MathWorks Math Modeling Challenge
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PREVIEW PAPER: ABOVE AVERAGE

The team's executive summary contained some results, but their overview of the results was incomplete. The team provided good reactions to the first two questions, but their response to the third was not as strong.

For the first question, the team stated the model but only provided their final results. It would not be trivial to replicate their results. The team incorporated access to internet resources into their model and noted that there should be a limiting value which was a nice addition to the model. Another positive in their approach to the first question is that they noted that the pandemic marked a significant change, and they limited the way they used the data after the start of the pandemic.

For questions two and three the team's presentation parallelled their approach to the first question. They clearly stated the model but did not provide sufficient details to make clear how they calculated their parameters. They clearly stated their results but no intermediate calculations are given. One interesting thing about their approach is they noted the correlation between education and income and reduced the number of factors. Also, they recognized their predictions for Barry were unexpected and explicitly discussed their results rather than simply moving on.

**from among the screened sample of papers examined during pre-triage work.*

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REMOTE WORK : FAD OR FUTURE

EXECUTIVE SUMMARY

The question this year is very much relevant to our future career. Since the beginning of the COVID-19 pandemic, remote working seems to have become more and more common and more widely discussed as an alternative to conventional working styles, and while the widespread awareness of remote working is helping to bring such work style into discussion in urban and suburban areas, from Seattle, WA in USA to Barry, Wales in the UK, increase in percentage of workers working remotely can be observed in the past few years, albeit in different levels.

Question 1 looks at how each industry and in different regions around the globe may react to changes in workstyle. For the sake of simplicity, we assumed variables such as proportion of workforce and relative proportion of workers in each industry to be constant, with the actual number of workers in each industry in each region increasing linearly, and mainly discussed the effect of more and more jobs going remote in the upcoming years, as internet access becomes more widespread. The results were very much in favour of remote workstyle, giving a global average of 30% of jobs becoming remote-ready in 2024 and 31% in 2027.

Question 2 focuses instead on among people who have full access to remote-ready jobs, how many of them are willing to work remotely in reality. In this part we have introduced around 5 variables and 1 constant. We tried to find relationship between willingness coefficient and each variable and combining them together into one equation. For the sake of simplicity, we used linear regressions for most of the predictions.

Question 3 is rather a conclusion of the previous two questions, we combined both data to find the percentage of workers who work remotely in 2021 and made predictions to workers in 2024 and 2027. The result defers from cities to cities. In general, we found the UK workers to be more in favour of remote learning as a result of long halting periods of national lockdown. British firms and companies have developed a more mature system of remote working environments compare to the US hence leads to increasing number of workers working in this manner.

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2 Questions

2.1 Question 1 – Ready or Not

Create a model which, for a given city, estimates the percentage of workers whose jobs are currently remote-ready. Apply your model to the cities below to make predictions for the percentage of remote-ready jobs in 2024 and 2027. You may need to account for how the inputs to your model will change over time.

US:	*Seattle, WA*	*Omaha, NE*	*Scranton, PA*
UK:	*Liverpool, England*	*Barry, Wales*	

2.1.1 Assumptions and Variable

The current remote-ready jobs, r, for each region can be calculated with equation below:

$$r = \frac{\sum J_i X_i}{eL},$$

Where J_i is the percentage of remote-ready jobs and X_i is the total number of workers in the industry i, e is the employment rate, and L is the total worker population. Each of the variables which r depends on can be modelled using the available data and some assumptions on their relationship with each other.

Number of jobs which are currently remote-ready would be equivalent to $\sum J_i X_i$.

Assumptions below were made for modelling:

1. There are no significant changes in policies or complications in terms of politics or economics; that is to exclude effects of possible pandemics in the future, change in national leadership and therefore employment policies, international warfare between major political powers, etc.
2. Social structure changes very little and proportion of workers in each industry does not vary significantly from the current level (ratio of X_i to total population remains constant).
3. Local population increases proportionally with the local job market size, so that employment rate does not vary dramatically; and employment rates in each region can be taken as constant throughout the past and future decades (constant e for each region).

4. Aging in population is neglected and overall proportion of workers in suitable working age range remains constant.

2.1.2 MODEL

Based on assumptions above, the variables have been reduced to J_i, the local percentage of remote-ready jobs in each industry, and X_i, size of local workforce in each industry (and by extension L).

One of the requirements for remote working is internet access in the local area. We used this relationship and modelled J_i according to

$$J_i = k_i I^n \ ,$$

Where I is an internet capacity index evaluated based on proportion of local adults with internet access, n is the order of relations and k_i is a constant unique to each region and each industry.

