M^3 Challenge Runner Up
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Problem: Lunch Crunch: Can Nutritious Be Affordable and Delicious?

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Lunch Crunch:
Can nutritious be affordable and delicious?

Team 2854

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1 Summary

The specter of runaway American obesity promises rising healthcare costs and a decrease in life expectancy. To combat nutritional defects in schools, Congress passed the Healthy, Hunger-Free Kids Act (HHFKA) in 2010. While the policy was designed to aid the school lunch system, critics argue that the law merely exacerbates the problem at the expense of taxpayers. In order to quantitatively reach a conclusion regarding the HHFKA, we analyzed the data surrounding concepts of individuals' having unique needs and the inability of the proposed system to satiate these needs. We then utilized this information in developing our own lunch plan that stayed within the budget, fulfilled nutritional requirements, and appealed to students.

To develop a comprehensive model which could accurately portray the caloric needs of school-aged children, we first determined the basic caloric needs of children, measured as basal metabolism rate (BMR) in $\frac{kcal}{day}$. Traditional methods are highly inaccurate when applied to children because they fail to take into account the extra caloric intake needed to support growth as well as the effects of body composition on BMR. We created a novel mathematical model to determine the BMR of a child aged between 5 and 19 as a function of gender, age and growth rate, height, weight, and body fat percentage. Age and gender were used to categorize the child into one of three age groups, and growth rate was determined in each of these age groups. Height, weight, and body composition were incorporated as correction factors as well. This basic caloric need was multiplied by lifestyle factors to represent the contributions that daily activity, other meals, and amount of sleep have on caloric intake. On average, the caloric intake of females ranged from $631.46\frac{calories}{lunch}$–$1,066.72\frac{calories}{lunch}$, while the male caloric intake ranged from $695.88\frac{calories}{lunch}$–$1,169.15\frac{calories}{lunch}$.

The next step in modeling the HHFKA was to determine what percentage of students would have their caloric needs met at lunch. Using the data generated from our updated caloric intake model, it was clear that the current meals provided by the HHFKA would not satisfy the immense caloric intake requirements of high school students around the nation. An analysis revealed that the current policy could only support students who received the suggested amount of sleep and had very little physical activity. However, utilizing a simple random sample revealed that there was a significant difference between the number of students who were sleep deprived and the number of those who got enough sleep. Once the sleep deprivation factor was applied to the daily caloric intake, it was evident that even if the students remained motionless throughout the day, they would not have enough calories to support them.

We then modeled the HHFKA program from a school's point of view, using a constrained optimization model. Each fundamental food group (fruits, vegetables, proteins, grains, and dairy) as well as sodium were constrained using the federal minimums and maximums. We created a model to quantify how likely a student would be to actually eat food by modeling how well it tasted—and therefore how appealing it was—based upon its chemical makeup. The model was then tested for budgets of \$7 and \$6 per week per student, for students in elementary school, middle school, and high school. Although we were able to produce quantified, generalized weekly meal plans, none managed to fulfill the caloric needs of the students: the largest deficit was 832 calories for the high school \$6-per-week-per-student budget. After performing a sensitivity analysis, it became clear that our limiting variables were the sodium maximum and the grain minimum.

Given the failure of the HHFKA to adequately provide for the nutritional needs of the students of America, we are left with no choice but to recommend it be discontinued. Currently, it only serves to drain the budget of local school boards and limit their nutritional options. If it must be continued, we recommend that either the maximum on sodium be lifted, or the minimum on whole grains be lowered. Nevertheless, the optimal solution is simply to discontinue the entire HHFKA.

Contents

2 Introduction

2.1 Background

In 2010, the Healthy, Hunger-Free Kids Act was launched with a goal of improving child nutrition by authorizing funding and setting policies for several core nutrition programs [1]. Launched against the rising specter of American obesity, it promised to "establish national nutrition standards for all foods sold in school during the school day" [2]. This clause has since become a major source of contention. Specifically, controversy arises over whether individual students' caloric needs are being met by the standards set by the HHFKA [3].

Proponents of the HHFKA claim it encourages both the direly needed better nutrition and overall fairness. By encouraging schools to comply with federal guidelines, the HHFKA puts the nutritional fate of the students of America in the federal government's unified and driving hands instead of in the disconcerted hands of the fifty states. It also increases fairness by installing a social safety net of sorts, identifying high-risk areas and helping students who live in them.

Critics of the HHFKA insist it hamstrings local officials, bankrupts schools, and fails to provide even the basics of nutrition to students in need. Local officials, so the argument goes, know what's best for their own school, and meddling from Washington only exacerbates an already bad situation. Furthermore, the schools cannot even afford to pay for these expensive new plans: a budget of $7 per student per week speaks volumes about the collective pocketbooks of America's school system. All this could be tolerated if it brought results, but the critics insist it does not.

2.2 Restatement of the Problem

The USDA has asked us to address many issues concerning the Healthy, Hunger-Free Kids Act of 2010, specifically to:

1. Develop a model to determine the number of calories students need to eat at lunch based on a variety of personal factors: activity undertaken, food consumed, etc.
2. Determine the percent of the student distribution that has their caloric needs fulfilled by the proposed lunch calorie limits.
3. Develop a lunch plan with a weekly budget of $7, and then $6, per student while maintaining nutritional standards.
4. Recommend further modifications to the lunch-plan system.

2.3 Global Assumptions

1. Schools will comply with the federal mandate to adopt the new lunch plans.

   We will assume that the caloric content of the school lunches will be equal to the requirements set by the government. Individual schools will not differ significantly from the requirements.
2. Students all over the country are, on average, only marginally different; there are no major cultural or dietary variations. While local varieties and variances unequivocally exist, in large enough numbers they will conform to the mean.

