M^3 Challenge Fourth Place (Meritorious Team Prize) - $7,500
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Moody's Mega Math Challenge

Team 218

Summary

The automobile has defined how American cities are built and how Americans travel since the early twentieth century. However, the High Speed Railway systems in the Northeastern United States and Europe have proven successful. As a result, the Obama Administration is taking steps to institute a countrywide High Speed Rail system through the American Recovery and Reinvestment Act of 2009. This call to action poses a problem of how to allocate our limited funds among the ten designated Rail Corridors.

In our first model, we set up a function that uses a proportion in population growth along with calculated values for passengers switching from flying planes to taking high speed trains until 2037. We took into account the largest metropolitan statistical areas and common flight routes as well as population growth in the corridors for our analysis. We then used integration to calculate total increase in passengers due to high speed trains and concluded that ridership numbers can increase dramatically by up to thirty million riders in a single corridor.

In our second model, we examined the cost of building and maintaining a High Speed Rail line in each of these corridors. First we determined environmental and personal safety factors that would affect the fixed prices of building infrastructure: sloping topography of the land, geographical obstacles, natural disaster prevention, humidity, urban centers that are not stops on its route, and safety specifications as directed by federal and state regulation. Then, we researched projected costs of rail lines per corridor. Finally, we used factors such as fuel, crew costs, liability, and additional wear and tear on the tracks to compute the cost of maintenance for each rail line.

In our third model, we used the fuel consumption of cars, planes, and trains to show that trains used the lowest volume of oil per mile of commute. We then used this fact to conclude that a switch to a High Speed Rail commuter system would reduce the United State’s dependence on foreign oil. It would also decrease greenhouse gas emissions and thus be a more environmentally friendly choice.

In our fourth model, we combined the results of the above three models to rank the ten corridors in terms of need for federal funding based on projected profitability implied from cost and ridership estimates. We used an equation that took each of these qualities into account to objectively do this.

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I. Introduction

A. Background

The rise of the automobile in the twentieth century reshaped modern American cities. What were once walkable and sustainable high density centers were abandoned for the low density counterparts similar to the prolific automobile infrastructure of the suburbs. Automobile dependency throughout the entire United States has driven a higher dependency on foreign petroleum resources and decreased economic opportunities in mass transit, as in bus and passenger rail services. Subsidies for automobile infrastructure, particularly on highways and parking lots, now account for about nine percent of the gross national product (Duany 94); these hidden costs are paid off through income, property, and sales taxes rather than a direct fuel tax and obfuscate the true cost of car dependency. On the other hand, for every billion dollars spent on building roads, seven thousand jobs in transit are lost (Duany 95). This is all, of course, without mentioning the negative environmental externalities of using cars, including the rise in atmospheric greenhouse gases and the limited supply of nonrenewable petroleum available upon the planet.

High speed rail promises to assuage certain economic problems faced by modern America. It can help to reverse the current recession by stimulating job growth in construction and by employing people to maintain and to run the trains and stations. The proliferation of this mode of public transportation promises to derail the country's dependence on foreign oil, requiring less energy than other modes of transport. Within the European Union, some of the world's most developed countries have seen great success in implementing high speed passenger rail, such as the TGV (Train à Grand Vitesse) in France (TGV, France). When the Acela high speed intercity passenger rail program was introduced at the end of 2000, the air-rail market from Washington, D.C. to New York City saw 37% of its passengers travelling between the two cities by rail. Since then, the average percentage of rail travelers has risen from 45% in 2001 to 61% in 2009 (DeHaven).

After the successes of High Speed Rail systems in the North East Corridor and in Europe, this mode of transportation is being pointed to as a model of efficiency and profitability. As a result, there have been many calls to institute this type of system in other parts of the country. President Obama's American Recovery and Reinvestment Act of 2009 (The American Reinvestment and Recovery Act of 2009) called for such a system to foster job growth in the construction and transit sections of the economy. This improvement of our infrastructure would also facilitate intercity commutes and reduce transportation costs of businesses. Rapid transportation will also allow businesses to spread out and expand their employee base. The President's plan called for ten distinct additions to the country's High Speed Rail infrastructure as located in Figure 1.

