First Honorable Mention—$2,500

High Technology High School – Team #64, Lincroft, New Jersey
Coach: Ellen D. LeBlanc
Students: Kathryn Silverio, Elizabeth Wendel, Yelizaveta Yermakova

Team ID#: 064

National Social Security Dilemma – SOLVED!

SUMMARY OF RESULTS

It is projected that if the current Social Security system remains unmodified, it will collapse in 2018, due to the exhaustion of funds. Many people have unsuccessfully tried to rectify this widely recognized problem. However, due to its political sensitivity and extensive media coverage and hype, very little has been done to correct this growing problem.

Upon examination of current statistics and extrapolation into the future, the team has determined that the system will undeniably collapse. As an alternative, the team has come up with an efficient model that incorporates various demographic and economic variables, such as population trends, income trends, cost of living trends, retirement trends, disability claims, and the current infrastructure of the Social Security Administration, all in relation to time. In order to solve the problem, the team first examined the original intentions and objectives of President Franklin Delano Roosevelt when he created the Social Security Administration. This objective was, primarily, to create a safety net for a small percent of the population, namely the elderly and disabled. Upon close examination of statistics from the year 1935, the team determined that, in fact, only the oldest 5 % of the population qualified for retirement benefits. In 2005, however, statistics show that more than 12% of the current United States population collects benefits from the system because of the age threshold. The team decided that in order to restore the integrity of the system, the main reform would be to keep the percentage of the population receiving the benefits, rather than the threshold age, constant.

The team's model is a working model that also takes into account the fact that the income tax rate should change as the population evolves with the threshold age. These reforms were tied together by way of a simple differential equation which, when solved, would exhibit the income tax rate percentage as a function of time.

In this way, this mathematical model not only will last the 75 years but will continue to work for the longevity of the United States, changing and evolving with the population, demographics, and economics. Also, because the model was oversimplified, due to the time constraints, it is fluid enough to be changed at any time to include more variables and exclude any correctable assumptions. Moreover, the method of developing this particular model can have a wider application beyond the realms of Social Security to include any economic problem currently existing in the United States.

INTRODUCTION

Issues with Current System

"One of America's most important institutions—a symbol of the trust between generations—is also in need of wise and effective reform. Social Security was a great moral success of the 20th Century, and we must honor its great purposes in this new century. The system, however, on its current path, is headed toward bankruptcy. And so we must join together to strengthen and save Social Security."

President George W. Bush
State of the Union Address
February 2, 2005

Numerous problems face the current Social Security System. The Social Security System was designed for a much different society facing different circumstances. As an example, in 1935, when Social Security was first implemented, most women were unemployed. However, today, over 60% of women work. In 1935, the average American did not live long enough to collect retirement benefits. Today, life expectancy is 77 years. There are also currently fewer workers to support retirees. In 1950, there were 16 workers for every Social Security beneficiary, whereas today there are only 3.3 workers for every Social Security beneficiary ("The Problems" 1).

These are just a few of many examples corroborating the fact that Social Security is heading toward bankruptcy. According to the Social Security Trustees, in 2018 Social Security will be paying out more than it collects. Every year afterward, the margin of debt will increase.
When today's generation of young workers retire, the system will be bankrupt. By 2034 the trust fund ratio is projected to have declined to the point of fund exhaustion (Snow 51).

According to the Social Security Trustees, as of 2004, the cost of doing nothing to fix our Social Security system was approximately $10.4 trillion (Snow 58). To keep the current level of benefits, all federal taxes would have to be doubled immediately and permanently. All state taxes would have to rise immediately and permanently by 20 percent (Bland 1). Neither of those solutions is viable or practical.

Bland, Drew. "Problems with Social Security." Opinions and Ideas. 13 Oct. 2004. 4 Mar. 2006 <http://www.dailyorange.com/media/paper522/news/2004/10/13/Opinion/Problems.With.Social.Security-751428.shtml?norewrite&sourcedomain=www.dailyorange.com>.

Snow, John W., et al. "The 2004 Annual Report of the Board of Trustees of the Federal OASID Insurance Trust Funds." Social Security Online. 4 Mar. 2006 <http://www.ssa.gov/OACT/TR/TR04/tr04.pdf>.