To further simplify the question, the five regions are further classified into urban and super-urban regions, with Barry, Omaha and Scranton classified as urban, Liverpool and Seattle classified as super-urban. For the same time period, the level of internet access in urban regions are similar with each other, and that of super-urban regions are similar. The order of relationship n is also taken to be the same for urban regions and super-urban regions.

Level of internet access generally improves with time, gradually approaching 100%. This is modelled with an exponential relationship

$$I = 1 - ae^{-bt},$$

Where a and b are constants and are different for urban and super-urban areas, and t is number of years, with the year of first data point (in this case 2014) arbitrarily defined as $t = 1$. We found the line of best fit at $a = 0.21, b = 0.10$ for urban areas, and $a = 0.19, b = 0.11$ for super-urban areas. This gave projections of internet capacity index for 2024 and 2027 for urban and super-urban regions.

Using data on internet accessibility and remote workforce in Liverpool and Barry, we calculated n to be 2.336 and 1.786 for super-urban and urban regions respectively, with notably high correlation coefficients. k_i is then calculated using J_i and I of 2020, completing model for prediction of J_i.

Projection for X_i and L is calculated based on current population and rate of population change. Rate of population changes is calculated from pre-existing data of population in the past decade.

Using the equations in **2.1.1**, we calculated percentage of remote-ready jobs in 2024 and 2027 for each of the five regions as required.

UK (2024):

Liverpool, England: 23.9% *Barry, Wales:27.4%*

USA (2024):

Seattle, WA: 38.8% *Omaha, NE: 34.8%* *Scranton, PA: 26.5%*

UK (2027):

Liverpool, England: 24.7% *Barry, Wales:28.0%*

USA (2027):

Seattle, WA: 40.3% *Omaha, NE: 36.2%* *Scranton, PA: 27.4%*

2.1.3 CONCLUSION

The percentage of remote-ready jobs increase over the years according to this model, which would be realistic when compared with the current trends. Overall, these values are rather moderate estimates for what the future of remote-working may look like, as we have, in many different ways, tried to eliminate the explosion of remote-working jobs brought by COVID (we are, after all, hoping that there will not be yet another pandemic in this decade). For this reason, we chose to extrapolate the X_i values and assumed a linear increase in size of workforce and used internet usage data between 2014 to 2020 instead of the more updated ones from 2020 to 2021. These decisions obviously impacted our final results, but since our results still suggest an overall increase in remote-ready jobs, we believe this is a rather fitting modest model.

This peaceful and undisturbed mindset is quite obviously overly optimistic – in our assumptions we stated the lack of global warfare as one of our basic assumption, and the current geopolitical climate clearly proves otherwise. The possibility of future conflicts would decrease level of internet access and proportion of workers in each industry, hence decreasing the number of remote-ready positions, while the possibility of yet another pandemic would drastically increase this number.

Variation in percentage of remote-ready positions in different regions seems to follow the same patterns in the present and in the future; places with more online work available today seems to have more online work available in 2024 and 2027, and vice versa. This can be easily explained by local companies' cultures, etc. The notable one would be to compare Liverpool and Seattle. While both are super-urban cities, Liverpool has the lowest number of remote-ready jobs out of the five regions while Seattle has the most, likely due to the industrial nature of Northern England and the internet-based companies situated in Washington, USA.

This brings up one of the variables we did not consider – as the main workforce shifts to the younger age groups (or rather age groups who are currently of younger ages), more people may wish to work in newer or 'smarter' industries with better salary or better working conditions, and less may wish to enter the labour-heavy careers simply due to development of AI and robotics. People may even be drawn to some industries because they offer more remote positions. These are all interesting and bold assumptions which we have not considered and would perhaps give much less conservative and more realistic results.

2.2 Question 2 – Remote Control

Not all workers who can work from home choose to do so or are permitted by their employer to do so. Create a model that predicts whether an individual worker whose job is remote-ready will be allowed to and will choose to work from home.

2.2.1 Assumptions and Variables

Model I

For this question we considered each worker on a more individual basis. Given the choice of whether to work offline or not, various different factors may come into play in the decision-making process. The variables which we chose to look at are as below:

1. Household income.
   Working remotely would require personal devices and, speaking from our personal experience of online schooling, rather stable internet connection. Household income could be a good overall measure of the resources available to the workers in the household.
   Mean household income and distribution of household income would be different for different regions and would change overtime as an effect of inflation and other economy-related events.
   Higher household income would lead to higher willingness to work remotely.
2. Education level.