Figure 1: A map of President Obama's planned expansion to the United State's High Speed Rail Infrastructure ("Vision for High Speed Rail in America")

Not all of these locations would receive the same level of economic benefit from the proliferation of a High Speed Rail system. In contrast to the all of the cities connected by high speed rail in the US, Europe, and Asia, some of the planned destination cities do not have intracity public transportation infrastructures that are developed enough to support an increased load of non-automotive travellers. The ease with which a commuter can reach his or her final destination upon arriving in a destination city will determine the willingness of this commuter to take a train. This paper aims to determine which of these locations will provide the most robust user base and therefore the best return on the public sector's investment.

B. Restatement of the Problem

In modelling which of the ten designated HSIPR locations are most deserving of high speed rail funding, several key factors must be considered. Primarily important is the question of whether or not a high speed rail system will be profitable enough to cover the costs of its implementation. Over the next twenty years, high speed rail travel must be able to compete with other forms of transportation such as driving and flying for its construction and maintenance to be economically viable. In addition, the oil consumption of trains should be considered compared with that of the current modes of transportation for a given area.

II. Analysis of the Problem and Model

A. Assumptions of the Model

- The price per mile of a ticket will be the same as a ticket for the Acela Express.
- Population will grow linearly within a corridor over the next 20 years.
- This population growth rate will be equivalent to that of the corridor's most populated city.
- The total ridership of the largest city within a corridor is equivalent to the total ridership within the corridor.
- People act rationally in economic situations, so they will take the cheapest mode of transport available to their destination.
- Price of a High Speed Rail ticket will be calculated in the same way as on the Acela Express.
- People that are current consumers of train transport will continue to do act in this way.
- Except for the increase immediately following high-speed train construction, the proportion of riders to non-riders in the population will remain constant.
- All high-speed trains will have a capacity of 300 seats, approximately the same capacity as the Acela Express ("Acela Express").
- On high-speed trains and aircraft, 70% of seats are occupied (based on estimates from Center for Clean Air Policy).
- Due to the proliferation of NEXUS cards that cheapen trips across the border for regular commuters, we are assuming that the cost of crossing the US/Canada border is negligible for commuters who cross the border regularly.
- Assume that there is a 20 minute wait time at rush hour for border crossing.
- It will take about the same amount of time to build each corridor as it took to build the Acela Express.
- Proposed total costs are as lowest possible. As such, the maximum mileage of rail that can be refurbished for high speed transit will be.

B. Addressing the Problem

In our first model, we gauge the potential effects on ridership of the changes in rail times and prices with respect to other modes of intercity transportation as caused by the institution of a high speed rail system.

In our second model, we asses the cost of building and maintaining a High Speed Train network compared with the costs associated with driving and flying the same commutes.

In our third model, we judge the impact that the proliferation of a high speed rail system will have on the United State's current dependence on foreign oil.

C. Design and Testing of the Models

1. First Model: Determining Future Ridership and the Effects of Changes in Rail Times

Assuming that, like the Acela Express, each train system will take five years to complete, we will model ridership estimates off of the extrapolation of population data starting five years from now and examining the projected ridership of the following twenty year period. We will then use a mathematical model to estimate quantitatively the ridership in each region.

