M^3 Challenge Fifth Place Team
Exemplary Team Prize: $5,000

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Problem: Lunch Crunch: Can Nutritious Be Affordable and Delicious?

****Note: This cover sheet has been added by SIAM to identify the winning team after judging was completed. Any identifying information other than team # on an M^3 Challenge submission is a rules violation.*

Team #3015
Lunch Crunch: Can Nutritious be Affordable and Delicious?

SUMMARY:

Unhealthy nutrition is a pressing yet often overlooked problem in America today. However, efforts to address this problem, such as the Healthy, Hunger-Free Kids Act of 2010, which promotes good nutrition, often face resistance because of the complex network of stakeholders at play. Students, parents, school districts, and all levels of government sometime desire differing goals in the situation, leading to stagnation of progress in the regard. In this report to the United States Department of Agriculture, we have addressed this problem in three different parts.

First, we created a mathematical model which predicts the amount of calories a specific student will need in a school lunch based on his or her personal attributes. We first considered the student's current body mass index (BMI) in relation to a target BMI value for the individual. By making this comparison, we could adjust the student's lunch calorie amount to push them toward a healthy BMI, whether it is a higher or lower one. We then incorporated other variables, including the amount of sleep a student obtains, their height and weight, their level of physical activity, and whether or not they ate breakfast that morning. Testing of this model shows that it can accurately predict how many calories a student will need throughout the day and during lunch.

Next, we detail a process and model to determine the percentage of students who will have their caloric needs met at lunch if they eat a standard school lunch. Overall, we saw that most of the demographics that we discussed in Part I displayed a relatively normal distribution. Age and gender were not included in the distribution modeling, because they are assumed to be uniform. We determined the percentage of the student population to which certain parameters of caloric need applied, and then used these in a model that collectively determined the amount of students that could be nutritionally covered under an 850-calorie lunch program. By treating each demographic separately, we were able to run a density-based normal distribution integral, which gave not only percentage of coverage based on demographic but on BMI within each demographic. We found that only just over 10% of students would have their lunch needs met. However, as our data for daily calorie needs matches well with USDA surveys, and lunch should account for one third of students' daily calories, we determined that school lunches are insufficient for most students.

Finally, we demonstrate a weekly lunch plan that stays within a budget of $7 per student per week, appeals to students in taste and quantity, and meets nutritional standards. First, we divided our search into the five food groups defined by the USDA. Using the Consumer Price Index to find retail prices of various food items in each category, we created a list of over 80 different possible lunch combinations that satisfied the school district's budget constraint. Then, in addition to the variety, we used student preferences to weight the menu options so that food that the students prefer would appear on the menu more often. By inputting all of this information into a program, the program would generate a monthly menu incorporating variety and student preferences while staying within the budget constraint and federal nutrition guidelines. When the budget is reduced by $1, we lose approximately half of our menu variety, but we still manage to maintain enough choices to ensure an appropriate amount of variety for the students.

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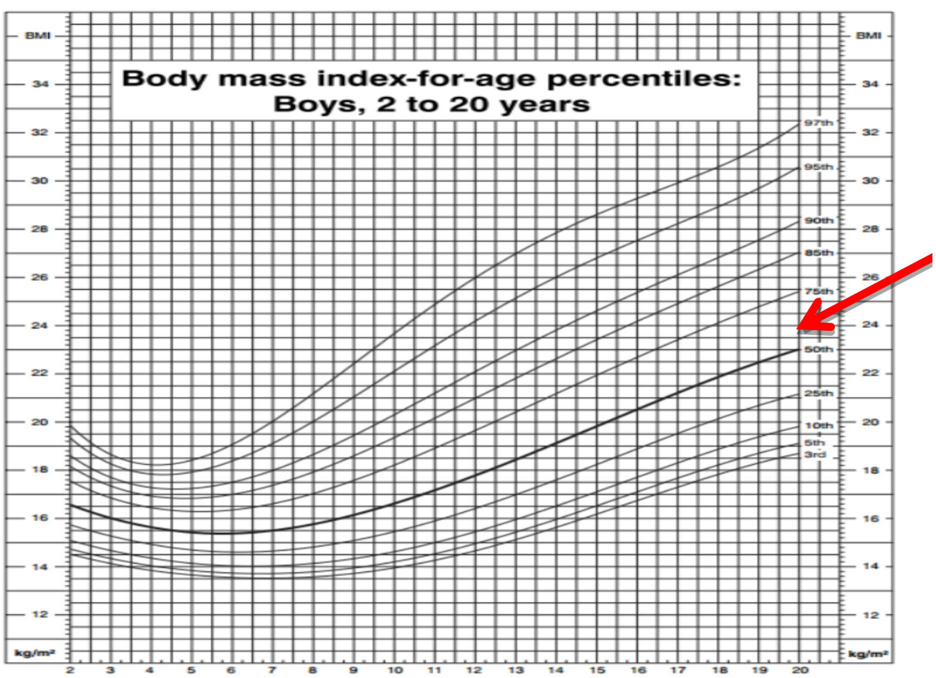
INTRODUCTION:

The search for the mythical "Fountain of Youth" illustrates the basic human desire to live as long as possible. While people may never find the secret to eternal life, they do have the keys to longevity in their hands. One path to maintaining vitality manifests in nutrition. Recognizing the long-term importance and impact of healthy eating, First Lady Michelle Obama spearheaded the efforts that culminated in the passage of the Healthy, Hunger-Free Kids Act of 2010. Unfortunately, as with many well-intended government programs, the Act ran into trouble during its implementation, including an increase in the cost of school lunches and a decrease in school lunch participation. As a result, the USDA has sent out a plea for help in analyzing the problem and finding a solution. We, as Team 3015, have stepped up and answered the call.

PART I: YOU ARE WHAT YOU EAT?

In this part of the problem, the USDA asked us to develop a mathematical model that comprehensively considers multiple attributes of American students to determine the necessary amount of calories that the student should eat during a school lunch. Our team determined a wide range of factors initially but decided that the only factors that we should consider for this section of the model should be those that directly affect a student's lunchtime caloric needs, not the availability of food and calories for a given student at any time during the day. This determines a baseline caloric need for a student with a given set of attributes to maintain a healthy style of living.