   Although we can see difference between different schools around the nation, there should be no major differences with respect to the variables that we are looking at.

3 Caloric Intake

3.1 Discussion

As humans, we eat to live. In order to get the energy that we need to carry through our day's work, we have to constantly eat food, as our bodies cannot convert solar energy directly into carbohydrates as plant bodies do. While the amount of food in the world is more than enough to feed the world's population, uneven distribution of the world's resources results in populations with too much food or too little food. On the one hand, if people do not get their necessary caloric intake per day, they will suffer from forms of malnutrition, such as marasmus [26] and kwashiorkor. On the other hand, too many calories result in weight gain, obesity, and possible heart issues.

To help those who lacked sufficient daily caloric intakes, the Obama administration set guidelines for school lunches around the nation. Yet are they providing what these kids need?

3.2 Basal Metabolism Rate

In 1918, the Harris–Benedict equations, which determined a person's basal metabolic rate (BMR) in kilocalories per day, were published [25]. By taking inputs of height, weight, and age into account, the equations were able to estimate the BMR—the amount of energy needed to sustain bodily functions. Scaling factors are applied to the equation to match experimental data. Revisions on these equations were made in 1984 and 1990 to calibrate the data with a larger experimental data set [15]; however, they still do not take variations in growth rate and measurement of body composition into account. Because our model is aimed at determining the necessary caloric intake for children, growth rate plays an extremely important role. The Harris–Benedict equation does not accurately predict that value, as it is not aimed specifically towards any age range and disregards the additional intake needed for children to grow, nor does it take into account lean body mass (LBM) percentage. While recent models, such as the Katch–McArdle equation, have been created in order to determine BMR from LBM, they then neglect to factor in age, height, weight, and gender [27].

3.2.1 Growth Rate and Age

Assumptions

1. We can discount the segment of the population that suffers from disorders which stunt or accelerate the normal rate of growth. The portion of the U.S. population which suffers from such diseases can be considered negligible. For instance, the fraction of the population that suffers from dwarfism is well below 1% [12]. Even considering the area under two tails of a distribution, the proportion falls well below our level of significance.
2. Rate of growth in children is assumed to follow average growth charts provided by the World Health Organization (WHO).
3. Larger and smaller rates of growth occur with similar frequency.
4. Age in children does not change energy requirements for tissue growth. Therefore, average energy expenditure for growth is constant relative to age.
5. Growth rate at a certain age can be summarized on a yearly basis. While the WHO's growth charts provide data regarding monthly growth rates of children, there is insufficient data and time to categorize children of ages 5–19 years by month.
6. Growth rate follows height trends seen in the World Health Organization height-to-age charts.

Consuming enough energy in the form of caloric intake is essential in maintaining normal growth in children. The growth rate, which is defined as the change in a child's height and weight over a certain period of time, is highly dependent on age. This can be seen in the WHO's Height-for-Age Charts [28]. Each gender needs its own model because of fundamental differences between them.

The Harris–Benedict equation to determine the BMR was defined as [25]

$$\begin{aligned} BMR_{men} &= 66.4730 + (13.7516 * weight(kg)) + (5.0033 * height(cm))(6.7550 * age), \\ BMR_{women} &= 655.0955 + (9.5634 * weight(kg)) + (1.8496 * height(cm))(4.6756 * age). \end{aligned} \tag{1}$$

However, this equation reflects the BMR of subjects with an age range of $n = 98$ [15]. While this is sufficient to determine the caloric intake based on age in adults, growth rate is a greater factor for caloric intake in children. Thus, BMR should be dependent on the rate of growth.

The average child has three main patterns of growth between 5 and 19 years of age: growth before, during, and after adolescence [28]. Because the patterns of growth are similar within each age group but very different between them, our group decided to create three separate models of BMR with respect to growth rate. This seemed more feasible than attempting to summarize three fundamentally different patterns within one BMR calculation.

Of both males and females, the height-vs.-age graph is relatively linear until the approximate age of adolescence. The height as a function of age is logarithmic during an interval of approximately 4 years during adolescence. After these 4 years, the rate of growth continues at a constant rate that is far smaller than the first constant value. Thus, the function for BMR contribution of age will be determined by a function which is split into the three age groups, separated into males and females.

Female Age Group 1: 5–11

The World Health Organization provides a Height-for-Age analysis of girls which indicates that girls grow approximately 5.983 cm per year between the ages of 5 and 11 [28]. This data can determine the percentage of growth in a year. According to a study conducted by the Food and Agriculture Organization of the United Nations, each percent cellular growth in cells, consisting of (1) energy used to synthesize tissues and (2) energy deposited into the tissues, expends an average of 3200 kcal [29].

Thus, the model to generate the growth rate factor in BMR among girls ages 5–11 is

$$BMR_{f(1)} = \frac{GR_{f(1)} * E}{h} = \frac{191}{h}, \tag{2}$$

where BMR is in kcal, GR is the constant growth rate of 5.983 cm/year among girls ages 5–11, E is the 3200 kcal energy expenditure for growth, and h is the height of the child in meters.

Female Age Group 2: 12–16

In the following age group, girls 12–16 years of age, the rate of growth decreases as age increases. There is ample data from the World Health Organization to determine the exact growth rate for children of each age. The average growth rate of a girl from ages 12–16 is given by Table 1, according to the World Health Organization [28].

Then BMR will be modeled with the equation

$$BMR_{f(2)} = \frac{GR_{f(2)} * E}{h}, \tag{3}$$

where GR is the varying growth rate according to Table 1, E is the 3200 kcal energy expenditure for growth, and h is the height of the child in meters.

