

MathWorks Math Modeling Challenge 2018

Marvin Ridge High School–

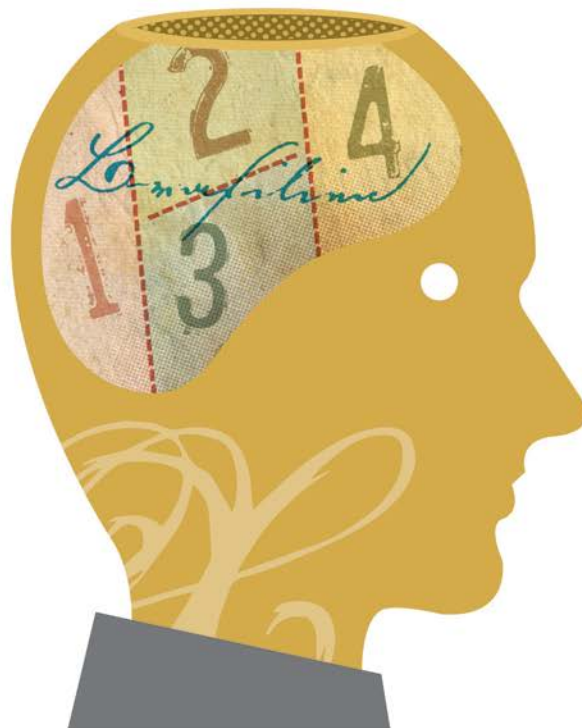
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MathWorks Math Modeling Challenge Runner Up

\$15,000 Team Prize



Better ATE Than Never

Executive Summary

Food insecurity is one of the largest factors contributing to the perpetuation of poverty in the United States. Defined by the USDA as “a lack of consistent access to enough food for a healthy, active life,” food insecurity is often overlooked in nations with a high standard of living such as the U.S., yet in 2016, 1 in 8 Americans were food insecure, a total of 41.2 million individuals^[1]. Growing awareness of the relationship between food insecurity and poverty is drawing attention to the need for reduction in food wastage^[2].

With roughly $\frac{1}{3}$ of the food produced for human consumption going to waste, nations must concern themselves with reducing waste and allocating saved food supplies to food insecure individuals^[3]. Residents of North America and Oceania waste the most food out of anyone in the world, at approximately 300 kg/capita annually^[4]. The high degree of food waste in the United States has prompted increased focus on food redistribution strategies. The development of quantitative models to analyze the feasibility and potential efficacy of such models are critical to assuring that redistribution strategies can succeed in meeting the needs of the food-insecure. In order to meet this need, we have developed a variety of models which aim to predict food wastage and food need, identify key target demographics to reduce food wastage, and develop a model strategy for addressing food insecurity in our community.

First, we developed a composite model which predicts total food waste and total food need for a given state (both in terms of dollars). The model uses data on food production to determine the total value of food wasted during distribution and consumption, and demographic data to calculate the total dollar worth of need required to lift the food-insecure population to a state of food security. Finally, the model subtracts need from wastage to determine whether the state wastes enough food to meet the needs of its food-insecure population, thus making a food redistribution strategy viable.

We then used data on food wastage by demographic to create a multivariate regression model to predict dollar worth of annual food wastage per household based on characteristics such as income and family composition. A multiplier based on individual caloric intake was also added to adjust waste estimates for household member ages. This model was created in Mathematica, and it can be used to identify key demographics which waste large amounts of food.

Finally, we developed a set of strategies to address the problem of food insecurity within our community, and compared their effectiveness using a benefit/cost ratio of each strategy. Information about grocery stores, food consumption, and food insecurity in our county were used to create strategies that maximized redistribution at minimum cost. Overall, our investigation revealed that redistribution is most effective in food-producing regions, that a high-income households with many people are key targets for reduction in food waste, and that a grocery salvage strategy holds promise for reducing food insecurity in our community.

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Restatement of the Problem

The problem asks us to do the following:

- Create a mathematical model which can predict (a) the amount of food waste in a state, and (b) the amount of food needed to feed the food-insecure population in that state, in order to determine whether or not redistribution of food waste could solve food-insecurity. We are to demonstrate the use of this model for Texas.
- Create a mathematical model which predicts total annual food waste for a household based on traits such as the age and number of its residents and its annual household income, and then demonstrate the use of this model for a given set of households.
- Identify a community of interest and develop a mathematical model to determine the most efficient strategies for redistribution of food; that is, the strategies which produce maximal food redistribution at minimum cost.

Part I: Just eat it!

A. Restatement of the problem

Using the number of food-insecure people in a state as well as the amount of food per person needed to bring that person to food security, the total cost of food needed to achieve food security can be determined. Similarly, using food waste per person (which varies by income bracket) and the population of the state, the total value of food waste can be determined. These two numbers can then be compared to determine the viability of a food redistribution strategy.

B. Assumptions

- The average number of meals needed to bring a person from food insecurity to food security is constant across all states. The difference in cost needed to achieve food security across states arises from the different costs of meals in different states.
- The cost of food is directly proportional to its weight.
- Food lost via agricultural production, post-harvest handling and storage, and processing cannot be redistributed as it is not fit for human consumption. Food lost via distribution and consumption can be redistributed as it is simply not purchased or consumed, respectively.

