Summa Cum Laude Team Prize — $20,000
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MOODY'S MEGA MATH CHALLENGE 2006
MODELING SOCIAL SECURITY, BENEFITS AND DETRIMENTS
A NEW SOLUTION

TEAM 057
MARCH 4th, 2006

MOODY'S MEGA MATH CHALLENGE 2006

SUMMARY

In order to identify the possible solutions to the current problems with the Social Security system, the first task we attempted was the development of a mathematical model of the current Social Security that agreed with the widely accepted conclusion that Social Security will go bankrupt before the middle of the current century.

After developing a series of abstract equations to model the system, we collected current data which we plugged into our model. Our model concluded that the current Social Security system, if left unchanged, will no longer be able to pay out its promised benefits by the year 2044. Using our model, we changed variables such as the combined Social Security tax rate and the age of retirement in order to find combinations that resulted in a Social Security system that lasted for more than 75 years. We found that raising the retirement age from 65 to 70 both allowed a decrease in taxes and ensured that Social Security would survive to provide for the current generation of workers.

To ensure that the solutions outlined in our paper will work, certain data must be taken regularly in the future. Our model must be continually rechecked against new data. These data include monitoring assets of the trust fund on a yearly basis, life expectancy functions in our model, population proportion functions in our model, and growth of the economy as compared to the functions of our model.

RESTATEMENT OF PROBLEM

The Social Security debate has and will play a significant role in shaping the politics of the 21st century. The current Social Security model, which depends on the "pay as you go" system, requires American workers to deposit a fixed payroll tax (about 16%) in the Social Security trust fund. The fund currently amounts to $1,686,800,000,000, but due to increasing demand it will run out in 50 or so years.

Our goal was to analyze the issues involved with Social Security and determine a feasible model that will guarantee the integrity of the system for at least 75 years. Furthermore we determined the viability of the current model and manipulated this model by varying factors such as the amount of Social Security taxation, the benefit structure, and the age for full retirement benefits. We also determined whether private investment accounts would be a meaningful solution to the Social Security problem.

List of assumptions and their justifications

1. Linear increase in average income

   The 2004 average income data provides a base for modeling the increase in average income over the years. Furthermore, since we received data scaled to 2004 values, we chose to maintain a standard and used the 2004 values as the base values in our model.

2. Total population follows exponential growth

   A typical population follows a logistic growth curve, but as the United States population is still in the growth phase it is unlikely the curve will begin to level off. It is estimated that the population's leveling-off and stabilization will occur at around 500 million individuals. Since the population is still around 300 million there is still a while to go. Therefore it is safe to assume that the US population can be modeled by a growth curve. Furthermore, the curve had an R value of .96 indicating that it closely fit the data.

3. Inflation

   Our income rates have been adjusted for inflation based on 2004 dollars and rates. However, these vales will not be 100% accurate, and thus our trust fund, input, and output values will be not be error-free. The Average Income Data scaled for 2004 inflation clearly fit an exponential growth curve. The curve had an R value of .93 indicating that it closely fit the data.

4. Do not account for economic fluctuation

We assume there is not a major war, depression, or alien invasion that would otherwise deplete our economic treasury.

5. Percentage of population which is disabled is constant

We assume there is not going to be a considerable natural disaster, nuclear explosion, or otherwise crippling experience which will significantly alter the disability rate of the American population.

6. Ideally, 95% of all workers within the age range of 20 and 65 can and do work. Some individuals out of this population do retire early and some cannot work, which decreases the total income of this age group; however, this is counterbalanced by the income of the young people under 20 who work, as well as the small population over 65 who work. Therefore the total income of this ideal situation that we used in our equations is accounted for and should have a relatively low error.

MODEL DESIGN

Identification of variables

The goal of our new Social Security model is to create a new solvent program, one where the total assets of the trust fund should be greater than 0, but not substantially over 0 because otherwise the government would be just taking money from the people and not using it for the people. We broke up the variables which will affect this net dollar amount into two categories: those affecting the input money going to the government, and those affecting the output money back to the people.

Initial Amounts:

1. Dollars already in trust fund
    a. As of 2004, the total assets in the Social Security trust fund total to $1,686,800,000,000.

Input:

1. Total population
2. Working population: This part of the population, in concordance with their annual income, helps to build the system which pays into the government which can use the money as it needs to pay out to its beneficiaries.
3. Distribution of annual income

Output:

1. Percentage of population which is retired: This population receives about 67% of the total Social Security dollar output by the government. As the total population will change, this function of the retired percentage will help us to output the right number of people who need the money, and thus the correct amount of money drawn from the account for these retired people.