"The Problems Facing Social Security." The White House. 4 Mar. 2006 <http://www.whitehouse.gov/infocus/social-security/200501/socialsecurity1.pdf>.

ANALYSIS OF PROBLEM

The object of the model is to revitalize the original purpose of the Social Security Act as first implemented by President Franklin Delano Roosevelt during the Great Depression: to provide a financial safety net for a small percentage of the population that cannot work due to old age and/or disability. When analyzing the statistics of the year 1935, it was discovered that the Social Security Plan provided this financial safety net for a very specific percentage of the population; namely, the oldest 5% and the disabled. Later on, as the century progressed, the healthcare system became more and more advanced, and the life expectancy rate for all people increased. This put a financial strain on the system that it was not calculated to bear at its inception.

President Clinton revised the Social Security System and increased the age of admittance to the retirement plan 2 years for people born after 1940. The team thought that he took a step in the right direction. A model that provides for the population in any respect should evolve correspondingly to the evolution of the population. The percentage of the population protected under Social Security—5%—was not chosen by President Roosevelt by accident. At its creation, the system was supposed to provide a financial safety net for this percentage of the population only: the oldest and weakest 5%. The team thought that this is what should be a constant in the system, not the age that qualifies the person for Social Security upon retirement. This is one of the factors that defined the equations used to model this situation.

Something else that was taken into consideration was that there are people over 65 years old who are still working full time and receiving their Social Security benefits. This, in effect, not only is wasteful for the system but defeats the purpose, drawing money away from people with need to people who just want extra money. One of the team's propositions is to eliminate this so that people who are over the determining age and are still working would not receive the benefits.

JUSTIFICATION OF MODELING USED

After thoroughly researching the problem, the team analyzed the situation and decided to use a deterministic mathematical model rather than a probabilistic model. The objective was to find an exact solution, so the team created a model that could be easily adjusted. The model is a working model and can be easily modified to take into account more or fewer variables, depending on the preference and desirable precision of the outcome. Also, this model would not call for very drastic changes that the general public would object to, and furthermore, it does not privatize the system, a very controversial issue.

ASSUMPTIONS

Specific Assumptions

#1

When Social Security was initiated, the age at which people could start collecting Social Security benefits was 65. Upon reviewing historical data, the team found that approximately 5% of the American population at this time was 65 or older. Therefore, we decided to preserve the 5%

threshold. By doing this we address one of the major flaws in the current system: setting a fixed age neglects changes in life expectancy and evolution of the healthcare profession.

"Population distribution by Age, Race, Nativity, and Sex Ratio, 1860-2004." U.S. Statistics: Population. Infoplease. 4 Mar. 2006 <http://www.infoplease.com/ipa/A0110384.html>.

"Table 27. Life Expectancy at Birth, at 65 Years of Age, and at 75 Years of Age, According to Race and Sex." Health, United States, 2005 . Center for Disease Control and Prevention. 4 Mar. 2006 <http://www.cdc.gov/nchs/data/hus/hus05.pdf#027>.

#2

The percentage of the population under 18 is assumed to be held constant. Children aged 0–17 composed 25% of the population in 2003, and it is projected that children aged 0–17 will compose 24% of the population in 2020. This change is so small that the team believes any change in the next 75 years in this percentage will be negligible. The team is also assuming that anomalies such as a baby boom will not occur.

"America's Children: Key National Indicators of Well-Being 2005 ." Child Stats. Forum on Child and Family Statistics. 4 Mar. 2006 <http://www.childstats.gov/americaschildren/index.asp>.

#3

The percentage of the population that is disabled and unemployed is assumed to be held constant. The percentage of the population that is disabled was 16.9% in 2004. In a National Health Interview Survey conducted in 1995, it was found that 73% of those with disabilities worked. The team is assuming that no random occurrences will cause these percentages to fluctuate and any change in the next 75 years in these percentages will be negligible.

"People Quick Facts." State and Country Quick Facts. 12 Jan. 2006. US Census Bureau. 4 Mar. 2006 <http://quickfacts.census.gov/qfd/states/00000.html>.

"Statistics About People with Disabilities and Employment." Office of Disability Employment Policy. July 2001. U.S. Department of Labor. 4 Mar. 2006 <http://www.dol.gov/odep/pubs/ek01/stats.htm>.