To predict future ridership, we generated the following function:

$$R = \frac{R_0 P(t)}{P_0},$$

where

$$R_0 = (C + R_p)(0.031 \times 1.016^x)^{-1}.$$

R is the total number of riders, R_0 is the number of people riding high-speed trains immediately after introduction, and R_p is the number of people riding planes along the corridor's route. $P(t)$ is the expected population within the largest Metropolitan Statistical Area (MSA) of the corridor up to a specified point in a year t (2000 is at $t = 0$), $P_0 = P(12)$ is the current population in the MSA (that is, in the year 2012, so $t = 12$), and C is the total Amtrak train ridership during fiscal year 2011. The expression $(0.031 \times 1.016^x)^{-1}$ (Aerlines) is the proportion of market share held by high speed trains versus airplanes, and x is the time in minutes of any given train ride; we will analyze train rides that connect two major cities within the corridor. In order to ensure that values were not skewed by the presence of major cities that connect several corridors together, for six of the corridors, we chose another major city that did not have the greatest MSA population; we have specified these within the table. The constant $\frac{R_0}{P_0}$ satisfies our condition that there the proportion of riders to non-riders will remain constant except immediately after the introduction of high-speed trains. Below are values for C and functions representing the population of the MSA. Being that the scope of time addressed is only two decades, a linear model for the population of the largest MSA in each corridor, which we assumed to be representative of the entire corridor's growth, was regressed from population statistics between 2000 and 2009.

Figure 2: Amtrak Ridership and Population Functions for the Largest MSA in each Corridor

Corridor	Largest MSA	Amtrak Ridership in Fiscal Year 2011 (in people)	Population Functions
1. Southeast Corridor	Washington, DC Atlanta	4,850,685 114,938	P = 67,579 t +4,862,822

2. California Corridor	Los Angeles Oakland	1,606,121 401,964	P = 37869 t + 1.3×10^7
3. Pacific Northwest Corridor	Seattle	672,485	P = 38,570 t +3,036,982
4. South Central Corridor	Dallas	54,498	P =137,376 t + 5,183,866
5. Gulf Coast Corridor	Houston	19, 637	P = 125,383 t + 4,717,779
6. Chicago Hub Network	Chicago Minneapolis/St. Paul	3,393,695 116,785	P = 47336 t +9.71×10^6
7. Florida	Miami	94,556	P = 56173 t + 5.77×10^6
8. Keystone Corridor	Philadelphia Pittsburgh	3,872,781 133,855	P = 30863 t + 2.6[illegible] $\times 10^6$
9. Empire Corridor	New York Buffalo	8,995,551 155,015	P = 71951 t + 1.12×10^7
10. Northern New England Corridor	Boston Montreal	2,296,311 84,581	P = 16104 t + 4406458

To find R_P, we analyze the most important airway within each corridor. The number of flights was retrieved from Google Flights for June 6, 2012, and we used a national average of 72.97 passengers per flight (calculated from 51.4 million passengers on 704,400 flights in November 2011) ("RITA BTS Airline System Traffic Down 0.1 Percent").

Figure 3: Total Passengers on One Primary Airway in each Corridor

Corridor	Flight	Number of Flights each Day	Number of Passengers Flying One Way per Day	Total Passengers per Year
Florida	Miami, FL to Tampa, FL	6	437.82	319,609
California	San Francisco, CA to San Diego, CA	14	1021.58	745,753
Pacific Northwest	Seattle, WA to Portland, OR	23	1678.31	1,225,166
Chicago Hub Network	Minneapolis, MN to Chicago, IL	36	2626.92	1,917,652
Empire	Buffalo, NY to New	22	1605.34	1,171,898

	York City, NY			
Keystone	Pittsburgh, PA to New York City, NY	17	1240.49	905,558
South Central	Dallas, TX to Tulsa, OK	14	1021.58	745,753
Gulf Coast	Houston, TX to Atlanta, GA	21	1532.37	1,118,630
Southeast	Atlanta, GA to Washington D.C.	29	2116.13	1,544,775
North New England	Boston, MA to Montreal, Canada	12	875.64	639,217

We now have enough data to calculate the number of riders in each corridor in each year.
Using this data and these functions along with the R model for ridership, we conclude that the following numbers of people will ride high speed trains at the following MSAs in the following years (starting in 2017, or $t = 17$, given the earlier assumption that it will take five years for the high speed rail system to be built):

