1,000	180	100%	9,000	

Thus we determined that an optimization of the problem via linear programming, specifically the simplex method, was an appropriate way to find the best solution, based on our assumptions. The running of the simplex method was done using a Java program called Interactive Operations Research Tutorial. The following is the setup of the simplex method before the iterations were performed. The 'max z' equation is the objective function which we are seeking to maximize. The variables are defined as follows:

X1 = percentage of the food waste dealt with the EAT1 method
X2 = percentage of the food waste dealt with the EAT2 method
X3 = percentage of the food waste dealt with the LIVESTOCK method
X4 = percentage of the food waste dealt with the COMPOST method
X5 = percentage of the food waste dealt with the LANDFILL method
X6, X7, X8, and X9 are slack variables which are required to deal with the inequalities.
X10 is an artificial variable used to deal with the equation and is dealt with using the BIG M method before the regular simplex method can proceed.

The equations are defined as follows:

Equation 1) deals with the monetary restraint.
Equation 2) deals with the restraint of volunteer hours.
Equation 3) deals with the limitation of promotional and educational effort available.
Equation 4) deals with the constraint imposed to reduce CO2 emissions.
Equation 5) ensures that all of the waste food is used by making the choices sum to 1.

The inequalities in the last line ensure that we do not perform any option in the negative direction; for example, we cannot take food out of the landfill.

Linear Programming Model:

Number of Decision Variables: 5

Number of Functional Constraints: 5

Max Z = 183 X1 + 91.5 X2 + 102 X3 + 3.75 X4 + 0 X5

subject to

$$1) \quad 1350 X1 + 450 X2 + 600 X3 + 0 X4 + 1000 X5 \leq 1000$$

$$2) \quad 540 X1 + 180 X2 + 296 X3 + 200 X4 + 120 X5 \leq 180$$

$$3) \quad 1.5 X1 + 1.5 X2 + 0.1 X3 + 0.2 X4 + 0 X5 \leq 1$$

$$4) \quad 10620 X1 + 3600 X2 + 4720 X3 + 0 X4 + 1800 X5 \leq 9000$$

$$5) \quad 1 X1 + 1 X2 + 1 X3 + 1 X4 + 1 X5 = 1$$

and

$$X1 \geq 0, X2 \geq 0, X3 \geq 0, X4 \geq 0, X5 \geq 0.$$

After performing 9 iterations of the simplex method the tableau is in its final form as there are no negative values remaining in the objective function. The final tableau resembles the following:

Bas Eq	Coefficient of										Right	
Var No	Z	X1	X2	X3	X4	X5	X6	X7	X8	X9	X10	side
											1M	
Z 0 1	109.2	0	0	48.59	0	0	0.558	38.7	0	-66.9		72.15
X5 1 0	-1.95	0	0	0.476	1	0	-5e-3	-0.45	0	1.651		0.225
X3 2 0	2.093	0	1	0.419	0	0	0.006	-0.23	0	-0.7		0.116
X6 3 0	1660	0	0	-775	0	1	2.112	282.2	0	-1253		408.9
X9 4 0	1160	0	0	-3212	0	0	-16.3	-549	1	153.5		5674
X2 5 0	0.86	1	0	0.105	0	0	-3e-4	0.682	0	0.047		0.659

D. Conclusions

From the final tableau, we have a Z-value of \$72.15, meaning that by performing the distribution recommended by the table we can earn \$72.15 per day. The values for X2, X3, and X5 are basic variables, non-zero, and equal to 0.659, 0.116, and 0.225, while X1 and X4 are non-basic variables and equal to zero. Putting these numbers back into context means that we will operate the EAT2, LIVESTOCK, and LANDFILL method in order to minimize costs. The EAT2 method will be used to deal with 65.9% of the food waste from the school, 11.6% will be dealt with by the LIVESTOCK method, and 22.5% will be dealt with by the LANDFILL method. This solution optimizes the return on expenditure. The slack variables describe the amount of resources remaining: X6, or the amount of money not spent is \$408.90, and X9, or the lbs of CO2 remaining, is 5674 lbs. Both the

volunteer hours and promotion and education resources are the limiting resources, and their shadow prices, or the amount of financial gain that we can expect to receive by increasing each by one hour and one 'unit', are \$0.558 and \$38.70, respectively. Although counterintuitive, given our limited resources, sending a portion of food to the landfill allowed for the greatest overall positive impact in repurposing our food waste.

E. Limitations

The model assumes that the assumption of certainty is upheld. In reality, for any practical real-life model is likely to have some potential variability in the constraint, resource, or cost-value coefficients associated with the variables. For example, if student senate were able to provide 1 additional 'unit' of promotion and education, the initial gain is at a rate of \$38.70.

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