MathWorks Math Modeling Challenge 2022

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M3 Challenge FINALIST—\$5,000 Team Prize

JUDGE COMMENTS

Specifically for Team # 15376—Submitted at the Close of Triage Judging:

COMMENT 1: Very nice executive summary. It had a good combination of approach and a summary of results. Very nice job of clearly stating your assumptions through out the paper, and giving detailed justifications along with citations. Excellent job with the strengths and weaknesses. This was a great paper. Well written, clear, and easy to follow. Well done!

COMMENT 2: The summary was well-written and provided a nice overview of the approached used. The team presented a well thought out discussion of their assumptions and the corresponding justifications. The paper provided a nice discussion of each of the models incorporated. In particular, the model for question 2 showed nice insights. The paper would have benefited from a broader discussion (with regards to definition) of the impact of remote working on particular cities.

COMMENT 3: Summary is present and adequate but missing some insight into the approaches employed to construct the models. Models are presented and some valid reasoning behind models is provided. Well-written.

COMMENT 4: This is an excellent essay, congratulations! You provide plenty of sources to sustain your arguments and assumption. The structure and presentation are very sharp. Some things that could be improved. In Q1, it isn't really clear how the pandemic enters into the model. Demographics could have been considered in Q2. Still, you have done a great job!



***Note: This cover sheet has been added by SIAM to identify the winning team after judging was completed. Any identifying information other than team # on a MathWorks Math Modeling Challenge submission is a rules violation. ***Note: This paper underwent a light edit by SIAM staff prior to posting.

Remote Work: Fador Future

Team 15376

February 27, 2022

o Executive Summary

The COVID-19 pandemic wreaked havoc on the modern world. Many thought that the pandemic would pass swiftly, with little effect on their lives, but they were soon proven wrong. COVID-19 spurred the rate of remote employees dramatically, revolutionizing the idea of working in an environment other than an office. Although the pandemic is subsiding, the rise of remote work is here to stay. We aim to predict the proportion of jobs in five cities in the US and the UK that can feasibly be done remotely in 2024 and 2027. Then, among workers whose jobs could feasibly be done remotely, we determine whether an individual employee will work from home given employee and employer preferences. Finally, we synthesized our previous results to calculate the number of employees in these cities that will work remotely in 2024 and 2027.

First, we predicted the proportion of workers whose jobs are remote-ready for five cities in the US and the UK in both 2024 and 2027. We divided all jobs into three main industries: Manual Labor, Human Services, and Corporate. After considering trends of remote feasibility within each industry, as well as the contribution of each industry to the overall labor force, we computed the percentage of remote-ready jobs in the cities of Seattle, Omaha, Scranton, Liverpool, and Barry (Wales). In both 2024 and 2027, the city of Barry had the percentage of remote-ready jobs at 40.93% and 45.16%, respectively.

Next, we designed a model that outputs whether an individual in a remote-ready occupation will be given the option to work remotely by their employer and, given this choice, choose to work from home. To create this model, we first determined that employee productivity is the primary factor that an employer considers when allowing employees to work from home. We then determined that the primary factor that employees will consider when given the choice to work remotely is their overall happiness at work, which is impacted by job satisfaction, anxiety of COVID infection, the need to look after family members, and a desire to socialize with others. Once we determined all of these factors impacting employee productivity and happiness, we were able to combine them into a single equation modeling whether an individual in a remote-ready job will work remotely.

Finally, we combined our predictions from Part I with our model from Part II to find the percentage of workers in each city that would work remotely. In order to do this, we used the result from Part I to find the remote-ready workers. We then used our model from Part II to determine the amount of these remote-ready workers who would actually work remotely. After simulating this process 1,000 times, we found that Liverpool would be impacted the most by remote work as 12.1% of all workers would work remotely in 2024 and 13% of all workers would work remotely in 2027.

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1 Part I: Ready or Not

1.1 Restatement of the Problem

In this problem, we formulate a model that predicts the remote-readiness of jobs in five different cities: Seattle, WA; Omaha, NE; Scranton, PA; Liverpool, England; and Barry, Wales. Using this model, we determine the percentage of jobs that will be remote-ready in 2024 and 2027 for each of these cities.

1.2 Assumptions

- 1. *All jobs lie within the following 3 industries: Manual Labor, Human Services, and Corporate.* We refer to these three categories as primary industries because they encompass the vast proportion of all jobs in the workforce.
- 2. *The remote-readiness within each primary industry is the same among all jobs within that industry.* The work that a job entails defines its feasibility to be remote-ready. Since the nature of work performed within each primary industry is similar, we assume that overall remote-readiness within each primary industry is equal.
- 3. All minor industries can be placed within each of the three primary industries as secondary and *tertiary industries.* While other industries do exist, they comprise a minor proportion of all urban jobs and are thus negligible in our model. We define secondary industries as general types of occupations that lie within a primary industry. We define tertiary industries as the specific occupations that lie within secondary industries.
- 4. The initial proportion of jobs that are remote-ready in a secondary industry is equivalent to the average of the initial proportions of jobs that are remote-ready in its corresponding tertiary industries. Little reliable data exists regarding the feasibility of remote work for secondary industries as a whole. As such, averaging reliable approximations of the proportion of jobs within a tertiary industry that can be done remotely can accurately represent the proportion for their corresponding secondary industry.
- 5. *Manual labor jobs have already reached their limit of remote readiness.* Jobs that lie within the Manual Labor category all require hands-on work and either have minimal or no components that can be done remotely. In the next five years, it is unrealistic that this industry's remote-readiness will change substantially.
- 6. *Citizens of the US and the UK share a similar opinion regarding healthcare and education.* The US and the UK share many similar values [35], and their sentiments regarding virtual schools and telehealth can be thought of as roughly identical for modeling purposes.
- 7. The number of jobs within the corporate industry that are remote-ready is proportional to the number of remote workers in the corporate industry. As the number of employees working remotely continually increases, the number of jobs that become more flexible to suit the needs of their employees should increase at approximately the same rate. This is because employers modify the conditions and requirements of jobs as remote work becomes increasingly commonplace in society.