Figure 4: Projected Ridership for the Next 20 Years

Corridor	P(12) Current population	Ridership - 2017 R(17)	Ridership - 2027 R(27)	Ridership - 2037 R(37)
1. Southeast Corridor	5,673,770	345,048	383,836	422,624
2. California Corridor	13,561,428	1,096,942	1,127,388	1,157,834
3. Pacific Northwest Corridor	3,499,822	1,792,954	1,980,228	2,167,502
4. South Central Corridor	6,832,378	724,056	856,340	988,624
5. Gulf Coast Corridor	6,222,375	89,221	105,554	121,887
6. Chicago Hub Network	10,279,092	1,228,542	1,283,844	1,339,146
7. Florida	6,445,136	343,092	371,746	400,400
8. Keystone Corridor	2,981,416	1,309,286	1,438,151	1,567,016
9. Empire Corridor	12,074,112	856,172	905,716	955,260
10. Northern New England Corridor	4,599,706	1,543,524	1,596,634	1,649,745

In order to address the issue of how changes in rail travel times affect choice of transport mode, we simply have to look at the difference of R_0 C in each of the corridors. In all ten cases, the difference is positive, meaning that more potential riders will choose high speed rail over plane flights.

We also compute the total increase in ridership over the four years. To find this, we integrate

$$I = \int_{17}^{37} [R(t) - R^*(t)]dt = \int_{17}^{37} \frac{R_0 - C}{P_0} P'(t)dt,$$

where $R^*(t) = \frac{C}{P_0}P(t)$ is the number that would have ridden the train even without the high speed addition. We calculate this by multiplying the current ridership by the ratio of the estimated future population to the current population. Letting $P(t) = at + b$ since P is linear gives us

$$I = \frac{R_0}{P_0}\int_{17}^{37}(at+b)dt = \frac{R_0}{P_0}\left[\frac{at^2}{2} - bt\right]_{17}^{37} = \frac{R_0}{P_0}(540a + 20b).$$

Since we have a, b, R_0, and P_0, we can calculate the total increase in each of the ten corridors.

Figure 5: Increase in Ridership Per Corridor

Corridor	Increase in Ridership
1. Southeast Corridor	4,967,269
2. California Corridor	14,343,802
3. Pacific Northwest Corridor	23,931,512
4. South Central Corridor	15,708,105
5. Gulf Coast Corridor	1,599,636
6. Chicago Hub Network	23,179,845
7. Florida	5,296,570
8. Keystone Corridor	25,670,225
9. Empire Corridor	14,736,886
10. Northern New England Corridor	30,152,224

Thus, we can show that ridership for each of the ten corridors. In some corridors, especially the Northern New England Corridor, this increase has been drastic; over the twenty-year period, we predict an increase of over 30 million rides on the Amtrak above what would have occurred with conventional low-speed trains.

2. Second Model: Cost of High-Speed Train Network

The cost of constructing a train network has two main components: a fixed cost of building the infrastructure (such as railway tracks) and variable costs that depend on usage of the rail system (such as fuel, operators, and additional trains).

The fixed cost of implementing a High Speed Intercity Passenger Rail Line is dependent on several factors that alter estimates within each corridor. Each rail line must be engineered in accordance to the sloping topography of the land, geographical obstacles, natural disaster prevention, humidity, urban centers that are not stops on its route, and safety specifications as directed by federal and state regulation. For example, the proposed cost of high speed track in the Florida corridor is between \$22 million to \$27 million per mile ("Final Environmental Impact Statement") but the proposed cost of high speed track in the California corridor is approximately \$67 million per mile (California High-Speed Train Business Plan). This difference can be largely attributed to topographical difficulties that would increase the cost of both planning and writing blueprints for the design of the railroads, as Florida's range in elevation is

344.5ft, while California's range in elevation is 14,787.2ft (Wolfram Alpha). ~~Geographical factors can increase the fixed cost of high speed rails in two ways. First, because~~

Rail lines can either be laid new or can be refurbished from tracks that are calibrated for older, slower trains and prepared for high speed use. In all ten corridors, there are varying mileages of existing track systems that can be reformatted for use by high speed models. It is true that some railroad tracks already in use are better maintained than others, and so many would have to be stripped and re-laid instead of simply reformatted, but even so, the land does not need to be purchased or cleared like when being built new.

