Volcanic Eruption Fallout
STUDENT RESOURCE BOOK

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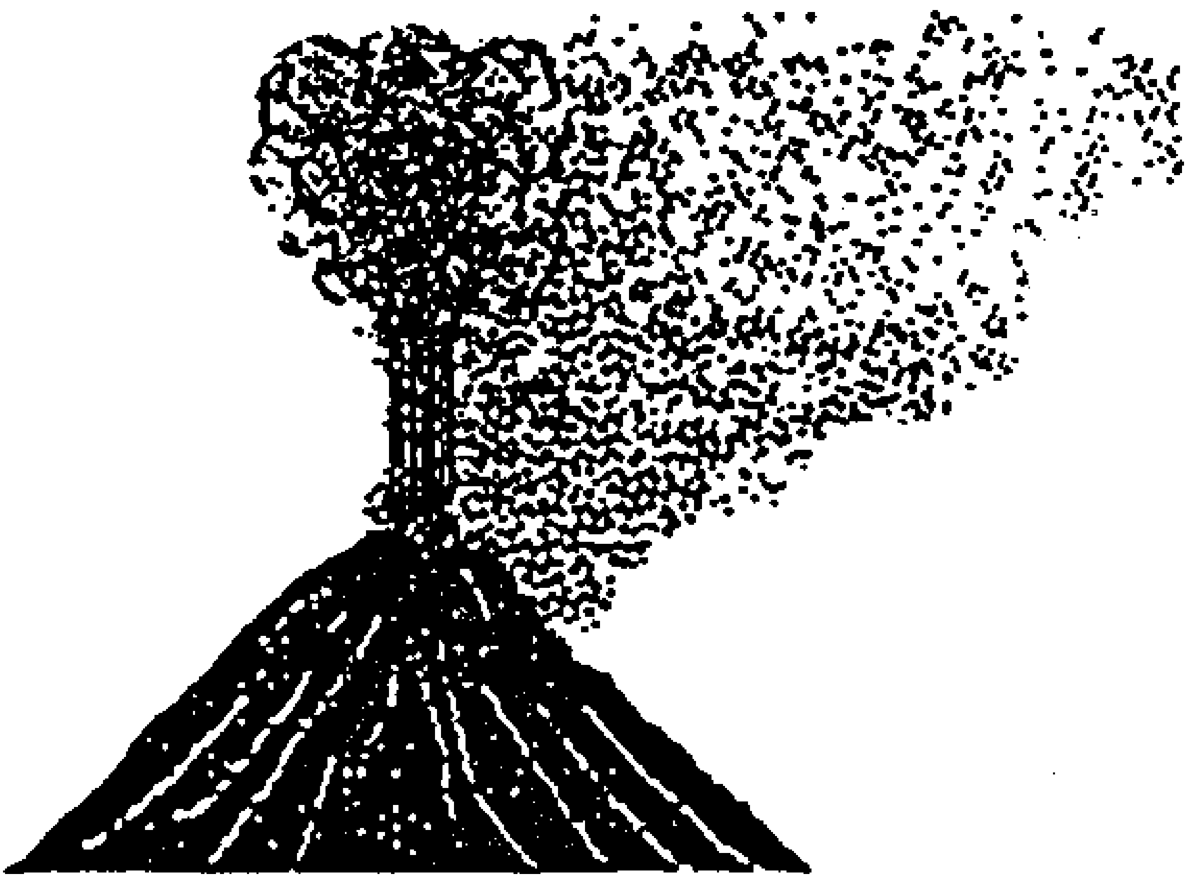

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I. Introduction

The surface of the Earth is constantly changing. Some changes are so slow that it takes a very long time to observe their effects. Some are so rapid that it is hard to escape from them. The series "Planet Earth" on the Educational Television Channel discusses these changes, how scientists believe they are caused, how scientists observe and try to predict them, and why it is important to study them.

Dr. William Rose is a geologist who specializes in the study of volcanoes and their interaction with our atmosphere. A geologist with this specialty is called a **Volcanologist.** In their pamphlet entitled "Volcano Watching," Robert and Barbara Decker write:

> Volcanoes are interesting but difficult features to study. They have their roots deep inside the Earth, and they fling their ash and gases high in the sky. They require study by geologists, chemists, physicists, meteorologists, and oceanographers -- all working together. That's part of the fun of it.

Scientists in each of the areas mentioned by the Deckers need to use mathematics. In this problem you have a chance to work together with Dr. Rose and his team of scientists, doing research related to volcanic eruption fallout. You too are part of the fun!

II. General Background

A. What Causes Volcanoes?

As long ago as 1910 an Austrian scientist suggested that North and South America had broken away from Europe and Africa and that the two land masses had slowly drifted apart. (Try fitting together the two land masses bordering the Atlantic and see how close the fit is. Even the mountain ranges seem like continuations of each other.) It was not till the 1960's that new discoveries led scientists to rethink and refine the old theory of continental drift. The present theory is referred to as **plate tectonics**. It is now generally believed that the Earth's crust is in the form of plates (six major ones at least) and that these plates drift slowly away from and into one another. (See Figure 1.)