C. Developing the Model

In order to compare food waste to food need, these parameters need to be expressed in terms of the same units. In this case, food waste and food need are best compared using units of dollars (\$); that is, the value of food wasted can be compared to the cost of food need.

Part A: Determining Food Security Need

The first task is to determine the dollar worth of food (i.e., cost) needed to bring a state's food-insecure population to a state of food security. To calculate this, the food-insecure population of the state is needed, as well as the annual cost per person to achieve food security.

$$\$Food\ Security\ Need = (\# Food\ Insecure\ Persons) * (\$ Need\ per\ person\ per\ year)$$

The number of food insecure persons can be easily calculated by multiplying the state's proportion of the population which is food-insecure (available at map.feedingamerica.org) by the state's total population.

$$\# Food\ Insecure\ Persons = (Proportion\ Food\ Insecure) * (Population)$$

Annual need per person was calculated in the following manner. First, we assumed that the average number of meals needed by an individual to achieve food security is relatively constant across various states, as listed above. This assumption is justified because the differing costs to achieve food security in different states arise from the cost of meals there. Variance of personal need across states comes from the cost of meals within that state. Additionally, feedamerica.org reports that, on average, food-insecure households experience food insecurity during 7 out of 12 months of the year, or about 30 weeks/year.

Cost Per Person Per Year

$$= (\# meals\ needed / week) * (meal\ cost\ w/in\ state) * (30\ weeks/year)$$

According to feedamerica.org, the average food-insecure American requires an average of \$17.38/week to become food secure. Average national meal cost is \$2.99, so the number of meals per week needed by the average food-insecure American to achieve security is $\$17.38/\$2.99 = 5.81$ meals. The meal cost per week is a parameter which will be input to the model when considering a particular state.

Thus the cost per person per year is as follows:

$$\begin{aligned} Cost\ Per\ Person\ Per\ Year &= (5.81\ meals/week) * (meal\ cost\ w/in\ state) * (30\ weeks/year) \\ Cost\ Per\ Person\ Per\ Year &= (174.30\ meals/year) * (meal\ cost\ w/in\ state) \end{aligned}$$

At this point, the total cost of food needed to achieve food security for all food-insecure individuals within a state for one year is given as follows:

$$\begin{aligned} \$ Food\ Security\ Need &= (\# Food\ Insecure\ Persons) * (\$ Need\ per\ person\ per\ year) \\ \$ F.S.N. &= (Proportion\ Insecure) * (State\ Population) * (174.30\ meals/year) \\ &\quad * (meal\ cost\ in\ state) \end{aligned}$$

Part B: Determining Food Waste

The second task is to determine the dollar worth of food wasted in year within a given state. We based our model for loss on the data provided by the Food and Agriculture Organization (FAO) of the United Nations, which provides information on food waste by weight through various processes on the path from production to table. As explained in our assumptions, we assume that food lost via agricultural production, post-harvest handling and storage, and processing is not edible and cannot be distributed. Only food currently being wasted through distribution (not being bought) and consumption (not being eaten) is fit for redistribution.

The percentage of food wasted through the two phases varies by the category of food. For a given category, the total dollar worth of food loss is given by the following:

$$\$ Wasted = (\$ Wasted \text{ via Consumption}) + (\$ Wasted \text{ via Distribution})$$

Additionally,

$$\$ Wasted \text{ via Consumption} = \text{State Receipts } (\$) * \text{Consumption Waste Percentage}$$

State Receipts are the total amount spent by consumers on the goods, and the waste percentage is the percent by mass of food that is not eaten by consumers. Thus this formula relies on the assumption that the value of food is directly proportional to its mass, as discussed above. State Receipts for Texas, and Waste Percentages for North America, are given by the FAO's spreadsheet.

$$\begin{aligned} \$ Wasted \text{ via Distribution} \\ = (\$ Available \text{ prior to distribution} * \text{Distribution Waste Percentage}) \end{aligned}$$

To determine how to calculate the dollar worth of food available prior to distribution, we first considered that state receipts (the amount of food purchased by all consumers in the state) are the product of the available food and the percent of food *not* wasted.

$$\text{State Receipts} = (\$ Available \text{ prior to distribution}) * (1 - \text{Distribution Waste Percentage})$$

Manipulating this equation for \$ Available yields the following:

$$\$ Available \text{ prior to distribution} = \frac{\text{State Receipts } (\$)}{1 - (\text{Distribution Waste Percentage})}$$

Thus,

$$\begin{aligned} \$ Wasted \text{ via Distribution} \\ = \frac{\text{State Receipts } (\$)}{1 - (\text{Distribution Waste Percentage})} * \text{Distribution Waste Percentage} \end{aligned}$$

Waste via distribution and consumption must be calculated for each category, as waste percentages and state receipts vary across categories. Therefore, the model for total dollar worth of food waste is:

$$\$ Wasted = \sum_{i=0}^n \left(\left(\frac{\text{State Receipts } (\$)}{1 - (\text{Distribution Waste}_n)} * \text{Distribution Waste}_n + (\text{State Receipts}_n * \text{Consumption Waste}_n) \right) \right)$$

where n is the number of categories, and the State Receipts_n , $\text{Distribution Waste}_n$, and $\text{Consumption Waste}_n$ are the total consumer expenditures and distribution/consumption waste percentages for each category.