2. Disabled population: This population receives about 13% of the total Social Security dollar output by the government. As the total population changes, the number of disabled people will change; however, we assume that the percentage of disabled people will remain constant.
3. Survivor population: This population receives about 20% of the total Social Security dollar output by the government. As a function of time, this population will change, which will change the total expense due to this population.
4. Life expectancy: As life expectancy increases, the retirees, the disabled, and the survivors will all live longer, and thus the government will be paying for the added years of these individuals. With this variable, the life expectancy as a function of time, we can recognize how these added years will add to the total government expense.

This net amount of money, which will need to be adjusted for inflation in both categories, will be a function of time with years (since 2004) as the units.

Research/background/diagrams of situation
Used data for annual employment statistics, annual retiree statistics, average income, and percentage of people over 65, 70

Table 1: Percentage of Population over 65 Years Old

Percentage of Population over 65 vs Time	
Year	**Percent of Population over 65**
1980	11.3
1985	11.9
1990	12.5
1995	11.6
2000	11.9
2005	12.4
2010	12.9
2015	14.5
2020	16.2
2025	18.2
2030	26.4
2035	20.3
2040	31.1
2045	20.5
2050	36.5

Graph 1: Percentage of Population over 65 Years Old

percent = 8.63/(1+(2.29*10^172)*exp(-.1967*time)) + 12.017

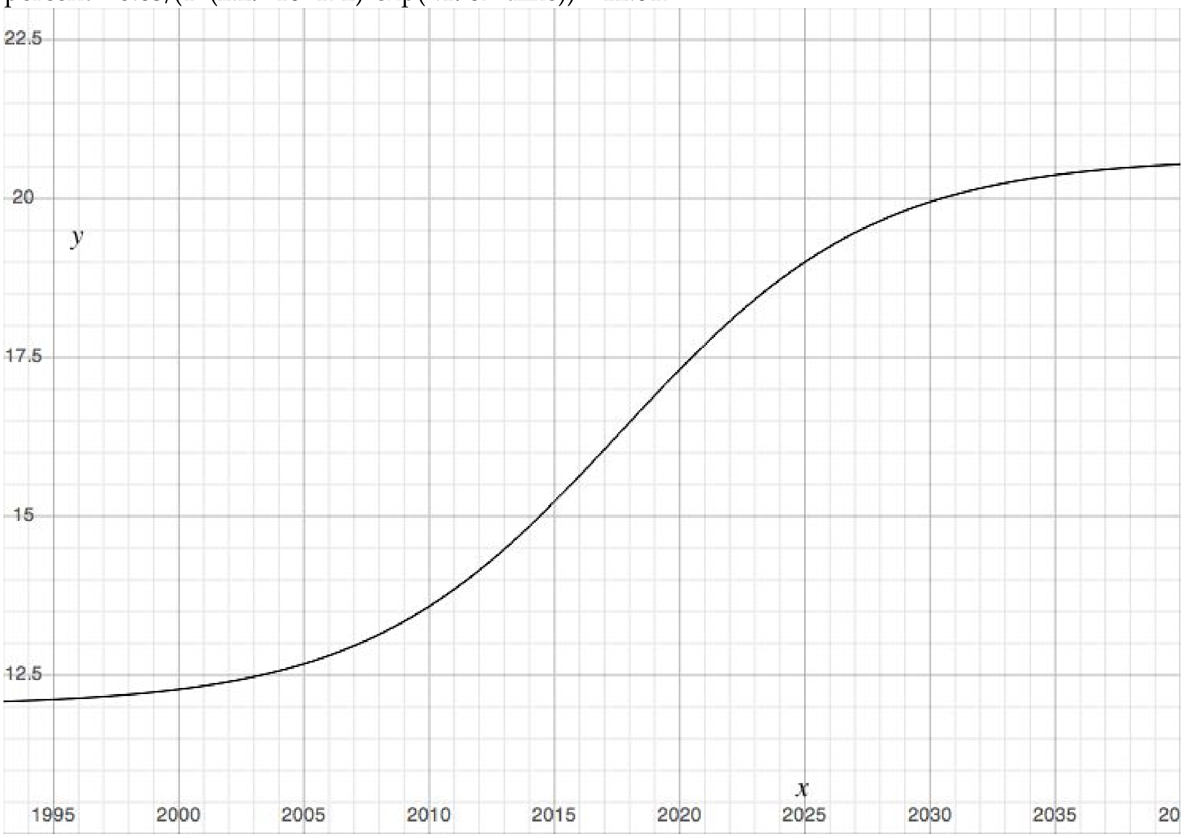


Table 2.
Percentage of Population over 70 Years Old

Year	% of Population over 70
1990	8.5
1991	8.7
1992	8.7
1993	8.8
1994	8.8
1995	8.9
1996	9
1997	9
1998	9
1999	9