Age (years)	Average Height (meters)	Growth Rate (cm/year)
12	1.517	6.2
13	1.567	5.0
14	1.600	3.3
15	1.618	1.8
16	1.626	0.8

Table 1: Age vs. growth rate in females

Female Age Group 3: 17–19

Between 17 and 19 years of age, girls experience a change in height which approaches zero [28]. This varies based on percentile: girls of above-average height grow less than those of below-average height. The difference in caloric intake due to height percentile is taken into account in the calculation of caloric intake based on height. Thus, the difference due to growth rate is negligible and does not need to be included as a correction factor in the model for BMR.

Male Age Group 1: 5–13

Using the same method employed in the model for girls during their stretches of linear growth, the average growth of boys, in cm/year, is approximately 5.551 [28]. Thus, the BMR of boys aged 5–13 can be determined by the equation

$$BMR_{m(1)} = \frac{GR_{m(1)} * E}{h} = \frac{178}{h}, \tag{4}$$

where GR is the constant growth rate of 5.551 cm/year among boys ages 5–13, E is 3200 kcal, and h is the height of the child in meters.

Male Age Group 2: 14–17

Again using the same method that was utilized to generate a model for girls, we determined the growth rate during this four year growth period for boys using the following table of growth rate with respect to age obtained from the WHO [28]:

Age (years)	Average Height (meters)	Growth Rate (cm/year)
14	1.637	7
15	1.694	5.7
16	1.731	3.7
17	1.753	2.2

Table 2: Age vs. growth rate for males 14–17

Then BMR will be modeled with the equation

$$BMR_{m(2)} = \frac{GR_{m(2)} * E}{h}, \tag{5}$$

where GR is the growth rate according to Table 2, E is the 3200 kcal energy expenditure, and h is height (m).

Male Age Group 3: 18–19

Between ages 17 and 19, girls experience a change in height which approaches zero. On the other hand, in boys, the average value of growth is still significant and must be taken into account in the model. Thus, using previous methods utilized for linear growth, we determined that the portion of BMR for growth in the third age group is

$$BMR_{m(1)} = \frac{GR_{m(3)} * E}{h} = \frac{32}{h}, \tag{6}$$

where GR is the growth rate of 0.389 cm/year among boys ages 18–19, E is 3200 kcal, and h is height (m).

3.2.2 Height and Weight

Energy required to perform vital body functions depends particularly on height and weight. This energy requirement varies between males and females.

Assumptions

1. Because people of all ages perform the same basic bodily functions, the energy required per unit height and weight, not taking growth rate into account, remains constant. This is supported by the Institute of Medicine (IOM).
2. As reported by the IOM, the energy required per unit height and weight is constant with respect to age. Discrepancies in energy can be attributed to growth rate and lean body mass calculations.

Height

The IOM Energy Requirement Estimation uses a chemical tracing tests to determine the effects of height and weight on BMR [30]. The IOM determined that weight factors more heavily into the calculation of BMR for males, while height factors more into BMR in females. The IOM determined that an average of 5.40 kcal/m*day are used in males, and an average of 7.26 kcal/m*day are used in females [30]. Thus, in females,

$$BMR_{height} = 540 * h, \tag{7}$$

and in males,

$$BMR_{height} = 726 * h. \tag{8}$$

Weight

Similarly, the same studies conducted by the IOM determined that males use an average of $15.91\frac{kcal}{kg*day}$, and females use an average of $9.36\frac{kcal}{m*day}$ [30]. Thus, in females,

$$BMR_{weight} = 15.91 * w, \tag{9}$$

and in males,

$$BMR_{weight} = 9.36 * w, \tag{10}$$

where w is the weight in kg.

3.2.3 Lean Body Mass (LBM)

Assumptions

1. The calories burned by a set amount of lean body mass is the same between genders. Thus, the difference between genders is accounted for by the difference in average body fat.
2. A mean value for average body fat percentage is constant and unaffected by age. A study conducted by the U.S. Department of Health and Human Services reported that the average body fat percentage range stays essentially constant in children aged 5–19 [31]. A mean of this range can be taken to determine average body fat percentage.

While height, weight, and age have a strong correlation with the amount of calories needed, the composition of a child's body, in terms of lean muscle or fat, must also be taken into account when calculating the BMR. One pound of lean muscle burns approximately 6.5 calories a day, while one pound of fat burns approximately 2.0 calories a day [32]. Thus, the amount by which a child's lean muscle mass exceeds the average amount must be multiplied by an average parameter of 4.5 and subtracted from his or her BMR.

Female LBM

In girls, the average body fat percentage is 14–21% [31]. Taking the mean of this range, we assume that the average girl will have a body fat percentage of 17.5%. Any body fat percentage value greater than or less than this value will be corrected based on

$$BMR_{LBM} = 4.5 * LBM = 4.5 * w * \left(.175 - \frac{\text{BFP}}{100}\right), \tag{11}$$

where w is the weight of the child in kg and BFP is body fat percentage.

Male LBM

In boys, the average body fat percentage is 9–15% [31]. Using a similar method and an average body fat percentage of 12%, the BMR value in boys will be corrected based on

$$BMR_{LBM} = 4.5 * LBM = 4.5 * w * \left(.120 - \frac{\text{BFP}}{100}\right), \tag{12}$$

where w is the weight of the child in kg and BFP is body fat percentage.

3.2.4 Final Equation for BMR

The final BMR was determined to be a sum of the four previously calculated values, to give the final equation

$$BMR = BMR_{\text{f(n) or m(n)}} + BMR_{height} + BMR_{weight} + BMR_{LBM}. \tag{13}$$

3.3 Lifestyle Factors

In addition to the physical factors previously proposed, there are certain lifestyle factors that will affect the necessary caloric intake of school children. When looking at the myriad of factors that could affect the amount of calories that need to be consumed, there are three major factors.