8. *The ratios in which secondary industries comprise a primary industry remain constant after 2021.* The employment of secondary industries varies, but these values are too volatile to accurately model.

1.3 Variables

Variable	Description	
w	Proportion of workers that are part of a specific industry	
r	Proportion of jobs that can be done remotely (remote-readiness)	
t	Time in years since Jan 1, 2013	

Table 1: Variables for Part I

Note: The subscripts *M* , *H*, *C*, *E*, *A* will correspond to Manual Labor values, Human Services values, Corporate values, Education values, and Healthcare values, respectively.

1.4 Model Development

In our model, we first predict the proportion of workers that each primary industry will have in 2024 and 2027. Then, we predict how remote-readiness changes within each primary industry. We then combine both predictions to arrive at our final prediction of remote-readiness percentage in these five cities.

We define the Manual Labor primary industry to consist of mining, logging, construction, manufacturing, trade, transportation, and utilities. We define the Human Services primary industry to consist of education, health services, leisure, and hospitality. We define the Corporate primary industry to consist of information, financial activities, and business services.

1.4.1 Industry Growth and Decline

In the past two years, all primary industries lost thousands of workers as a result of the COVID pandemic. In the years before, although lightly fluctuating with the economic cycles, job growth in the US [20][27][28] and the UK [16][3] across most industries was positive and constant.

Since job growth was generally observed to be constant before COVID, we used a linear regression to predict the amount of jobs held by each of the secondary industries in 2024 and 2027. After we obtained these values, we added each secondary industry to its corresponding primary industry. Once we had all of the projected workers for each primary industry, we were able to find the proportion. Below are the tables of our projections:

Proportion	Seattle	Omaha	Scranton	Liverpool	Barry
ω_M	0.369	0.381	0.457	0.273	0.193
w_{H}	0.326	0.314	0.341	0.456	0.725
w_{c}	0.305	0.305	0.202	0.271	0.082

Table 2: Proportions for 2024

Proportion	Seattle	Omaha	Scranton	Liverpool	Barry
w_M	0.374	0.381	0.458	0.269	0.186
ω_{H}	0.324	0.315	0.340	0.451	0.726
w_{c}	0.302	0.304	0.202	0.280	0.088

Table 3: Proportions for 2027

1.4.2 Remote-readiness Growth and Decline

We treat Manual Labor, Corporate, and Human Services separately.

Per Assumption 5, the proportion of manual labor jobs that can be conducted remotely will remain constant in our model, i.e., r_M remains constant, which can be calculated via the data provided of tertiary industries [17] and Assumption 4.

Human services comprises leisure, healthcare, and education [14]. The nature of leisure work makes it difficult to be done remote (similar to Manual Labor) [15], so the proportion of leisure jobs that can be done remotely remains constant over time. These values can be calculated via the data provided of tertiary industries [17] and *Assumption 4*.

Regarding healthcare, the feasibility of telehealth corresponds directly with consumer demand for telehealth as opposed to traditional healthcare, which in turn allows for health workers to work remotely. 98% of telehealth users were satisfied with healthcare [18]. We use *Assumption 6* to apply this statistic to the UK. In addition, 50% of telehealth users claimed that a bad experience would ruin their experience with a telehealth software [2]. So, we can model the proportion of telehealth users that switch to in-person health as $0.02 \cdot 0.5 = 0.01$. Furthermore, 75% of in-person health users would be open to switching to a telehealth alternative [31]. So, for a given healthcare customer, their decisions for the nature of their healthcare can modeled by the following Markov Chain, with each transition representing a year.



We then simulated results for t = 11 and t = 14, given that for t = 7, 19% of people have tried a telehealth visit [31].