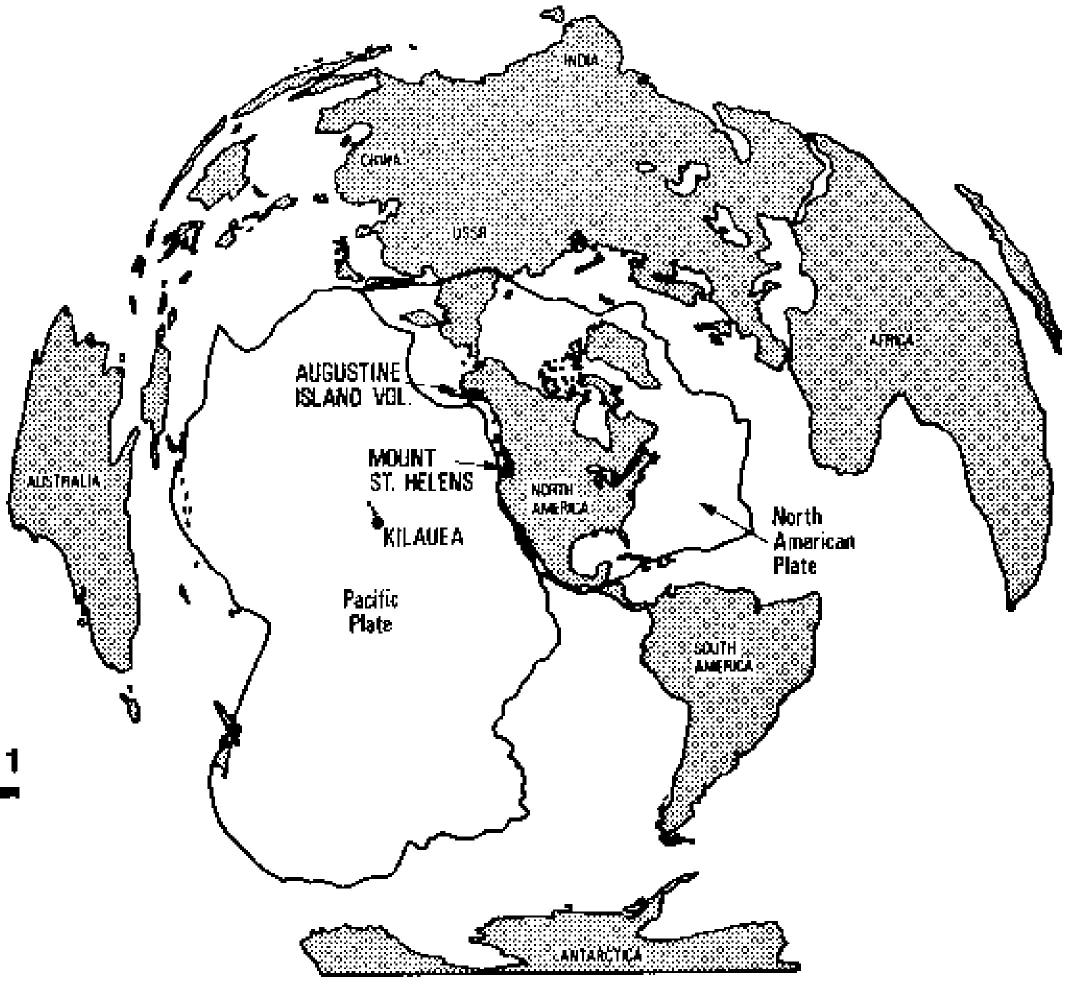


Figure 1

The hot, weak layer of the Earth (upon which the plates drift about) appears to be the source of the molten rock, called **magma**, that issues from volcanoes when they erupt. When magma reaches the surface it is called **lava**. As the lava pours out it is heaped on the Earth's surface in a conical mass forming a mountain with an internal part consisting of one or more passageways leading to the surface from a source of magma far below.

Most volcanoes occur at boundaries between plates, either where they spread apart or where they collide. Some volcanoes, however, occur within the plates at places called "hot spots."

If you live in Hawaii, you are familiar with Kilauea. This volcano occurs at a mid-plate "hot spot" and is characterized by relatively calm eruptions of lava rivers which can run for tens of kilometers down gently inclined slopes.

If you live in Washington State, or even in Idaho, Montana, Oregon, or Wyoming, you will have had personal experience with the recent activity of Mount St. Helens, a volcano formed where an ocean plate collided with a continental plate. The eruption from this volcano was violent and explosive, destroying some of the mountain and much of the surrounding area.

Volcanologists are still working on the answer to why volcanoes differ. Two major factors appear to be: the amount of gas released, and the ease or difficulty with which the gas escapes from the molten rock.

B. What Happens During and After an Eruption?

The hot molten rock and the gases trapped within it come to the surface of the volcano. As they escape into the atmosphere the molten rock is broken up by the gases. If the broken lava has a very large surface area, it quickly transmits its heat to the air. A rapid transfer of heat from the lava to the air causes the air to expand. This carries material upward in a high convective column. In this process, then, there is a rapid transfer of heat from the lava to the air, the air expands, the column goes up, and that brings the volcanic material very quickly to high levels in the atmosphere. This is the first and most spectacular phase of the volcano's interaction with the atmosphere.

Eventually the column stops rising. What makes it stop? When the heat is gone the air no longer expands. The column stops growing and for the moment everything is in equilibrium at that level.

Gravity and wind now enter the picture and begin to broaden the effect of the volcanic action. The ejected particles start to fall and at the same time the wind blows them parallel to the Earth. Usually wind speed is greater than the speed at which the particles fall. Depending on the direction and strength of the wind, the particles are blown various distances and in a variety of directions. However, the distance the particles travel before they land depends on many factors, not just the strength of the wind. The size and the density of the particles are two important factors. It may be that some of the particles are so fine that they never fall to Earth. The density and the viscosity of the air are also important. These properties change as the height above the Earth changes. These factors will be discussed in detail in the next section.

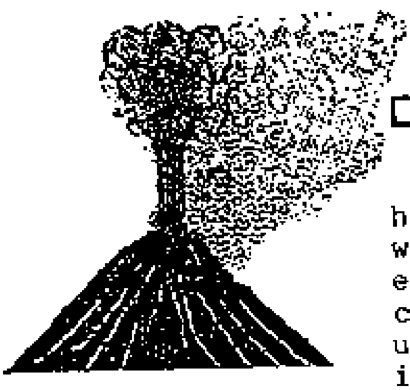
The type of eruption described here is a vigorous vertical ejection, powered by the explosive release of pressured gas which was dissolved in the magma before the eruption. Figure 2 is a schematic sketch showing this type of eruption and its cloud of ejected material.