3.3.1 Assumptions

1. The physical activity constant is about normally distributed.

   If we were to look at the distribution of how much people exercise, we would see a majority of the population around an average value. If we were to shift to the left or right of the value, we would still see a large number of people. However, if we were to move a bit further away from this central value, the number of people who exercise much more or much less than the average would drop dramatically with either extreme. This suggests a normal distribution, where the majority of the data (68% of the data) falls within one standard deviation of the mean. In addition, the magnitude of the slope dramatically increases as we move away from the mean but flattens again at a sufficiently large value.

3.3.2 Physical Activity

To model the impact of physical activity on the necessary daily caloric intake, we introduced a new constant, $k_{activity}$. In many previously used models, including the Katch–McArdle formula and the Harris–Benedict formula, the number of calories needed per day is determined by multiplying the basal metabolic rate (calculated with various formulas) by a scaling factor for level of activity, ranging from 1.2–1.9. In general, the level of activity is simply estimated, but our group decided to analyze it further.

To do this, we must first understand what the values 1.2 and 1.9 represent. 1.2 represents people who have unusually low amounts of exercise or no exercise at all, while 1.9 represents unusually high amounts of exercise. If we assume that the distribution of how much people exercise is about normally distributed, we can express these boundaries in mathematical notation. By assuming normal distribution, we say that unusual values fall two standard deviations or more away from the mean:

$$Bounds = \mu \pm 2 * \sigma, \tag{14}$$
$$1.2 = \mu - 2 * \sigma, \tag{15}$$
$$1.9 = \mu + 2 * \sigma. \tag{16}$$

By solving the following system of two equations, we can say that the constant $k_{activity}$ as a normal distributed value centered at $\mu = 1.55$ with a standard deviation of $\sigma = 0.18$.

Now, to determine how far away each person is from the mean of $k_{activity}$, we have to break down exercise. In general, we need two major types of exercise: high-intensity (tug-of-war and other events that would require quick bursts of energy) and low-intensity (medium-paced jogging and other endurance events). The U.S. Department of Health and Human Services recommends that children get about an hour of exercise per day. In addition, throughout the week, children should get an equal distribution of high-intensity and low-intensity exercises.

Due to this, we can say that the time spent on exercise is similarly a normal distribution, with a mean of $60\frac{minutes}{day}$ and a standard deviation of the aggregate of standard deviations of both high- and low-intensity workouts. Because only variances add, we will express the total standard deviation as

$$s_{total} = \sqrt{s_{low}^2 + s_{high}^2}. \tag{17}$$

As a result, the final z-score will be

$$z = \frac{time - \bar{x}}{s_{total}} = \frac{time - 60}{15.8}. \tag{18}$$

Because the z-score is independent of the value of the standard deviation and mean, we can use this z-score to estimate the value of $k_{activity}$ by simply transferring it over:

$$k_{activity} = z * 0.18 + 1.55. \tag{19}$$

If we combine all of the equations, we find that the value of $k_{activity}$ as a function of exercise time per day, we get

$$k_{activity} = \frac{time - 60}{15.8} * 0.18 + 1.55. \tag{20}$$

3.3.3 Other Meals

The caloric needs of a child for lunch also depend on how many calories the child has already consumed or will consume throughout the rest of the day through a combination of breakfast, dinner, and snacks. The question arises: "How many calories should the school lunch provide?"

For many low-income students, the provided school lunches may provide up to 50% of their daily caloric intake [8]. On the other hand, it has been shown that eating too many calories during the course of one meal will increase the amount of calories converted to fat and cause a feeling of sluggishness for the next few hours [9]. The amount of calories provided by the school meal must fall between the two extremes, providing enough to keep students going but not enough to make them lethargic.

Ultimately, we cannot simply remedy a loss of breakfast or dinner with a surplus of lunch.

3.3.4 Sleep

The amount of sleep that each person gets per night affects both performance and the needs of the person on the next day. It follows that the amount of sleep obtained would greatly affect the caloric needs of school children during lunch. Speaking in terms of the general trend, it is clear that less hours of sleep correspond with a much greater caloric need. However, in a study done by the University of Pennsylvania, it was shown that the amount of sleep deprivation did not significantly affect the caloric needs of a person [10]. In the cases of the control subjects, the caloric intake per day was 2489.4 ± 759.2 calories. However, after a period of extended wakefulness, one day of sleep deprivation, two days of sleep deprivation, etc., it was clear that the caloric intake jumped to about 3000 calories per day but was independent of the number of days of sleep deprivation.

As such, the sleep factor in our model, k_{sleep}, only has two values. If the child is not getting enough sleep per night, $k_{sleep} = 1.2$, indicating that this child needs more calories to function properly. The value of the constant does not vary due to the fact that the caloric intake is independent of how many hours of sleep the child has lost. On the other hand, if the child is getting enough sleep, the value of $k_{sleep} = 1$.

3.4 Final Equation

Based on all of the previously mentioned factors, we can estimate that the necessary number of calories per student can be determined by the following function:

$$\text{Number of Calories} = k_{activity} * k_{sleep} * BMR, \tag{21}$$

where $k_{activity}$ represents the impact that activity will have on caloric intake, k_{sleep} represents the impact that sleep will have on caloric intake, and BMR represents the basal metabolic rate previously calculated.

To determine the average number of calories for lunch only, we will divide the final value by three, as the total number of calories should be equally distributed across the three meals.