“Map the Meal Gap - Union County.” *Feeding America*.

Cash receipts, commodity ranking, and share of state receipts for the top 44 crops in Texas (excluding those that are not food crops, such as cotton lint)

https://data.ers.usda.gov/reports.aspx?ID=17843#Paf12231d97954ad1bdac6be756269fbe_2_18iT0R0x43

	Rank	Commodity	State receipts (in thousands of dollars)	Share of State receipts (percent)	Total Loss (thousands)	Total Lost (actual dollar amount)
		All commodities	20,897,606	100	3,014,702	3014702000
		Animals and products	13,154,912	62.9	2,002,599	2002599000
		Crops	7,742,694	37.1	1,012,103	1012103000
						0
cereals	8	Sorghum	480,609	2.3	139,573	139573000
	12	Wheat	297,693	1.4	86,452	86452000
	14	Rice	128,209	0.6	37,233	37233000
	36	Oats	9,618	0	2,793	2793000
	44	Rye	3,929	0	1,141	1141000
						0
roots and tubers	15	Potatoes	113,522	0.5	42,601	42601000
	28	Carrots	18,368	0.1	6,893	6893000
						0
oilseeds and pulses	16	Peanuts	109,663	0.5	5,494	5494000
	17	Pecans	94,020	0.4	4,710	4710000
	9	Cottonseed	439,982	2.1	22,044	22044000

Figure 1: Excel spreadsheet of Texas agricultural data used to calculate waste. The loss for each item was calculated based on our \$ Wasted model, with the waste percentage parameters determined by the values reported by the FAO.

Part C: Combining the Models

A state has enough food waste to feed its food-insecure population if the value of food wasted exceeds the dollar worth of need. That is, if surplus food value is defined as the value of food wasted minus the dollar worth of need, a positive surplus food value indicates that the state has enough food waste to meet the needs of its food insecure population. Thus, our final model for Question 1 takes the following form:

$$Surplus\ Food = \$\ Wasted - Food\ Security\ Need\ (FSN)$$

$$\$Wasted = \sum_{i=0}^n \left(\left(\frac{State\ Receipts\ (\$)}{1 - (Distribution\ Waste_n)} \right) * Distribution\ Waste_n \right) + (StateReceipts_n * ConsumptionWaste_n)$$

$$\$F.S.N. = (Proportion\ Insecure) * (State\ Population) * (174.30\ meals/year) * (meal\ cost\ in\ state)$$

D. Validating the Model

Part A: Validating the Food Security Need model

Our model to calculate food security need was

$$\$F.S.N. = (Proportion\ Insecure) * (State\ Population) * (174.30\ meals/year) * (meal\ cost\ in\ state)$$

The FSN varies linearly with the proportion of food-insecure people in the state and the state’s population. This is consistent with the idea that each additional food-insecure individual contributes, on average, the same amount of additional need. The FSN also varies linearly with

the state's average meal cost, which also makes sense, considering that the amount of food need is dependent on the cost of food in the state.

The model is also consistent with states' reported annual food budget shortfalls (i.e., the total additional amount of money needed to bring the food-insecure population to a state of food security). For Texas,

$$\begin{aligned} \$F.S.N. &= (.157) * (2.786 \times 10^7 \text{ people}) * (174.30 \text{ meals/year}) * (\$2.59/\text{meal}) \\ \$F.S.N. &= \$1,970,000,000 \end{aligned}$$

The reported annual food shortfall for Texas, according to feedingamerica.org, is about \$2,000,000,000, meaning that our model is about 98.5% accurate in this case.

Similarly, for North Carolina,

$$\begin{aligned} \$F.S.N. &= (.165) * (1.015 \times 10^7 \text{ people}) * (174.30 \text{ meals/year}) * (\$2.84/\text{meal}) \\ \$F.S.N. &= \$829,000,000 \end{aligned}$$

With a reported food budget shortfall of \$844,000,000, our model is 98.2% accurate for NC. See Part E, Results of the Model, for additional examples.

Part B: Validating the \$ Wasted model

On the other hand, our model for food waste was

$$\begin{aligned} \$Wasted &= \sum_{i=0}^n \left(\left(\frac{\text{State Receipts } (\$)}{1 - (\text{Distribution Waste}_n)} * \text{Distribution Waste}_n \right) + (\text{StateReceipts}_n * \right. \\ &\left. \text{ConsumptionWaste}_n) \right) \end{aligned}$$

For a given category, the value of food wasted varies linearly with state receipts (consumer expenditure). This is logical as the value of food waste in a state should be proportional to the value of food purchased. The portion of the equation represents money wasted during the consumption phase varies directly with consumption waste percentage as expected; the situation is more complicated for the distribution phase as the state receipts variable accounts for expenditure *after* distribution loss. Within the range of 0 to 1 (the range in which the Distribution Waste Percentage variable is valid), the component of the model that represents food wasted via distribution varies as a rational function which is continuously increasing. This behavior makes sense as an increase in distribution percentage means an increase in food waste. This nonlinear behavior is also explained by the fact that an increase in distribution waste percentage means that more food was available in the first place (as state receipts remain constant) *and* that more food is wasted via distribution.