C. What Is Ejected During an Eruption?

Many types and sizes of solid fragments make up the ejected material. Volcanologists call this volcanic debris **pyroclastics**, a good word to describe it since the word means literally "fire fragments." Pyroclastics are classified according to size. Volcanic **ash** refers to fine particles similar to flour up to coarser particles about the size of rice; **cinders** can be pieces as large as golf balls; **blocks** include the larger chunks, some of which can be the size of houses.

As the debris from explosive volcanic eruptions falls back to Earth it forms distinct layers blanketing the slopes of the surrounding land. The coarser fragments fall first and nearest, while the fine ash is blown away and falls last, sometimes at great distances. All the debris that falls to Earth is called **fallout**.

Figure 2



Eruption Cloud

D. Why Is it Important to Study Fallout?

The most dramatic and terrifying volcanic hazard is the glowing avalanche. This happens when the volcanic debris produced by an explosive eruption is too heavy to rise. Then a glowing cloud of debris can pour down a mountain at speeds up to 100 km/hr, flattening and burning everything in its path.

Much less spectacular but also more far-reaching is the effect of the drifting cloud (Figure 2). The area affected by volcanic ash can be very large and very irregular in shape, depending on the winds which often blow in different directions at different altitudes. Ash which reaches the ground can damage many things such as vegetation, water drainage, and automobiles. Fine ash suspended in the air makes breathing dangerous.

Volcanic ash can cause problems even before it reaches the ground. A recurring problem in recent years has been the sudden encounter between aircraft and volcanic eruption clouds. Instruments on airplanes cannot detect such clouds and visual detection, particularly at night or in bad weather, is not always possible.

Records show that volcanic ash can cause significant damage to windows and surfaces of both small and large planes. Radio communications can be affected adversely. Fine particles in the drifting cloud accumulate electrostatic charges; lightning occurs frequently and can be a hazard. Ash particles can be very cohesive in eruption clouds. The material is often wet and can load on the body and wings. Silicate particles in the eruption cloud are perhaps the most critical materials since they can melt inside aircraft engines.

E. What Is a Mathematical Model?

In an effort to understand and predict the behavior of the world around us, physical and social scientists study physical phenomena, economic behavior, and social trends. In doing so they try to develop equations and formulas which describe these things as they have happened in the past and which can be used to predict future behavior. Such a set of equations and formulas is called a **mathematical model**.

In particular, the geologists, physicists, mathematicians, and others who study volcanoes want a mathematical model that will describe the behavior of volcanic fallout so that its rate of fall and location can be predicted. In creating a mathematical model a variety of assumptions must be made. Before a model can be applied with confidence it is important to study the assumptions made and also to **test** the model to see whether it agrees with observed experimental data.

Some mathematical models can be tested by laboratory experiments. If you have had the chance to do these in school you know that to perform a successful experiment you plan ahead carefully, keep careful track of the quantities you know, and make careful measurements of the quantity you are observing. You are there at the beginning and you study the experiment until the end.

With a volcano it is quite different. You are studying an experiment that began long before you were born, and sometimes is so violent it is not possible to collect the relevant data.

F. Why Study the Augustine Volcano?

At the northwest corner of North America lies Alaska, the largest state in the United States. (See Figure 3.) In 1959 Alaska became the 49th state to enter the Union. Alaska is famous for its scenic beauty. It has hundreds of snow-capped mountains, including Mt. McKinley, the highest in North America.

Figure 3

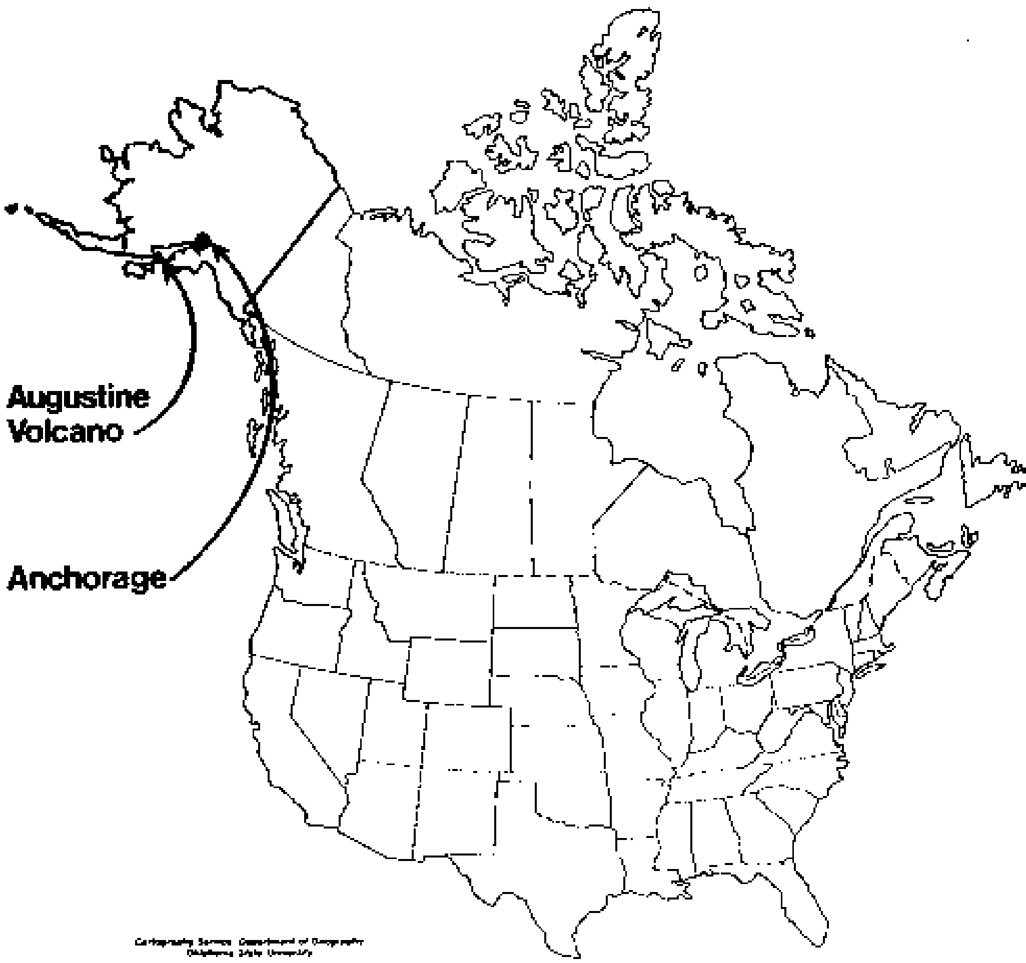
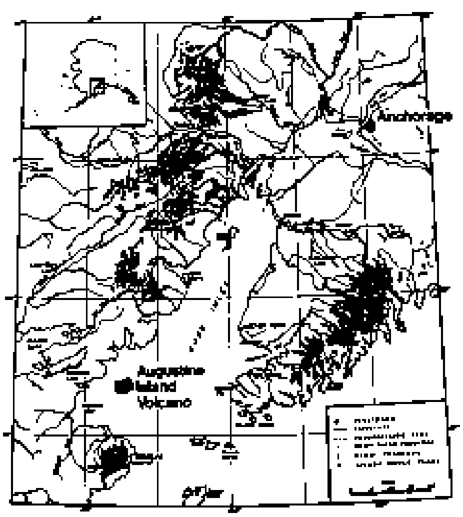