By using data provided by [23] for kids with average proportions:

Age	Average Caloric Intake for Males	Average Caloric Intake for Females
10	884.91	874.07
11	932.14	931.55
12	979.63	1,023.77
13	1,064.11	1,070.10
14	1,117.91	1,023.77
15	1,149.74	1,070.19
16	1,156.67	1,034.97
17	1,026.19	1,052.46
18	1,168.25	1,031.92
19	1,169.15	1,066.72

Table 3: Caloric intake for ages 10–19 for lunch

We can only calculate the statistics for males at or above the age of 10 because the formula for body mass index becomes inaccurate when the height and weight drop below a specific minimum. As a result, the statistics for males under 10 become inaccurate and were not displayed.

For all of these cases, the average data from the CDC were used. To calculate the caloric needs under the best conditions, it was assumed that the subjects got the necessary amount of sleep and they did not need to move during the day. For the worst conditions, the opposite were assumed. The subjects were sleep-deprived and needed to move quite a bit. In all of these cases however, it is clear that the current calorie limits will not work. However, the amount of calories estimated makes quite a bit of sense. Although we need around 2000 calories to live, in America, many people consume much more than that to keep going throughout all hours of the day.

It is clear that the standards for calories per lunch meal were based off of average data provided by the CDC. It is most likely that three factors were overlooked or falsely assumed. The first was that students would get more than enough sleep each night. In addition, his standards assume that students have little physical activity throughout the rest of the day. These assumptions would minimize the value of the lifestyle factors, but both of these assumptions are clearly false. The BMR values that were used could have been inaccurate and based on the needs of humans of all ages rather than only children. As a result, the standards provided are too low.

3.5 Sensitivity Analysis

In order to determine which variable plays the biggest role in the necessary caloric intake of a kid, we took one data point and shifted the values of each variable by 5% shifts to see what would happen to the necessary caloric intake.

We did not shift constants such as k_{sleep} or $k_{activity}$, as those cannot be changed. For sleep, there are only two possibilities, the student either gets enough sleep or does not get enough sleep. For $k_{activity}$, the 1.2 and 1.9 factors are already the two extremes, in the case where the person does not move or gets extreme amounts of exercise. As such, it would not make sense to shift these values.

These results carry significant implications for the effectiveness of the entire lunch plan system. For both females and males, weight and height proved the most sensitive inputs, leaving waist size in the dust. This suggests that weight is not related to obesity but is instead an intrinsic function of height. Combined with the relatively low effect waist size had, this suggests the amount of food eaten during lunch is a response to one's body type rather than one gorging oneself. Logically,

Factor	Shift %	Shift in Caloric Intake for Females (%)	Shift in Caloric Intake for Males (%)
Weight	5.00	2.81	1.82
	-5.00	-2.73	-1.79
Height	5.00	2.47	3.09
	-5.00	-2.34	-2.93
Waist	5.00	-0.75	-0.69
	-5.00	0.74	0.67

Table 4: Sensitivity analysis for variables

it follows that school lunches do not encourage unhealthy eating habits, at least in relation to waistband size; instead they support rapidly growing adolescents.

4 Who's Getting Enough

In order to determine the proportion of high-schoolers who are getting enough calories from the standard lunch meal, we must find the proportion of high-schoolers who get enough sleep. This will greatly affect the amount of calories needed. From a simple random sample of 500 students from the census [24], we can see the number of students who are not getting enough sleep ($\leq$ 8 hours).

Grade	Gender	Ratio of Sleep Deprived vs. Non–Sleep Deprived
9	Female	1 : 0
9	Male	1 : 1
10	Female	57 : 4
10	Male	33 : 8
11	Female	34 : 3
11	Male	37 : 9
12	Female	146 : 13
12	Male	102 : 2

Table 5: Sleep deprivation in high-schoolers

In order to test if there is a significant difference between the number of high-schoolers that are sleep deprived and those who get enough sleep, a two-sample t confidence interval was created. The 95% confidence interval spans from $-88.2 \leftrightarrow -4.6$. Because 0 does not fall in this interval, we are 95% confident that the difference in means of the two data sets is not 0. As a result, there is clear evidence that more high-schoolers are sleep deprived. If we use this in our calculations of caloric intake necessary to support function, none of the students in high school can be supported with the 750–850-calorie meal plan proposed by the HHFKA (even if the kids remain inactive for the rest of the day).

5 Lunch Plans

5.1 Assumptions

1. **Schools will follow the USDA's menu [22]** because it is a federal guideline. Even if there are slight variations, the proportion and nutritional value will be similar.
2. **The unit cost of each dish is fixed per location and per portion size.** Prices of food are largely influenced by market forces outside the control of the school food services. These

prices are set by the distributor, not by the schools. In addition, we assume that the unit cost of the food does not change based on the quantity purchased. At the bulk levels of food required to feed large schools, the amount of food purchased should not significantly impact the unit cost.

5.2 Discussion

We developed a model to design the optimal lunch plan for current students—attending grades from kindergarten to eighth—to satisfy the threefold constraints of budget, nutritional standards, and student appeal. Schools are constrained to spending \$7.00 a week; both the federal government [13] and our own model constrain the optimal caloric intake; students need to like the food on their plates, or else they will not eat [14].

