Moody's Mega Math Challenge 2007
Minimizing Risk...
Maximizing Portfolio Profit

Team #76
March 4, 2007

Summary

Our goal in this paper was to design a portfolio to maximize the profit made after investing in a set of securities for one year. We pursued two methods of portfolio design, one based on qualitative analysis of the company indicators and one based on the Markowitz model. Our qualitative analysis depended on the five economic indicators which represent the health of the company. We used these indicators to predict future growth and future stock price at the end of one year.

We tested this model against a fully quantitative model based on quadratic programming with the Markowitz model. The Markowitz model uses a covariance matrix to take the inherent volatility of the stock marketplace into consideration. We developed our model to be testable in a condition where we must justify investments against a risk-free possibility, such as a government treasury bill, and found that certain combinations of stocks with negative covariance can work together to minimize risk and maximize possible profits. With this model, we also showed that reaching an arbitrary level of profit gain is possible, even up to a 100% gain, while minimizing risk exposure with a mathematical function.

After using both methods we compared the two portfolios against each other to determine the relative performance. Obviously, since we do no know how the market will perform in the next year, we cannot make a definitive prediction about how the portfolios will perform. Both models exhibited relatively wide investment diversity by choosing companies making different products and in different areas. The qualitative model had the advantage about being able to incorporate information about future product and growth opportunities which could not be quantified in numerical data. By changing the input parameters to our Markowitz model we can input the amount of profit we want to make and find a portfolio with the least risk that will enable us to achieve that.

The Problem

The aim of every stock market investor is to make a profit through the buying and selling of company shares. Investors use different methods and indicators to determine which companies would be the most likely to bring a profit to the shareholder with an acceptable level of risk. Our goal is to determine a method to select the best combination of up to six stocks using the given $30,000 to maximize the net profit after 1 year. We used the indicators of stock quality and risk to make our choices of stock. We used two models, the value investment theory and the Markowitz model.

The value investment theory involves qualitatively assessing stocks' fitness based upon market metrics, such as free cash flow, return on invested capital, P/E ratio, etc. Based upon how each stock performs in these metrics, the decision is made whether to invest in the stock or not. The Markowitz model, by contrast, attempts to quantify many factors involved in investment in the stock market. It uses covariance, how one stock changes in relation to another stock, to calculate risk associated with investing in certain stocks. Given certain parameters it can give us the best investment path while taking risk and reward into account.

Assumptions

1. The investors are rational and will not attempt to assume too much risk upon the purchase and sale of stocks. The irrationality of stockholders is a random variable that is almost impossible to account for, and risks will probably balance each other out anyway.
2. The market is generally rising, as stated by the prompt.
3. As a portfolio manager our goal is to maximize the return with an acceptable level of risk. To achieve this goal we assume that we can use government treasury bills as a risk-free investment when the stock options we have available are too risky. We also assume that the covariance of the treasury bills with the stock market is zero for our calculations in the Markowitz model.
4. The number of shares outstanding over a year remains relatively constant, because a company will not sell a large portion of its shares and, similarly, will not buy a large portion. Since the companies are rather stable, we can assume that the companies do not need to perform any significant changes in the number of outstanding shares.
5. There is no transaction cost and thus no incentive to buy more expensive stock. As such, the commission cost is not relevant to the long-term growth of a stock since we will be making a purchase only once in the year.
6. The current price-to-earnings ratio is relatively constant from this year to the next. This value is the best estimate we have to predict stock prices in the future.

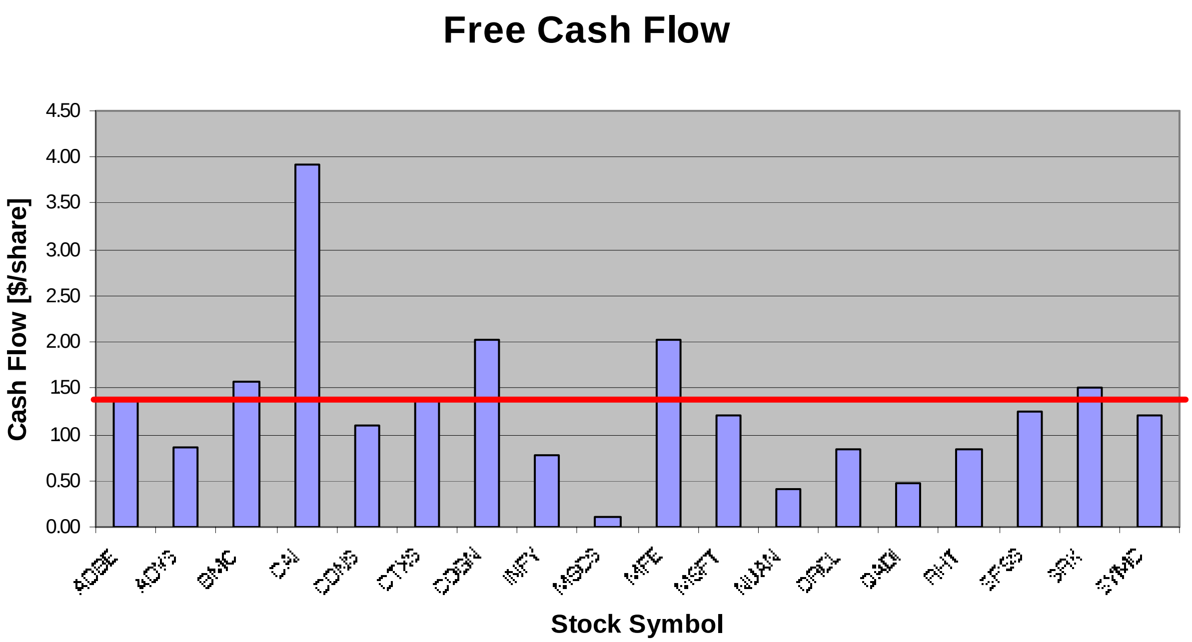
Value Investment Theory

The decision of which stock to purchase is often based on values of the indicators of the stock: free cash flow (FCF), return on invested capital (ROIC), P/E ratio (price-to-earnings ratio), P/S ratio (price-to-sales ratio), and beta (β).