Figure 4



Cook Inlet and Augustine Island

The Aleutian Mountain Range in southwest Alaska contains glacier-covered peaks and active volcanoes. These form the background for the Cook Inlet Area. (See Figure 4.) At the western end of Cook Inlet is Augustine Island, the home of the Augustine volcano. At the eastern end, about

280 km northeast of Augustine Island is Anchorage, the largest city in Alaska. This area of Alaska is the scene of our study.

The Augustine volcano is active and erupts from time to time. The most recent eruptions were March 27 to April 4, 1986. Compared to some volcanic eruptions, these were relatively small. From the scientific point of view, however, they presented a very good setting for study. The ash clouds were observed and described by satellites, local air traffic, weather stations and local residents. Weather conditions were good for observation. Particles of volcanic debris were collected in the air and on the ground. There are data on the sizes, shapes, and compositions of the particles. We know when and where the material fell. From the satellite images and seismic data it is possible to make a good estimate of when the ash was erupted and what height it reached in the atmosphere. The eruptions were not so violent as to prevent taking measurements close to the site. Because of the wealth of data available, Augustine's eruptive activity provides an excellent opportunity to compare observed data and theoretical data concerning the rate of fallout of volcanic ash.

As part of Dr. Rose's team we want to use the data collected at the Augustine volcano to test the mathematical model. In preparation for our work we must become familiar with some definitions and with the mathematical model we are testing.

III. Specific Background and Data

A. Fallout

Fallout refers to any kind of polluting particles falling through the atmosphere. In our case the reference is to volcanic debris.

As you read through this material you will find a number of letters that may not be familiar to you. Scientists like to use Greek letters to represent quantities they are defining. In the next paragraph meet the Greek letter Mu (pronounced "mew" and written μ). You already know this one if you belong to the high school mathematics honorary society, Mu Alpha Theta.

A large percentage of the eruptive material from the Augustine volcano was very fine. Samples of the fallout collected were examined. The diameter of the particles was about 22 micrometers (μm). A micrometer is 10^{-6} meters or 10^{-4} cm. Thus the average diameter of the ash particles from Augustine was

$$d = 22 \times 10^{-4} \text{ cm.}$$

You can realize how small this is when you think that the dot over an "i" has a diameter of about 400 μm, and the smallest grains of flour are 50-100 μm. We are primarily concerned here with fine-grained volcanic ash.

Fallout rate is a measure of the amount of fallout per unit of time. There are two principal ways of measuring amounts. Both of these will be important in our study. One is in terms of volume. For example, we might measure fallout in cubic meters per hour (m^3/hr) or in cubic centimeters per second (cm^3/s). On the other hand we can measure fallout in terms of mass. For example we might use kilograms per second (kg/s) or metric tons per day (t/d). A **metric ton** is 10^6 grams, or 10^3 kg.

The **density** of a substance is its mass per unit volume. In order to change from measuring in terms of mass to measuring in terms of volume, or vice versa, the density of the substance must be known.

(Meet the Greek letter Sigma, written σ, and looking like an o with a tail on it.)

In the case of the ash from Augustine the density on the average was

$$\sigma = 2.5 \text{ g/cm}^3.$$

From time to time in this section we introduce **Try Out Problems (TOPs)**. These are short questions to test your understanding of what you have read. Your teacher has answers for them in the Teacher Resource Book.

TOP 1.
(a) Find the mass of 1 m^3 of rock if its density is 2.5 g/cm^3.
(b) A spherical piece of pumice has radius 1.9 cm and mass 14.4 g. Find its density in g/cm^3.

(c) The density of a rock particle is 2.5 g/cm^3. Express this in g/mm^3.