Regarding education, we use NCES data regarding enrollment in virtual schools as a proportion of all students in the US [19], and we will use *Assumption 6* to apply this to the UK. Remote education workers are employed by virtual schools, and the enrollment of these schools corresponds to the feasibility of remote employment for education workers. While online schools saw a surge in enrollment during COVID-19, this number began returning to normal levels during the 2021-22 school year [36], and thus the available NCES data can be utilized, with year on the x-axis and the proportion of all students in the US that are enrolled in an online school on the y-axis. After

applying a cubic regression, the following equation is obtained with $R^2 = 0.95$:

$$r_A = 0.00019t^3 - 0.00106t^2 + 0.0016t + 0.00399 \tag{1}$$

Currently, 37% of jobs in the Corporate primary industry can be performed remotely [21]. Furthermore, we know that in 2020, there are 78.5 million workers that work remotely, and the projected number of remote workers in 2024 is 93.5 million. Using these two data points, we see that the number of workers working remotely is projected to increase by a factor of 19.1% in the next four years. As per *Assumption 7*, we can assume that the number of jobs in the Corporate industry that will be remote-ready in 2024 will also increase by 19.1%. Thus, we can estimate that for a given *t*, the percentage of jobs in the Corporate industry that will be remote-ready is

$$r_C = 0.37 \cdot 1.191^{(t-7)/4} \tag{2}$$

So, we can see that in 2024, the estimated percentage of corporate jobs that will be remote-ready is 37 \cdot 1.191 = 44.07%. Similarly, in 2027, the estimated percentage of jobs that will be remote-ready is 37 \cdot 1.191^(7/4) = 50.25%. These values can be utilized for all cities, as corporate remote readiness is independent of location.

Per Assumption 8, r_H can be calculated via the r values for its the secondary industries of human resources with a separate weighted average. All industry-wise r values are shown in the tables below.

Proportion	Seattle	Omaha	Scranton	Liverpool	Barry
r_M	0.08	0.07	0.09	0.09	0.16
r_H	0.49	0.49	0.52	0.42	0.47
r_{C}	0.44	0.44	0.44	0.44	0.44

Proportion	Seattle	Omaha	Scranton	Liverpool	Barry
r_M	0.08	0.07	0.09	0.09	0.16
r_H	0.55	0.55	0.58	0.45	0.52
r_{C}	0.50	0.50	0.50	0.50	0.50

Table 4: Remote-readiness for 2024

Table 5: Remote-readiness for 2027

1.5 Results

Combining our predictions for industry proportion and remote-readiness across industry through the use of a weighted average, we arrive at the following results regarding overall remote-readiness across all five cities.

Year	Seattle	Omaha	Scranton	Liverpool	Barry
2024	32.35%	31.47%	30.67%	33.70%	40.93
					%
2027	35.91%	35.32%	31.87%	36.67%	45.16
					%

1.6 Strengths and weaknesses

Employment cycles between periods of growth and recession because of how closely it is tied with the economy [12]. Thus, a linear model was a strong choice to model job growth across industries because it was able to ignore economic fluctuation to determine the overall trend of industry growth. Another strength of our model was that we utilized the principle of derived demand extensively with healthcare and education. Derived demand is a principle that states that the demand for a good is proportional to the demand for the resources required to make that good, including labor. Without substantial demand for remote forms of healthcare and education, the remote forms of those occupations will not exist, making remote work unfeasible. Utilizing derived demand allows for our model to account for real-world economic decisions made by firms in these industries.

One weakness of our model was that the linear correlation for Barry's industry growth was lower than expected. This can be attributed to the city's overall lack of growth in employment, which ended up increasing variation among workers in each industry instead of pointing towards clear trends. Another weakness of our model was that our healthcare Markov chain's transition weightings may have been biased towards remote healthcare. As a result, cities with more healthcare workers would overestimate the *r* values of remote workers in a more dire fashion.

2 Part II: Remote Control

2.1 Restatement of the Problem

In this problem, we are instructed to predict whether or not a worker whose job is remote-ready will actually work from home. We will consider an employee's own choice as well as their employer's choice.

2.2 Assumptions

- 1. An employee's willingness to work remotely is determined by their happiness gain from staying home. An employer's willingness to allow remote work is determined by changes in employee work output. Employees seek to maximize their personal happiness, while an employer's focus is to maximize the economic output of the business, i.e., employees' work output. [7][34].
- 2. *A person's waking hours are divided into 2 main portions: active work and free time.* Free time includes recreation, eating, and other leisurely activities. Active work includes time a person dedicates to their job, including commute time and time spent at work.
- 3. *A person is maximally happy during free time.* Free time allows a person to do whatever they wish; hence they will choose an activity that maximizes their happiness.
- 4. The factors influencing a person's happiness at work are job satisfaction [25], anxiety of COVID infection [10], the need to look after family members [5], and the desire to socialize with others [37]. According to the sources linked above, these are the dominant factors we will include in our model.
- 5. The only factor that employers consider when deciding whether to give their employees the choice to work remotely is worker productivity. The main goal that employers have is to maximize their employees' productivity, so it is fair to assume that employers will not give their workers the choice to work remotely if they cannot reach a certain level of productivity.
- 6. The only factors that impact an employee's work output are their WiFi speed [1], their care taking responsibilities [6], and overall happiness [13]. According to the sources linked above, these are the dominant factors we will include in our model.
- 7. *Employees sleep for 7 hours each day, leaving 17 working hours per day.* The majority of adults in both the US and UK sleep this amount each night [7][34].
- 8. *Employers will not prevent in-person work*. As reflected in the growing push to return to inperson work throughout all industries [24], we assume that employers have determined that on a macroscopic scale, in-person work generates more economic output. As a result, we assume that companies' policies will not restrict workers from working in person.
- 9. *The amount of work that a remote worker does is the same as an in-person worker.* Because we only consider remote-ready workers in this section, we assume that remote workers' quantity of work will not be changed as a result of transitioning remote.
- 10. *Company WiFi speeds are always adequately high.* WiFi capabilities vary based on company, but given our limited time we will not be considering variability in companies' WiFi speed.