Moreover, our team realizes the ongoing and growing issue of obesity in America and saw this opportunity as one to revamp the school food system to help combat this problem. To do this, we decided to include in our model a parameter that includes the difference between a student's current body mass index (BMI) and the healthy BMI for that age group, as determined by the United States Government. The model's output of caloric need would apply slight pressure in a direction that either increased or decreased a student's BMI at a healthy rate toward the target BMI of the age group.



Example of BMI percentile chart used for boys. If a student had a BMI above or below the 50th percentile, "pressure" was applied by adjusting caloric intake to gradually move the student to the healthy range in a responsible manner. Numerous sources cite a BMI between the 5th and 85th percentile as healthy, but to simplify the model, the 50th percentile was used as a marker for the ideal goal.

Figure 1: BMI Percentiles for Age Groups from Center for Chronic Disease Prevention and Health Promotion

Main Assumptions and Justifications:

- Body mass index (BMI) is a normalized comparison of height and weight and is a standard for determining a person's physiological health. We found that the healthy range was the 5th to 85th percentile [6], but as including this range would complicate the model excessively, we assumed that the target BMI would be at the 50th percentile of a given age's range. This is justifiable since applying pressure towards the 50th percentile technically should not detriment one's health.
- The caloric needs of a man differ from that of a woman by approximately 10%, as men burn more calories on average at that rate—a female's metabolic activity is 90% that of a otherwise comparable male [1]. We also assumed that this rate difference would remain constant for all ages and activity levels, which is justifiable since no evidence exists that demonstrates a dramatic fluctuation of this value across these variables. Moreover, this allowed our model to maintain simplicity in this regard.
- Sleep deprivation leads to an increase in caloric need by an average of 5% [2]. We assumed that this point fell at the average sleep deprived value of Americans (2 hours less than needed), and used that to fit the logarithmic submodel for sleep deprivation and its effects on calorie intake [3]. Many sources demonstrate such correlation between sleep deprivation and increased calorie intakes.
    - Our team determined that the values of sleep deprivation did not have an effect on caloric increase that depended on age, as we normalized the deprivation variable by using sleep needed for various ages.
- A person's baseline metabolic rate is primarily determined by his or her weight [4], so our group assumed that weight would be the dominant factor accounted for in this calculation. Height is accounted for in BMI.
- Students' physical activity levels (PAL) can be divided into three different levels for simplicity: low/none, moderate, or high/vigorous. This is justifiable since physical activity varies so widely that exactly pinpointing modelling values for them would be nearly impossible. Moreover, many sources similarly divide physical activity levels like this [6].

Part I Model:

We first determined that the variables that would significantly influence a student's caloric *need* during lunch would include a student's baseline metabolic rate (determined by weight primarily), age, physical activity level (PAL), sleep, BMI (weight to height ratio), gender, and whether the student eats breakfast. We began our model with a process flow chart to outline the cascading effects of variables on caloric need at lunchtime, as shown below (Figure 2).

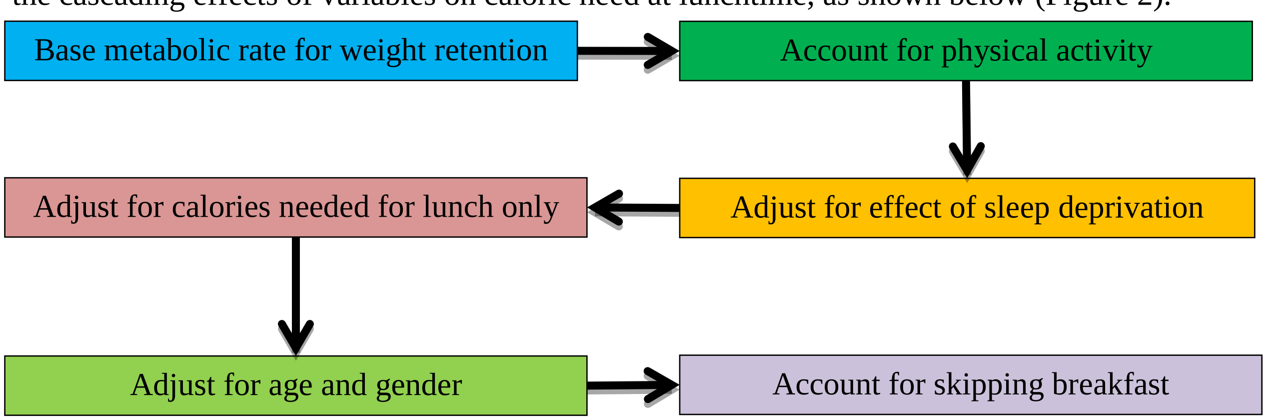


Figure 2: Flow Chart of Model Creation

We first determined the baseline metabolic rate, BMR, to be approximately equal to 10 times a person's weight [4], shown by Equation 1. This BMR is the metabolic rate needed for the body to carry our normal processes such as breathing and pumping of the heart but not other activities such as digestion and movement.

Equation 1

$$BMR \approx 10 \cdot w$$

$$w = weight\ (lbs)$$

Physical activity levels (PAL) of an individual were accounted for next. Calories burned and demanded by physical activity varied with weight [5], so the active metabolic rate (AMR) would be dependent on BMR, PAL, and weight, shown by Equation 2:

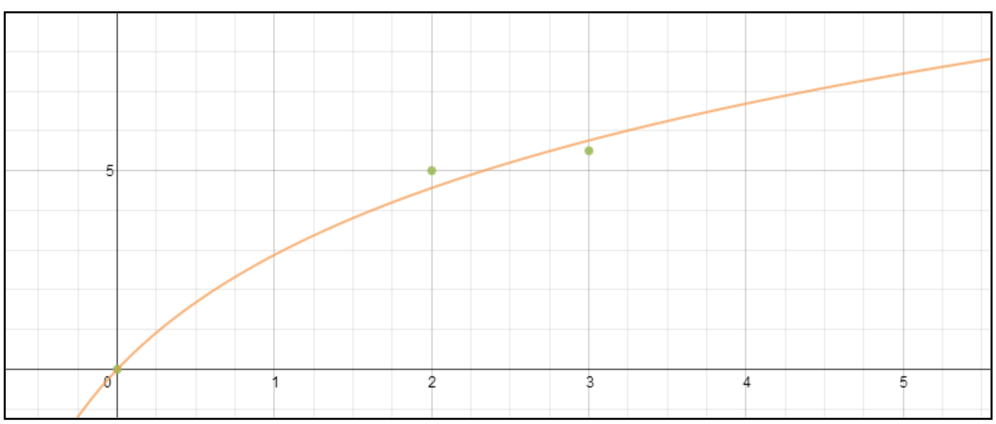
Equation 2

$$AMR = BMR + (PAL \cdot w)$$

PAL is quantized as either low/no activity, moderate activity, and vigorous activity, which were assigned coefficient values of 0.60, 2.31, and 4.00, respectively, correlating with trends observed in data from a Harvard Medical School study relating body weight to calories burned in various levels of exercise [5]. The specific values depended on the slope of graphs based on the data relating calories burned vs. weight (not shown) multiplied by constants based on the average duration of activity at a given PAL, determined from standards from United States Estimated Energy Requirements [6].