2000	9
2001	9.2
2002	9
2003	9.1
2004	9
2005	9
2010	9
2015	9.7
2020	11
2025	12.6
2030	14.1
2035	15.4
2040	15.7
2045	15.9
2050	15.8

Source: United States Census Bureau

Graph 2.
Percentage of Population over 70 Years Old
percent = 7.099/(1+(1.059*10^180)*exp(-.20475*time)) + 8.86

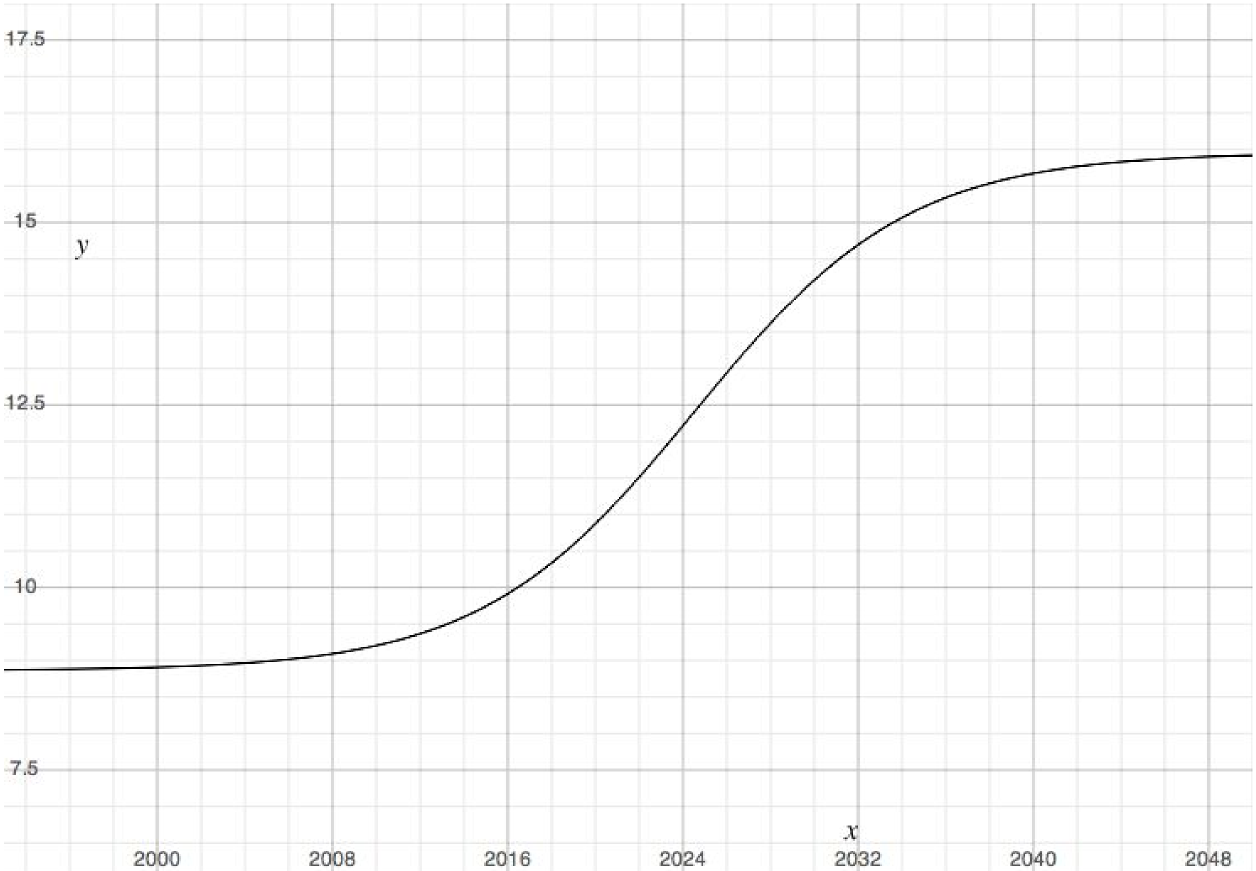