Part C: Validating the overall model

Our final model took the following form:

$$\text{Surplus Food} = \$Wasted - \text{Food Security Need (FSN)}$$

$$\begin{aligned} \$Wasted &= \sum_{i=0}^n \left(\left(\frac{\text{State Receipts } (\$)}{1 - (\text{Distribution Waste}_n)} * \text{Distribution Waste}_n \right) + (\text{StateReceipts}_n * \right. \\ &\left. \text{ConsumptionWaste}_n) \right) \end{aligned}$$

$$\begin{aligned} \$F.S.N. &= (\text{Proportion Insecure}) * (\text{State Population}) * (174.30 \text{ meals/year}) * \\ &(\text{meal cost in state}) \end{aligned}$$

If surplus food is positive, then the state's wasted food is enough to meet the needs of the food-insecure population. The overall model depends on six variables. Three vary by category (state receipts, distribution waste percentage, and consumption waste percentage) and three vary only by state (proportion of population food-insecure, state population, and meal cost in state).

Below is a sensitivity analysis showing how small changes in the variables affect the overall surplus food calculation for Texas.

State Receipts*	Surplus Food (\$)	Percent change (Change-Original)/(Original)
10% reduction	743,231,000	-28.9%
No change	1,044,702,000	0%
10% increase	1,346,172,000	+28.9%
Distribution Waste Percentage* **	Surplus Food (\$)	Percent change (Change-Original)/(Original)
10% reduction	966,922,000	-7.4%
No change	1,044,702,000	0%
10% increase	1,122,482,000	+7.4%
Consumption Waste Percentage* **	Surplus Food (\$)	Percent change (Change-Original)/(Original)
10% reduction	784,949,000	-24.9%
No change	1,044,702,000	0%
10% increase	1,304,455,000	+24.9%
Proportion Food-Insecure **	Surplus Food (\$)	Percent change (Change-Original)/(Original)
10% reduction	1,237,567,000	+18.5%
No change	1,044,702,000	0%
10% increase	842,648,000	-19.3%
State Population	Surplus Food (\$)	Percent change (Change-Original)/(Original)

10% reduction	1,237,567,000	+18.5%
No change	1,044,702,000	0%
10% increase	842,648,000	-19.3%
Meal Cost In State	Surplus Food (\$)	Percent change (Change-Original)/(Original)
10% reduction	1,237,567,000	+18.5%
No change	1,044,702,000	0%
10% increase	842,648,000	-19.3%
Model Parameter: 174.30 years/year	Surplus Food (\$)	Percent change (Change-Original)/(Original)
10% reduction	1,237,567,000	+18.5%
No change	1,044,702,000	0%
10% increase	842,648,000	-19.3%

*This variable varies across categories of food; the same change was applied to the variable in all categories

**Reductions and increases to this variable were calculated multiplicatively, NOT via addition. At 10% reduction of a variable of 20% would yield 18%

The sensitivity analysis of the model reveals that predictions of food surplus (waste – need) are relatively sensitive to changes in model parameters. Changes in variables such as state receipts (consumer expenditure) and meal cost produce significant changes in the output. However, variations in model parameters never came close to making the surplus negative; even major changes in model parameters would still show that Texas wastes enough food to feed its food-insecure population.

E. Results of the Model

The results of the model for various states are shown in the table below.

State	\$ Wasted Food	FSN (Food security need)	Surplus Food (\$ Wasted Food-FSN)
Texas	\$3,015,702,000	<i>\$1,970,000,000</i>	\$1,045,702,000
North Carolina	\$1,735,389,000	<i>\$829,000,000</i>	\$906,389,000
Rhode Island	\$3,835,000	\$69,943,000	-\$66,108,000

States marked in green waste enough food to meet the needs of their food-insecure population, while those marked in red do not. As major agricultural states, Texas and North Carolina wasted many more dollars' worth of food than necessary to meet their food security needs. This indicates that a food redistribution strategy could be viable in principle for large agricultural states. For smaller, nonagricultural states like Rhode Island, however, need exceeded wasted food value. Consumption waste in Rhode Island was not enough to meet need. This indicates that a food redistribution strategy may be less effective in nonagricultural regions without transport from outside regions.

F. Strengths and Limitations

Our model's strengths lie in its simplicity and accuracy. It takes a simple list of parameters which are readily available through FeedAmerica and the US Department of Agriculture's Economic Research Service, and easily generates estimates of the value of food waste, food security need, and surplus food. Estimates of food security need match state-reported values closely. The model is also easy to interpret; a positive surplus food value means that a state could feed its food-insecure residents using its current food waste.

One limitation of our model's limitations is its moderate sensitivity to variations in certain parameters. For example, if the estimated proportion of a state's population that is food-insecure is off by 10%, food surplus changes by 18.5%. However, this sensitivity arises from inaccuracy in data reported by states and not from flaws in the model. Additionally, even major changes in model parameters do not come close to changing the sign of the food surplus.