Free Cash Flow

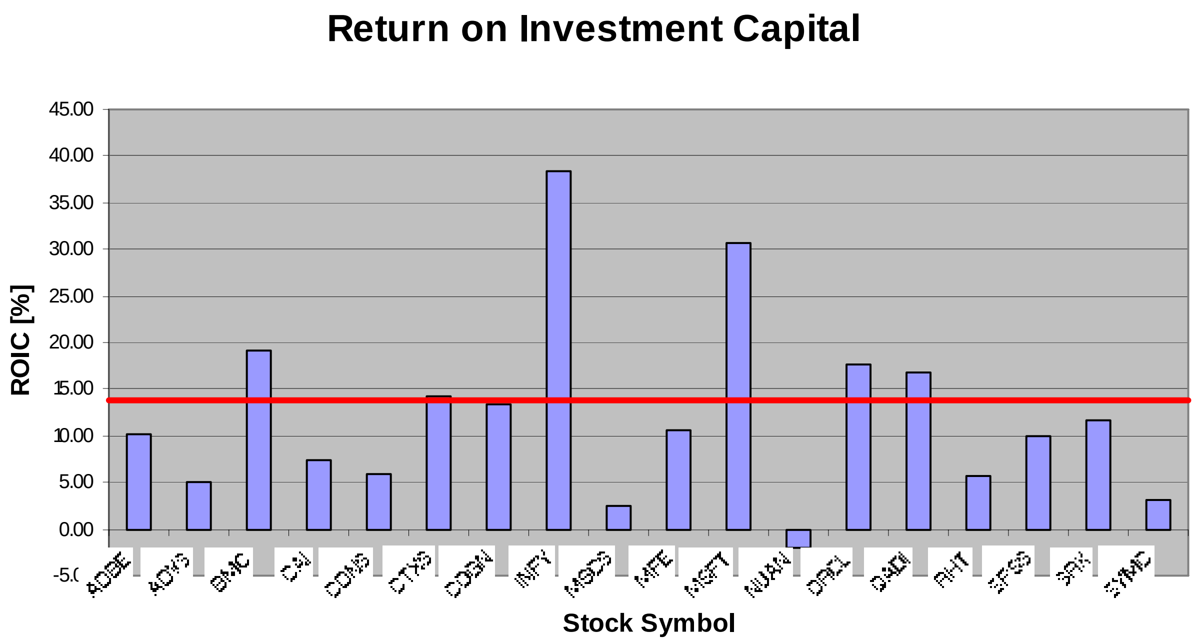
Free cash flow is determined by the net income plus amortization and depreciation, minus capital expenditures and dividends. A company may report strong earnings (total revenue minus total expenses) by manipulating what qualifies as an expense, but FCF can reveal how much of that money is actually left over after additional company spending. For our purposes, FCF indicates a strong company capable of generating growth and real money. An aggressive company which has strong FCFs even after heavy spending indicates a company with strong potential for growth. Deceptively, conservative companies may also achieve a large FCF by spending only a small portion of their earnings. A small or negative cash flow indicates a company which is spending its savings or borrowing money. A negative FCF, however, may have two implications. Young company's often have negative FCFs because they are spending heavily now with the hope of rapid growth in future revenue. However, a mature company with a low FCF usually signals a company has low prospects for future growth.

In analyzing the companies' potential for growth based solely on their FCF, CACI International, Inc. (CAI), with its $3.92/share, towers above the rest. CACI has made over thirty acquisitions in recent years. Such a large FCF is testimony to its strength as a company in light of heavy spending. The next five largest company FCFs are from Cognos Inc. (COGN), McAfee (MFE), BMC Software (BMC), SRA International (SRX), and Adobe Systems Inc (ADBE).



Return on Investment Capital (ROIC)

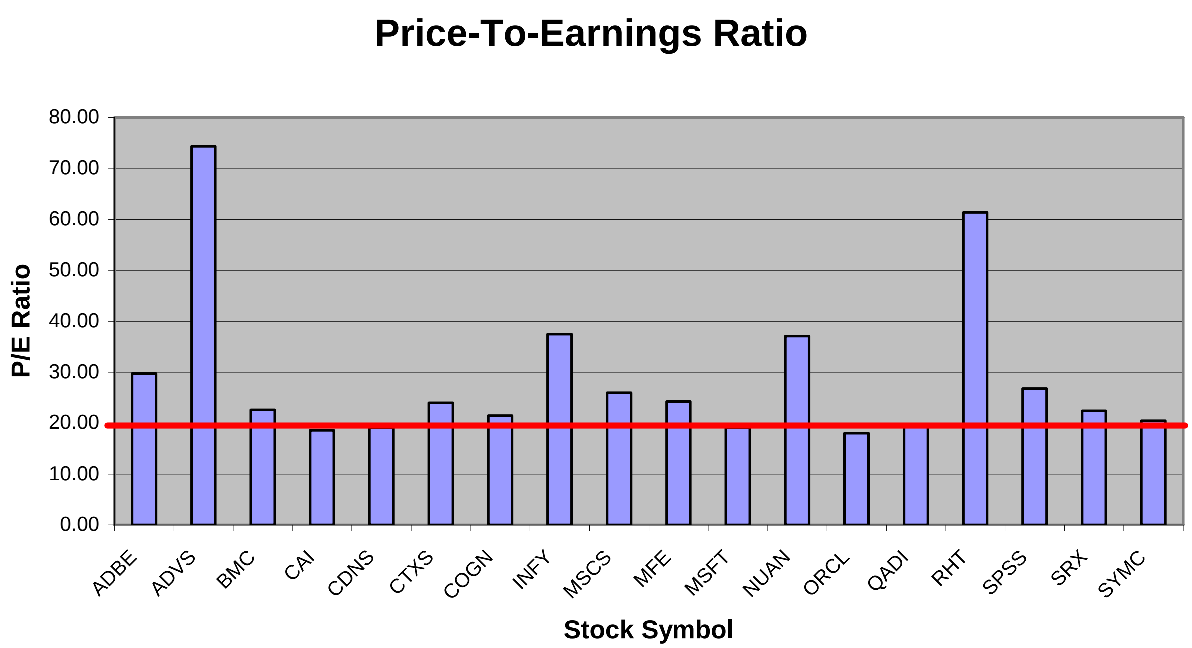
The return on invested capital represents how well a company can allocate its investments to make a profitable return. A high ROIC shows competent management which makes investment decisions which pay off over time. A high ROIC would indicate a strong company whose stocks would most likely increase over time. However, while ROIC is considered one of the most reliable performance indicators for finding quality investments, it is rarely used because of the difficulty in obtaining the ROIC value. However, to make best use of the ROIC, one must look at the trends of the ROIC over the past few years. If there is an upward trend, it suggests the company is successfully capitalizing new investments and thus will have a strong performance in the coming year. Unfortunately, we were unable to find historical ROIC values, so we made our decision based on the current ROIC values.



Looking at the current ROIC values, the six companies with the highest ROIC are Citrix Systems Inc. (CTXS), QAD Inc. (QADI), Oracle Corporation (ORCL), BMC Software (BMC), Microsoft Corporation (MSFT), and InfoSys (INFY).