The cost of building the railroads in each corridor is dependent on distance of the proposed lines in each corridor. In the second column of the following table are total costs of building the railroad systems in each corridor as proposed by different governmental organizations and cited by the American High Speed Rail Alliance's website. Total mileage of track is recorded in the third column. Cost per mile of track was calculated from total cost divided by total mileage. This figure does not account for new or refitted tracks, since these are different for each corridor, and the difference does not matter in terms of cost of implementing a full high speed passenger transit system. A global assumption is that tracks would be reformatted where possible instead of building new tracks, to lower total cost as much as possible.

Figure 6: Cost Per Mile in Each Corridor

	Total Cost	Total Miles of Track to be Built	Cost Per Mile of Infrastructure to be Built
California	$40 billion	1715 miles	$23.32 million/mile
Florida	$4.28 billion	360 miles	$11.90 million/mile
Southeast	$5.05 billion*	1964 miles	$0.4058 million/mile
Keystone	$.145 billion	259 miles	$0.5598 million/mile
Gulf Coast	$12 billion**	1022 miles	$12 million/mile
Northern New England	$13.0953 billion	751 miles	$17.44 million/mile
Pacific Northwest	$17.9 billion	310 miles	$57.74 million/mile
South Central	$24.95 billion	1186 miles	$10.11 million/mile
Chicago	$7.7 billion	2468 miles	$3.12 million/mile
Empire	$8.2 billion	463 miles	$17.71 million/mile

**Construction costs for rails in South Carolina and Georgia cannot yet be determined. $5.05 billion is the average between the two proposed values, $2.6 and $7.5 billion.*

***Cost for implementing Gulf Coast high speed rail transit has not been proposed. For our model we used the cost per mile for Florida high speed rail because of their topographical similarity and lack of existing rail lines that can be converted.*

A high-speed train car costs about $30-36 million for a 350-seat train ("Maintenance and Operation"), so we will estimate the cost of a 300-seat train at $30 million. Since a train travels an average of 310,000 miles per year, we have a capital cost of $96.77 per train-mile. A train has a capacity of 300 seats but is normally, by our assumption, only 70% full, so it transports on average 210 passengers. Thus, the capital cost of constructing trains is 46 cents per passenger-mile, or $96.60 per train-mile.

The other variable costs (fuel, crew costs, liability, and additional wear and tear on the tracks) vary somewhat between the three types of trains proposed (diesel, electric, and Maglev). All trains had crew costs of about $4.33 per train-mile, passenger services requirements of $1.66 per train-mile, wear and tear of $1.56 per train-mile, and passenger liability of 1.3 cents per passenger-mile (or $2.73 per train-mile), all in 2008 dollars ("Operating Costs"). Adding these together gives $10.28 per train-mile. Furthermore, there are maintenance costs of $11.49 per train-mile for diesel trains, $13.11 for electric trains, and $8.69 for Maglev trains ("Operating Costs"). We multiply these prices by 1.0527 to correct for 5.27% cumulative inflation since 2008 (Bodansky). Diesel trains use 2.42 gallons per train-mile ("Operating Costs"), so at an average price of $4.051 per gallon (Wolfram|Alpha), diesel trains have fuel costs of $9.803 per train-mile at current prices. The Acela train, an electric high-speed train, uses up to 30.3 kW/seat ("Acela Express"), which equates to 43.29 kW/passenger assuming 70% occupancy, and travels at up to 150 miles/hour, so it consumes .288571 kWh/passenger-mile. Electricity prices vary widely and fluctuate hourly but can be approximated by about $0.10/kWh, so the energy cost of an electric train is about $0.0289/passenger-mile, or $6.06/train-mile. Maglev trains use approximately 1.341 the energy of conventional electric trains, as calculated from old price data of $2.61 and $3.50 per train-mile for costs using old energy prices data ("Operating Costs"). We will use our updated cost of $6.06/train-mile for conventional electric trains and the conversion factor of 1.341, so Maglev trains have fuel costs of $8.13/train-mile. This data is summarized in the table below.