5.3 Model

To fit all of these constraints, we developed a constrained linear optimization model. The federal government's Healthy Plate [16] divides up food intake into the categories of fruit, vegetables, proteins, grains, and dairy. The USDA guidelines for schools [13] also mandate a minimum amount of consumption in each of these categories depending on grade level. Collectively, these constraints can be represented with the equations

$$\begin{aligned}
2.5\frac{cups}{week} &\leq \Sigma_{i=1}^{5}\text{fruit}_i,\\
3.75\frac{cups}{week} &\leq \Sigma_{i=1}^{5}\text{vegetables}_i,\\
8\frac{oz}{week} &\leq \Sigma_{i=1}^{5}\text{proteins}_i \leq 10\frac{oz}{week},\\
8\frac{oz}{week} &\leq \Sigma_{i=1}^{5}\text{grains}_i \leq 9\frac{oz}{week},\\
5\frac{cups}{week} &\leq \Sigma_{i=1}^{5}\text{dairy}_i,\\
&\Sigma_{i=1}^{5}\text{sodium}_i \leq 3200\frac{mg}{week},
\end{aligned} \tag{22}$$

with the values of USDA guidelines for the school grades K–5. Other grades can be constrained similarly with the USDA data at the source. Equation 22 shows the values of the corresponding servings. Each equation sums the total of each food group—fruits, vegetables, proteins, grains, or dairy—distributed by the schools in an entire week, from $i = 1$ fo 5, where i is the day of the week from $n = 1$ (Monday) to $n = 5$ (Friday). Each parameterized variable, such as $fruit_i$, is the amount of the corresponding category served on the ith day of the week. The value of the numerical bounds are from the USDA's recent nutrition standards for school districts for the new national school lunch program [13]. Note that only the protein, grains, and sodium categories contain upper limits, while others contain only a lower limit.

An additional constraint was required to enforce the given school budget of \$7.00 per week per student via the equation

$$\Sigma_{i=1}^{5}\text{cost} \leq \frac{\$7.00}{week * student}. \tag{23}$$

With the constraints given, a linear optimization model is constructed to maximize the amount of calories distributed via school lunches across a whole week. A data set of potential entree and side dish selections was compiled using the USDA's proposed school meal plan options. The meals offered in the plan span a broad range of styles and tastes and offer a significant amount of variety in choice. The optimal meal plan will consist of a linear combination of these potential options, where the constant factor of each term represents the number of times that item will appear in the cafeteria per week. These constant factors must be positive but need not be whole number values, as it is possible for an item to appear as a reduced portion size on the menu. There is an upper limit to the constant values, as it is undesirable for one food to be repeated too many times in any given week. Since in the USDA's proposed school meal plan each item is repeated a maximum of 2 times a week, our linear optimization model uses this as its upper bound. By summing up the terms of the linear combination, a value of the amount of calories contained in the plan can be determined and maximized.

5.3.1 Caloric Scaling Factor

Maximizing the output of the linear combination of calorie values as-is rests on the assumption that all of the calories offered will be consumed by students. In reality, not every student will eat all of the food on their plate. A recent study on the effects of the USDA's new regulations found that a significant percentage of food is discarded uneaten by the students [14]. As a result, the amount of calories served to a student is not reflective of the actual amount of calories consumed. To account for this waste, we introduced a unitless caloric scaling factor P representing the average proportion of food actually consumed by a student. Instead of maximizing solely the linear combination, we instead maximize the value of the expression

$$\Sigma(P \times calories), \tag{24}$$

where Σ represents a summation over all terms of the linear combination, or a summation over all the menu options.

Although each individual student will have his or her own preferences regarding the amount of food they consume, this overall probability approach serves as a general measure of food consumption across the whole of the student body. Since it is highly unlikely that all students will consume the whole of a given dish and the proportion consumed drops off rapidly with decreasing attractiveness of the food, the value of P varies as a rising exponential decay function of the form

$$P = 1 - e^{-\text{taste}}, \tag{25}$$

where the taste coefficient, *taste*, is reflective of the overall attractiveness of the food. This form may allow a food to experience a negative probability of being eaten, but if it does it will have no consequence for the model: any food scoring that low would not have been picked in the first place.

Of the five components comprising the sense of taste (sweetness, sourness, bitterness, saltiness, and umami), sweetness, saltiness, and bitterness are the most relevant [17]. Vegetables are generally more bitter than other foods [18], primarily due to their relatively high calcium content [20]. Calcium, therefore, contributes negatively to the overall taste of the food. Sweetness is strongly related to sugar content, which thus contributes positively to the taste coefficient. Saltiness is a positively tasty characteristic as well [21]. With the three variables of sugar, sodium, and calcium content, the taste coefficient can then be modeled as

$$\text{Taste} = A * \frac{sugar}{cup} + B * \frac{sodium}{cup} + C * \frac{calcium}{cup}, \tag{26}$$

where A, B, and C are the coefficients relating sugar, sodium, and calcium content to the taste coefficient. These values were obtained via calibration with existing study data on the rate of food waste per food group category [14]. We averaged the sugar, sodium, and calcium content—obtained from FDA labels [19]—of the items from the USDA's recommended menu [22] and stratified the averages by food group. We then set the P equations equal to the percentages from Cohen's study and solved the system of linear equations

$$\begin{aligned} 0.403 &= 1 - e^{-(A*3.58+B*194+C*35.3)}, \\ 0.569 &= 1 - e^{-(A*22.4+B*22.7+C*15)}, \\ 0.736 &= 1 - e^{-(A*1.29+B*276.6+C*8.05)} \end{aligned} \tag{27}$$

to determine the values $A = 0.0445$, $B = 0.00513$, and $C = -0.0181$ for the coefficients. A and B have positive values, and C is negative, confirming the original correlation estimate. The final equation for P becomes

$$P = 1 - e^{-(0.0445*\frac{sugar}{cup}+0.00513*\frac{sodium}{cup}-0.0181*\frac{calcium}{cup})}. \tag{28}$$

Since the values of the variables *sugar*, *sodium*, and *calcium* vary per dish, P must be calculated individually for each type of food on the menu.