#4

Children are assumed to have no net income. The team justifies this assumption with the fact that the number of employed high school students is generally negated by the many unemployed college students.

#5
The team is assuming that all widows in the Social Security system are either working and collecting a pension for their dead spouse or retired and collecting a pension for themselves. In other words, they aren't collecting Social Security pensions for both themselves and their spouse.

#6
The team is assuming that the cost of administration and processing of claims decreases as a function of time. The team is instituting the administrative change of providing an incentive for use of online transactions instead of the postal service ($20 per postal transaction). The team is then assuming that more people will use the online choice as time increases. Therefore, since online processing saves money, the cost of administration will decrease over time.

#7
The team is assuming that K(t) = -(t/84) + (2089/84). This was calculated by taking K to be 1 in 2005 and (1/6) in 2075. This was determined because at t=2005, A(2005) = A_0, and at 2075, administrative costs will be at a minimum.

General Demographic and Economic Assumptions

Immigration
The team is assuming that there would be no major fluctuation in the number of immigrants in America over the next 75 years. Therefore, the team's model does not take into account immigration.

Unemployment
According to information from the American Academy of Actuaries, "unemployment doesn't have a significant impact on system finances." Therefore, the team decided no to include unemployment data in our model.

"Assumptions Used to Project Social Security's Financial Condition." Issue Brief. Jan. 2004. American Academy of Actuaries. 4 Mar. 2006 <http://www.actuary.org/pdf/socialsecurity/assumptions_0104.pdf>.

DESIGN OF THE MODEL

The solvency of the Social Security system is modeled by the amount of money going in minus the amount of money withdrawn. Let F(t) equal the amount of money in the trust fund as a function of time. Let L(t) equal the amount of money entering the fund as a result of taxation of the workforce. Let W(t) equal the amount of money withdrawals. Therefore, F(t) is a function of L(t) and W(t):

$$F(t) = L(t) - W(t). \qquad \text{(Eqn. 1)}$$

To define the amount of money put into the trust fund, it is necessary to take into account average income as a function of time, population as a function of time, the percentage of the population under age 18, the percentage of the population that is disabled and unable to work, and the tax rate for Social Security (currently 6.20%). The team found that statistic from the following source:

Social Security (United States). Wikipedia. 4 Mar. 2006 <http://en.wikipedia.org/wiki/ Social_Security_(United_States)#Current_operation>.

Let I(t) equal income as a function of time and P(t) equal population as a function of time. C is defined as the percentage of the population under age 18. C is assumed to be constant (see Assumption #2). D is defined as the percentage of the population that is disabled and unemployed. D is also assumed to be constant (See Assumption #3). Let x(t) equal the income tax rate of every working individual in the US.

To define a value for C, we used data for C in 2003 and a projection for C in 2020. In 2003, C was 0.25; in 2020, C is projected to be 0.24 (America's Children). Therefore, we decided that 0.25 was a reasonable value for C.

To define a value for D, we used the established disabled percentage of the population from 2004 and the percentage of disabled people who are not employed. In 2004, the percentage of the population that was disabled was 16.9%. The percentage of disabled people who were not employed was 73%. Therefore,

$$D = 0.169 \cdot 0.73 = 0.125.$$

The amount of money entering the fund is equal to the number of contributors times their average contribution. The entire workforce contributes to the Social Security fund; the workforce will be the total population minus the total number of retirees, children, and unemployed disabled people (see Assumptions #1 and #4). The average contribution will be the average income times the tax rate. In other words,

$$L(t) = I(t) \cdot x(t) \cdot P(t) \cdot [1 - 0.05 - C - D]. \qquad \text{(Eqn. 2)}$$

There are three ways to collect from Social Security: retirement benefits, disability benefits, and survivors benefits. Administration costs will also be withdrawn from the fund. Let E(t) equal total benefit withdrawals and A(t) equal processing costs:

$$W(t) = E(t) + A(t). \qquad \text{(Eqn. 3)}$$

The total amount paid out in retirement benefits will be the number of qualified seniors ($0.05 \cdot P(t)$) times Z(t), where Z(t) is the average retirement benefit payout. Likewise, the amount paid out in disability benefits is $D \cdot P(t) \cdot B(t)$, where B(t) is the average disability benefit payout, and the amount paid out in survivors benefits is $V \cdot P(t) \cdot S(t)$, where V is the percentage of the population that is widowed and S(t) is the average survivors benefit payout (see Assumption #5). Therefore,

$$E(t) = 0.05 \cdot P(t) \cdot Z(t) + D \cdot P(t) \cdot B(t) + V \cdot P(t) \cdot S(t). \qquad \text{(Eqn. 4)}$$