2.3 Variables

Variable	Description	
Н	Happiness coefficient (hrs)	
Р	Productivity (work/hr)	
Т	Time spent working (hrs)	
W	Total work in a day	
τ	Time to commute to work (hrs)	
N_t	Number of toddlers (ages 0-3) at home	
С	COVID anxiety coefficient	
J	Job satisfaction coefficient	
S	WiFi speed (MB/s)	
T_c	Time spent taking care of children	
f	Fraction of remote-ready employees that will work remote	

Table 7: Variables for Part II

2.4 Model Development

Our problem is split up into two sections: deriving an employee's inclination to work remotely, and deriving their employer's willingness to allow remote work. By *Assumption 1*, we will gauge these values with a happiness coefficient H and a work output value W, respectively, which we will find in the following sections.

2.4.1 Deriving *H*

We seek to find the coefficient H describing an employee's happiness, with larger values of H indicating more happiness. This coefficient will be used to compare their relative happiness in a remote versus an in-person work experience.

Per Assumption 2, we consider happiness for free time and active work time. Since a person's total happiness depends not only on how happy they are at a given time but also how long they are happy for, our value for H will be given by

$$H = h_{\text{free}} \cdot t_{\text{free}} + h_{\text{work}} \cdot t_{\text{work}}, \tag{3}$$

where $h_i \in [0, 1]$ is a dimensionless value describing a person's level of happiness at a given point in the day. Below is a visual representation of this equation:



Figure 1: Visual representation of Eq. (3)

A person's day is divided into free time (blue) and work (red) periods. Given values of relative happiness h_i in each period, the total happiness one feels in a day is equivalent to spending time

 $h_i \cdot t_i$ being completely happy; this is essentially the value of *H*.

We now derive these values for remote and in-person work.

Remote work

When working remotely, the time to commute $\tau = 0$, so the active working time as defined in *Assumption 2* is $t_{\text{work}} = T$. Per *Assumption 7*, there are 17 waking hours in a day, so the time spent on free time is $t_{\text{free}} = 17 - T$.

Per Assumption 3, an employee's happiness is maximal during free time. Hence, $h_{\text{free}} = 1$. By Eq. (3), we yield

$$H_{\rm free} = h_{\rm free} \cdot t_{\rm free} = 17 - T \tag{4}$$

Job satisfaction is directly related to h. If a person is fully satisfied with their job, then active work is akin to free time, i.e., h = 1. On the other hand, a highly unsatisfied worker will not enjoy their work at all, i.e., h = 0. Hence, the value for h during active work is proportional to a person's job satisfaction J. The other factors influencing happiness at work (listed in *Assumption 4*) are not present, since a person has minimal risk of COVID infection at home, is readily available to care for family members, and does not have the ability to socialize with coworkers. As a result, for values of $J \in [0, 1]$ (on the same scale as h), h = J. We therefore yield

$$H_{\rm work} = h_{\rm work} t_{\rm work} = h_{\rm work} T = JT.$$
(5)

By Eq. (3), the total happiness coefficient for remote work is

$$H_{\text{remote}} = H_{\text{free}} + H_{\text{work}} = 17 - (1 - J)T.$$
(6)

Since $J \le 1$, we observe that a longer work day T decreases happiness and higher job satisfaction J increases happiness, matching logical predictions.

In-person work

We now find *H* for in-person work. As an employee must now commute to work, the active working time defined in *Assumption 2* is now $t_{\text{work}} = \tau + T$. As a result, $t_{\text{free}} = 17 - T - \tau$.

Since an employee will still have maximal happiness during free time,

$$H_{\rm free} = h_{\rm free} \cdot t_{\rm free} = t_{\rm free} = 17 - T - \tau. \tag{7}$$

However, an employee's happiness during work h_{work} differs from the remote scenario as it is influenced by the factors in *Assumption 4*. We analyze each factor to determine its impact on *h*:

• *COVID anxiety* : We define a coefficient of COVID anxiety *C*, with C = 0 indicating low anxiety and C = 1 indicating high anxiety. As a greater value of *C* must reduce an employee's happiness in an in-person work environment, we assume a negative linear relationship:

$$h \propto 1 - C \tag{8}$$

• *Looking after family members*: The family members that employees will need to look after are primarily young children less than the age of 3 who cannot yet attend school (toddlers).

Hence, N_t , the number of toddlers in a family, reflects obligations at home that will impact an employee's happiness while working in-person. We estimate that the theoretical maximum number of toddlers that parents will have at any moment is 3. Taking care of 1 child results in a high increase in one's anxiety at work—and a major decrease in their happiness—but adding another child increases this anxiety by a smaller amount. As a result, we estimate an exponential dependence of h on N_t of the form

$$h \propto e^{-aNt}$$
. (9)

Since 3 children will theoretically minimize the value of *h*, we estimate that $(N_t, e^{-aNt}) = (3, 0.05)$ is a point correlating our values. This returns $a = -0.998 \approx -1$.