Price-to-Earnings Ratio

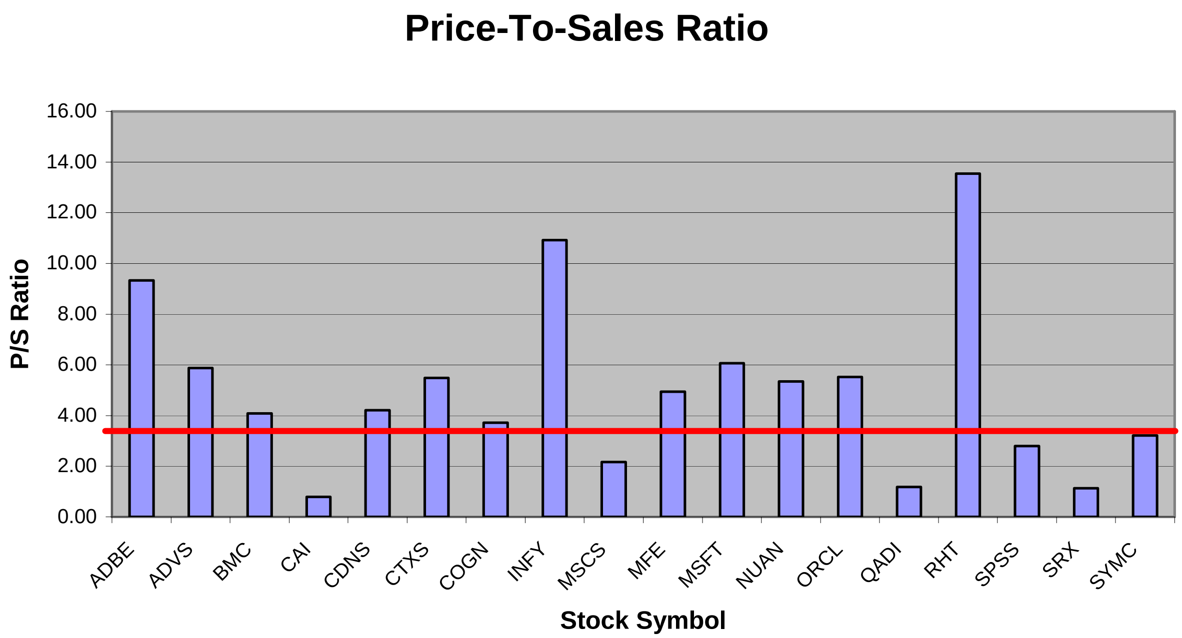
The price-to-earnings ratio compares a company's stock price against its per-share earnings. Usually, a high P/E suggests an expectation of higher earnings growth in the future compared to companies with a lower P/E. However, a higher P/E also means there is more risk in buying the corporation. A lower P/E ratio means the stock is a better value. It is also important to note that in general P/E values of a company can only be compared to the P/E values of companies in the same industry. However, this limitation does not affect us since all the companies we can invest in are in the computer software/services industry. The P/E ratio is also known as the price/earnings multiple or, simply, the multiple, because it also shows the amount investors are willing to pay per dollar of earnings.



Among the eighteen companies given, the six stocks with the lowest P/E ratio are Oracle Corporation (ORCL), CACI (CAI), CDSS (CDNS), Microsoft Corporation (MSFT), QAD Inc. (QADI), and Symantec Corporation (SYMC).

Price to Sales Ratio

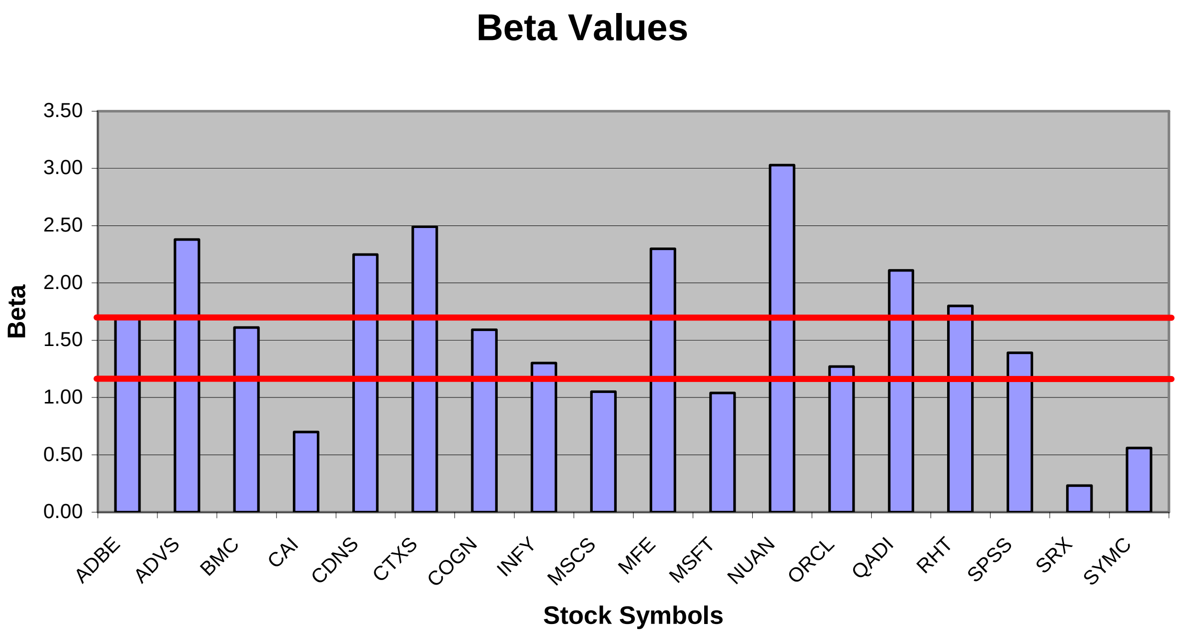
The price-to-sales ratio is the ratio of the market capitalization to the company's revenue for the past year. P/S ratios are most relevant when comparing within the same sector; for this example, all of the companies are computer software/services companies, and thus the P/S ratio has meaning. A lower P/S ratio is good because this means that the company is not overvalued and has a larger potential for growth. Since sales are harder to manipulate than earnings, the P/S ratio is regarded by many to be a truer measure of a company's performance than the P/E ratio.



The six stocks in the P/S ratio with the lowest P/S ratios are CACI (CAI), SRA International (SRX), QAD Inc. (QADI), MSC Software Corp. (MSCS), SPSS Inc. (SPSS), and Symantec Corporation (SYMC).

Beta

Beta (β) represents the overall volatility of a stock as compared to the market, or as compared to the specific portfolio or basket. In this case, we are measuring beta of a stock with respect to the NASDAQ. If beta is close to one, then the stock returns in question will tend to follow the returns of the broader stock market. That is, if the stock market rises, then so will the stock. In broad terms, if a stock has a beta coefficient of less than one, it changes less than the stock market over a period of time, whereas a beta coefficient greater than one indicates greater movement over a period of time than the market. Since our investment outlook is a medium range of only 1 year, we want a relatively modest volatility of around 1.50. Too large of a value could result in unacceptable risk, since the stock may be fluctuating down when we have to sell, resulting in a loss. The top six stocks closest to a beta of 1.50 are Cognos Inc. (COGN), SPSS Inc. (SPSS), BMC Software (BMC), Adobe Systems Inc. (ADBE), Infosys (INFY), and Oracle Corporation (ORCL).