Next accounted for was the effect of sleep deprivation on a student's metabolic calorie need. Sleep deprivation will increase a person's metabolic rate, and thus calorie need [6], by an average of 5% [2]. However, we assumed that the amount of extra calories needed to compensate will vary depending on the amount of sleep deprivation, which we took as the amount of sleep less than the needed nightly sleep for an age group. Using data from various sources, we derived a logarithmic sub-model that produced an output of the percent increase in calories depending on the degree of sleep deprivation (Figure 3).

Percent Calorie Intake Increase Needed



Hours of Sleep Loss (Ideal Number of Hours – Hours Slept)

Figure 3: Needed Calorie Intake Increase as Function of Sleep Loss

We assumed a logarithmic model with a y-intercept at (0,0), for having the necessary hours of sleep would lead to no deviation in calorie need. Moreover, the logarithmic function could model the leveling off of effects as sleep deprivation increases, which was seen in our research. The model yielded Equation 3 for sleep deprivation factor (SDF),

Equation 3

$$SDF = 1 + .04155\ln(S_N - S_A + 1)$$
$$S_N = Sleep\ needed\ (hours)$$
$$S_A = Sleep\ achieved\ (hours)$$

where the S_N (sleep needed) values are given based on age groups: 11 hours for 3–6 year olds, 10.5 hours for 7–12 year olds, and 8.5 hours for 13–18 year olds [7]. The model would then entail multiplying the active metabolic rate by the sleep deprivation factor to yield an adjusted calorie intake (ACI):

Equation 4

$$ACI = AMR \cdot SDF.$$

Our team assumed that if a student gets more sleep than is needed, this would provide no effect to the calorie needs, and in this case, the SDF would equal 1. Thus, the window for the logarithmic submodel is detailed as $0 < t < h$, where 0 is ideal sleeping conditions and h is no sleep achieved by the student at all.

Next, we adjusted the model to apply pressure for a healthy BMI. We did this by using a factor called ideal mass deviation (IMD), given by Equation 5:

Equation 5

$$IMD = (BMI_T - BMI_C) \cdot 22\ lbs \cdot h^2$$
$$BMI_T = 50th\ percentile\ BMI = Target\ BMI\ (\tfrac{kg}{m^2});\ BMI_C = Current\ BMI\ (\tfrac{kg}{m^2}); h = height\ (m).$$

This value would then be added to the adjusted calorie intake to give a total amount of calories needed to work towards a given BMI.

Our group next began the final adjustments of the model for age and gender. Based on numerous studies, we found that metabolic rate consistently slows with age, without regard to fitness [8]. Using data from the United States health.gov website on general calorie intake by age for various activity levels, our group determined that the effect of age on metabolism followed an age adjustment factor (AAF) shown in Equation 5:

Equation 6

$$AAF = 3.62 \cdot 0.952^{age}$$

age=age of student (years)

This exponential model represents the constantly decaying metabolic rate of a human being. Intuitively, this decaying exponential model fits the situation and resources found on the subject; however, outside of age values for students (older age values), we realize that the model will lose accuracy since it will continue to decay. However, since this model only concerns predicting values for students who have young ages, interpolation using this function can be safely used.

We then multiply this to the sum of the IMD and ACI, giving ACI_{new}:

Equation 7

$$ACI_{new} = AAF(IMD + ACI).$$

Gender was then taken into consideration. We assumed that the data portrayed in a US National Library of Medicine study showing that women exhibited metabolic rates at approximately 10% below males to be universally true, without any dependence on other factors [1]. Therefore, we used a binary condition for gender, where if the student is male, the new ACI_{new} is multiplied by 1, and for a woman, it is multiplied by 0.9, representing 90%.

This new value represents the daily calorie intake of any person. To adjust this for the lunch-only calorie intake, we multiply the value by 1/3 because the United States government suggests that lunch consist of one third of daily calorie values [9]. However, we acknowledge that many students skip breakfast and must rely on lunch for a higher portion of the total calories. Because breakfast consists of a smaller meal portion than lunch or dinner, and because dinner is suggested to account for nearly half of daily calorie intake [9], we assumed it to account for one fourth of daily calories. We then assigned another binary condition, where if the student skipped lunch, we add 1/4 to the 1/3 lunch factor by which we multiply the daily calorie intake. This yields a final equation, given by Equations 8 and 9.

Equation 8

$$Calorie\ at\ Lunch = \left(\frac{1}{3} + \frac{B}{4}\right)(ACI_{new})(Gender)$$

B = breakfast (0 for breakfast eaten, 1 for breakfast not eaten)

Gender = 1 for male, 0.9 for female

Equation 9

$$(\{[(PAL \cdot w) + (10 \cdot w)] \cdot [1 + 0.04155 \ln(S_N + S_A + 1)]\} + [(BMI_T - BMI_C) \cdot h^2 \cdot 22]) \cdot [3.62(0.952)^{age}] \cdot (gender) \cdot (\frac{1}{3} + \frac{B}{4})$$

(Variables defined in equations above)

PART I: TESTING AND VALIDATION OF MODEL

The model was then tested by comparing output values of daily calorie intake for an "average" student (meaning BMI=ideal 50th percentile BMI) for ages between 5 and 17 to data found from the USDA's 2010 Dietary Guidelines for Americans report. We found it simpler to test our models outputs for daily calorie intake in place of calorie intake for lunch since that data was more readily available and reputable. Moreover, the daily calorie values should directly correlate to the lunch values, since the lunch values are theoretically one third of the daily values.