Figure 2: The graph of e^{-Nt} . Note the largest decrease is between $N_t = 0$ and $N_t = 1$.

• *Socializing* : According to the Our World In Data survey [37], people in the US report socializing as increasing their happiness by 10.6%. As a result of socializing with coworkers in person, we further adjust *h* by multiplying by 1.11.

Combining all these factors,

$$h_{\rm work} = 1.11(1 - C)Je^{-Nt} \tag{10}$$

which gives

$$H_{\text{work}} = h_{\text{work}} \cdot t_{\text{work}} = h_{\text{work}}(\tau + T) = 1.11 J e^{-Nt} (1 - C)(\tau + T).$$
(11)

The total happiness coefficient can be derived by summing
$$H_{\text{free}}$$
 and H_{work} above:

$$H_{\text{in-person}} = 17 + [1.11 J e^{-Nt} (1 - C) - 1] (\tau + T)$$
(12)

2.4.2 Deriving W

We seek to find the work output of employees between remote and in-person options. We will focus on the work output W that an employee accomplishes in one day, which is given by

$$W = PT \tag{13}$$

This value will later be used to compare employees' productivity when working remotely versus in person, which, per *Assumption 1*, will help employers decide whether to give their employees the option to work remotely. We first consider the remote case, W_{remote} .

Remote work

In the remote case of Eq. (13), $W_{\text{remote}} = P_{\text{remote}}T_{\text{remote}}$. Starting with P_{remote} , happiness and WiFi speed are contributing factors by *Assumption 6*. We will derive the effect of each factor on P_{remote} below:

• *Happiness*: according to a Forbes study conducted in 2021 [13], being happy increases productivity by 13%. Since a person's happiness throughout the day impacts their productivity at work, we use H_{remote} from the previous section as a metric for happiness. Linearly varying the increase of productivity due to happiness,

$$P_{\text{remote}} \propto 1 + 0.13 H_{\text{remote.}}$$
 (14)

WiFi speed : WiFi connection strength directly impacts employee productivity as employees log onto their work server and attend remote meetings. When WiFi speed is slow such that it is the limiting factor in an employee's productivity, it is directly related to productivity; P_{remote} ∝ S. Above a certain critical speed S_c, however, an employee's productivity is no longer limited by WiFi speed but rather their own work ability, thus causing productivity to plateau. This relationship is demonstrated by the given graph:



Figure 3: Relationship between P_{remote} and S

Combining all of our factors yields a piecewise function for *P*:

$$P_{\text{remote}} = \begin{bmatrix} S(1+0.13H_{\text{remote}}) & S \le S_c \\ P_0 \cdot (1+0.13H_{\text{remote}}) & S > S_c \end{bmatrix}$$
(15)

where P_0 is some constant. Now, T_{remote} depends on the amount of time T_t a remote worker will spend looking after toddlers. This time is cut out of their work time, so

$$T_{\text{remote}} = T - T_t. \tag{16}$$

By Eq. (13), we yield

$$W_{\text{remote}} = \begin{cases} \bigcup_{l \in P_{0}}^{L} \cdot \frac{S(1+0.13H_{\text{remote}})(T-T_{t})}{S_{c}} & S \leq S_{c} \\ \sum_{l \in P_{0}}^{L} \cdot (1+0.13H_{\text{remote}})(T-T_{t}) & S > S_{c} \end{cases}$$
(17)

Following logical predictions, greater happiness, faster WiFi speeds, and less time to care for toddlers increase the work done by an employee.

In-person work

When working in person, the influence of WiFi speed is absent, by *Assumption 10*. In addition, a lack of people to care for effectively yields $T_{\text{in-person}} = T$. However, happiness is still a relevant factor in influencing productivity. Using the same constant P_0 as above,

$$W_{\rm in-person} = P_0(1+0.13H_{\rm in-person})(T).$$
 (18)

Summing up our equations,

$$H_{\text{remote}} = 17 - (1 - J)T$$

$$H_{\text{in-person}} = 17 + [1.11Je^{-Nt}(1 - C) - 1](t + T)$$

$$W_{\text{remote}} = \begin{bmatrix} S(1 + 0.13H_{\text{remote}})(T - T_t) \\ S_c \end{bmatrix} S \le S_c$$

$$F_0 \cdot (1 + 0.13H_{\text{remote}})(T - T_t) \qquad S > S_c$$

$$W_{\text{in-person}} = F_0(1 + 0.13H_{\text{in-person}})(T)$$

We now seek to find whether or not an employee will work remotely. First, an employee needs approval from their employer. According to [23], employers have tolerated dips in work output by 20% in the past. Hence, if a remote employee's work output is 80% or above of their in-person work output, they will be allowed to work remotely.

After getting approval, we will also consider whether or not an employee chooses to work in person. Because an employee chooses to maximize their happiness, they will work remotely if their happiness is greater for remote work.

As a result, an employee will work remotely if and only if

Wremote	> 0 0 and	Hremote	
Win-person	2 0.8 anu	$\overline{H_{\text{in-person}}}$	≥ 1

for the values of each variable above.

2.6 Strengths and weaknesses

The strength of our model is that it robustly incorporates the most prevalent factors in terms of what employees consider when debating between in-person and remote work. Because our model is highly complex, its accuracy is higher than that of a simpler model. Adding to our complexity is recognizing that work output W and the happiness of employees H are not independent of one another; W depends on H.