Conclusion for Value Investment Theory

Upon analyzing all eighteen stocks under these five criteria, we have decided upon diversifying our portfolio in six different stocks. We acknowledge that the eighteen companies come from similar sectors and hold overlapping interests and, as a result, strong sales from one company may negatively impact another company. In choosing the following six, we also looked into the growth potential within the next year for these companies by examining their markets, business plans, and past trends. Another positive factor of this selection of stocks is the geographical location of the companies. Not all the companies selected are U.S.-based. The location of these companies range from the United States to Canada to India. This geographic diversification insures some stability by minimizing risk through the varying countries' economies. Our portfolio is as follows:

CACI International

CACI International Inc. (CAI) serves both federal and state governments in the United States and also offers consumer and market analysis. Its services cater to major markets in areas such as defense, intelligence, and homeland security—all promising markets for future growth. Furthermore, a recent report has shown that the United States government is allocating more money for private contracting companies, which represents another growth opportunity for this company. Its large FCF and exceptionally low P/S ratio are strong cues for investment, compensating for its mediocre P/E ratio and lackluster ROIC and beta value.

Cognos Inc.

Cognos Inc. (COGN) focuses upon business intelligence and performance management software. Cognos works with businesses both as a whole and in particular areas looking for ways to enhance performance. Its Cognos 8 architecture provides flexibility in dealing with its customers, and its recent acquisition of Celequest Corporation is highly promising. Cognos exhibits a strong FCF at $2.03/share along with a high ROIC. Additionally, the P/E and P/S ratios are fairly attractive and the beta value is in the ideal range at 1.59.

Infosys Technologies Ltd.

Infosys Technologies Limited deals with information technology services. It is one of India's largest IT companies boasting revenue of over $2.2 billion. At a glance, its FCF of $0.78/share is among the lowest. Yet in examining its best-of-the-group ROIC, one may conclude that the extra money from Infosys earnings are well spent, and continued growth is probable. The P/S and P/E ratios are seemingly undesirable, but such numbers reveal that the Infosys is growing and investing large amounts of money. Its beta falls within our desirable range, and all in all, Infosys demonstrates strong growth potential as it invests heavily.

QAD, Inc.

As the United States becomes more involved with international companies and trades, globalizing its market to boost the economy, more software and applications to ease communication among customers, suppliers, and partners worldwide become increasingly in demand. QAD, Inc. provides such enterprise applications on two levels: the Enterprise level, which provides software for intra-enterprise functions, and the Extended Enterprise level, which provides communication for supplier-management and customer-management functions. As such, global manufacturers can collaborate with whomever they need to in order to insure prompt delivery of the requested product. QAD, Inc. was selected as one of our stocks because it was one of the top stocks in the ROIC, P/E ratio, and P/S ratio categories. This then suggests that this company is efficient in managing its money and has a promising future. The low P/E ratio suggests low risk in the future and the low P/S ratio shows that the company is not overvalued. Thus, QAD, Inc. has large potential for growth and is a strong, quality company in which to invest.

BMC Software, Inc.

BMC Software, Inc. is an independent software vendor that provides software solutions for the management of information technology infrastructure. It also provides applications for enterprise systems, applications, databases, service management, though it focuses on service management. Among the five indicators, BMC Software, Inc. appeared in the top six list in FCF, ROIC, and beta. These indicators show that BMC Software, Inc. is not only efficient in managing its investment capital but also has a large amount of free cash to spare. The risk value, beta, is also low. Thus, BMC, as an efficient and well-managed company, is a good choice for investment.

Oracle Corporation

Oracle Corporation is an enterprise software company that develops, manufactures, distributes, and services database, middleware, and business management software. All of the software is available and functional for all business, from small and mid-size to global enterprises. They are also able to run on low-cost machines, which is ideal for all companies to use. This is one of the reasons that Oracle Corporation is promising for the future as a versatile provider of software for businesses, especially in the United States, which has thousands of businesses of all sizes. Oracle Corporation was also selected based on its ranking in the top six in the categories of ROIC and P/E ratio. Again, this shows that this company is efficient in managing its finances and allocating the appropriate amount into its operations. Its high ROIC value also indicates that their stocks will increase over time. Since its P/E ratio is low, there is another indicator that Oracle Corporation is a strong company. As such, we have selected Oracle Corporation as another member of our portfolio.

Distribution

We concluded the following distribution of funds based solely on the collaborative analysis of the five indicators: FCF, ROIC, P/E ratio, P/S ratio, and beta.

Company	Portion of Funding	Allotted Money (est.)	Prices per share	Number of Shares	Initial Leftover Funding	Allotted Money Used
CAI	~30%	~$9,000	$46.41	193	$42.87	$8,957.13
COGN	~20%	~$6,000	$39.78	150	$33.00	$5,967.00
INFY	~15%	~$4,500	$53.10	84+2	$39.60	$4,566.60
QADI	~15%	~$4,500	$8.07	557+1	$5.01	$4,503.06
BMC	~10%	~$3,000	$29.96	100	$4.00	$2,996.00
ORCL	~10%	~$3,000	$16.71	179+1	$8.91	$3,007.8

Total Spent: $29997.59
Money Left: $2.41

In reviewing the recent evidence concerning relatively high ROIC and relatively low P/E as strong indicators of stock value, we chose to invest in the same six stocks in light of the other three indicators. We considered reallocating the amount of funding to each stock, but as we considered the companies' ROIC and P/E values, we noticed that those with high ROIC values often had relatively high P/E values, in comparison to the other six companies. For instance, InfoSys had a very high ROIC, but its P/E value was also rather high. Similarly, CACI sported a relatively low and ideal P/E value, but had an extremely low ROIC value. Thus, we concluded that the current allocations still represent the optimal ranking of stocks according to the five indicators as well as the recent evidence that relatively high ROIC and low P/E indicate higher stock values.

Markowitz Model

Since we were not convinced that the current metrics (FCF, ROIC, P/E, P/S, and beta) provided a thorough picture of the performance of these stocks, we wished to combine their recent performance histories (how the stock has performed over the last year) into a unified model. Under the modern portfolio theory, championed by Harry Markowitz and William Sharpe, the return of a single asset like a stock can be modeled mathematically as a random variable, with a group of stocks (a portfolio) modeled as a weighted average of these stock investment returns.

Though our objective was to maximize the net profit from a purchase of technology stocks in a market which we can assume is generally rising, we cannot neglect the inherent risk associated with any investment in individual stocks. An investment in any one of these eighteen companies must be justified against an equivalent investment in a hypothetically risk-free asset like a short-dated government security. The current rate of return for a one-year government treasury bonds is 5.05% in March 2007 (source: http://www.forecasts.org/1yrT.htm). We used this so-called no-risk investment as a benchmark against which to measure the advantage we would gain by investing in any of the stocks. We included treasury bonds as another viable investment option along with the eighteen stocks provided to us.