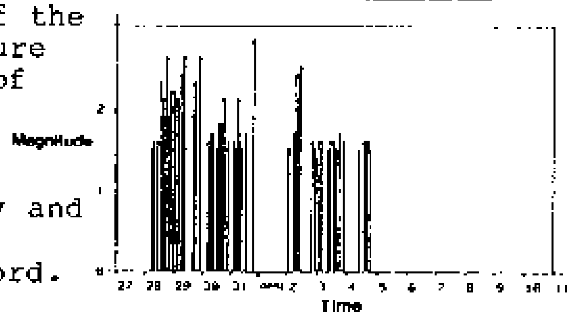
(d) Assume the density of a rock is 2.5 g/cm^3 and its mass is 6.398 t. Find its volume.

B. Eruption Rate

The **eruption rate** of a volcano is the quantity of material ejected into the atmosphere per unit of time.

The seismic record gives an indication of the strength of the volcanic eruptions. (See Figure 5. This record was recorded by a University of Alaska seismic station 28 km north of the volcano.) In the case of Augustine it was possible to determine the eruption rate by measurement during times of moderate intensity and to estimate the peak eruption rate by extrapolating from this using the seismic record.

Figure 5



Magnitudes of Seismic Moment
28 March - 4 April

The estimated peak eruption rate at the Augustine volcano, March 27 and 28, 1986, was 7.5×10^7 t/d.

COLUMN HEIGHT

The **column height** is the distance the volcanic column rises above the crater of the volcano usually at its peak level of eruption. Volcanoes vary a great deal in height. Table 1 gives the maximum column height for some past volcanic eruptions as well as the maximum rate of eruption in m^3/s.

TABLE 1

Eruption	Maximum rate of eruption (m^3/s)	Maximum column height (km)
Mount St. Helens, 1980	2×10^4	22(o)
Hekla, 1947	2×10^4	24(o), 21(c)
Agung, 1963	3×10^4	23(o)
Bezymianny, 1956	2×10^5	45(o), 42(c)
Santa Maria, 1902	4×10^4	34(c), 29(o)
Hekla, 1970	6×10^3	16(o), 15(c)
Ngauruhoe, 1975	2×10^3	10(o), 10(c)
Taupo, 160 AD	1×10^6	>50(c)

A column height identified as (o) means estimated from observation; (c) means calculated from a formula like the one discussed in the next paragraph.

TOP 2.
(a) Calculate the estimated peak eruption rate of Augustine in m^3/s. Recall that the density of ejected material was 2.5 g/cm^3.
(b) Compared with the eruption rates listed in Table 1, where does Augustine rank?

WILSON'S EQUATION

Wilson's equation is a mathematical model of the relationship between the column height of an eruption and the eruption rate. This equation was derived by fitting the best possible curve to observed data. It states

$$H = 236.6\ Q^{1/4}$$

where H is the column height in **meters**, and Q is the mass eruption rate in **kilograms per second** (kg/s). The number 236.6 depends on the units used and on the way heat is transmitted to the air.

Figure 6

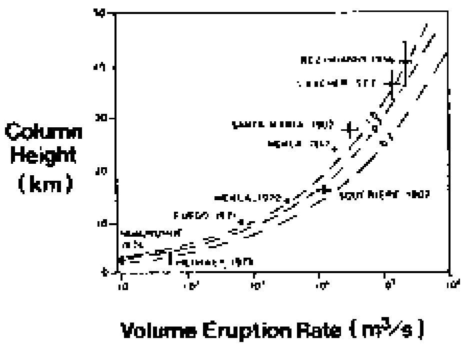


Figure 6 shows a plot of column height as a function of eruption rate. You will notice in Figure 6 that the horizontal scale which represents eruption rate is marked in uneven intervals labeled 10, 100, 1000, ... The quantity plotted there is not the actual eruption rate but the power of 10 that gives that rate. If you have studied logarithms you know that this is a logarithmic scale and what is plotted is $\log_{10}$(eruption rate). This type of plot is frequently used when there is a big difference between values to be plotted.

C. Vertical Speed of Fallout

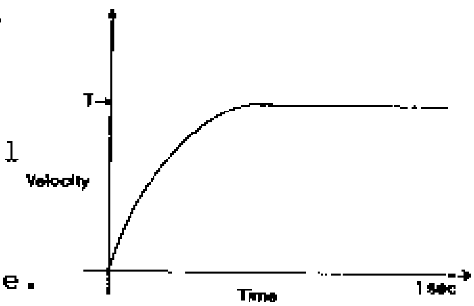
As the particles of volcanic material fall through the atmosphere they are subject to acceleration due to gravity and also to resistance from the atmosphere.

TERMINAL VELOCITY

An object is in free fall if it is falling in a vacuum where there is no air resistance. In the case of free fall, the acceleration due to gravity determines the speed of an object during its fall to Earth. This acceleration near the Earth's surface is 9.8 meters per second per second. For objects with considerable mass relative to their surface area, gravity plays a dominant role and we approximate the falling of objects by considering it to be free fall. With constant acceleration the speed increases at a constant rate.

In the case of fine volcanic ash falling through the atmosphere, free fall does **not** give a good representation. Factors other than gravity are very important. The shape, size, and rotation of the falling particles, the density of the particle, the drag exerted by the atmosphere, -- all these tend to affect the rate of fall. Very soon the acceleration due to gravity is counteracted by the resistance of the atmosphere so that the speed at which the particle falls becomes constant. This constant rate of fall is called the **terminal settling velocity**, or simply **terminal velocity**. When a small particle is released in air it quickly reaches its terminal velocity. This condition happens when the drag force of the air on the particle is exactly equal and opposite to the force of gravity. The particle then continues to fall toward the Earth at this constant speed unless different atmospheric conditions cause that speed to change.

Figure 7



Terminal Velocity

Figure 7 indicates a plot of velocity on the vertical axis and time on the horizontal axis. Notice that for a very short time the speed is increasing in a straight line. Then it increases more slowly and finally reaches a place where it levels off to a constant. The length of time it takes to reach this terminal velocity is so short that it can be neglected, and the terminal velocity can be considered to be the speed with which the particle falls.