Furthermore, our model is personalized to each individual with the inclusion of the factors J, C, and N_t . This makes it more accurate than a model that would only consider macroscopic generalizations for a populace.

While we were able to personalize the model to each individual, we were not able to do so for companies. In *Assumption 10*, we assumed that companies would have adequate WiFi to support the optimal productivity of their workers. However, in a real scenario, this would not be true as the WiFi capabilities of companies varies. If given more time, we would include this factor in our model as a way to increase its accuracy.

3 Part III: Just a Little Home-work

3.1 Restatement of the Problem

In this problem, we synthesize our answers from Parts 1 and 2 to create a model which will predict the percentage of workers that will work remotely in a given city. Using this model, we predict the percentage of employees that will work remotely in 2024 and 2027 in the cities that we assessed in Part 1. Finally, we rank these cities by the extent of the impact that remote work will have on each city.

3.2 Assumptions

- 1. *Only children aged 0-3 need care-giving time.* Children older than 4 usually go to preschool/school during the day, thus they don't need their parents to care for them during the workday.
- 2. *The average number of children per family is constant regardless of city.* We can assume that the average number of children that families have in the UK and US is unaffected by the city in which they are located.
- 3. *Each employee works 8 hours a day.* An eight-hour workday is the traditional workday in both the US and the UK.
- 4. *Employees' WiFi speed relative to the WiFi speed required by their jobs stays constant.* Although WiFi capabilities will increase in the future, the WiFi demands of each industry also grow. Thus, we treat current WiFi speed as a benchmark for the future. For example, someone having 50 megabits/s of WiFi in 2020 might have 80 megabits/s of WiFi in 2024, but the work they are able to perform remains the same because of the increased demand for power. Thus, we can treat WiFi speed as a constant.
- 5. *The standard deviation of a parameter can be represented by 10% of the parameter's mean.* Making the standard deviation 10% of the mean is a common technique utilized in statistics when a lack of data is present. As such, we assume this standard deviation so we can incorporate variance into our model.
- 6. *COVID-19 will have negligible impact in 2024 and 2027* As we see the pandemic numbers waning, we can assume COVID will no longer be a significant factor because new technology and vaccines will be developed in the future.

3.3 Variables

The variables used in this simulation will be the variables listed in both Parts I and II, as our model will ultimately synthesize both data sources to provide and compare the numerical data in the five cities, as well as the two variables from the previous results.

Variable	Description	
w	Proportion of workers that are part of a specific industry	
r	Proportion of jobs that can be done remotely (remote-readiness)	
S	WiFi speed (MB/s)	
J	Job satisfaction coefficient	
N_t	Number of toddlers (ages 0-3) at home	
τ	Time to commute to work (hrs)	
T_t	Time spent taking care of toddlers (hrs)	
F	Final percentage of workers who go remote	

Table 8: Variables for Part III

3.4 Model Development

We combine our results from Parts I and II to run a Monte Carlo simulation (linked in references).

All of our data that is plugged in to our Monte Carlo simulation with equations found in Parts I and II are found in our data references. We explain each variable here:

- Variables w, r, S, J, and τ are found in our reference tables. They were sourced from [4], [22], [26], [11] and [29] and our results from previous sections.
- By Assumption 3, we have T = 8.
- The value of S_c from Part II is found to be 50 MB/s [30].
- To calculate T_t , our model in Part II only considered the amount of time the employees spend taking care of toddlers during their work time, not during the entire day. Thus, we scaled down our values [33] [32] by $T/17 = \frac{8}{17}$ to give us the time parents spend taking care of toddlers during only their work hours.
- To calculate N_t , we found the average number of children per household [9] [8]. Since the distribution of children's ages should be uniform, we divided the numbers found by six as only toddlers—children ages 0-3—are relevant towards our model. However, N_t encompasses all households who have toddlers, and some household might have more than one toddler. Thus, we know that the probability of having one toddler is 1/6, then having two toddlers is 1/36, and three toddlers is 1/216. We then utilized weighted probability in our code to generate the different number of toddlers for each employee.

Because we are finding a percentage of workers that will be remote, we used a standard population of 1000 people in each simulation. We first found the proportion of workers in each city who had the potential to work remotely for each of the three primary industries. With this proportion of workers, we simulated whether or not each individual worker would return to the workplace using the model given by Part II. In the end, we tabulated the number that worked remotely and found the corresponding percentages. We then ran 1000 simulations per city in each year.

Below are the graphs of our Monte Carlo simulations for remote worker percentage in each of the five cities for 2024 and 2027.





Figure 4: Graphs for our Monte Carlo simulations

The median percentage for each simulation is tabulated as follows:

City	F_{2024}	F_{2027}
Seattle	10.4%	11.6%
Omaha	9.7%	10.9%
Scranton	8.4%	9.3%
Liverpool	12.1%	13%
Barry	11.5%	12.7%

Table 9: Median percentages from Monte Carlo simulation

Thus, from our Monte Carlo simulation, we can rank the predicted percentage of workers who will work remotely in each city in descending order as Liverpool, Barry, Seattle, Omaha, and Scranton. Liverpool will have the highest rate of remote employees in both 2024 and 2027, while Scranton will have the lowest rate of remote employees in both 2024 and 2027. As such, Liverpool is impacted the most by remote work.