Figure 7: Cost For Each Type of Train Propulsion System (in dollars)

	Diesel	Electric	Maglev
Equipment maintenance	11.49	13.11	8.69
Other non-fuel costs	10.28	10.28	10.28
Subtotal (2008 dollars)	22.77	24.39	18.97
Subtotal (2012 dollars)	23.91	25.68	19.97
Fuel costs (2012 dollars)	9.80	6.06	8.13
Total costs (per train-mile)	33.71	31.74	28.10

Thus, the variable costs are $96.60/train-mile/year in capital costs and $26.37-$33.71/train-mile in additional costs.

The following table summarizes the miles of track to be built for diesel powered, electricity powered, and Mag-Lev high speed trains. The values in this table were taken from the United States Government

Accountability Office's Request to Congressional Requesters from March 2009, "High Speed Passenger Rail."

Figure 8: Miles of Track to Be Built with Respect to Type of Power

	Miles of Track for Diesel powered High Speed Trains	Miles of Track for Electricity powered High Speed Trains	Miles of track for Mag-Lev Trains
South East	1673	251	40
California	926	520	269
Pacific Northwest	310	0	0
South Central	0	1186	0
Gulf Coast	1022	0	0
Chicago	2468	0	0
Florida	360	0	0
Keystone	0	259	0
Empire	463	0	0
North New England	751	0	0
TOTALS	7973	2216	309

In accordance with this table, the total number of miles of track to be laid is 10,498 miles. Of this, 76% will be diesel powered. We will simplify Model 4 by calculating using only diesel powered high speed trains.

3. Third Model: Reduction of Dependency on Foreign Oil

We stated in our previous model that electric high speed trains averaged .288571 kWh/passenger-mile and Maglev trains averaged a factor of 1.341 more energy consumption, or .386973 kWh/passenger-mile. One barrel of oil, when burned, produces 6.12 GJ, or 1700 kWh of thermal energy, and fossil fuel-based electrical generation is generally only 33.2% efficient ("Energy Units"), a barrel of oil would produce 564.4 kWh of electricity. Thus, traditional electric and Maglev trains use the equivalents of 5.1135×10^{-4} and 6.85718×10^{-4} barrels of oil, respectively.

By 2025, cars are expected to have an average fuel efficiency of 23.08 miles per gallon, and, on average, a car transports 1.6 passengers (Center for Clean Air Policy). Thus, a car can transport $23.08\times 1.6 = 36.928$ passenger-miles per gallon of gas, so it uses $1/36.928 = 0.0271$ passenger-miles per gallon. Since there are 42 gallons of gasoline in 1 barrel, this equates to $0.0271/42 = 6.453\times 10^{-4}$ barrels of oil per passenger-mile. However, refining a gallon of gasoline requires about 6 kilowatts of electricity (Gateway Vehicle Electric Club), which is equivalent to

$6/564.4 = 0.0106$ barrels of oil. Thus, the total energy consumption of driving is $0.0271(1/42 + 0.0106) = 9.3333 \times 10^{-4}$ barrels of oil.