Given a sample group of workers, there is a clear positive correlation between their level of education and probability of working remotely. This of course may be due to higher levels of education tend to prepare workers for more analysis-based careers which can be conducted via the internet, which would not the focus of this question, but the correlation may also be cause by the longer time spent in education leading to higher familiarity with devices and remote-working tools, and hence increased willingness to work online.

While average education levels have been increasing significantly in the past, given that the question only asks for predictions of 2 and 5 years in the future, it is safe to assume that rise in education levels is negligible.

3. Time spent on commuting between working place and home.
   Working remotely means saving the time and money to commute between working place and home. Therefore, rather obviously, the longer it takes someone to travel from home to working place, the more likely that person would choose to work remotely. Thus, we think there is a directly proportional relationship between those two variables.
4. Median age of the local demographics.
   From daily life and all forms of media, it is easy to arrive at an answer that is rather stereotypical, albeit very true – younger generations are generally more open to change and technology. We conclude that the shift the median age of local work force may have significant impact on people's willingness to go remote.
   We assumed that the aging or de-aging of a local population is linear and extrapolated from present-day data.

Model II

We made another model to find the importance of each element (income, commute time, age). In this model, education level is excluded because we can assume the education level is related to the income and it was difficult to move 4 variables in the limited time. We made this model with just 3 cities in the US because of the lack of the data of the UK.

2.2.2 MODEL

We introduced a rather simplistic model for a 'willingness to work remotely' index *w* for each region, given by equation

$$w = k\frac{HET}{A},$$

where *H* is the household income, *E* is a quantised measure the level of education received, *T* is the time taken for commute, *A* is the median age of the local population and *k* is a proportionality constant, which is different for each region, a table is displayed at appendix.

Method used to find predictions for each variable:

- For H, household income, we have found data of median household income for each city during 2019 and 2020, and in theory we have ignored the effect of the pandemic and used linear regression to predict the outcome of the median household income in 2024 and 2027 of each city respectively. Assuming economic growth to be stable in both countries. The graph could be found in appendix.
- For variable E, the quantized measure of level of education, we found the education coefficient for each city and assumed the education level to be constant over years. The reason for this surprising assumption is that after doing some research on local applicants on the UCAS system (UK application system), we concluded that education level will stay roughly the same during years as when the population grows, the number of university places generally increases in proportion. So does the A-level grades as it follows normal distribution of the population. Hence, we have made the conclusion that education level stay roughly the same (similar logic applies to most OECD countries).
- For variable T, which is the mean time taken to commute, we found a coefficient to use for each city to simulate the gradual technological improvement of each city, we have ranked them in order and gave them coefficients between 0 and 1.
- The last variable A, median age, we used linear regression to simulate the median age in 2024 and 2027.

We adjusted the value of *k* so that the value of *w* would be equivalent to percentage of people who choose to work remotely when given the opportunity. The predictions are as below:

UK (2024):

Liverpool, England: 54.6% *Barry, Wales:49.88%*

USA (2024):

Seattle, WA: 17.65% *Omaha, NE: 14.34%* *Scranton, PA: 22.32%*

UK (2027):

Liverpool, England: 54.6% *Barry, Wales:56.15%*

USA (2027):

Seattle, WA: 17.88% *Omaha, NE: 14.53%* *Scranton, PA: 22.61%*

Model II

We started by considering 10000 imaginary people in the cities using histograms of each element. They have elements chosen randomly following the probability based on the histograms. For each person, we created an arbitrary index in order to decide they work remotely or not, adding up the commute time (min, t), the income ($K, h), and the minus ages(a) over 16.

If the index is greater than the certain value, we assumed that they work remotely. Needless to say, it didn't work. In order to provide the actual value of the percentage of the people who works at home given in the data spread sheet, we multiplied some values by t and h(which are fixed). We wrote a program so that we obtain the most valuable set of values (seattle.py, scraton.py, ohama.py, and define.py).

$$xt + yh + a \geq 250$$

As the result, the best values for x and y are 3.9 and 4.8 respectively (Figure 1,2). In Figure 1 and 2, x and y axis show x and y respectively, and z axis show the sum of difference between the actual percentage and the modeled percentage.

2.2.3 CONCLUSION

When compared with itself, the regions all showed an increase in willingness to work remotely between present days, 2024 and 2027, save for Liverpool which maintained the same level of willingness between 2024 and 2027. This is not particularly surprising judging by the current trend in remote working.