3.6 Strengths and weaknesses

The Monte Carlo simulation was able to successfully synthesize the models developed by the previous two parts to create a final projection. Additionally, the Monte Carlo simulation allowed us to present the expected outcome of remote worker percentage in each city and all potential variation of this percentage in a comprehensive yet concise manner.

However, a weakness of our model was that we weren't able to find data on the variance of some factors. As a result, a standard deviation of 10% of the mean had to be assumed for some of our factors in order to run the Monte Carlo simulation effectively. In addition, the lack of data specific towards each city made it difficult to find exact values for each of our factors. So, some data points from different cities were assumed to be equal even though this may not be the case in reality.

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Reference tables for Problem 3

Industry	w	r	S	J	N_t	τ	T_c
Manual Labor	0.369	0.08	119.89	0.48	0.32	0.53	0.83
Human Services	0.326	0.49	119.89	0.58	0.32	0.53	0.83
Corporate	0.305	0.44	119.89	0.62	0.32	0.53	0.83

Table 10: Seattle 2024

Industry	w	r	S	J	N_t	τ	T_c
Manual Labor	0.374	0.08	119.89	0.48	0.32	0.53	0.83
Human Services	0.324	0.55	119.89	0.58	0.32	0.53	0.83
Corporate	0.302	0.50	119.89	0.62	0.32	0.53	0.83

Table 11: Seattle 2027

Industry	w	r	S	J	N_t	τ	T_c
Manual Labor	0.381	0.07	46.4	0.48	0.32	0.32	0.83
Human Services	0.314	0.49	46.4	0.58	0.32	0.32	0.83
Corporate	0.305	0.44	46.4	0.62	0.32	0.32	0.83

Table 12: Omaha 2024

Industry	w	r	S	J	N_t	τ	T_c
Manual Labor	0.381	0.07	46.4	0.48	0.32	0.32	0.83
Human Services	0.315	0.55	46.4	0.58	0.32	0.32	0.83
Corporate	0.304	0.50	46.4	0.62	0.32	0.32	0.83

Table 13: Omaha 2027

Industry	w	r	S	J	N_t	τ	T_c
Manual Labor	0.457	0.09	42.56	0.48	0.32	0.32	0.83
Human Services	0.341	0.52	42.56	0.58	0.32	0.32	0.83
Corporate	0.202	0.44	42.56	0.62	0.32	0.32	0.83

Table 14: Scranton 2024

Industry	w	r	S	J	N_t	τ	T_c
Manual Labor	0.458	0.09	42.56	0.48	0.32	0.32	0.83
Human Services	0.340	0.58	42.56	0.58	0.32	0.32	0.83
Corporate	0.202	0.50	42.56	0.62	0.32	0.32	0.83

Table 15: Scranton 2027

Industry	w	r	S	J	N_t	τ	T_c
Manual Labor	0.273	0.09	66	0.48	0.28	0.98	1.18
Human Services	0.456	0.42	66	0.58	0.28	0.98	1.18
Corporate	0.271	0.44	66	0.62	0.28	0.98	1.18

Table 16: Liverpool 2024

Industry	w	r	S	J	N_t	τ	T_c
Manual Labor	0.269	0.09	66	0.48	0.28	0.98	1.18
Human Services	0.451	0.45	66	0.58	0.28	0.98	1.18
Corporate	0.280	0.50	66	0.62	0.28	0.98	1.18

Table 17: Liverpool 2027

Industry	w	r	S	J	N_t	τ	T_c
Manual Labor	0.193	0.16	104	0.48	0.28	0.32	1.18
Human Services	0.725	0.47	104	0.58	0.28	0.32	1.18
Corporate	0.082	0.44	104	0.62	0.28	0.32	1.18

Table 18: Barry 2024									
Industry	w	r	S	J	N_t	τ	T_c		
Manual Labor	0.186	0.16	104	0.48	0.28	0.32	1.18		
Human Services	0.726	0.52	104	0.58	0.28	0.32	1.18		