G. Summary

Using information on national food wastage and a state's agricultural production, we created a model which could determine the value of a state's total food wastage. Using demographic information on states' food-insecure populations and additional money they need, we created a model which could determine the cost of food needed to achieve food security for all citizens. By combining the two models, we created a model which can predict whether or not a state wastes enough food to feed its food-insecure population. For major agricultural states, food surplus was high; these states usually waste more than enough food to feed their food-insecure populations. For less agricultural/smaller states, however, food surplus is negative, and food redistribution would not be an effective strategy without outside involvement.

Part II: Food Foolish?

A. Restatement of the problem

Using historical data about how individuals from different income brackets, ages, etc., develop a mathematical model which can predict household annual food waste based on demographic characteristics. For purposes of comparison, the model should express food waste in terms of the monetary value of the food.

B. Assumptions

- The proportion of food that an individual wastes outside the home is constant across various demographics.
- The proportion of food wasted within a given category (cereals, meats, etc.) is constant.
- Individual food waste is proportional to caloric consumption.

C. Developing the Model

In order to predict food waste per household, we developed a multivariate function in Mathematica based on demographic data such as household income and composition. We used data from the Bureau of Labor Statistics, which records money spent on each of six categories of food (cereals, meats, fish, dairy, fruits and vegetables, and out-of-home purchases) for different household makeups/incomes^{[5][6]}. We calculated the dollar worth of food in each category wasted by these households by multiplying the value purchased in each category by the average percentage wasted according to the the FAO. This yielded the response variable, food waste per household, for our multivariate models^[4].

At this point, we began developing the models using household income and number of people in each household as predictor variables and dollar worth of food wastage as the response. We tried several different regressions and found the most effective model to be a quadratic regression of the form $\text{predictedWaste} = ax^2 + bx + cy^2 + dy + e$, where a , b , c , d , and e are constants, x is the annual household income, and y is the number of people in the household. We created six such models (one for each category); the total dollar worth of food wastage for the household is given by the sum of their outputs. The Mathematica code for these regressions is located in Appendix A.

As an example, the multivariate model for cereal waste is shown below:

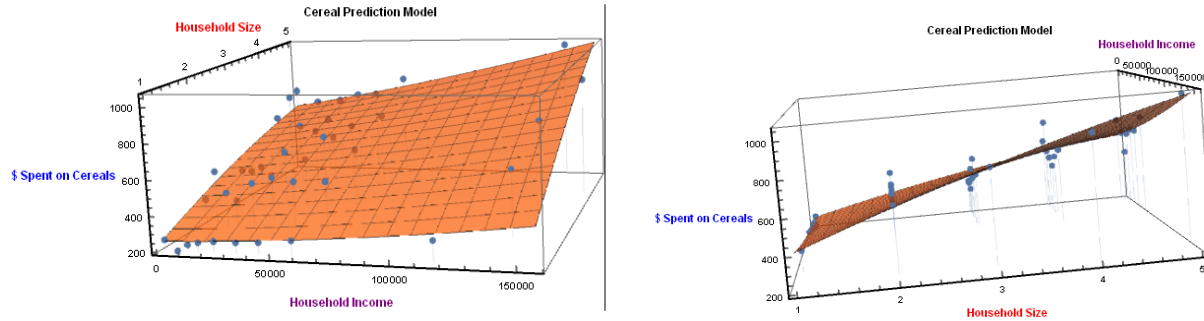


Figure 2: Food waste tends to be highest in large and high-income households

These models do not take into account the various ages of the people within the household. To adjust for the age distributions of the household members, we used a multiplier based on caloric intake. People who eat less food tend to purchase less food, and thus waste less food. The proportion of food an individual eats compared to the national average is given by:

$$\text{Caloric proportion} = (\text{Caloric Intake by Individual}) / (\text{National Mean Caloric Intake})$$

To calculate national mean caloric intake, we determined the average caloric data for each age bracket using data from health.gov^[7]. These age group-specific averages were used as estimates for Caloric Intake by Individual in the above equation. We then multiplied the average caloric consumption for each age group by that group's proportion in the U.S. population, using data from the US Census^[8]. This yielded the national mean caloric intake, 2090 calories/day.

To determine the contribution to household waste by the individual, we multiply the caloric proportion calculated above by the base prediction given by our multivariate model. The base prediction per person is the household prediction over the number of people in the household.

$$\text{Individual Contribution} = \frac{\text{Household Prediction}}{\text{Number of People}} * \frac{\text{Individual Caloric Intake}}{2090 \text{ calories/day}}$$

Thus the total waste of the household is the sum of these individual contributions, as shown:

$$\text{Total household waste} = \frac{\text{Household Prediction}}{N * (2090 \text{ calories/day})} * \sum_{i=0}^N \text{CaloricIntake}_i$$

where Household Prediction is the total household prediction given by the multivariate predictor model (which is not adjusted for age), N is the total number of people in the household, and CaloricIntake_n is the individual's caloric intake based on their age bracket.

D. Validating the Model

Because we used nonlinear regression, we could not use r² to evaluate our model. Instead, we used standard error of the residuals to give an estimate of the average error in our calculations. The values for our standard error the residuals for each model, as well as the standard deviation of the response variables used to train the models, are given in the table below.