D. A Formula for Terminal Velocity

Before we can proceed to write a formula for calculating terminal velocity we must discuss some of the properties of air which affect this.

You have probably noticed already that we are emphasizing the **units** of the quantities considered. When doing calculations that refer to real situations you can get ridiculous answers if you are not careful to keep units correct. You can handle units as if they were algebraic quantities, and often you can check your work by examining the units. For example, suppose d is measured in cm, T in cm/s, and σ in g/cm^3. The units of the product $d^2T\sigma$ can be found by writing:

$$\text{cm}^2 \times \frac{\text{cm}}{\text{s}} \times \frac{\text{g}}{\text{cm}^3} = \frac{\text{g}}{\text{s}}.$$

The product $d^2T\sigma$ has units g/s.

VISCOSITY

Viscosity is the property of a fluid or gas which makes it resist flowing. The resistance is caused by the attraction between the molecules of the fluid or gas. You may be familiar with viscosity as an important property of oils. Viscosity affects the lubricating ability of oils. A "heavy" oil has high viscosity and is needed for lubricating a heavy load. A "light" oil, one with a low viscosity, is used to lubricate a light load.

The viscosity coefficient, commonly referred to as **viscosity**, is a measure of the viscosity of a liquid or gas. It is usually measured in grams per centimeter-second, that is, g/cm-s. Note that, in the notation g/cm-s, the "-" is a hyphen and is to be interpreted as multiplication. Thus in the product (diameter)(viscosity) the units are

$$\text{cm} \times \frac{\text{g}}{\text{cm} \times \text{s}} = \text{g/s}.$$

Viscosity depends only on molecular structure and temperature. It is independent of pressure. The viscosity of a gas increases as the temperature rises.

The viscosity of air at 0°C is 1.71×10^{-4} g/cm-s. Its viscosity at 20°C is 1.81×10^{-4} g/cm-s. The viscosity of the air is represented by the Greek letter Eta, pronounced "ate uh" and written η.

TOP 3.
The terminal velocity (T) at 20°C is 5 cm/s. Calculate the ratio T/η. (Include the units.)

THE REYNOLDS NUMBER

A key to understanding how particles fall through the air is the **Reynolds number, Re.** It describes the type of flow occurring around a particle which is moving through a fluid such as air. The Reynolds number is dimensionless, that is, it has no units.

The Reynolds number is given by the formula

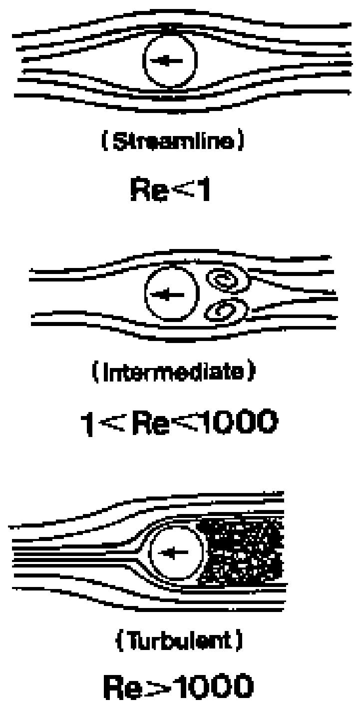
(1) $$Re = \frac{d\,T\,\rho}{\eta}$$

where d is the diameter of the particle, T is its terminal velocity, ρ is the density of the atmosphere, and η is the viscosity of the atmosphere. The units chosen for d, T, ρ, and η must be compatible so that Re will be dimensionless. (See TOP 4.)

Again we use a Greek letter, Rho (pronounced row as in "row your boat" and written ρ).

In formula (1) the numerator represents inertial force and the denominator represents viscous force. Inertia is the tendency of matter to retain its velocity along a straight line. Figure 8 shows three types of flow. If the Reynolds number is large, inertial forces are much greater than viscous forces and turbulent flow results. If the Reynolds number is small viscous forces dominate and streamline flow results. In between, for $1 < Re < 1000$, the flow is said to be intermediate.

Figure 8



Sketch of Flow Types

TOP 4.

(a) Suppose diameter is measured in cm, terminal velocity in cm/s, density in g/cm^3, and viscosity in g/cm-s. Show that Re is dimensionless.

(b) Calculate Re if d = 22 μm, T = 0.04 m/s, $\rho = 0.5 \times 10^{-3}\ g/cm^3$, and $\eta = 1.33 \times 10^{-4}$ g/cm-s. What type of flow does this represent?

DRAG COEFFICIENT

A convenient measure of the resistance offered by air is the drag coefficient. This is related to the Reynolds number and is given by expressions that depend on the size of the Reynolds number. In our case we have very small particles so the Reynolds number is likely to be small. **If the Reynolds number turns out to be smaller than 1,** the drag coefficient is given by the formula:

(2) $C = 24/Re.$

Like the Reynolds number, the drag coefficient is dimensionless when the units are chosen properly.

EQUATION FOR GRAVITATIONAL SETTLING

Equation (3) is the basic equation describing the falling of objects to the Earth through air. It relates terminal velocity, drag coefficient, and other constants describing gravity and the properties of the atmosphere. For this equation to apply we assume that the particles are rigid spheres falling independently of each other.

$$(3) \quad V \sigma g = \frac{1}{2} \rho C A T^2$$

where:

V = volume of the particle = $(4/3)\pi(r)^3$ cm^3

σ = density of the particle = 2.5 g/cm^3

g = acceleration due to gravity = 980 cm/s^2

ρ = density of the atmosphere (g/cm^3)

C = drag coefficient

A = cross-sectional area of the sphere = $\pi(r)^2$ cm^2

T = terminal velocity (cm/s)

The left hand side of equation (3) represents the force due to gravity, that is, mass times acceleration. The right hand side of the equation represents the resistance of the air. Both the density of the atmosphere and the drag coefficient vary with altitude.