Table 3: Percentage of Population between the Ages of 20 and 65

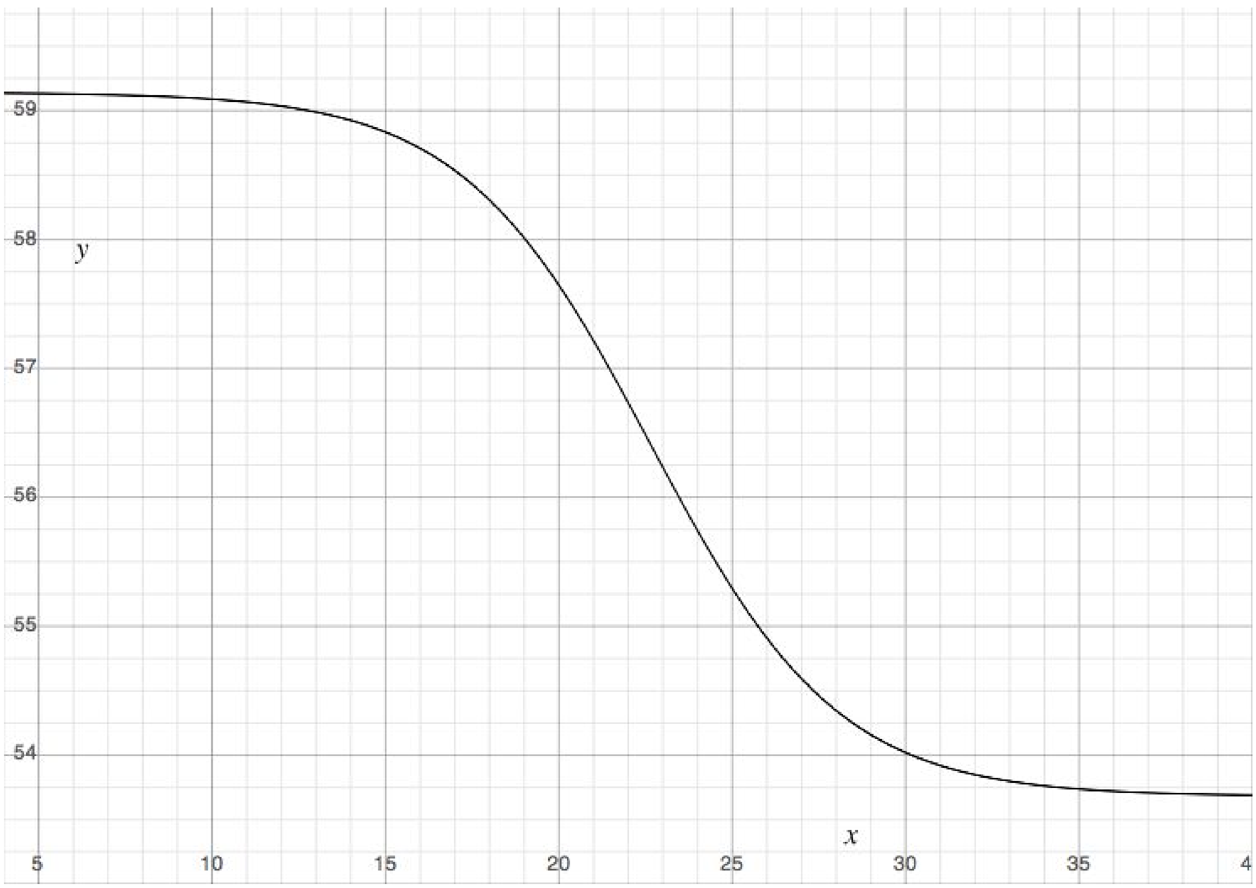
Percentage of Population 20-64 vs. time	
Year	**Percentage of Population**
1980	57
1985	58.6
1990	58.6
1995	58.5
2000	59
2005	60
2010	60
2015	59.1
2020	57.4
2025	55.4
2030	54.1
2035	53.9