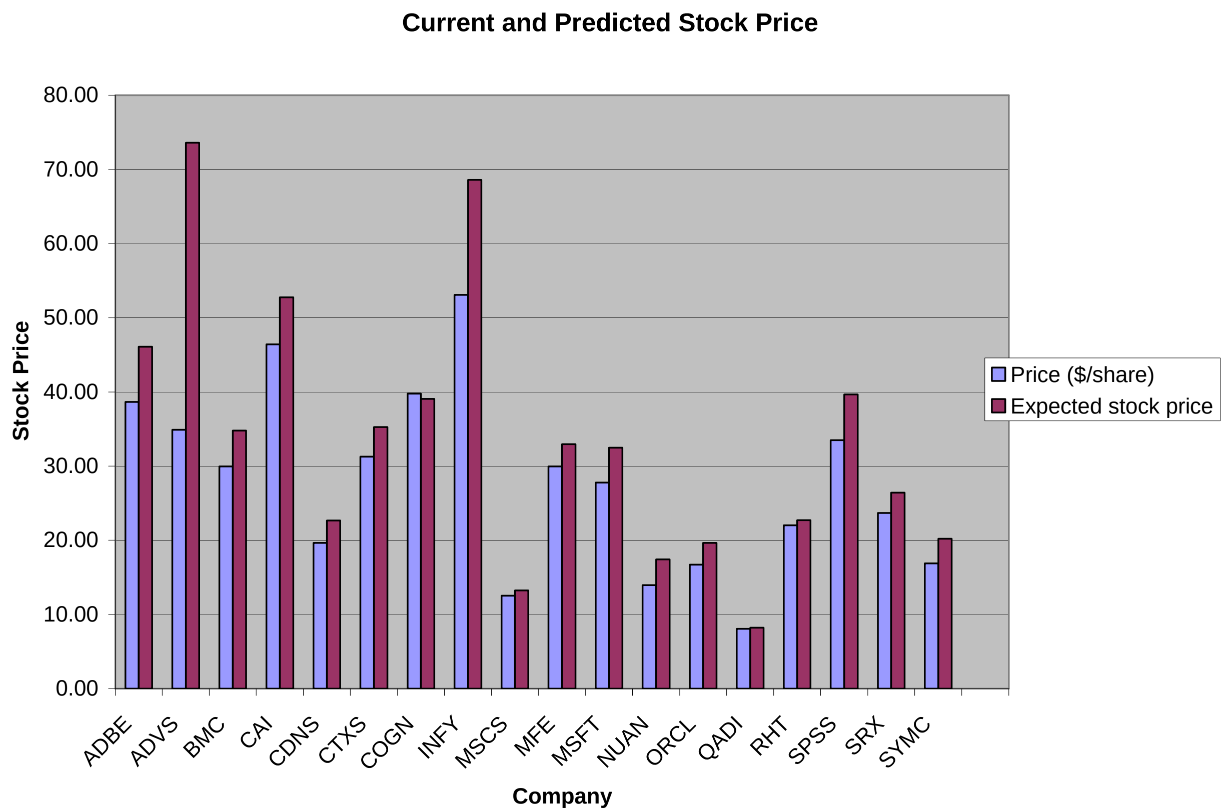
The Method Behind the Model

Our goal was to develop a Markowitz model which would find the optimal allocation of our $30,000 capital among the available technology stocks and the one-year T-bill. The Markowitz model is a type of quadratic programming optimization model, which balances the two goals of maximizing net profit and minimizing the risk of not obtaining a profit. Thus, the objective is to minimize the variance of a portfolio's total return, with the constraints that the expected growth of the portfolio is at least some target level and that we don't invest more capital than we have.

Quadratic programming is a special type of mathematical optimization problem, in which the problem can be formulated with an *n* by *n* matrix called Q, and our aim is to minimize an objective function which is second order (quadratic) in all variables and subject to an inequality and equality constraint. Quadratic programming problems can be solved with a number of numerical optimization methods, including the active set, conjugate gradient, and interior point methods. We utilized Maplesoft's Maple 10 program, which contains a QPSolve command that solves such optimization problems using the active set method.

As a quadratic program, we first model the decision matrix X as the set $\{x_1 ... x_n\}$ of the amount of capital (in dollars) we allocate to each stock possibility. In our case, $n = 19$ since we have eighteen technology stocks and the T-bills. We are subject to the constraint $\sum x_i \le b$, where *b* is the budget of $30,000 we are allocated.

In order to make predictions about the future, we need to have an expectation for the rate of return that can be anticipated from an investment in any given stock. To find these expectations, we used the P/E ratios provided and future earnings per share estimates that we found online. We assume that the P/E ratio is relatively constant because the company will keep making money at about the same rate that it is now. This is a necessary assumption, because there is no way to predict future earnings reliably. We also assume that the current price of the stock is not an anomaly, i.e., that there is not an irrational stock panic or speculative bubble surrounding any stock on our list. To get the future stock price, we multiplied the future earnings estimate for the next year (averaged from predictions by various stock analysts) by the current P/E ratio. This is the best indicator we have of the stock price at the time we sell it. We then applied the formula $R = \frac{(Expected - Current)}{Current}$, finding the proportion of the current stock price that we expect to gain from a year investment in each stock.



With our expected percent return on each stock's value, which we called R, we could calculate the expected profit from each stock i by multiplying the money allocated to i (called x_i) by the expected percentage gain: $x_i \cdot R_i$.

By adding up all of these expected profits, we can find the expected total gain from our investment portfolio. Since our model will allow us to arbitrarily set the profit we wish to obtain from our transaction (called g), the inequality constraint for our quadratic programming model will be $\sum x_i \cdot R_i \geq g$.

Our final constraint is that all of the money allocations in the set $\{x_1 ... x_n\}$ are non-negative, since it is impossible to invest a negative amount of money in a stock.

Finally, we needed some way to estimate the relative risk involved in additional investment in each stock. Since we are limited to a small number of stocks that are not only in the same sector, but within the same industry, these stocks are extremely likely to move up and down with high (positive or negative) correlation to each other. For example, a gain in the stock of Adobe corporation, which produces a number of imaging solutions, can indicate either that Adobe is stealing market share from its competitors or that the consumer market has experienced a greater demand for imaging solutions. Thus, we would expect that its competitor Nuance (which also produces imaging solutions) would see a marked decrease in its share price as it lost market share to Adobe, or a similar increase in light of increased demand.