Figure 4: Predicting Daily Calorie Needs for Male Students with Low Activity Level

Age	Weight (lbs)	Predicted Daily Calories Needed	Actual Average Calories Needed (health.gov source)	Difference (Residual)	Percent Difference
5	41.8	1254	1200	-53.7	-4.5
6	46.2	1319	1400	80.6	5.8
7	50.6	1376	1400	24.1	1.7
8	57.2	1481	1400	-81.0	-5.8
9	61.6	1519	1600	81.3	5.1
10	70.4	1653	1600	-52.6	-3.3
11	77	1721	1800	78.9	4.4
12	93	1931	1800	-131.4	-7.3
15	110	2071	2100	28.9	1.4
17	140	2390	2400	10.1	0.4

Shown above in Figure 4 is a table of calculations used to test the model created. Figure 4 specifically depicts the situation of low activity male students over various ages. Predicted values were subtracted from actual values from the USDA source, with residuals being plotted in Figure 5 below. Similar tables were made for each type of situation with male and female students at low, moderate, and high levels of activities. For each situation, low residuals were seen, as they fell within the margin of error of the given statistics. Moreover, no discernible or obvious pattern in seen in the residual plots, supporting the created model's accuracy and feasibility. Finally, almost all percent differences for each test set were less than 10%, strongly supporting the accuracy of the model in regards to predicting calorie intake for a student not only during the day but also for the given problem of lunch.

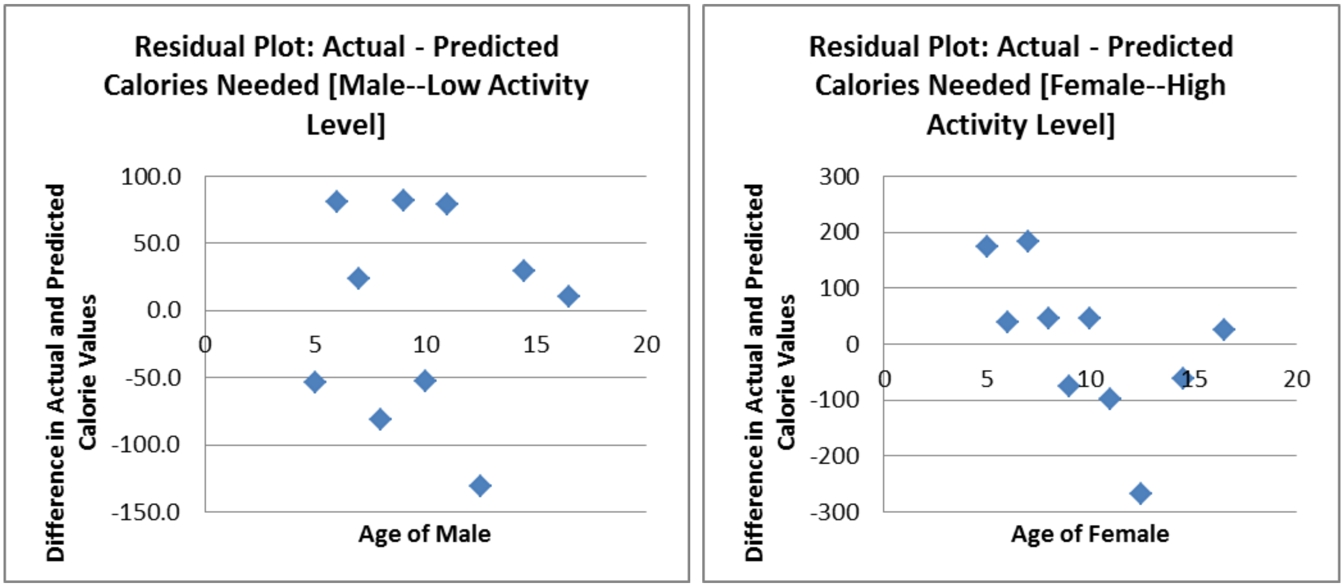


Figure 5: Needed Calorie Intake Increase as Function of Sleep

Extensive testing was then conducted on the rest of the model, including plugging in values for sleep deprivation levels, whether breakfast was eaten or not, and deviation from ideal BMI levels. Testing was also done using the "Census at School" Random Sampler data provided [11]. Yet, since there was no concrete data to compare these values to, testing was considered in terms of relative feasibility. Due to space constraints, these tables are not shown. However, testing does show feasibility of values produced for the output of lunch calories needed for a student based on his or her certain attributes. Testing can continue in the future as more data in collected by outside sources on variables that are included in our model; this will allow for a standard of comparison.

PART I: SENSITIVITY ANALYSIS OF MODEL

For sensitivity analysis of the model, inputs were changed in value and the resulting output values were considered to see how drastically these changes would affect the final results. First, the deviation in the BMI from ideal conditions for the age group of the student was changed (from –10 to 10, covering from the 1st to the 99th percentile in Figure 1).

Our model can be seen to be not very sensitive to even large changes in the BMI level of the student. Even in extreme cases, our model stays within the range of a normal lunch calorie level of 750–850 calories as determined by government standards [10]. Other variables were also tested, including weight, height, physical activity level, and sleep levels. Small changes in these variables also showed no dramatic changes in the output value of the model, demonstrating that our model is not unfavorably sensitive to small changes. However, the binary variable of whether the student has eaten breakfast or not was seen to cause the model to be sensitive: changes from "yes" to "no" in this variable caused relatively large changes in the output of the model. Intuitively, this makes sense, since breakfast will account for 25% of a student's calorie needs [10], so missing this meal will cause lunch values to be changed by a large percentage as well.