2040	53.7
2045	53.6
2050	53.5

Graph 3. Percentage of Population between the Ages of 20 and 65

Note: Different scale used (years starting at 2000)

percent = 5.46/(1+(2.35*10^-4)*exp(.3687*time)) + 53.68

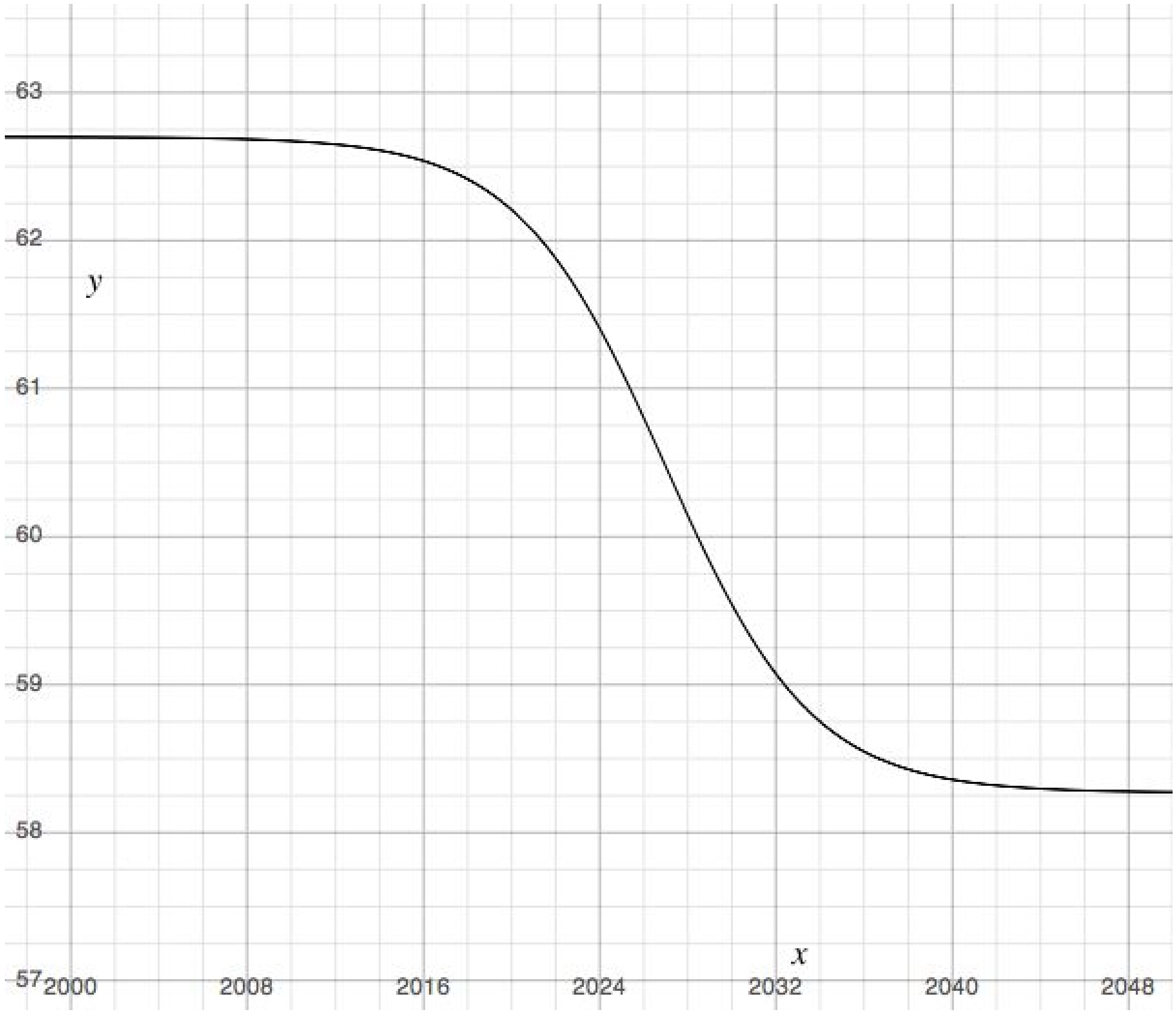


We sourced data which showed that approximately 95% of Americans between the ages of 20 and 65 were working. Therefore we took the total population between these ages and took 95% of each individual value.