Category	Standard Error of the Residuals (\$)	Standard Deviation of Waste Values (\$)
Cereals	46.23	193.70
Meats	280.58	305.56
Fish	16.37	42.21
Dairy	30.06	141.47
Fruits and Vegetables	66.18	257.06
Out-of-Home Purchases	264.30	1190.61
Overall*	395.31	505.07

*The overall standard error/standard deviation were computed using the square root of the sum of the variances for each category

For each category, the standard error of the residuals is smaller than the standard deviation of the response variable, indicating that our predictions are typically accurate within a standard deviation of the response variable, annual household waste (\$). It is important to note that these measurements involve standard error and standard deviation *before* the outputs of each of the six model are summed and the age distribution multiplier is applied.

E. Results

Here are the predictions of household waste for the households given in the problem:

Household	Annual Income (\$ / year)	Predicted Waste, unadjusted (\$ / year)	Predicted Waste, age adjusted (\$ / year)
Single parent and toddler	20,500	681.14	561.91
Family of Four (Two Parents and Two Teenage Children)	135,000	1,733.79	1,928.66
Elderly Couple	55,000	848.11	812.60
Single 23-year-old	45,000	631.51	678.89

Unsurprisingly, the family of four had the greatest predicted waste. The single parent and toddler with a low family income had the least predicted waste. Based on these test data, key target demographics for waste reduction are households with (a) a large household income and/or (b) many people.

F. Strengths and Limitations

One of the strengths of our model is its predictive accuracy. Based on the standard error of the residuals, our predictions are typically within one standard deviation of the mean for a given category. Of course, some of this predictive power is lost when the categories are combined, but the overall standard error remains less than the overall standard deviation of the original data. Another strength is the model's ease of use; taking simple inputs of household income, number of family members, and age of family members, it automatically computes an estimate of total household income.

One limitation of the model is that it is based on average data about the behaviors of various demographics rather than individual behavior, so it may not account for individual variations. Additionally, it operates on the assumption that food waste proportions within a given category of food are constant.

G. Summary

Using data on food expenditures based on household composition, as well as information about food wastage for different categories of food, we developed a dataset which used household income and number of members as predictors and total dollar worth of annual food waste as the response. We then used this dataset to produce a multivariate regression model within Mathematica. Following this, we developed a caloric multiplier which adjusts individual food waste based on caloric consumption. Together, the caloric multiplier and multivariate model can be used to estimate total dollar worth of annual household food waste based on demographic information.

Part III: Hunger Game Plan?

A. Restatement of the Problem

Analyze the economic environment surrounding the food retail industries, producers, and local food-insecure consumers of Union County, North Carolina in order to develop a food redistribution strategy. We will investigate multiple strategies for maximizing food redistribution at minimum cost, and assess the comparative costs and benefits of these strategies.

B. Assumptions

- Every grocery store in Union County makes \$14,000,000 per year, the national average annual sales income for grocery stores in the US^[9].
- Grocery stores in Union County North Carolina are unable to sell 10% of their inventory, representing the 10% average wastage by grocery stores in the United States^[10].
- The cost of diesel fuel is constant at \$2.55/gallon^[11].

- All of the food that is marked down by the grocery or collected by our employees is bought or utilized, respectively.
- All food that would have been thrown out of the grocery store is now donated.
- The farthest that any truck driver would drive in one day is the length of the perimeter of Union County and a diagonal across union county

C. Developing the Strategy

Part A: Strategy I - Salvage Groceries

According to MarketWatch, nearly \$900 million worth of expired food is discarded from the supply chain each year^[12]. Salvaging groceries that are near their expiration date can both minimize costs for grocery retailers as well as increase net benefits for society, particularly those individuals facing food insecurity. The first task in the creation of such a strategy is quantifying the amount of gross sales for an average grocery store each year. To calculate this, we observed a 10% wastage rate for grocery stores and used the average income of a grocery store in the US to determine the average possible gross sales of a grocery store in Union County North Carolina^[10].

$$\begin{aligned} \text{Average Possible Gross Sales for Grocery Store} & * 0.90 \\ & = \text{Average Gross Sales for Grocery Store} \\ \text{Average Possible Gross Sales for Grocery Store} & = \frac{\text{Average Gross Sales for Grocery Store}}{0.90} \end{aligned}$$

With average gross sales being \$14,000,000, we obtain an average possible gross sale for grocery stores of \$15,556,000^[9]. Upon obtaining the possible gross sales, we calculate the amount wasted for a single grocery store. This is done by finding 10% of the possible gross sales.

$$\begin{aligned} \text{Average Amount Wasted by Grocery Store} \\ & = \text{Average Possible Gross Sales for Grocery Store} * 0.10 \end{aligned}$$

With average possible gross sales equal to \$15,556,000 we obtain an average amount wasted per store of \$1,555,600. To determine the total amount wasted by grocery retailers in Union County, we multiplied the average amount wasted by a grocery store by 24, which is the number of grocery stores Union County^[13].

$$\begin{aligned} \text{Amount Wasted by Grocery Stores in Union County} \\ & = \text{Average Amount Wasted by Grocery Store} * 24 \end{aligned}$$