Administration costs are currently measured to be 60% of a cent for every dollar of benefits. The initial value of administrative costs is therefore

$$A_0(t) = 0.006 \cdot E(t). \qquad \text{(Eqn. 5)}$$

We assume that administrative costs will decrease with time (see Assumption #6). We define A(t) as

$$A(t) = 0.006 \cdot K(t) \cdot E(t), \qquad \text{(Eqn. 6)}$$

where K(t) is a measure of how quickly the administrative costs. We used two basic inferences (see Assumption #7) to determine

$$K(t) = -(t/84) + 2089/84. \qquad \text{(Eqn.7)}$$

Therefore, combining Eqns. 3, 4, 6, and 7,

$$W(t) = [0.05 \cdot P(t) \cdot Z(t) + D \cdot P(t) \cdot B(t) + V \cdot P(t) \cdot S(t)] \cdot \{1+0.006 \cdot [-(t/84) + 2089/84]\}. \text{ (Eqn. 8)}$$

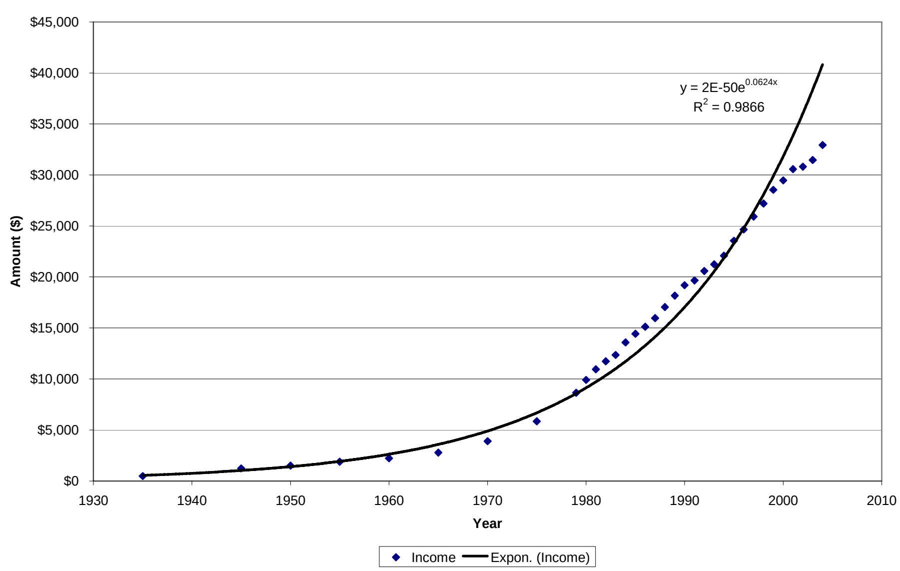
Combining Eqns. 1, 2, and 8,

$$F(t) = I(t) \cdot x(t) \cdot P(t) \cdot [1 - 0.05 - C - D] -$$
$$[0.05 \cdot P(t) \cdot Z(t) + D \cdot P(t) \cdot B(t) + V \cdot P(t) \cdot S(t)] \cdot \{1+0.006 \cdot [-(t/84) + 2089/84]\}. \quad \text{(Eqn. 9)}$$

We obtained functions for I(t), P(t), Z(t), B(t), and S(t) from regression analysis:

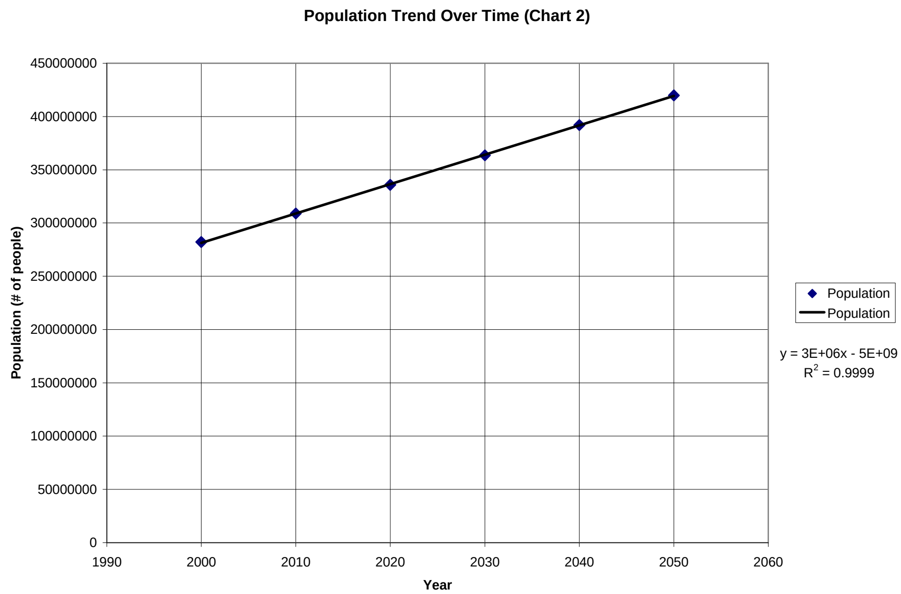
$$I(t) = 2 \cdot 10^{-50} \cdot e^{0.0624 \cdot t} \qquad \text{(see Chart 1)},$$
$$P(t) = 3 \cdot 10^{6} \cdot t - 5 \cdot 10^{9} \qquad \text{(see Chart 2)},$$
$$Z(t) = 10^{-50} \cdot e^{0.062 \cdot t} \qquad \text{(see Chart 3)},$$
$$B(t) = 10^{-44} \cdot e^{0.0552t} \qquad \text{(see Chart 3)},$$
$$S(t) = 5 \cdot 10^{-54} \cdot e^{0.066 \cdot t} \qquad \text{(see Chart 3)}.$$

Average Per Capita Personal Income (Chart 1)



I(t) was determined to be $2 \cdot 10^{-50} \cdot e^{0.0624 \cdot t}$ with an exponential regression using Excel with data from the following source:

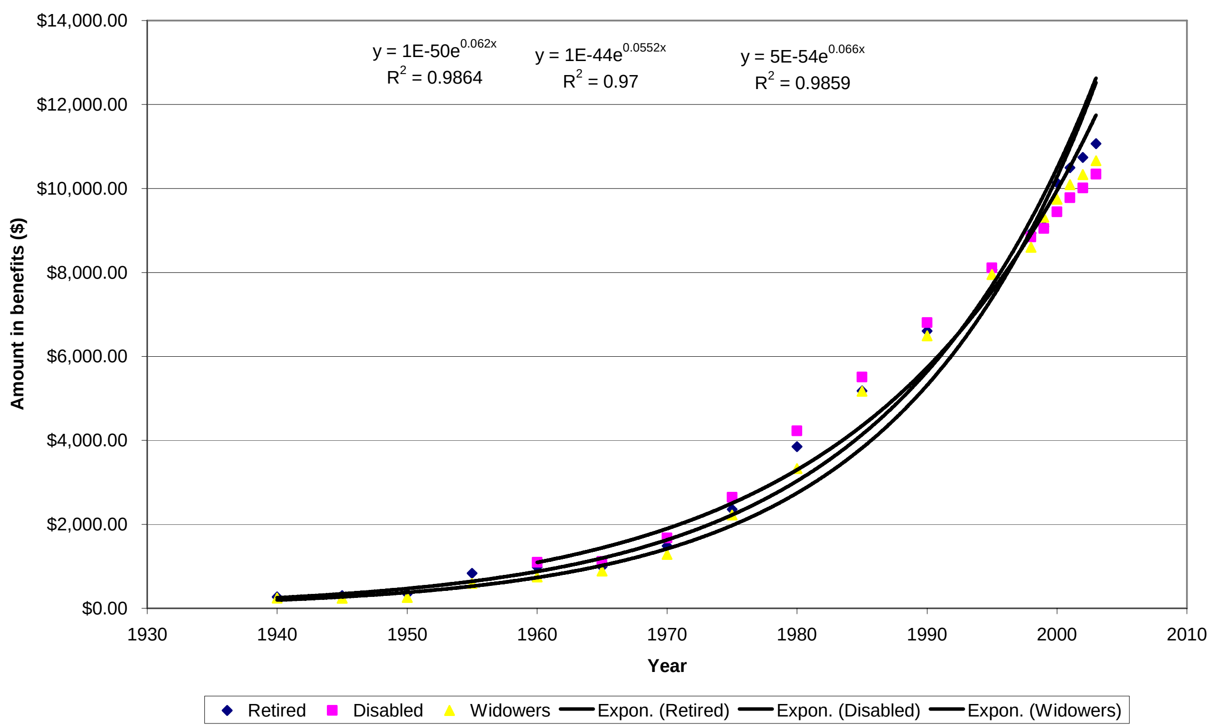
"Per Capita Personal Income." Poverty and Income. Infoplease. 4 Mar. 2006 <http://www.infoplease.com/ipa/A0104547.html>.