The energy consumption of diesel trains and planes are calculated in a similar matter as automobiles. A diesel train consumes 2.42 gallons/train-mile, or 0.011524 gallons/passenger mile, so the total oil used is $0.011524(1/42 + 0.0106) = 3.9688 \times 10^{-4}$ barrels of oil per passenger-mile. Similarly, an airplane has an efficiency of 64 seat-miles/gallon (Scott McCartney) and we assume that the average airplane is 70% full, air travel has an efficiency of 44.8 passenger-miles/gallon, or 0.022321 gallons/passenger-mile. Thus, $0.022321(1/42 + 0.0106) = 7.6876 \times 10^{-4}$ barrels/passenger-mile. We summarize the results below:

Figure 9: Energy Consumption of Each Type of Transportation

Mode of Transportation	Energy Consumption (barrels of oil equivalents per passenger-mile)
Electric train	5.1135×10^{-4}
Maglev train	6.85718×10^{-4}
Diesel train	3.9688×10^{-4}
Automobile	9.3333×10^{-4}
Airplane	7.6876×10^{-4}

Thus, any of the three forms of high-speed train significantly reduces energy demands (and thus America's demand for foreign oil) compared to automobile and airplane transport.

4. Fourth Model: Ranking Corridors

The following train ticket fares were obtained for trips on the Acela Express leaving Washington, D.C. at 5 AM (the earliest time available) and at 7 AM (the most expensive time, likely due to rush hour). Since the trains travel at about 80 mph on average, we obtained distances by multiplying travel times by 80/60.

Figure 10: Ticket Price and Trip Distance At Commuter Hours

	Trip Time (minutes)	Trip Distance (miles)	5 AM	7 AM
To Baltimore	40	53.33	$40	$53
To Philadelphia	90	120	$106	$142
To New York City	165	220	$169	$218
To New Haven	265	353.33	$181	$233
To Boston	400	533.33	$190	$244

(Amtrak.com)

The following two equations, acquired through regression, may be used to model the relationship between price *P* (dollars) and time travelled *t* (minutes). At 5 AM:

$$P_5 = -201.026 + 68.010 \ln t.$$

At 7 AM:

$$P_7 = -249.701 + 86.002 \ln t.$$

It is important to note that shorter distances are overall less profitable for Amtrak than higher distances. We attribute this to the phenomenon of economies of scale.

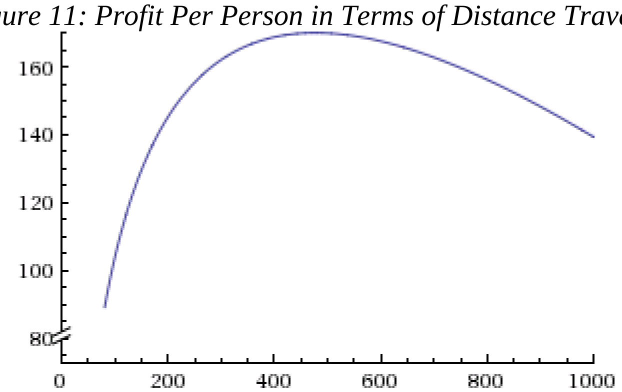
Transforming P_7 to minutes using $x = 4t/3$, $t = 3x/4$, gives us

$$P_7 = -249.701 + 86.002 \ln (3x/4).$$

From model 2, we know that the variable cost of a passenger train is $96.60 in capital per train-mile-year and $33.71 per train-mile. Since a train lasts about 40 years and real interest on municipal bonds averages about 3% per year (Mankiw), we can treat the capital cost as a 40-year mortgage at 3% interest, which would have an annual payment equal to 4.296% of the principal (Bankrate.com). Thus, the variable cost of a diesel train is $37.86/train-mile, which, upon dividing by the 210 passengers, gives us a variable cost of 18.03 cents per passenger-mile. We can subtract this from price received to obtain Amtrak's variable profit per person (R represents total ridership):

$$\Pi / R = -249.701 + 86.002 \ln (3x/4) - .1803x.$$

Figure 11: Profit Per Person in Terms of Distance Travelled



Since R represents total increased ridership over the period, we can multiply R by Π / R to obtain

$$\Pi = R(-249.701 + 86.002 \ln (3x/4) - .1803x).$$

In order to place corridors in order of importance, we created an index on the concept of

$$\text{index } I = \text{benefit to society } B - \text{cost to society } K,$$

again based on the largest MSA within the rail corridor. Given that the purpose of this model is to create a comparative index for which corridor is most deserving of high speed rail (the higher the index, the more deserving it is), we summed the values ignoring units.