Table 19: Barry 2027

0.62

0.088 0.50 104

Corporate

0.28 0.32

1.18

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Code

```
1 % variable extraction from table
2 T = readtable ('liverpool_2024 - Sheet1.csv','PreserveVariableNames',true) %Reading
       data from Csv
w_m = T\{1, "w"\};
4 f = 0.1; %standard deviation factor
5 w_h = T\{2, "w"\};
6 \text{ w_c} = T\{3, "w"\};
7 r_m = T\{1, "r"\};
s r_h = T\{2, "r"\};
9 r_c = T\{3, "r"\};
10 S = T{1, "S"};
11 J_m = T\{1, "J"\};
12 J_h = T\{2, "J"\};
13 J_c = T{3, "J"};
14 N_t = T\{1, "N_t"\};
15 tau = T\{1, "tau"\};
16 T_c = T\{1, "T_c"\};
17 % simulation for 1000 people
18 \text{ pop} = 1000;
19 % we find the number of remote-ready workers in each sector
20 rem_m = ceil(pop * w_m * r_m);
21 rem_h = ceil(pop * w_h * r_h);
22 rem_c = ceil(pop * w_c * r_c);
23 counttracker = [];
24 % finding number of people with each number of kids
25 \text{ N0} = 1 - N_t;
26 \text{ N1} = 36 * \text{N_t} / 43;
27 N2 = 6 * N_t / 43;
28 N3 = N_t/43;
29 %simulation 1000 times monte carlo
30 for i = 1:1000
       count = 0;
31
32 %simulation for manual laborers
  for j = 1:rem_m
33
       S1 = normrnd(S, f^*S);
34
       J_m1 = normrnd (J_m, f*J_m);
35
       %Weighted probability for # of children
36
       var = [0 \ 1 \ 2 \ 3];
       W = [N0, N1, N2, N3];
38
       N_t1 = randsample(var,1,true,W);
39
       %Variable setup with Standard Deviation
40
       tau1 = normrnd(tau, f*tau);
41
       T_c1 = normrnd(T_c, f^*T_c);
42
       H_r1 = 17 - (1 - J_m1) * 8;
43
       H_{i1} = 17 + (1.11*J_m1*exp(-N_t1)-1)*(8+tau1);
44
       W_r1 = (1+0.13*H_r1)*(8-T_c1);
45
       W_r2 = S1/50*(1+0.13*H_r1)*(8-T_c1); %Work rate without 50Mbps of "ifi
46
       W_{i1} = (1+0.13*H_{i1})*8;
47
       %Piecewise where WiFi speed is larger/less than 50Mbps.
48
       if S1 >= 50
49
           W_r1/W_i1;
50
```

```
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```

```
if W_r1/W_i1 > 0.8
51
                 if H_r1/H_i1 >= 1
52
                      count = count + 1;
53
                 end
54
55
            end
       end
56
       if S1 < 50
            W_r2/W_i1;
58
            if W_r2/W_i1 > 0.8
59
                 if H_r1/H_i1 >= 1
60
                      count = count + 1;
61
                 end
62
63
            end
       end
64
   end
65
   %Simulation for Service sector
66
   for j = 1:rem_h
67
       S1 = normrnd(S, f^*S);
68
       J_h1 = normrnd(J_h, f^*J_h);
69
       var = [0 1 2 3];
70
       W = [N0, N1, N2, N3];
71
       N_t1 = randsample(var,1,true,W);
72
       tau1 = normrnd(tau, f*tau);
73
74
       T_c1 = normrnd(T_c, f^*T_c);
       H_r1 = 17 - (1 - J_h1) * 8;
75
       H_{i1} = 17 + (1.11*J_h1*exp(-N_t1)-1)*(8+tau1);
76
       W_r1 = (1+0.13*H_r1)*(8-T_c1);
77
       W_r2 = S1/50*(1+0.13*H_r1)*(8-T_c1);
78
       W_i1 = (1+0.13*H_i1)*8;
79
       if S1 >= 50
80
            W_r1/W_i1;
81
            if W_r1/W_i1 > 0.8
82
                 if H_r1/H_i1 >= 1
83
                      count = count + 1;
84
                 end
85
86
            end
       end
87
       if S1 < 50
88
            W_r2/W_i1;
89
            if W_r2/W_i1 > 0.8
90
                 if H_r1/H_i1 >= 1
91
                      count = count + 1;
92
93
                 end
94
            end
       end
95
96
   end
   %Simulation for Corporate
97
   for j = 1:rem_c
98
       S1 = normrnd(S, f^*S);
99
       J_c1 = normrnd(J_c, f^*J_c);
100
       var = [0 1 2 3];
101
       W = [N0, N1, N2, N3];
       N_t1 = randsample(var,1,true,W);
103
```

```
tau1 = normrnd(tau, f*tau);
104
       T_c1 = normrnd(T_c, f^*T_c);
       H_r1 = 17 - (1 - J_c1) * 8;
106
        H_i1 = 17+(1.11*J_c1*exp(-N_t1)-1)*(8+tau1);
107
        W_r1 = (1+0.13*H_r1)*(8-T_c1);
108
        W_r2 = S1/50*(1+0_13*H_r1)*(8-T_c1);
109
        W_{i1} = (1+0.13*H_{i1})*8;
       if S1 >= 50
111
            W_r1/W_i1;
112
            if W_r1/W_i1 > 0.8
                 if H_r1/H_i1 >= 1
114
                      count = count + 1;
115
                 end
116
            end
117
118
       end
       if S1 < 50
119
            W_r2/W_i1;
120
            if W_r2/W_i1 > 0.8
121
122
                 if H_r1/H_i1 >= 1
                      count = count + 1;
123
                 end
124
            end
125
126
       end
128 end
   frac = 100 * count / 1000;
129
130 % finding proportion
131 counttracker(end + 1) = frac;
   end
132
133 %graphing results
134 histogram(counttracker, [0:0.5:25])
135 hold on
136 title ("% of workers remote in Liverpool, 2024")
137 ylabel ("Frequency")
138 xlabel (" Percentage ")
139 \text{ ylim} = ([0 \ 30]);
140 yl = ylim;
141 mu = median(counttracker);
142 xline(mu, 'Color', 'g', 'LineWidth', 2);
143 string = "Median = " + mu
144 legend ('', string)
```