TOP 5. Check the units on each side of equation (3).

E. Calculating Terminal Velocity

With the formulas of Section D and appropriate atmospheric data, we can calculate terminal velocity. If you look carefully at equations (1), (2), and (3) you will see that T is involved in both (1) and (3). As they come from the physicist, the equations are not in a form to make direct calculation of T easy. One of the jobs of the mathematicians on our team is to put these formulas together in such a way that we can solve for T in terms of known quantities.

In addition to an expression for T, we need some of the data collected at the Augustine volcano.

ATMOSPHERIC DATA

We have seen that temperature, viscosity, and atmospheric density all vary with altitude. Before we can list atmospheric data we will need to take account of this in some way. We certainly cannot use different data for every meter of altitude. In order to adjust to the changes in these quantities, we think of the atmosphere as consisting of a succession of layers. The layers are chosen so that in each layer the atmospheric properties can be assumed to be constant. The eruptions under consideration were less than 8 km in height and the mouth of the volcano is 1.2 km above Anchorage. We might consider nine atmospheric layers, each 1 km in thickness.

We will consider particles that fall through the first nine atmospheric layers. Changes encountered in the atmospheric properties may cause the value of T to be slightly different from one layer to the next.

Table 2, Section G, contains atmospheric information for each of the nine layers.

F. Horizontal Distribution of Fallout

So far we have considered only the way the ash particles fall to the ground in still atmosphere. Such a situation is highly unlikely. The atmosphere was not still at Augustine during the time in which we are interested.

WIND CONDITIONS

Data on wind speed, wind direction, and other atmospheric properties were collected by balloons sent up every 12 hours from stations in the neighborhood of Augustine Island. Balloons rise and drift with the wind. A miniature radio transmitter called a radiosonde is carried aloft by the unmanned balloon. It has instruments for broadcasting the humidity, temperature, pressure and wind direction.

The wind **direction** recorded is the azimuth direction. **Azimuth** is the angle in degrees, measured clockwise from the north. The direction recorded is the direction **from which the wind is** blowing.

The wind **speed** is measured in knots. A **knot** is a speed of 1 nautical mile per hour. A nautical mile (nm) is a little longer than our usual mile. To be precise:

1 nautical mile = 1852 meters.

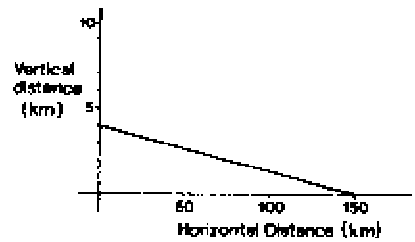
TOP 6.

(a) A small plane flies at about 150 knots. Express this in km/s.

(b) Assume the wind direction has azimuth 330°. Draw an arrow representing the wind on an xy-plane in which the y-axis points north. Would you expect this wind to be warm or cold?

The wind speed and wind direction may both vary with altitude (height above sea level). The path taken by the volcanic ash as it falls to Earth is influenced by both of these. At the same time that the particles are falling, the wind is driving them horizontally in some direction.

Figure 9

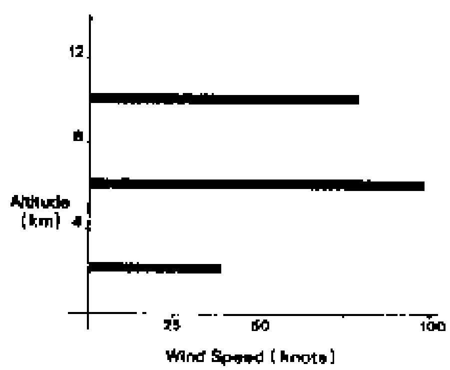


Path of a Particle

For example, suppose that it takes a particle 2 hours to fall from 4 km above the surface to the ground. At the same time the wind speed is 40 knots. As the particles fall they are also blown horizontally by the wind. The horizontal distance is 2 x 40 = 80 nautical miles, that is, 80 x 1.852 km = 148.160 km. The path of the particle can be approximated by combining the vertical and horizontal motion. (See Figure 9.)

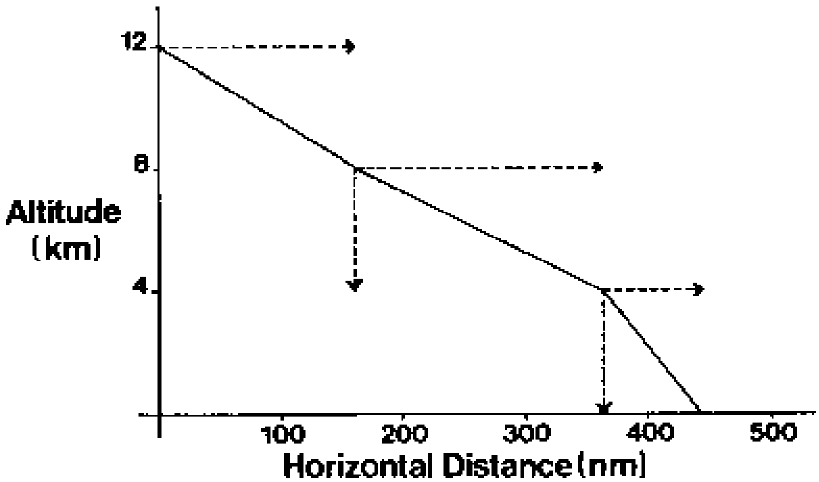
Usually the wind speed changes with height. For example, suppose that between 0 and 4 km the wind speed is 40 knots, between 4 and 8 km the wind speed is 100 knots, and between 8 and 12 km the wind speed is 80 knots. We can represent this graphically as in Figure 10, in which the lengths of the arrows indicate the speed of the wind. This figure does not show the direction of the wind, so such a representation is of value primarily in cases where the wind direction is steady. To show both speed and direction a three-dimensional figure would be needed.