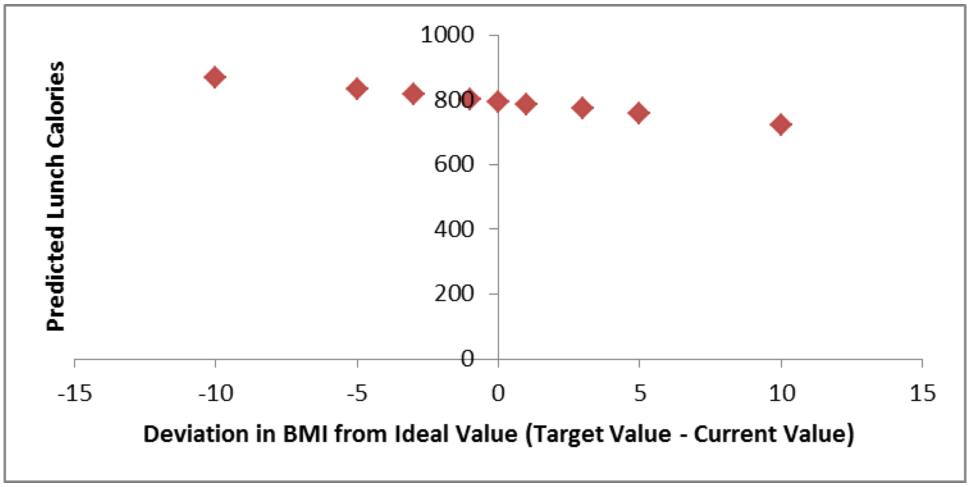


Figure 6: Changes in Lunch Calorie Prediction as Function of Deviation of BMI from Ideal Value

Sensitivity analysis was also done on one the assumptions made in the model: that females had a value for their metabolic rate of 90% of that of men, which led to their calories being multiplied by this amount each time. Although quite evident and obvious, we tested how changing this value in small amounts would affect the final output; it was observed that the final value predictably changed in direct correlation. However, we believe this assumption to be strong, so this value should not change much in practice.

PART II: ONE SIZE DOESN'T NECESSARILY FIT ALL

In this part of the problem, we first used the attributes from our model and determined the distribution of high school students in the United States among each of these attributes. While the caloric needs of an *average* student are met based on the guidelines of the Healthy, Hunger-Free Kids Act of 2010, many *individual* students have several aspects (identified in Equation 9) where their caloric needs are greater than the calories of the school lunch. After modeling this, we were then able to determine the percentage of students who actually have their caloric needs met by a school lunch. We have thus divided this section into these two separate parts.

First, we developed a model of the distribution of U.S. high school students based on the factors of caloric needs.

ASSUMPTIONS AND JUSTIFICATIONS : SECTION ONE OF PART II

- We first defined finding the distribution as the percentage of students in each category, so our model is based on the percentage of the student population.
- Furthermore, we assumed that the class size for each grade in high school was the same size and exposed to a similar environment, so that age has relatively no impact on the distribution other than BMI. This is a fair assumption since data show no obvious differences between the number of people in each grade [16].
- The distribution of the BMI curve is a normal distribution, as shown in Figure 8 when we analyzed data of BMI that was based on gender and age [12]. For example, the following figure is the distribution curve of 15 year old males (given standard error, mean, number of trials, and percentile BMI as comparison). Furthermore, since height and weight directly correlate to BMI, they were not included in the model.

- Because the BMI data fits a normal distribution curve, determining the percentage of students for a specific BMI is impossible (since the probability for a single point for an area under a curve is 0). Instead, we used BMI within the range of standard deviations as the percentage of high school students. For simplicity's sake, we also regarded each category as independent of one another and that ratio for males:females was 1:1.

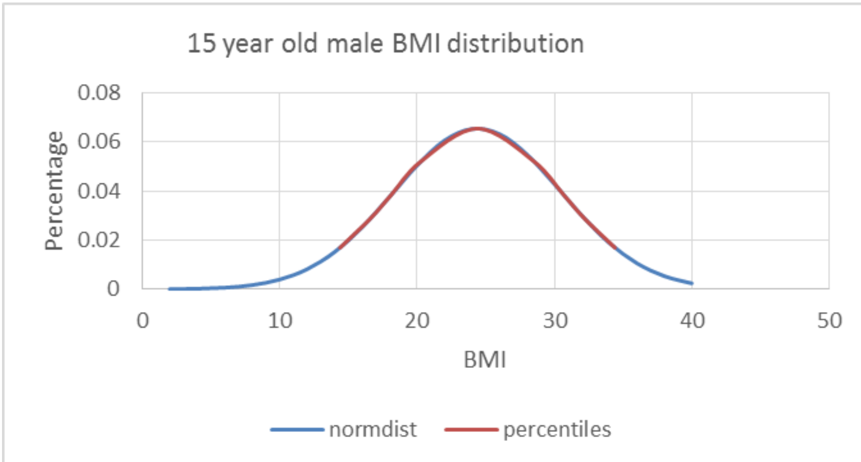


Figure 7: Showing BMI normal distribution

With these assumptions, we narrowed the categories for caloric needs at lunch to BMI, sleep, PAL, and breakfast. Since the model will be used to determine the percentage of high school students for each given category, it will be:

Equation 10

$$\%\,HS\ Student\ Population = \%BMI * \%Sleep * \%PAL * \%Breakfast * Gender * Socioeconomic\ Status$$

For example, the percentage of high school students who have a BMI within a *z*-score of 0 and 1, sleep between 5 and 6 hours, are not physically active, had breakfast, and is female would be: $0.34 * 0.10 * 0.184 * 0.8 * 0.5 = .0025024 = .25\%$

Note that the percentage of students who did not meet requirements for daily physical exercise was 18.4% [13] and that 80% of students eat breakfast [14]. We also added the factor of socioeconomic status, which wasn't included in Equation 9; this is determined to be that 21% percentage of students live in poverty [15].

Sensitivity Analysis of Section One of Part II

Since we determined that all variables were independent of each other and that they are defined discretely and categorically, changing any of the values by extremely small increments would be inapplicable. Furthermore, since all variables are multiplied together, changing any of their values by a small amount would result in miniscule change in the final answer. We tested these small changes in the inputs and observed such small changes in the final output as expected.

Section Two of Part II

Our team was also assigned to determine the approximate the amount of students that our model predicts would be nutritionally covered by the average school lunch. This means that the caloric value of the average school lunch is greater than the caloric need of a student. The average school lunch consists of up to approximately 850 calories [10].

Using our model from Part I of the problem and the distributions established in the earlier section one of Part II, we calculated the percent of each demographic covered by the average school lunched, and proportioned that with the percent composition of the total student population based on the used demographics.