Table 4:

Graph 4: Percent of Population between 20 and 70 Years Old

percent = 4.43/(1+(1.1355*10^-263)*exp(.2987*time)) + 58.27



Identification of Relationships between Variables

Define: Year as t
Define: Total dollar input to Social Security as a whole as IN
Define: Total US population as a function of time as P(t)
Define: Total percentage of working population as a function of time as WP(t)
Define: Total percentage of retired population as a function of time as RP(t)
Define: Average annual income as a function of time as AI(t) in 2004 dollars
Define: Social Security payroll tax as a constant, set as 12.4% (divided evenly between employee and employer)
Define: Total dollar output from Social Security fund to Social Security beneficiaries as OUT
Define: Primary Insurance Amount, the amount of money a retired beneficiary gets per month, as PIA

Define: The Average Indexed Monthly Earning, the basis for which the PIA is calculated, as AIME(t)
Define: The variable to scale benefits with the growth of the economy as SCALE
Define: Dollars as d
Define: Payroll tax in decimal form as PAY

AIME(t) = AI(t)/12
Yearly earnings, AI(t), are divided by 12 to find monthly earnings, AIME(t).

SCALE (d,t) = [AI(t)]/[AI(2004)]*d
SCALE represents the change in annual income as a function of time from the baseline year (2004), multiplied by the dollar amount the change represents.

PIA(t) = .9[SCALE(612,t)] + .32[(SCALE(3689,t)-SCALE(612,t)] + .15[AIME(t)-SCALE(3689,t)]
The PIA is calculated in strata:
PIA = (90% of first $612 of AIME) +
(32% of the AIME between $612 and $3689) +
(15% of the AIME above $3689).
The threshold numbers ($612 and $3689), known as "bend points," are adjusted for the growth in average annual income according to the SCALE function defined above, so that growth in wages is matched by a growth in Social Security benefits.

IN = .95[P(t)*WP(t)*AI(t)]*PAY
The total income of US workers equals the total working population (the total population times the percentage of working population) times the average annual income. This in turn is multiplied by the payroll tax rate (which currently is 16%), to find the total intake of Social Security. The total working population is multiplied by .95 to match our assumption that only 95% of able-bodied workers actually work (since some are victims of unemployment, disability, etc).

OUT = {[PIA(t)*(P(t)*RP(t)] + SCALE(862,t)*DP(t)}12
The PIA multiplied by the total number of retired persons (P(t)*RP(t)) equals the total amount of money distributed to Social Security beneficiaries. However, Social Security also pays disabled workers an average of $862 a month. The payout is scaled in the same way that the "bend points" of the PIA are calculated. Since all of these functions measure dollars in terms of months, the total outlays are multiplied by 12 in order to calculate annual outlays.

\$ in trust fund = 1,686,800,000,000 + IN - OUT

Data:

Table 5: Tests of Our Model

Constants:
Disabled Population = 2%
Survivors = 2%

Retirement Age	Tax Rate	First Year Bankrupt
65	16.00%	2044
65	17.00%	2057
65	18.00%	**2083**
65	18.90%	**Indefinite**
70	10.00%	2038
70	12.00%	2062
70	13.00%	**2094**
70	13.70%	**Indefinite**

Graphs

Derived Functions

The Average Indexed Monthly Earning (AIME) is the base used for calculating how Social Security benefits are distributed according to what is known as the Primary Insurance Amount (PIA). The equation is as follows:

PIA = (90% of first \$612 of AIME) +
(32% of the AIME between \$612 and \$3689) +
(15% of the AIME above \$3689),

Out = (PIA(t) × RP(t) + grow(862,t) × DP(t)) ×12,

PIA(t) = .9(grow(612)).

Using source code developed by our model incorporating all of these equations, the projections indicated that the Social Security trust fund would run dry by the year 2044 with the current retirement age of 65 and a relatively low tax rate of 16%. As this would not provide for our generation as well as future generations, we established that the current system was indeed not viable and resolved to discover the variables whose alteration would prove to extend the life of the present Social Security system.

Through testing our model through several examples shown below, we established that in order to optimize the current model, we would need to change the variables of current retirement age (age at which an individual begins to receive full retirement benefits), as well as alter the currently existing Social Security taxation rate of 12.4%.