5.3.2 Calculations

The full model to be evaluated via linear optimization consists of

- maximizing the output of equation 24 containing the calorie count and caloric scaling factor,
- constraining the linear combination by the constraints given in equations 22 (USDA guidelines) and 23 (cost budget), and
- bounding the values of the linear coefficients by the range $[0, 2]$ as stated.

The linear optimization was performed in Python using the scipy.optimize module of the SciPy library. The Sequential Least Squares Programming method was selected as the optimization algorithm in order to incorporate constraints and bounds on the multiple variables.

The model was run for each of the three school grade ranges specified by the USDA and also output by the first model given in this paper: grades K–5, grades 6–8, and grades 9–12. In addition, the model was run for two different proposed weekly student food budgets: \$7 per student per week and a lower budget of \$6 per student per week. The resulting food calorie totals $calories_{food}$ were compared to the required amounts as determined by Model 1. This caloric deficit, the shortage in the amount of calories provided by school lunch, is calculated via the equation

$$deficit = \overline{calories}_{intake} - calories_{food}, \tag{29}$$

where $\overline{calories}_{intake}$ is the average of the necessary caloric intakes for the given age/grade range, as determined by Model 1 and shown in the table. Age was approximately mapped to grade via the equation

$$age = grade + 5, \tag{30}$$

and the value of $\overline{calories}_{intake}$ was determined via the equation

$$\overline{calories}_{intake} = \frac{\Sigma_{age=x}^{y}\,(calories_{intake,age,male} + calories_{intake,age,female})}{(y-x)\times 2}. \tag{31}$$

The results of these 6 total model runs and their corresponding comparisons are shown in Table 6. Calories per Day represents the output of the model, while Average Caloric Intake represents the value of $\overline{calories}_{intake}$.

Grade Range	Weekly Budget ($)	Calories per Day	Average Caloric Intake	Deficit
K–5	$7	423	761	338
K–5	$6	388	761	373
6–8	$7	462	992	530
6–8	$6	420	992	572
9–12	$7	356	1083	727
9–12	$6	251	1083	832

Table 6: Results of the linear optimization model run across grade levels and various student food budgets

	K-5 / $6 BUDGET	9-12 / $7 BUDGET
Entrees	➢ 2 Turkey Sub Sandwiches ➢ 0.3 Servings of Burrito ➢ 2 Hamburgers ➢ 0.7 Servings of Baked Fish Nuggets	➢ 1.5 Turkey Sub Sandwiches ➢ 1.1 Servings of Spaghetti ➢ 0.3 Servings of Burrito ➢ 2 Hamburgers ➢ 2 Servings of Baked Fish Nuggets
Sides	➢ 2 Servings of Green Pepper Strips ➢ 1.3 Servings of Cooked Green Beans ➢ 1.2 Servings of Broccoli ➢ 2 Servings of Cauliflower ➢ 0.5 Servings of Canned Peaches ➢ 2 Servings of Sweet Potato Fries ➢ 1.9 Servings of Whole Wheat Rolls ➢ 2 Servings of Canned Pears	➢ 2 Servings of Green Pepper Strips ➢ 2 Servings of Cooked Green Beans ➢ 2 Servings of Broccoli ➢ 2 Servings of Cauliflower ➢ 2 Servings of Kiwi Halves ➢ 0.7 Servings of Cooked Corn ➢ 1 Serving of Canned Peaches ➢ 0.3 Servings of Sweet Potato Fries ➢ 2 Servings of Grape Tomatoes ➢ 2 Servings of Whole Wheat Rolls ➢ 2 Servings of Canned Pears ➢ 1.9 Servings of Whole Wheat Rolls
Beverages	➢ 2 Servings of Skim Milk ➢ 0.3 Servings of Low Fat Milk	➢ 2 Servings of Skim Milk ➢ 0.4 Servings of Low Fat Milk

Figure 1: Weekly food plans for K-5 / $6 budget and 9-12 / $7 budget

The generated weekly food plans for two representative model runs—K-5 / $6 budget and 9-12 / $7 budget—are shown in Figure 1. The other 4 food plans are omitted due to space concerns, but each follows similar trends and patterns in terms of food distribution. The food plans summarize the number of servings of given foods that should be given each week in order to satisfy the USDA and budget constraints while providing the maximum amount of calories to students as possible. In use in a school, the food plan will need to be divided up by day of the week, and the food and drink portions divided up appropriately.

5.4 Sensitivity Analysis

In order to help determine which constraints have the greatest impact on the results of the model, a sensitivity analysis was performed on the parameters of the model. Given that the grades 9-12 / $6 output has the highest caloric deficit of 832 calories, the analysis was performed specifically on this model run in order to search for potential remedies for the deficit. Each of the various constraints and parameters of the model were adjusted upward and downward by 20%, and the model was rerun to evaluate the relative effect of each of the variables on the model output. Since some of the constraints were upper bounds and others lower bounds, increasing certain variables resulted in a lower output, while increasing others resulted in a higher output. The graph in Figure 2 reflects only the positive spread of the model output as a result of the 20% positive and negative shift.