P was also determined the same way with a regression in Excel. The data came from the following source:

"Projected Population of the United States, by Age and Sex: 2000 to 2050." U.S. Interim Projections . U.S. Census Bureau. 4 Mar. 2006 <http://www.census.gov/ipc/www/usinterimproj/natprojtab02a.xls>.

Trend in Annual Benefits for the Retired, Widowed, and Disabled Part of the Population Over Time (Chart 3)



Z, B, and S were determined likewise with exponential regressions in Excel. The data for all three of them came from the following source:

"Average Monthly Social Security Benefits, 1940–2003." U.S. Economy and the Federal Budget. Infoplease. 4 Mar. 2006 <http://www.infoplease.com/ipa/A0780010.html>.

One variable in the model remains undefined: x(t). This function represents the tax rate as a function of time. Therefore, tax rate will change as the demographics of the country change, enabling the model F(t) to change as the population changes. This gives F(t) a distinct advantage over the current system because it is a working model. Currently the equation at hand, F(t), includes three variables, so it cannot be solved directly and calls for optimization. Since the problem at hand is to basically find the maximum budget allowed while limiting the range of the interest rate x which varies with time, the derivative of F(t) was taken and set to produce the differential equation:

$$\left[1.683\cdot 10^{-54}\cdot t^2-2.9831787\cdot 10^{-50}\cdot t+4.46768755\cdot 10^{-47}\right]\cdot(1.06822671717)^t+$$
$$\left[6.64285714286\cdot 10^{-51}\cdot t^2-1.17734071429\cdot 10^{-46}\cdot t\pm 1.76225821429\cdot 10^{-43}\right]\cdot$$
$$(1.063962344728)^t+\left[1.474549714285\cdot 10^{44}\cdot^2 t^2-2.6128187495998\cdot 10^{48}t+3.906574600142\cdot 10^{51}\right]\cdot$$
$$(1.0567519439307)^t-5.7534\cdot 10^{-41}\cdot e^{0.0624\cdot t}\cdot\frac{dx}{dt}-0.0624\cdot 5.734\cdot 10^{-41}\cdot x\cdot e^{0.0624\cdot t}+$$
$$3.45204\cdot 10^{-44}\cdot e^{0.0624\cdot t}\cdot\frac{dx}{dt}+0.0624\cdot 3.45204\cdot 10^{-44}\cdot x\cdot t\cdot e^{0.0624\cdot t}+3.45204\cdot 10^{-44}\cdot t\cdot e^{0.0624\cdot t}\cdot\frac{dx}{dt}=0.$$

This equation contains only two variables and one of the variable's derivatives, so once the differential equation is solved for x, x is purely represented as a function of time. Regrettably, time and skill constraints placed upon the team did not allow it to solve this differential equation for x(t) and come up with a final solution. The differentiable equation is not separable but it is a first order differential equation that the team could solve had it been given more time.

To differentiate the equation, it was divided into parts of one and two variables. The one variable part was differentiated with respect to time using a TI-89, and the other parts had to be done by hand in the following fashion.