We obtain a total amount wasted of approximately \$37,333,000 within the county. Using map.feedingamerica.org, we determined that the additional money required to meet the food needs in Union County is \$12,088,000^[14]. To ensure that the grocery stores are able to make as much revenue as possible while selling their almost expired products at a lower price, our strategy would take the gross amount of almost expired goods and price them so that the total savings to food insecure consumers would be \$12,088,000. We now obtain a constant that will be multiplied by the total losses of the grocery stores in order to determine what the markdown prices would be in order to satisfy the needs of the food insecure population while allowing stores to make the most revenue possible.

$$\begin{aligned} \text{Constant for Price Mark Down} \\ = 1 - (12,088,000 / \text{Amount Wasted by Grocery Stores in Union County}) \end{aligned}$$

We obtain a constant of approximately 0.67 when considering the amount wasted or lost is \$37,333,000. This constant is then multiplied to the total amount wasted to obtain a value representing the additional revenue earned by the grocery stores cumulatively.

$$\begin{aligned} \text{Grocery Store Revenue} \\ = \text{Constant for Price Mark Down} * \text{Amount Wasted by Grocery Stores} \end{aligned}$$

Taking 0.67 as the constant and \$37,333,000 we obtain that the revenue made by the grocery stores cumulatively is \$25,013,110. Now, on average each grocery store in the county makes an additional revenue of \$1,042,213. Plus, all consumers, assumed to be food insecure based on their eligibility for aid programs, receive a cumulative savings of \$12,088,000, amounting to the additional financial representation of food required for becoming food secure. Our strategy maintains that perishables are salvaged three days prior to their expiration date, while nonperishables are salvaged two days prior to expiration date. At this point, they are still safe to eat^[15].

Part B: Strategy II - Food Collection for Our County

Our second strategy takes a more active approach to reducing food loss. In this strategy, food that would be thrown out is instead donated to the county and picked up by full-time truck drivers. This food would then be distributed to a facility where it would be able to be picked up, for free, by people who are food insecure. Because all food that would be thrown out is now donated, the benefit of this strategy totals the total loss grocery stores take, which is \$37,333,000. However, the cost is a function of lost sales and running costs of our strategy.

$$\begin{aligned} \text{Total Cost/year} &= \text{Lost Sales/year} + \text{Running Costs/year} \\ \text{Lost Sales} &= 37,333,000 * p \end{aligned}$$

where p equals the true proportion of food donated that would have been sold.

To run our strategy, we would need a building and transportation. Thus:

$$\text{Running Costs/year} = \text{Building Costs/year} + \text{Transportation costs/year}$$

We found a building in Monroe, North Carolina, where we could rent 10,000 square feet of space for \$47,500 per year^[16]. This building would provide us with enough space to store the food while being economical enough for our plan.

The transportation costs are more complicated. We need a truck to drive, truck drivers, and gas to fuel the truck. Therefore

$$\text{Transportation Costs/year} = \text{Truck Cost/year} + \text{wages} + \text{gas cost/year}$$

The median truck driver salary in the United States is \$40,260^[17]. We would need 2 full time employees to drive the trucks and run the facility, and we would hire at the median rate, so $\text{wages} = 2 * \$40,260 = \$80,520$.

We plan on buying a new truck that would last us ten years. This truck, along with the trailer, would cost Union County \$150,000. This truck would obtain 6.5 miles per gallon^[18]. We plan on keeping the truck for 10 years. Therefore

$$\text{Truck Cost/year} = \$150,000 / 10 \text{ years} = \$15,000/\text{year}.$$

For gas, we assumed that Union County was a square with an area of 640 mi²^[19]. This meant that the size of each side for Union County was $\sqrt{640} \approx 25.3 \text{ miles}$. This means that $\text{maximum distance/trip} = 4 * \sqrt{640} + \sqrt{2} + \sqrt{640} \approx 137 \text{ miles}$.

Next, we would need the drivers to go on their routes three times per week to restock the facility.

Thus, $\text{maximum} \frac{\text{distance}}{\text{week}} = 137 * 3 = 411 \text{ miles}$. Because the price of diesel gasoline is \$2.55 in our model, and the mpg of our truck is 6.5, $\text{total gas/week} = 411 \text{ miles} / 6.5 \text{ mpg} * 2.5 \text{ gallons} = \$161/\text{week}$ and $\text{total gas/year} = \text{total gas cost/week} * 52 = \$8380/\text{year}$.

Therefore, $\text{Transportation Cost / Year} = \$15,000 + \$80,520 + \$8380 = \$103,860$

And $\text{Running Costs / Year} = \$47,500 + \$103,860 = \$151,400/\text{year}$.

Therefore, $\text{Total Cost / year} = \$37,333,000 * p + \$151,400$

where p equals the true proportion of food donated that would have been sold.

D. Results: Costs and Benefits of Strategy

Part A: Strategy I - Salvage Groceries

The benefit/cost efficiency of model 1 is $\frac{\text{Total Benefit of Consumers} + \text{Total Benefit of Stores}}{\text{Total Cost to Stores}}$, which is calculated to be $\frac{12,088,000 + (37,333,000 - 12,088,000) * (1-p)}{12,088,000 * p}$, where p equals the true proportion of food marked down that would have been sold at the original price point, $0 < p \leq 1$. The numerator of this equation is the total amount of benefit, in dollars, that is saved by consumers. The number is the total amount of money needed to bring all food insecure families in Union County to food security, which, in our model, is the total benefit that consumers would receive.^[14]

Added to this number is the amount of new sales that the grocery stores are making due to this policy. Because $0 \leq (1 - p) < 1$, $12,088,000 \leq 12,088,000 + (37,333,000 - 12,088,000)(1 - p) < 37,333,000$. This means that the maximum amount of benefit for the grocery salvage strategy is less than \$37,333,000.