From the sensitivity analysis, it is determined that the two variables with the greatest constraining factors on the model are the USDA minimum grains and maximum sodium requirements. The values of A, B, and C on the graph represent the three scaling factors within the model of P, the caloric scaling factor. According to the sensitivity analysis, shifting these variables—representing

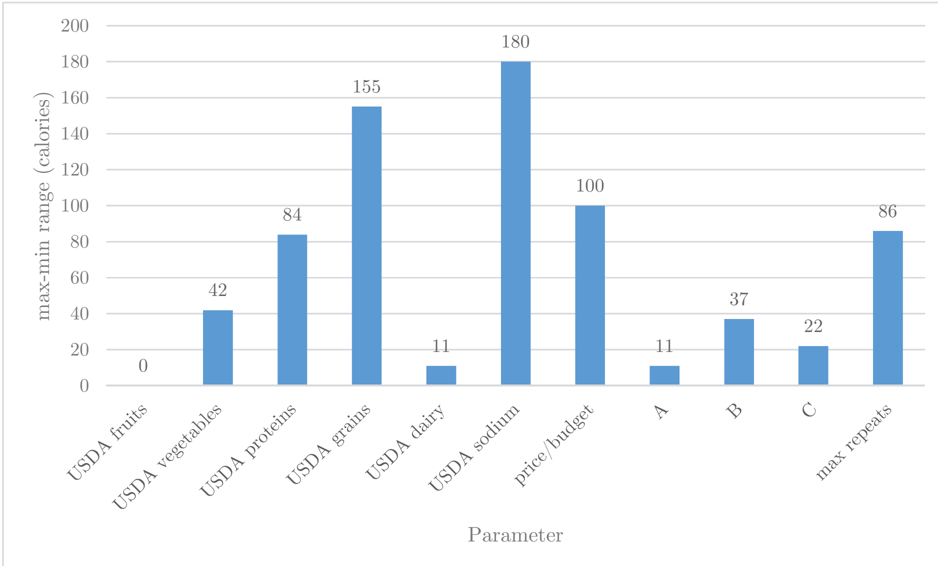

Figure 2: Sensitivity analysis of Model 2 constraints and parameters, with 20% positive/negative shift

a shift in perception and increase in awareness of nutrition by students—will have a small impact on the amount of calories they consume, but not nearly as significant as the impact of the USDA constraints. Allowing for additional repeats of menu items per week will allow for more healthful items to be served more frequently, but this could lead to decreased attractiveness to students and result in correspondingly lower values of the caloric scaling factor P.

5.5 Discussion

It is immediately clear from the model results in Table 6 that, given the current constraints, it is not possible to offer students even a significant portion of the amount of food that is required to sustain their diets. Given the results of the sensitivity analysis, the requirements on sodium and grains must be reevaluated in depth. The severe constraint imposed by the sodium cap can be interpreted in two ways. In one sense, the USDA limit is too strict and limits the flexibility of school districts in implementing attractive student meal plans, and may thus be a contributing factor why the new school lunch program has been so poorly received nationally by students. Yet on the other hand, it shows how prevalent the use of sodium is in the production of foods, particularly the highly processed and frozen foods employed by school districts to minimize costs of production.

The created model can be generalized to solve for an optimal food plan given any set of input food menu options. Any menu item can be entered into the system, given only that the variables of fruits, vegetables, proteins, grains, and dairy servings and sugar, sodium, and calcium content are known. In a school district, the model can be utilized to take the offerings of a food distributor and narrow down the specific options that will best satisfy the needs of their own students.

6 Further Studies

A more thorough analysis of additional variables in this study would reveal that other factors may affect the caloric needs of students. Many medicines and supplements affect both the nutritional and caloric requirements for children, and illnesses could have a potential effect on both calorie expenditure and ability to perform physical activity. In addition, common food allergies and diet restrictions were avoided when generating a meal plan, but further exploration of allergies and diets, including nut allergies, lactose intolerance, and gluten-free diets, could produce a more comprehensive meal plan. Exploring the relationship between genetics and daily nutritional needs would further elucidate this complex problem. However, due to time constraints, this element was left out of the model. Our group attempted to compensate for this by incorporating the influence of body composition on caloric intake needs. Additionally, a study relating stress levels and caloric needs would provide additional information.

Due to time constraints, the problem of obesity in the U.S. was not tackled, even though the inclination of overweight individuals toward weight loss would have an impact upon the model. Further studies on the effect of advertisement and brand recognition on the appeal of food to children could also help to produce a more effective meal plan. In addition, familiarity could play a role in the appeal of a lunch plan for students. This is a result of the exposure effect, a psychological phenomenon where preference for a stimulus increases with exposure [7].

We could also rework our model by challenging the assumption that people all over the country were essentially the same, even though local variations on dietary culture could prove incredibly significant. In the future, we would like to observe the effects regional differences have on the outcome of the model, as well as differing socioeconomic conditions. However, given the utter paucity of the current results, it is unlikely the addition of either of these two would have changed the conclusion.

7 Recommendations and Conclusion

In our investigation, we have found that the HHFKA is at best an ineffective measure to implement urgently needed nutritional policies. At worst, it is actively harming both the students and schools it affects. Primarily, the basal metabolic rate, the minimum number of calories needed to be consumed in order to maintain vital body functions, was calculated for children between the ages 5 and 19 as a function of gender, growth rate, height, weight, and body fat percentage. Then this value was multiplied by lifestyle scaling factors of physical activity, calorie intake during other meals, and amount of sleep. As a result, it was determined that at the current caloric needs, no high schooler would be satisfied with the lunches provided by the Obama administration. Assuming the best case scenario (adequate amounts of sleep and no movement during the day), the lunches barely meet the caloric requirements. However, considering most high-schoolers are severely sleep deprived, the number of students who have their caloric intake needs satisfied is negligible compared to the population.

On a national level, we strongly recommend that the Obama administration stop enforcing the HHFKA. Rather than implementing beneficial nutritive policies, it is actively encouraging harmful. If it insists on continuing, it should encourage the creation of low sodium foods, and champion increasing the tastiness of existing foods to reduce the amount of food which is wasted. Nevertheless, these measures are at best stopgap, and we wholeheartedly recommend the discontinuation of the program.

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