ANALYSIS

Tests and Results

With the current retirement age of 65 years, we found through our model that as the tax rate increases, the solvency of the Social Security model lengthens substantially, and once the tax rate gets higher than 18.9%, the Social Security system should last indefinitely. However, we predicted that this increase in tax rate would not be received positively by the general population, and instead chose another option: increasing the retirement age to 70 years. Using this option in our model, only a 13.7% tax rate would be required in order to create a Social Security system which would last indefinitely according to this model. Although this increase in the retirement age required to receive benefits would also be negatively by the general population, we feel that either of these two solutions would be a small price to pay for the security of the future of the general population. In addition, each of these changes would not completely uproot the system, unlike other solutions which involve extreme economic changes. In summary, to optimize the current Social Security model, the government should increase the retiring age to 70 as well as increase the tax rate; however, one of these two solutions would be satisfactory, as shown in our data table.

Analysis of model (constraints, strengths, weaknesses)

Due to difficulty finding data, we assumed the percentage of the population which is disabled to be a constant. However, there may be several factors influencing this rate. Because life expectancy is going up, disability and aging which may be addressed under Social Security will increase. However, as medicine and healthcare technology improve, we assumed that new methods, medicines, and discoveries will serve to reduce a good portion of this.

Additionally, a huge point of discussion for our group was the incorporation of privatization in our model. However, since this solution is at first economically risky, as well as being hugely politically sensitive, we have decided not to use it as a solution as it is a bit precarious to ever put into effect as the strain on the current economical system is substantial within the first few years until the payback occurs. However, we did come up with an equation to incorporate this solution within our model:

IN = [WP(t)*AI(t)]*0.062+0.062[(AI(t)*WP(t))*e^(0.0125t)],

OUT = same as previous model,

$ in trust fund = 1,686,800,000,000 + IN – OUT.

CONCLUSION

In order to keep the Social Security trust fund solvent, we recommend two courses of actions for Congress:

1. Increase the payroll tax rate.

2. Increase the normal retirement age.

Using our model, when we increased the payroll tax rate from 16% to 18%, the Social Security trust fund remained solvent until 2083. When the payroll tax rate was raised to 18.9%, the trust fund remained solvent indefinitely.
Similarly, when we increased the normal retirement age from 65 to 70, the Social Security trust fund remained solvent indefinitely as long as the payroll tax rate was kept at a minimum of 13.7%

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Appendix

Source Code:

```
StartingYear = 2004
BendPoints = [612, 3689]

def AverageIncome(time):
return (8*10**-13)*math.exp(0.0192*time)

def AIME(time):
return AverageIncome(time)/12

def scale(dollars, time):
return AverageIncome(time)/AverageIncome(StartingYear) * dollars

def PIA(time):
return .9*scale(BendPoints[0], time) + .32*(scale(BendPoints[1], time) - scale(BendPoints[0], time)) + \
.15*(AIME(time) - scale(BendPoints[1], time))

def RPone(time):

# retired population, age >= 65
percent = 8.63/(1+(2.29*10**172)*math.exp(-.1967*time)) + 12.017
return TotalPopulation(time)*(percent/100)

def RPtwo(time):

# retired population, age >= 70
percent = 7.099/(1+(1.059*10**180)*math.exp(-.20475*time)) + 8.86
return TotalPopulation(time)*(percent/100)

def WPone(time):
```

```
# working population, between age 20 and 65
time -= 2000
percent = 5.46/(1+(2.35*10**-4)*math.exp(.3687*time)) + 53.68
return TotalPopulation(time + 2000)*(percent/100)*(95/100)

def WPtwo(time):

# working population population, between age 20 and 70
percent = 4.43/(1+(1.1355*10**-263)*math.exp(.2987*time)) + 58.27
return TotalPopulation(time)*(percent/100)*(95/100)

def DisabledPopulation(time):
return TotalPopulation(time)*0.02

def TotalPopulation(time):

return (2.79*10**6)*time - 5.3*10**9

def In(time):
# change this variable as desired
tax = 0.1369
# replace WPtwo to WPone to set the retirement age to 65
return WPtwo(time)*AverageIncome(time)*tax

def Out(time):
# replace RPtwo with RPone to set the retirement age to 65
return 12*(PIA(time)*RPtwo(time) + scale(862,time)*DisabledPopulation(time) +
scale(862,time)*Survivors(time))

def Survivors(time):
return TotalPopulation(time)*0.02

def Run():
year = StartingYear
TrustFund = 1686800000000
for x in xrange(100):
TrustFund += In(year) - Out(year)
# scale the trust fund for increase in income, as a result of the yield of bonds
TrustFund *= AverageIncome(year)/AverageIncome(year - 1)
print "Year: " + str(year) + " Trust Fund $: " + str(TrustFund)
year += 1

if __name__ == '__main__':
Run()
```