Beneficial characteristics include the predicted cumulative increase in ridership and the Transit Score at the train station, which is an index that determines the quality of public transit at a specified location (WalkScore.com). Costly characteristics include the capital outlay required to complete the project.

We choose the following measurement because ridership leads to increased revenue and will cut foreign oil, as well as helping people, which depends on the Transit Score. We estimate that the value of being able to enter a city with good public transportation has a value of $20 per person in combined economic activity stimulated and personal satisfaction. Since the transit score is measured out of 100, we divide the transit score by 5 to convert to a dollar value of $0-20 per person.

Profit is entered into the index equal to its cost, as the index is roughly equal to the economic value of the rail to a city. Profit is roughly equal to $150 per person for the lengths of rides that most passengers will choose, as shown in Figure 11.

The reduction in oil consumption from planes to diesel high-speed trains is about 4×10^{-4} barrels of oil per passenger-mile, which, since most trips are less than 250 miles, the reduction in oil is less than 0.1 barrels. Since we could reduce our future demand for foreign oil by stockpiling foreign oil, the cost of reducing oil consumption should not exceed the current price of oil ($106.70/barrel, or $10.67/barrel), far less than the revenue received. Letting I be the cumulative increase in ridership gives us

$$Index = (I \cdot Transit\ Score/5) + (I \cdot 150) + (I \cdot 10) - Cost.$$

This leaves us with the final rankings of:

Figure 12: Ranking of Corridors

Keystone Corridor	4.383
Southeast Corridor	0.082
Florida	-3.399
Chicago Hub Network	-3.528
Empire Corridor	-5.556
Northern New England Corridor	-7.668
Gulf Coast Corridor	-11.715
Pacific Northwest Corridor	-13.592
South Central Corridor	-22.342
California Corridor	-37.418

D. Recommendations

Based on each region's projected ridership figures, the cost associated with construction and maintenance, and the promise of reduction of greenhouse gasses and dependence on foreign oil resources, we recommend, in the order of most deserving to least, that the following corridors receive the most intensive aid for High Speed Rail programs: Keystone Corridor and the Southeast Corridor.

We also recommend that the following corridors, also listed from most deserving to least, do not receive as much funding due to their region's high costs of building a High Speed Rail coupled with low projected ridership: Florida Corridor, Chicago Hub Network, Empire Corridor, Northern New England Corridor, Gulf Coast Corridor, Pacific Northwest Corridor, South Central Corridor, and California Corridor. Although these corridors would likely still benefit from a High Speed Rail system, it is not enough to offset the costs over the first twenty years.

However, there were some potential sources of error in our analysis. As shown by the rising costs in California's plan to build a high speed railroad, the capital costs may differ from the predictions shown here, and they are likely higher than we predicted. If so, our two recommendations may actually be unprofitable for us. Many of the mileages for the routes were also estimated, as the plans for future routes are not yet final. We did not account for fluctuations in the price of a ticket depending on the demand, the ridership.

III. Conclusion

The models created a fairly accurate comparison of the profitability and usefulness of High Speed Rail systems in each of these corridors. The first model estimated the future ridership of the trains and thus assessed their profitability and usefulness to the public. The second model estimated the cost of building and maintenance of a rail system in each of these regions. The third model analysed a rail system's impact on the United States' dependence on foreign oil. The fourth model used a ranking system using the results of the previous models to objectively rank the corridors based on how much they deserve federal funding for these products.

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