This relationship in the field of statistics is called the covariance between two random variables. We thus entered data for the price of each stock over the last year (from March 4, 2006 to March 4, 2007) into the Maple program and found the covariance of each pair of stocks as a matrix. This historical data is summarized in the graph below. The thick red line is a representation of the movement of the entire set of eighteen stocks base on their market capitalization and stock price relative to the total market capitalization of the index. The percent contribution of each share is

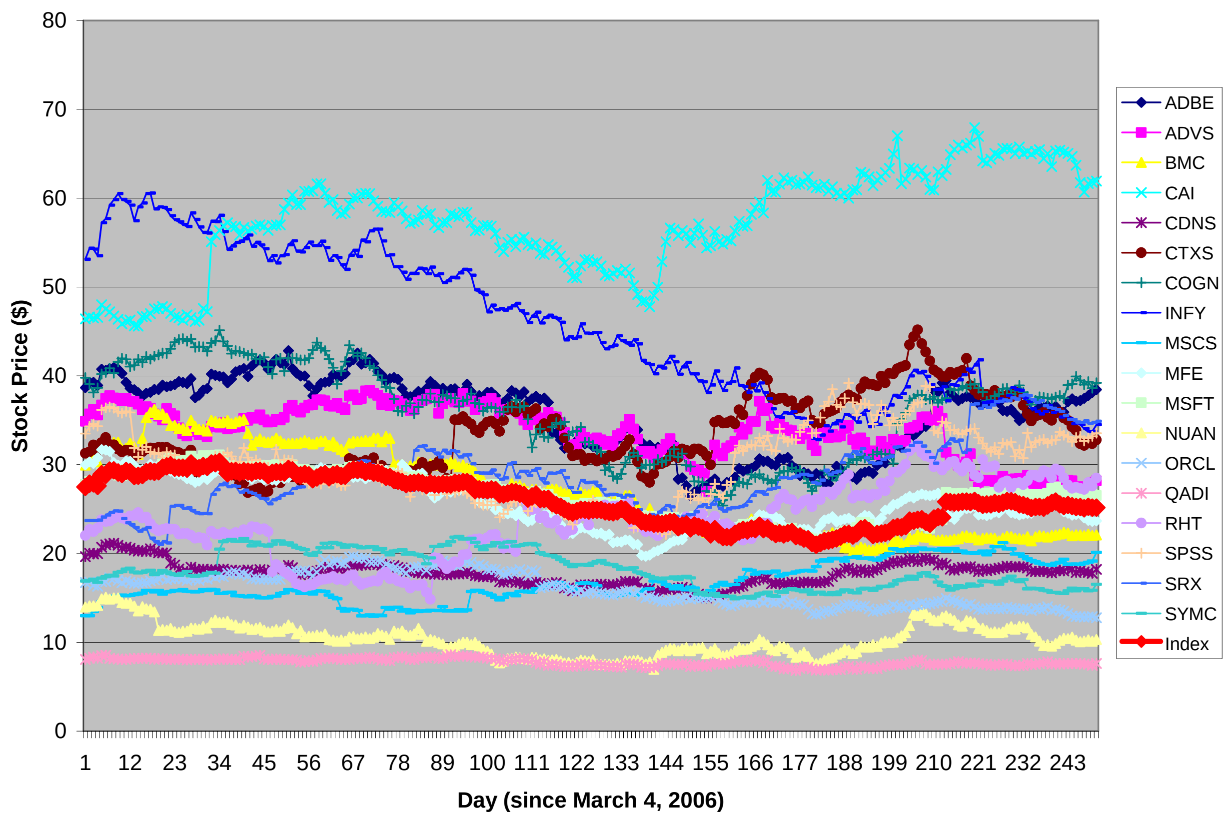
$$percent = \frac{shares \cdot (share\ price)^2}{total\ market\ cap} = \frac{market\ cap \cdot share\ price}{total\ market\ cap}.$$

The total index is represented by

$$I(d) = \sum_i \frac{OutstandingShares_i \cdot Price_i(d)}{\sum_j OutstandingShares_j \cdot Price_j(d)},$$

where d is the day.

The covariance matrix, which we called Q, was the constraint matrix of risk for our Markowitz model. Investing in two stocks which have negative covariance is considered a positive balance, as it increases diversification by hedging the risk of a fall in one stock by a stock which tends to rise during the same time. Even though an asset's expected return may be negative, its covariance with other assets may be sufficiently negative to reduce the variance of total return and decrease the overall risk. Thus, its optimal allocation under this model would be positive.



Our second-order objective function is therefore $X^2 \cdot Q$ (where one occurrence of the X-matrix was transposed for matrix multiplication). The result, when expanded, was a quadratic function in every stock variable.

Testing the Model

In order to test our model, our primary variable was the profit gain g from the investment that we wanted to ensure. Our Markowitz model will then use the QPSolve function of Maple to determine the local minimum point on our objective function that has the least possible risk. Our expectation is that for lower profit gains, the Markowitz model will instruct us to invest most of our money in T-bills, which are the safest way to make a small amount of money. The more profit that we wish to make, the more risk we will have to assume—and the more the Markowitz model will allocate money to increasingly risky stock investments.

We could also change the amount of money that we wanted to spend to be less than the \$30,000, but since we have assumed that T-bills have no risk of loss, there is no incentive to keep any money in cash form over the year instead of investing it into a T-bill. Therefore, our equality constraint is $\sum x_i = b = \$30{,}000$.

To investigate this hypothesis, we tried our model at the point where $g = \$1{,}000$. Our result was, expectedly:

$$[\,advs = 8.07793566946316090\ 10^{-28},\ bmc = 0.,\ nuan = 0.\\ ,\ orcl = 0.,\ adbe = 0.,\ cdns = 0.,\ mscs = 0.,\ cai = 0.\\ ,\ tbill = 30000.,\ mfe = 0.,\ srx = 0.,\ ctxs = 0.,\ cogn = 0.,\ infy = 0.\\ ,\ msft = 0.,\ symc = 0.,\ rht = 8.07793566946316090\ 10^{-28}\\ ,\ oadi = 0.,\ spss = 0.\,]$$

The numbers listed are the dollar amounts to be invested in each type of stock or bill. Since we only wanted to make a tiny amount of profit on our investment of \$30,000, the model confirmed that the risk is minimized by investing the entire budget in risk-free T-bills. Notably, since the model is an approximation method and not exact, it suggested that we invest a tiny fraction of a cent (on the order of 10^{-28}) in ADVS and RHT.

Next, we assumed that we would want a more reasonable return of 10%, or \$3,000. With $g = \$3{,}000$, our result was:

$$[\,advs = 1269.83956407610958,\ bmc = 0.,\ nuan = 0.,\ orcl = 0.\\ ,\ adbe = 0.,\ cdns = 0.,\ mscs = 443.077442006085732,\ cai = 0.\\ ,\ tbill = 27687.2449647882858,\ mfe = 0.\\ ,\ srx = 189.466172109124954,\ ctxs = 0.,\ cogn = 0.,\ infy = 0.\\ ,\ msft = 54.4444025132373782,\ symc = 0.\\ ,\ rht = 355.927454507154892,\ oadi = 0.,\ spss = 0.\,]$$

Now, most money was still being invested in the T-bills, but some was being allocated to the stocks in our technology basket. The stocks that came out on top were ADVS, MSCS, SRX, and RHT. These stocks have relatively low volatility and negative covariance with each other, guaranteeing a high level of diversification, as seen in the Appendix B.