Figure 10



Wind Profile

Figure 10 shows only wind speed. Suppose we extend Figure 9 and assume that the volcanic material falls at a constant vertical speed of 2 km/hr throughout the 12 km fall, with wind conditions (all in the same direction) illustrated in Figure 10. Between 12 and 8 km the particles have fallen 4 km and moved horizontally 160 nm. From 8 to 4 km, the particles again fall 4 km and travel horizontally 200 nm. From 4 to 0 km the particles fall 4 km and travel horizontally 80 nm. Figure 11 illustrates the path of these particles.



Path of Particles from 12 km

Figure 11

The actual wind pattern at Augustine on March 28, 1986, is listed in Table 3, Section G.

G. Data

Augustine volcano is situated on Augustine Island in the Cook Inlet, 280 km southwest of Anchorage. The crater of the volcano is 1.2 km above sea level.

The Augustine volcano showed increased eruptive activity from March 27, 1986, until April 4, 1986.

Significant fallout began at Anchorage about 22 hours after eruption intensified.

Examination of the ash indicated that the particles were 22 μm in diameter.

The density of the fallout particles was 2.5 g/cm^3.

Table 2 lists the measured atmospheric conditions: temperature, density, and viscosity.

Table 3 lists the measured wind conditions: speed and direction.

TABLE 2

ATMOSPHERIC DATA

Altitude km	Temperature °C	Atmospheric Density (ρ) g/cm^3	Atmospheric Viscosity (η) g/cm-s
8-9	-48	$0.46x10^{-4}$	$1.48x10^{-4}$
7-8	-47	$0.52x10^{-4}$	$1.49x10^{-4}$
6-7	-44	$0.61x10^{-4}$	$1.50x10^{-4}$
5-6	-38	$0.69x10^{-4}$	$1.53x10^{-4}$
4-5	-33	$0.80x10^{-4}$	$1.55x10^{-4}$
3-4	-26	$0.87x10^{-4}$	$1.58x10^{-4}$
2-3	-22	$0.97x10^{-4}$	$1.61x10^{-4}$
1-2	-17	$1.09x10^{-4}$	$1.63x10^{-4}$
0-1	-12	$1.23x10^{-4}$	$1.66x10^{-4}$

TABLE 3

WIND SPEED AND DIRECTION

Altitude km	Direction Azimuth degrees	Speed knots
8-9	229.5	50.5
7-8	223.5	58
6-7	224	58.5
5-6	222	54.5
4-5	221	51
3-4	217.5	36
2-3	211.5	28
1-2	201.5	17.5
0-1	231	11

IV. The General Problem

What can we learn about the interaction between a volcano and the atmosphere? This interaction has been studied for many years. The observations related to the eruptions at the Augustine volcano can add to our understanding of the fallout pattern of fine volcanic ash.

V. The Particular Problems

Each of the following problems refers to the Augustine eruptions that occurred March 27 to April 4, 1986.

A. Preliminary Problems

1. Study Phase 1, the height of the eruption column.
   (a) The peak eruption rate was estimated to be 7.5×10^7 metric tons per day. Use Wilson's equation to calculate the maximum column height in kilometers.
   (b) Add your result to the graph of Figure 6.

2. Study Phase 2, the wind pattern.
   (a) Draw a wind profile (like the one in Figure 10) for Augustine Island, March 28, 1986. Use the data in Column 3, Table 3. Represent the speed of the wind by horizontal arrows. Make the length of the arrows proportional to the speed.
   (b) Find the prevailing direction of the wind by averaging the azimuth directions in Column 2, Table 3. Show this direction graphically in a plane. Remember that the wind direction is the direction from which the wind is blowing.

B. The Problem

How does the observed data compare with the data predicted by the mathematical model discussed in Section III?

The following procedure is suggested:

1. What is the formula for terminal velocity?

Terminal velocity is an important key to the distribution of fallout. Use the equations which describe the mathematical model to find an expression for T in terms of the known quantities.

2. What does the model tell us?
   (a) Consider the atmosphere from sea level to a height of 9 km as a series of 9 layers, each 1 km thick. Calculate the terminal velocity of fallout in each of these layers.
   (b) Use the terminal velocity in each layer as the rate of vertical fall. Calculate the time for a particle to fall through each layer. Find the total time of fall.
   (c) For each layer, calculate the horizontal distance travelled by a particle as it falls. Find the total horizontal distance.
   (d) On graph paper make a representation of the path of a particle as it falls from the maximum height to the ground. Represent heights on the vertical axis, and represent the path by a series of line segments, one for each layer, in which the line segment shows the motion resulting from combining the vertical and horizontal motion. (You will need to use different scales on the vertical and horizontal axes.)

3. Analyze the data calculated from the model.
   (a) Will particles falling from the 8-9 km layer reach the ground at Anchorage? Will they reach the ground anywhere in 22 hours?
   (b) If a particle reaches the ground in 22 hours, from what atmospheric layer will it need to begin its fall? How far will it have travelled horizontally?
   (c) If a particle reaches the ground in Anchorage, from what atmospheric layer will it need to begin its fall? How long will it take to reach the ground?

4. Compare the model predictions with the real data.
   (a) Does the mathematical model predict the actual time of fall and distribution of the particles? Explain why or why not.

(b) We can assume that the measured data is "correct" in the experimental sense of the word, that is, at least within 10%. As a scientist where will you look for any discrepancy between the predictions of the model and the actual data? What recommendation will you make to Dr. Rose?

C. Computer Problem

Write a computer program that will calculate and list T in each layer, the time it takes to fall through that layer, and the horizontal distance travelled. Also calculate the total time and total horizontal distance. In this program, or an additional one, calculate and list the Reynolds number for each layer. Include in your program any additional calculations that you think will help you in analyzing your work as suggested in Section V B3.