The denominator is the amount of sales, in dollars, that the stores are losing out on due to lowering the prices of goods close to expiration. This solution has no cost other than the direct sales cost to the stores themselves, so that is the only part of the denominator.

Part B: Strategy II - Food Collection by County

The efficiency of this strategy is calculated to be $\frac{37,333,000}{37,333,000 * p + 151,000}$ where p equals the true proportion of food donated that would have been sold at the original price. The numerator is the total benefit, which is the amount of money donated by the stores, equal to the monetary value of the amount of food that was thrown out previously. This is fixed at \$37,333,000. The denominator contains two major parts. The first is the loss that stores undergo for not selling their goods, which is the value of goods given away multiplied by the proportion of those goods that would be sold. The second part is the total fixed costs per year for the strategy, which is \$151,000.

Part C: Comparison

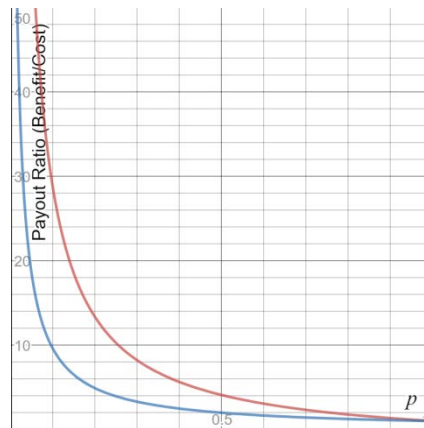


Figure 3: Graphs of net benefit/cost for both strategies, where red is grocery salvage and blue is food collection and redistribution.

As shown by the graph, grocery salvage is superior to food redistribution at all levels of p within $0 < p \leq 1$. However, both strategies are very similar when $p > .9$. Before this point, grocery salvage has considerably increased benefit/cost ratios.

It is worth noting that the overall benefit of food redistribution is greater than the overall benefit of grocery salvage throughout the entire interval, because food redistribution has an overall benefit of \$37,333,000 while grocery salvage approaches but can never reach that amount. However, food redistribution is significantly less cost effective societally than grocery salvage, so food redistribution is inferior to grocery salvage based upon efficiency.

E. Strengths and Limitations

Part A: Strategy I - Salvage Groceries

The Salvaging Groceries strategy is strong due to its ability to have a net positive impact on both local grocery firms and consumers, especially those who are food insecure. The strategy effectively redistributes retail waste due to expiration dates by marking down prices, thereby increasing revenue for grocery retailers and increasing savings for consumers requiring financial aid. It also has extremely low costs, consisting only of decreased revenue.

The strategy's limitations lie in its inability to effectively convert wasted product into complete revenue for retailers. While having the prospect of making a revenue of \$37,333,000, retailers are only able to convert revenue of about \$25,013,000 in Union County. Another limitation is the strategy's impact on food insecure groups. While resolving the minimum need in regards to food, the strategy fails to enable food insecure groups to achieve sustainable food security. Further, the strategy relies on the assumption that all food is sold, which is unlikely.

Part B: Strategy II - Food Collection by County

The strengths of this strategy lie in its ability to coordinate distribution of wasted food to households likely facing food insecurity. While not helping grocery retailers, this strategy effectively prices the cost of wasted food as 0, thereby enabling poverty-stricken families to obtain free food resources at only the expense of fuel required to travel to the county's distribution center.

One limitation faced by this strategy is its ability to please grocery stores. With nearly wasted food being picked up by the county for free, grocery stores receive no additional revenue on the food resulting in the same amount of revenue as if they were wasting 10% of the product they put on sale. Another limitation faced by this strategy is its reliance on a transport system coordinated by two individuals in order to maximize the net benefit to society. With two individuals being the sole workers responsible for the proper flow of the acquiring and distributing food system, it is somewhat unreliable in the real world due to high expectations for a small workforce in the county's Food and Nutrition Service. Lastly, this solution has many running costs, which make it significantly more expensive for the county.

F. Summary

Utilizing data regarding wasted food, average grocery store revenue, and food insecurity within Union County, we were able to create two strategies to lower the amount of food insecurity within our community. We determined that the strategy of salvage groceries was more effective than actively moving food because of its lack of running costs, lower costs to local businesses, and its ability to help grocery stores in communities that have a large amount of food insecure people.

Ultimately, our suite of models shows that food distribution strategies are most effective in agricultural states, and that grocery salvaging strategies specifically are effective at meeting the needs of food-insecure individuals. In particular, high-income households with many members are key targets for reducing food waste, and raising awareness among such demographics is a good way to reduce waste. Producers and consumers must work together to reduce food waste and develop and implement effective redistribution strategies to address food-insecurity in the United States.

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