With a much higher return of 33%, or \$10,000, we needed to take on a much higher amount of risk, as confirmed by our model:

$$[advs = 7854.19285928557020,\ bmc = 0.,\ nuan = 0.,\ orcl = 0.\\ ,\ adbe = 0.,\ cdns = 0.,\ mscs = 2740.51603018579636,\ cai = 0.\\ ,\ tbill = 15695.1818192460706,\ mfe = 0.\\ ,\ srx = 1171.88336082310684,\ ctxs = 0.,\ cogn = 0.,\ infy = 0.\\ ,\ msft = 336.748711841134764,\ symc = 0.\\ ,\ rht = 2201.47721861832542,\ oadi = 0.,\ spss = 0.]$$

Now stocks like ADVS, MSCS, RHT, SRX, and MSFT were being added to our portfolio in much higher proportions, and the T-bills accounted for only half of our investments.

Assuming that we wanted to make an incredibly high return on our investment of 100%, we set $g = \$30{,}000$ and tested the model there:

$$[advs = 26849.0878237021352,\ bmc = 0.,\ nuan = 0.,\ orcl = 0.\\ ,\ adbe = 0.,\ cdns = 0.,\ mscs = 0.,\ cai = 853.550746956713738\\ ,\ tbill = 0.,\ mfe = 0.,\ srx = 0.,\ ctxs = 0.,\ cogn = 0.,\ infy = 0.\\ ,\ msft = 0.,\ symc = 0.,\ rht = 2297.36142934115106,\ oadi = 0.\\ ,\ spss = 0.]$$

Now, risk has been assumed across the board, and ADVS is taken up with almost our entire model. Investment in T-bills is completely nonexistent, replaced by investment in RHT and CAI as well.

Conclusion

We started with the objective of maximizing our profit from a \$30,000 investment in up to six stocks from a basket of eighteen technology companies. By utilizing a number of performance indicators, especially the P/E ratio, we attempted to identify companies with stocks that were undervalued by today's marketplace and would likely be revalued in the future at a higher rate, thereby maximizing our gains.

However, our mathematical model has shown this investment strategy to be relatively shortsighted. The market of the real world, to be sure, is highly volatile as a whole and cannot be relied upon to revalue an undervalued stock appropriately. By investing in any stock, we are exposing ourselves to a high level of risk, and that risk exposure must be met with a proportional likelihood in reaching a great return on the investment.

We developed a model based on the research of Harry Markowitz and the modern portfolio theory that attempted to take the risks of the marketplace into consideration. By forcing our stock choices to justify themselves over an equivalent investment in T-bills, we ensured that our risk exposure would not be without a high likelihood of making a great return on our investment. From this model, we concluded that certain combinations of stocks with a negative covariance to each other would form optimal portfolios, thereby showing mathematical evidence that strategies of diversification can help reduce risk. We can, through our model, achieve an arbitrary level of profit gain while simultaneously minimizing our risk exposure. If we want to reach gains of 30–100%, we must take on significantly more risk, but we can still invest intelligently.

Appendix A – Source Code

```
/*
 * Index.java
 *
 * Created on March 4, 2007, 1:46 PM
 *
 * To change this template, choose Tools | Template Manager
 * and open the template in the editor.
 */

package moodys;

/**
 *  Represents our index of the 18 stocks
 *  @author Jonathan Newman
 */
public class Index {

    public final double[] index;

    /** Creates a new instance of Index */
    public Index(StockData sd) {
        index = makeIndex(sd);
    }

    private static double[] makeIndex(StockData sd) {
        StockData.Stock[] stocks = sd.stockData;
        double[] indexVals = new double[stocks[0].adjClose.length];
        double[] marketCaps = new double[stocks.length];
        double totalMarketCap;

        for(int day = 0; day < stocks[0].adjClose.length; day++) {
            totalMarketCap = 0;
            for(int j = 0; j < stocks.length; j++) {
                marketCaps[j] = stocks[j].sharesOutstanding *
stocks[j].adjClose[day];
                totalMarketCap += marketCaps[j];
            }
            for(int j = 0; j < stocks.length; j++) {
                indexVals[day] += stocks[j].adjClose[day] *
marketCaps[j]/totalMarketCap;
            }
        }
        return indexVals;
    }

    public String toString() {
        StringBuilder sb = new StringBuilder();
        for(int i = 0; i < index.length; i++)
            sb.append(index[i]).append(", ");
        return sb.toString();
    }
}
```

```
/*
 * StockData.java
 *
 * Created on March 4, 2007, 11:07 AM
 *
 * To change this template, choose Tools | Template Manager
 * and open the template in the editor.
 */

package moodys;
import java.util.*;

/**
 *  Encapsulates all the stock market data that we have for these stocks
 *  @author Jonathan Newman
 */
public class StockData {

    private static final String summaryData = "/Documents and
Settings/jn/Desktop/Moody's Mega Math Challenge/m3challengedata07.csv";
    private static final String stockPath = "/Documents and
Settings/jn/Desktop/Moody's Mega Math Challenge/Stock Prices/";

    public final Stock[] stockData;

    /** Creates a new instance of StockData */
    public StockData() {
        stockData = readData();
    }

    /**
     *  Encapsulates all the data for a particular stock
     */
    public static class Stock {
        public String ticker;              // Ticker symbol
        public String name;                // Company name
        public double pricePerShare;
        public double cashFlowPerShare;
        public double ROIC;                // Return on invested capital
        public double PEForward;           // Price to earnings ratio
        public double PS;                  // Price to sales ratio
        public double beta;                // Measure of volatility
        public String industry;            // Industry classification
        public double sharesOutstanding;   // Total number of shares
outstanding
        public long[] volume;              // The trading volume per day
        public double[] adjClose;          // Closing price each day
(adjusted)

        public String toString() {
            StringBuilder sb = new StringBuilder();
            sb.append("[").append(ticker).append(": ").append(name).append(",
")
             .append(this.pricePerShare).append(", ").append(cashFlowPerShare)
             .append(", ").append(ROIC).append(", ").append(PEForward)
```

```
            .append(", ").append(PS).append(", ").append(beta).append(", ")
            .append(industry).append(", ").append(sharesOutstanding)
            .append(", ").append(", [");

            for(int i = 0; i < volume.length; i++)
                sb.append(volume[i]).append(", ");
            sb.delete(sb.length() - 2, sb.length() - 1);
            sb.append("], [");
            for(int i = 0; i < adjClose.length; i++)
                sb.append(adjClose[i]).append(", ");
            sb.delete(sb.length() - 2, sb.length() - 1);
            sb.append("]");
            return sb.toString();
        }
    }

    /**
     *  Inputs data from text data files
     *  @return The computer understandable version of the stock data
     */
    private static Stock[] readData() {
        ArrayList<Stock> stocks = new ArrayList<Stock>();
        String[] lines = io.FileInput.getInput(summaryData).split("\n");

        for(int i = 1; i < lines.length; i++) {
            Stock s = new Stock();
            String[] stockInfo = lines[i].split(",");
            s.ticker = stockInfo[0];
            s.name = stockInfo[1];
            s.pricePerShare = Double.parseDouble(stockInfo[2]);
            s.cashFlowPerShare = Double.parseDouble(stockInfo[3]);
            s.ROIC = Double.parseDouble(stockInfo[4]);
            s.PEForward = Double.parseDouble(stockInfo[5]);
            s.PS = Double.parseDouble(stockInfo[6]);
            s.beta = Double.parseDouble(stockInfo[7]);
            s.industry = stockInfo[8];
            s.sharesOutstanding = Double.parseDouble(stockInfo[9]);

            String[] lines2 = io.FileInput.getInput(stockPath + s.ticker +
".csv").split("\n");
            s.volume = new long[lines2.length - 1];
            s.adjClose = new double[lines2.length - 1];
            for(int j = 1; j < lines2.length; j++) {
                stockInfo = lines2[j].split(",");
                s.volume[j - 1] = Long.parseLong(stockInfo[5]);
                s.adjClose[j - 1] = Double.parseDouble(stockInfo[6]);
            }

            stocks.add(s);
        }
        Stock[] stocksArray = new Stock[stocks.size()];
        return stocks.toArray(stocksArray);
    }

    /**
     *  Outputs a human readable version of all the stock data
     */
```

```
    public String toString() {
        StringBuilder sb = new StringBuilder();
        for(int i = 0; i < stockData.length; i++)
            sb.append(stockData[i]).append("\n");
        return sb.toString();
    }
}

/*
 * Main.java
 *
 * Created on March 4, 2007, 10:47 AM
 *
 * To change this template, choose Tools | Template Manager
 * and open the template in the editor.
 */

package moodys;

/**
 *
 * @author Jonathan Newman
 */
public class Main {

    /** Creates a new instance of Main */
    public Main() {
    }

    public static void main(String[] args) {
        StockData sd = new StockData();
        Index ind = new Index(sd);
        System.out.println(sd);
        System.out.println(ind);
    }
}
```