ASSUMPTIONS AND JUSTIFICATIONS: SECTION TWO OF PART II

- The above data showed that the BMI curves of the American public displayed relatively normal distribution. We assumed that this would be true of all demographics and that a change in one variable would not affect the distribution of the BMI.
- We measured the percent of a demographic by using a height chart, calculating the heights for which various BMI values would fit the model under 850 calories. We assumed that the distribution of height was not affected by the other variables in our model.
- We assumed that, like many other variables, the amount of exercise that a person will participate in follows normal distribution.

THE MODEL: SECTION TWO OF PART II

To begin modeling the data across the United States, our team created a program in Microsoft Excel which would input the variables from our model in part one of the problem, and output the lunchtime caloric need. The program ran this for multiple BMI values, each for a variety of heights. From the data, we determined the limiting height for each BMI value, above which the caloric need would exceed 850 calories at lunchtime. We then compared this data to the height distribution chart for a given age to determine the percent of students at a given BMI that the model predicts school lunches would properly feed [17, 18]. After repeating this for BMI values from 15 to 40, our team graphed the percentage of students covered versus BMI and observed a clearly linear trend. For the baseline student, which we decided to be an 18-year-old male with low physical activity and proper sleep and who eats breakfast, the trend in percent of students covered versus BMI could be modeled by the equation

Equation 11

$$\% = 1.089 - 0.114BMI$$

Our group then ran the model for various sleep deprivation levels, physical activity levels, and ages, for both genders and for all grade levels. We noticed that the linear trend in the above equation remained intact for all possibilities. After examining the data, we concluded that changing one of the parameters of the model would alter the above equation by a certain added or subtracted value. For each successive sleep deprivation level, the percent of students covered dropped by 20%; for each successive physical activity level, by 60%; for age 17, by 7.5%; for ages 16, 15, and 14, by 32.5%; and it increased for women by 20%. When a student skipped breakfast, they always needed more than 850 calories to maintain a healthy caloric input. This adjust the above equation to

Equation 12

$$\% = 1.089 - 0.114BMI - C$$

C = Constant decreasing value (assigned to each parameter), where the constant C is equal to the percent drop noticed by the parameter divided by 100 for percentage adjustments.

To use this data to calculate the amount of students that the model believes school lunches will cover, we combined the above equation with a normal distribution graph of BMI to determine the percentage of students covered for each demographic. To do this, we evaluated an integral that weighted the normal distribution with the density function modeled by the above equation, where BMI is the dependent variable. The integral is shown as

Equation 13

$$C_d = \int_0^{40} \left(\frac{1}{8\sqrt{2\pi}} e^{-\frac{(x-25)^2}{128}} \right) \cdot (1.089 - 0.114x - C)dx$$

C_d = Coverage for a given demographic

where the exponential function represents a normally distributed BMI function with extremities 0 and 40, a standard deviation of 8, and a mean of 25 [12]. This yields the percentage of each specific demographic (with a specific decreasing constant value) that will be covered under an 850-calorie lunch. The demographics, in this case, were each unique combination of physical activity level, gender, age, sleep level, and breakfast choice. The second part of the integral was the above equation, which gave the percentage of students covered at given BMI. We ran this simulation using the varying values of C for each parameter, which yielded different percentage coverage for each demographic. As C increased, the coverage decreased, because less students at any given BMI were covered under 850 calories. To determine the total coverage of school lunches under the model, we used a final summation formula that weighted each demographic coverage value based on the prevalence of the demographic in students, shown as

Equation 14

$$Total\ coverage = \sum C_d p_d$$

C_d = coverage of a demographic (%)

P_d = prevalence of a demographic (%).

This model yielded a total coverage value of only *10.43%*. Our group acknowledges that this value was particularly, even frighteningly low. However, we noted that if school lunches provide only 850 calories, and lunch should account for one third of the total daily calories of a student, then a student, on average, should be receiving 1950 calories a day [10]. This value is extremely low, especially for older students, who should, according to a USDA study, be getting 1000 more calories than that a day in many cases [6]. Therefore, it would follow that students should actually get up to 1000 calories at lunch. Our model was much more consistent with the USDA study's values for daily caloric needs, rather than the values set forth by the 850 calorie school lunches. Therefore, our group concluded that school lunches did not meet the average student's need for calories.

PART III: THERE'S NO SUCH THING AS A FREE LUNCH

In this part of the problem, the USDA tasked us with developing a lunch plan that simultaneously balances the desires of the school district, the students, and the federal government. Our team first defined just what it meant to satisfy each party. For the students, we defined an appealing lunch as tasty and variable, for people do not want to eat the same meal daily. For the school district, we used the given value of a $7 weekly budget to determine a cost of $1.40 for lunch each day. For the federal government, we determined that the nutritional standards consist of meeting the portion sizes given by the USDA on the "Choose My Plate" website.

Main Assumptions and Justifications

- The cafeteria service would not impact the appeal of the lunch service to students. This is justifiable because the budget given to us only included provisions for food; thus, we could not influence the level of service, and so must assume it is constant in all schools across the district.
- The food served would be prepared in a manner that maintains its nutritional value. This is justifiable because schools will not deliberately prepare food that will detrimentally affect the students' health under a lunch plan intended to meet federal nutritional standards.
- We define "variety" as enough options so that every meal of a week has at least 2 items different from the adjacent days.

Part III Model:

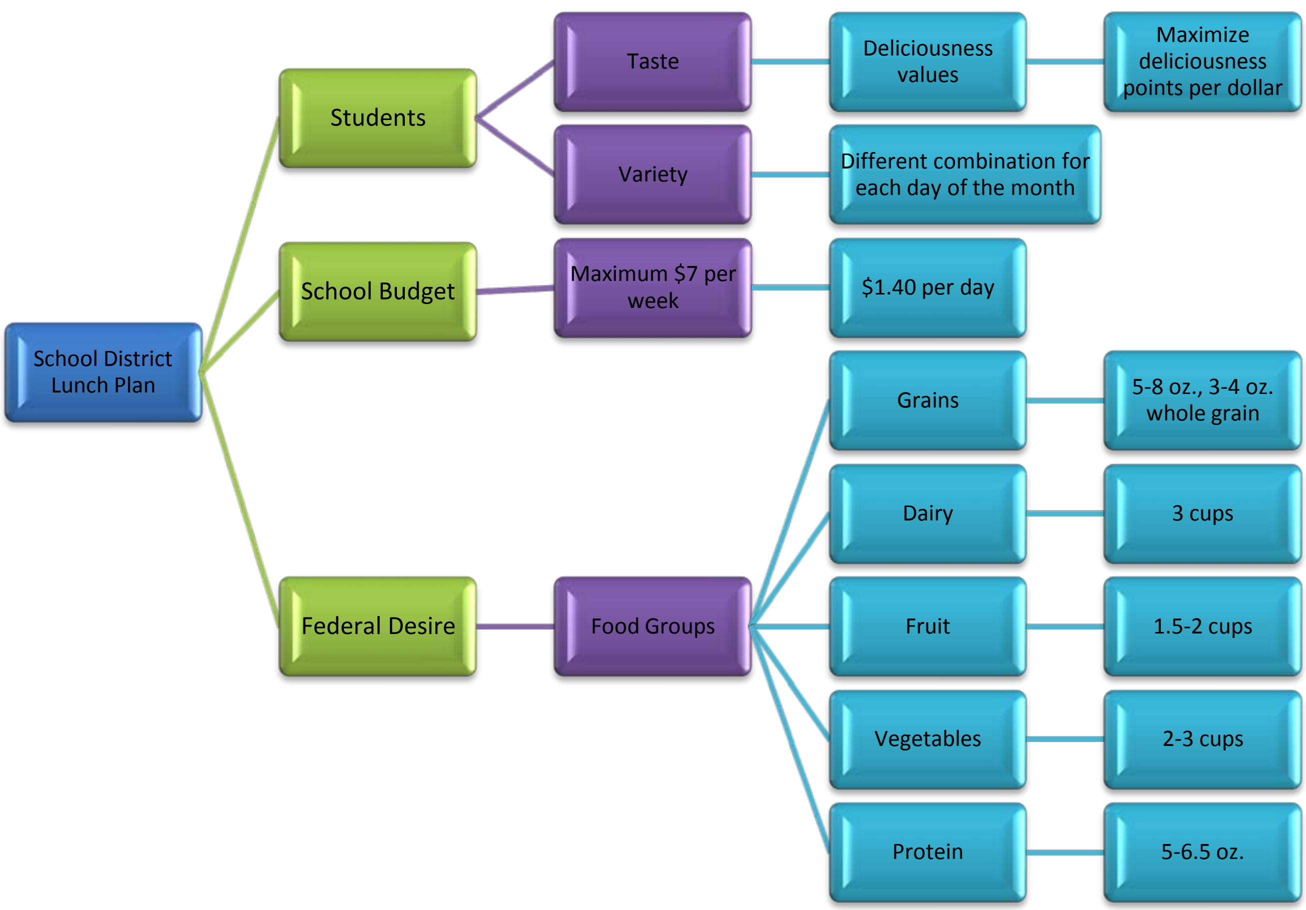


Figure 8

We first separated our analysis into the five food groups defined by the USDA in order to ensure that our lunch plan acceded to federal guidelines [19]. Next, we divided each group into a variety of different types of food in each group to appeal to students. Then, using data from the Consumer Price Index, we found the price per unit of each item [20]. We found conversions from purchased weight to yield amounts and used them to convert our prices into units matching the USDA serving sizes [21]. Next, we used the price per unit item to determine the cost of each item in order to fulfill the required servings according to USDA standards [22]. Finally, we used one third of that value because lunch should account for one third of a daily diet [9]. Below are representative calculations for apples and bananas:

Variables:
PP = Price per Pound
PD = Price per Daily Serving
Y=Cups per Pound Yield
UA = Average Cups per USDA serving

Equation 15:

$$PD = \frac{PP * UA}{Y}$$

Fruits	Price per Pound (PP)	Cups per Pound Yield (Y)	Average cups per USDA serving (UA)	Price per Daily serving (PD)	Price per Lunch serving
Apples	1.278	3	1.75	0.7455	0.2485
Bananas	0.595	2	1.75	0.520625	0.173541667

Using these costs, we found that there were 86 different lunch combinations at a maximum cost of $1.40.

In order to quantitatively analyze the appeal of the lunch program to students, we used a survey of students on their preferences for a variety of foods and assigned each item a “deliciousness” value. This value was based upon student’s responses to the survey, with each student voting “Favorite” counting for 1 deliciousness value, a vote for “Like some” counting for 0.5 deliciousness values, a vote for “Dislike a little” counting for –0.5 deliciousness values, a vote for “I will not eat this food” as –1 deliciousness value, and votes for “It depends” and “I do not know” as 0 deliciousness values [22]. Some products, such as apples, have multiple forms, such as whole apples and applesauce. For these food items, we took the average value of the deliciousness values of each final product to determine the main component’s deliciousness value. Rather than simply base decisions on overall deliciousness value values, we calculated the deliciousness value per dollar value for each type of food in order to balance the interests of the students in maximizing deliciousness values and the school district in minimizing costs. The sample calculations for apples and bananas are shown below:

DV = Deliciousness value
n = Number of subproducts derived from one main product.
DVT = Deliciousness value total
DVC = Deliciousness values per cost per pound
DVD = Deliciousness values per cost per daily serving

Fruits	Price per Pound	Deliciousness Values per Cost of One Pound	Cups per Pound Yield	Price per Daily serving	Deliciousness Values per Cost of Daily serving
apples	1.278	484.9705	3	0.7455	831.3779
bananas	0.595	756.2832	2	0.520625	864.3236

All of this data was inputted into a program designed to determine the daily lunch menus for each month. Each possible combination represents an individual data point, weighted based on its total deliciousness value per cost of daily serving, which is the sum of the component deliciousness values per cost of daily serving. Thus, the program would choose lunch menu choices with a preference towards the most efficient menu combinations—those which maximize student enjoyment while simultaneously minimizing costs, and always staying within federal guidelines. However, in order to account for individualized tastes and to create more variety, the program would choose two menu combinations for each day. The school would then split its daily resources to prepare both menu combinations. Whenever overlap in menu selections occurs, the school will simply prepare a full portion of that particular option. For example, the program might choose an initial menu combination of apples, potatoes, turkey, milk, and bread and a second combination of bananas, potatoes, chicken, milk, and rice. The cafeteria would prepare half a platter of apples/bananas, turkey/chicken, and bread/rice, but they would prepare full platters of potatoes and milk.