To obtain a value for x(t), the team would have to solve the differential equations:

$$\frac{d}{dt}[-5.7534\cdot 10^{-41}(e^{t})^{0.0624}x] = -5.7534\cdot 10^{-41}\cdot e^{0.0624\cdot t}\cdot\frac{dx}{dt} - 0.0624\cdot 5.7534\cdot 10^{-41}\cdot x\cdot e^{0.0624\cdot t},$$

$$\frac{d}{dt}[3.45204\cdot 10^{-44}\cdot t\cdot (e^{t})^{0.0624}x] = 3.45204\cdot 10^{-44}\cdot e^{0.0624\cdot t}\cdot\frac{dx}{dt} + 0.0624\cdot 3.45204\cdot 10^{-44}\cdot x\cdot t\cdot e^{0.0624\cdot t} + 3.45204\cdot 10^{-44}\cdot t\cdot e^{0.0624\cdot t}\cdot\frac{dx}{dt}.$$

Once a function for x(t) is obtained, the model F(t) will show the amount of money in the trust fund as a function of time.

Using the generated models for I(t), Z(t), B(t), and S(t) to predict current values of I, Z, B, and S:

I(2005) = 43299.33 (Value 1),
Z(2005) = 97083.86 (Value 2),
B(2005) = 116406.19 (Value 3),
S(2005) = 14762.46 (Value 4).

The current population, P(2005) = 295,734,134 (Census Bureau) (Value 5).

Through research, the team found x(t) to be 0.0620.

Therefore, if nothing changes, using Eqn. 9 and Values 1–5

$$F_0(t) = 793915643133\cdot[0.62663-R(t)] - (1+0.006)[4.350\cdot 10^{12}+2.871\cdot 10^{13}\cdot R(t)],$$

where R(t) is the percentage of the population that is 65 or over as a function of time. Using regression analysis of U.S. Census Bureau projections,

$$R(t) = -0.0002\cdot t^3 + 1.3198\cdot t^2 - 2668.2\cdot t + 2\cdot 10^6. \quad \text{(Eqn. 10)}$$

Therefore, the model for the current system with no changes is

$$F_0(t) = 793915643133\cdot[0.62663-(-0.0002\cdot t^3 + 1.3198\cdot t^2 - 2668.2\cdot t + 2\cdot 10^6)] - (1+0.006)[4.350\cdot 10^{12}+2.871\cdot 10^{13}\cdot(-0.0002\cdot t^3 + 1.3198\cdot t^2 - 2668.2\cdot t + 2\cdot 10^6)]. \quad \text{(Eqn. 11)}$$

Simplifying,

$$F_0(t) = 5935438728.77 \cdot t^3 - 3.917 \cdot 10^{13} \cdot t^2 + 7.918 \cdot 10^{16} \cdot t - 5.935 \cdot 10^{19}. \qquad \text{(Eqn. 12)}$$

To test the revised Social Security system, compare F(t)—the model of the revised Social Security system—with $F_0(t)$—the model for the current Social Security system.

SUGGESTION FOR TESTING THE MODEL

The models F(t) and $F_0(t)$ predict the amount of money in the trust fund as a function of time. The time period over which the system is solvent is represented by the time period where F(t) and $F_0(t)$ are positive. To test F(t), it is necessary to compare a graph of F(t) with a graph of $F_0(t)$. If F(t) is positive for a longer period of time than $F_0(t)$, the revised system is better than the current system.

REFLECTION (STRENGTHS/WEAKNESSES)

Strengths:

The model can't fail in reality because of the fact that the team is instituting a constraint on the model that only a certain amount of the population can benefit from the Social Security Plan. This also means that the model will evolve with the population as it changes in reality, keeping the spending constant.

The model has a variable "x" that stands for the interest rate of the income tax on the population salary that is allotted for Social Security. The model allows this interest rate to change as the population evolves.

Furthermore, one of the constraints of the model eliminates unnecessary spending by saying that people who are working full time and are over the determinant age cannot collect benefits. The team believes that this measure will restore Social Security's original purpose to provide a financial safety net for people of old age and/or with disability.

The model also takes into account the internet revolution and how, over time, more and more claims will be filed online rather than by more traditional methods, such as the U.S. Postal System. This ultimately save the government money by decreasing administrative spending.

Weaknesses

Unfortunately, although the team came up with a viable, executable solution, it did not have enough time to fully solve it to reach its goal: x as a function of t.

The model hasn't been fully tested and is fairly simplistic because it is based on many assumptions and doesn't take into account additional variables that should be implemented in any professional economic model.