Appendix B – Covariance Matrix

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S
Company N	ADBE	ADVS	BMC	CAI	CDNS	CTXS	COGN	INFY	MSCS	MFE	MSFT	NUAN	ORCL	QADI	RHT	SPSS	SRX	SYMC
ADBE	18.603	5.4592	15.331	-3.265	3.3129	-7.554	20.26	26.503	-4.1007	9.528	10.734	4.5758	5.5023	1.397	-7.077	-0.836	1.2492	6.1025
ADVS	5.4592	9.0305	10.082	-6.797	1.1145	-4.043	4.4864	17.868	-4.472	4.969	3.1709	1.0031	4.6287	0.796	-8.163	-1.834	-5.482	4.1115
BMC	15.331	10.082	23.973	-15.99	2.3537	-14.66	17.896	39.632	-8.4747	10.95	11.421	3.4201	8.0731	1.587	-14.44	-5.362	-9.432	7.4321
CAI	-3.265	-6.797	-15.99	32.36	-0.314	12.233	-3.2994	-28.91	7.6788	-3.269	-4.889	-0.697	-4.368	-0.736	8.1874	8.8241	18.09	-2.6424
CDNS	3.3129	1.1145	2.3537	-0.314	1.5841	0.3348	4.5179	4.739	0.0631	2.761	1.7151	1.9724	0.4773	0.244	0.2835	3.0797	0.4167	0.2816
CTXS	-7.554	-4.043	-14.66	12.233	0.3348	16.759	-7.6817	-22.76	7.1735	-4.283	-6.861	0.002	-4.881	-0.741	12.486	8.903	7.5504	-4.813
COGN	20.26	4.4864	17.896	-3.299	4.5179	-7.682	26.903	30.751	-3.0593	12.12	12.675	6.5506	5.3016	1.495	-5.918	2.3647	1.6485	5.7133
INFY	26.503	17.868	39.632	-28.91	4.739	-22.76	30.751	69.568	-14.281	19.09	19.264	7.1158	13.824	2.847	-23.31	-8.706	-16.29	12.504
MSCS	-4.101	-4.472	-8.475	7.6788	0.0631	7.1735	-3.0593	-14.28	4.9519	-2.847	-3.429	0.0239	-3.634	-0.599	7.7605	5.221	4.8443	-3.1938
MFE	9.528	4.9687	10.945	-3.269	2.7608	-4.283	12.122	19.085	-2.8465	8.281	6.1041	3.9889	3.547	0.938	-5.268	2.9289	-1.73	2.9116
MSFT	10.734	3.1709	11.421	-4.889	1.7152	-6.861	12.675	19.264	-3.4288	6.104	7.4558	2.5494	3.8112	0.893	-5.9	-2.064	-0.997	3.8558
NUAN	4.5758	1.003	3.4201	-0.697	1.9724	0.002	6.5506	7.1158	0.0239	3.989	2.5494	3.3608	0.5417	0.392	0.381	4.0178	-0.082	0.1032
ORCL	5.5023	4.6287	8.0731	-4.368	0.4773	-4.881	5.3016	13.824	-3.634	3.547	3.8112	0.5417	3.6937	0.636	-6.481	-3.531	-2.529	3.5472
QADI	1.3972	0.7962	1.5868	-0.736	0.2443	-0.741	1.4953	2.8466	-0.5987	0.938	0.8933	0.3917	0.6363	0.178	-0.997	-0.281	-0.341	0.5864
RHT	-7.077	-8.163	-14.44	8.1874	0.2835	12.486	-5.9184	-23.31	7.7605	-5.268	-5.9	0.381	-6.481	-0.997	16.485	8.5991	6.6286	-5.7439
SPSS	-0.836	-1.834	-5.362	8.8241	3.0797	8.903	2.3647	-8.706	5.221	2.929	-2.064	4.0178	-3.531	-0.28	8.5991	16.331	4.5304	-4.3309
SRX	1.2492	-5.482	-9.432	18.09	0.4167	7.5504	1.6485	-16.29	4.8443	-1.73	-0.997	-0.082	-2.529	-0.34	6.6286	4.5304	14.937	-1.0807
SYMC	6.1025	4.1115	7.4321	-2.642	0.2816	-4.813	5.7133	12.504	-3.1938	2.912	3.8558	0.1032	3.5472	0.586	-5.744	-4.331	-1.081	4.4986