Equation 16:

$$DVC = \frac{DV}{PP} \quad DVD = \frac{DVC*Y}{UA} \quad DVT = \frac{\sum DV}{n}.$$

PART III MODEL SENSITIVITY ANALYSIS:

The school district's ability to provide a variety of healthy and appealing meals to the student population fluctuates a significant amount depending on the budget as well as price of the commodities. This correlation is highly observable from the $1 from the weekly budget. When the budget drops from $1.40 to $1.20 a day, the school district loses over half of the lunch options. Yet, the options that the school can provide not only depend on the budget, but the quantity and price of the items themselves. While a $.10 increase for an apple may seem insignificant, this inflation in price could drastically limit meal options.

STRENGTHS OF MODELS

PART I:

- Our model accurately considers many variables that affect a student's caloric need during lunch. However, these variables are considered without overcomplicating the model, allowing for easy use and testing.
- The model is not overly sensitive to small changes in most inputs. Thus, it can offer accurate predictions and avoid large mistakes such as too little food during lunch for a student.

- The model was rigorously tested with real values. This lends to greater credibility for the model, and offers room for improvement in the future as more testing is done with the model.

PART II:

- With our model for the first section of this problem concerning distributions: this part's simplicity in function and design is a strength. Since its inputs are few and common to find through research, the model can be readily used and tested.
- The model treats each demographic as a separate entity and allows for an overlap of a normal distribution curve with a density function to more accurately predict the varying coverage of meal plans with BMI and with other variables simultaneously.
- The model does not lose any of the comprehensiveness that we set forth in part one, and very accurately matches the USDA's recommendations for daily caloric needs.

PART III:

- This model not only just accomplishes our goal of meeting the desires of the students, school district, and federal government but also provides other perks. The combination of foods from all the different food groups presents a myriad of options to keep the students interested while staying within the budget. By providing two different types of lunches with food from every category in both meals, we can service the tastes of more people.
- By utilizing "deliciousness points" we can compare different foods within a food group to determine the "tastiest foods" as ranked by the students, and best appeal to the children.
- By creating a ratio between a food's "deliciousness points" and the price of a serving, we can compare different foods to determine which commodity provides the "biggest bang for the buck."

WEAKNESSES OF MODELS

PART I:

- The model is sensitive to changes in the "breakfast" variable in our equation, since it is binary and only has two values. A change in this value can dramatically alter final output values; however, these values are usually within healthy levels. Further improvement in this area can be done by normalizing this variable in respect to quality/calories of breakfast in order to reduce drastic changes.
- Modeling of physical activity levels was severely simplified. Improvements can be done to add more detail to the PAL calculation, which could increase accuracy of the final output, since physical activity is an essential part of the equation.
- It would have been more accurate to create a range of healthy BMIs from the 5th percentile to the 85th percentile, which studies have shown are the healthy value range. Incorporating this range into the model would have made calorie suggestions more accurate, since students can obviously be at a healthy BMI without being exactly at the 50th percentile as we had to assume.

PART II:

- Equation 10 is reliant on the variables found for equation 9, so any other variables that are determined to affect caloric intake will not be reflected in the model. Furthermore, the assumption that all factors are independent indicates that other factors that could relate to one another, such as height, weight, and gender, won't be accounted for in the model. As a result, the model works best as a general indicator of population distribution.
- The model idealizes relationships between relationships of the demographics. While some variables affect the initial model set forth in part one in a logarithmic manner, versus additive or multiplication, the model assumes that the overall effect on the percent coverage will follow a linear trend for each set demographic, and that the linear trend can be adjusted by an added constant rather than one that is multiplied or added by another means.
- The model assumes that many variables follow normal distribution, and that they do not play an effect on the distribution of BMI. This may not be completely true, and may mitigate some effects of changing variables and demographics.

PART III:

- Even though we manage to provide items from different food groups, not all foods in the same food group have similar nutrients. For example, a potato consists mainly of starch, and is less nutritious than broccoli, which is high in fiber and vitamins. Furthermore, students hardly eat the same amount every day, much less every month. If a student plays a sport during autumn but not winter or spring, he or she will usually eat more during the fall season.
- The prices we calculated with in this model are mostly ingredients and not the final product. No schools sell just ground beef as school lunch but instead, for example, combine it with other ingredients such as lettuce and tortillas to create tacos. Therefore, if the meals are prepackaged somewhere else and shipped to the school, the prices in this model will differ from actual prices. However, because premade meals include the manufacturing, shipping, and packing costs, the actual price of food is higher than if schools made the food themselves.

CONCLUSION:

With childhood habits playing such a large influence in habits later in life, instilling a healthy diet in American children proves a paramount task for the American people. Not just the parents but the government and people as a whole should concern themselves with what children eat at school. By determining the most influential variables in the caloric intake of children from kindergarten to twelfth grade, our team modeled the necessary caloric intake per day and per school lunch for students. The data used to determine the portion of calories from lunch was taken from the USDA itself, and the daily caloric needs that our model displayed was consistent with USDA suggested values. However, realizing that not every student can be properly nourished by a school lunch, especially with budgetary constraints, our group set out to model the distribution of the demographic characteristics that we previously set forth. We found that most displayed normal distribution. We used this data to determine the percentage of students that certain parameters would affect, and we used this to develop a model which would calculate the total percent of students that could be covered by an 850 calorie lunch. We broke the model into separate demographics, which could be modeled with independent BMI-dependent density functions over a normal distribution curve. Summing these together, when multiplied with each demographic's respective prevalence in the student population, yielded a total coverage of 10%. This is very low, but our group quickly realized that the 850-calorie school lunch provided too few calories to meet the USDA suggested intake, which our model followed very accurately. Therefore, we suggest to the USDA to investigate whether schools provide adequate lunch food to meet caloric needs of our students, as our model suggests that they may not. Then, in order to formulate a viable lunch plan for a school district to implement, we based our model off of the USDA serving-size recommendations, and then, using the prices of various items, determined the price of different food combinations. The viable food options that were within the budget were then entered into a program that would select two meal options a day for the children to eat. Because we used "deliciousness points" to weight the student's preference, the more preferred meals are weighted to be chosen by the program more often so that the students can eat what they desire more often.

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