Comparing the regions against each other yields more interesting results. While in question one Seattle showed significantly higher proportion of jobs ready to go remote, here it seems that the workers in Seattle are also amongst the least willing to work remote. Based on the model we suspect this could be due to the very urban environment leading to short commute time and speaking in a more realistic frame of view this could also be a case of beautiful office buildings and good work environment making people enjoy spending time in offices. Speaking of which, the enjoyment in one's current working style is a variable we did not take into consideration due to the lack of data available. One's enjoyment in their current office or home working life may impact how much they are willing to change that.

We also neglected the possibility of working for a company in location A while living in location B – we personally know quite a few people who wish to do this in the future, and even a few who already live this lifestyle. This could hugely impact our model as the commute time would no longer be true (we used data on average commute time within the region), and the matter of local age group also becomes practically irrelevant.

Interestingly, Barry, Wales, perhaps the most rural region showed the most interest in going remote and is rapidly becoming more interested in doing so. We found this to be quite surprising and almost contradictory of the result we achieved in question 1.

2.3 Question 3 – Just a Little Homework

Synthesize your models from the first two questions to create a model which, for a given city, estimates the percentage of workers who will work remotely. Make predictions for the same cities you considered in Q1 for 2024 and 2027 and use those predictions to rank the cities in terms of the magnitude of impact that remote work will have on the city.

2.3.1 Assumptions and Variables

We approached this question as a combination of the two previous questions, and we essentially combined the results of the two previous questions. In question 1 we gave a very modest prediction for number of jobs which could go remote, in question 2 we answered the percentage of workers who would go remote if given the chance. These should provide enough data for estimating number of workers who would actually work remotely in the not-too-distant future of 2024 and 2027.

We tried to apply Model Ⅱ to this problem using additional assumptions of distribution of income, commute time and ages, in order to compare the two models. However, we couldn't because we ran out the time.

2.3.2 Results

Using percentage of remote-ready jobs, *r*, attained from question 1 and 'willingness to work remotely' index *w* in question 2, the overall percentage of remote workers in the future can be modelled as

$$Total\ percentage\ of\ remote\ workers\ = w \times r$$

Results are as below.

UK (2024):

Liverpool, England: 13.0% *Barry, Wales:13.6%*

USA (2024):

Seattle, WA: 6.9% *Omaha, NE: 5.0%* *Scranton, PA: 5.9%*

UK (2027):

Liverpool, England: 15.2% *Barry, Wales:15.7%*

USA (2027):

Seattle, WA: 7.2% *Omaha, NE: 5.3%* *Scranton, PA: 6.2%*

2.3.3 Conclusion

The results here are very much a mixture of trends spotted in question 1 and 2. Ranking the regions in how much remote working affects the local workforce and economy, the order would be as below

1. Barry, Wales

   This was perhaps the most surprising result for us. The effect of remote working is shockingly large in the Barry, with nearly a sixth of its working population estimated to be working online before the end of the decade.

2. Liverpool, England

   This results appears to be a compromise between the few remote-ready jobs and 'willingness to working remotely' index higher than 50%.

3. Seattle, WA

For Seattle and its reputation of being technologically advanced and liberalness, this is a very conservative result, with under 10% going remote by the end of the decade. This can however be compensated by its huge population.

4. Scranton, PE
5. Omaha, NE

Scranton and Omaha show the least development in percentage of remote workers, and are, according to our computations, least affected by the remote-working frenzy. This is quite surprising as the two regions have shown mediocre results in both question 1 and 2, only to end up at bottom of this list here. We suppose that compared to the distinct characteristics of other regions driving their work force remote, these cities are relatively ordinary and therefore are less susceptible to such changes.

3. Evaluation

We have reviewed each model separately in the conclusion sections in the different questions, and therefore here we decided to note down some observations we have made along the way.

Previously in this report, we have not focused particularly on how remote working may have different implications for people in different industries. It is fairly obvious that some industries would be happier to go online due to the nature of the work entailed, such as finance and information, which largely depend on individual work and rely heavily on computational devices, whereas in comparison mining and construction workers are far less likely to work online because of the physical labour required for these industries.

It is worth noting though, that while sounding almost too much alike science-fictions, the development in robotics and AI could potentially push workers in some industries towards other industries, and that would in turn greatly impact the proportion of remote workers. This is something that we have not taken into consideration due to the lack of data available, and that we wished to attain a more moderate and conservative result.

In the search for such moderacy, we also made quite a few bold assumptions. From the almost over-abuse of linear interpolation used for question 1 while finding X_i and in question 2 while modelling how the variables H, E, T and A varies with time, to the way some variables, which are clearly not constant, are taken to be constant due to the short time-frame, we do realise that there is much room for improvement, but regardless of the roughness we can ensure that our solutions are somewhat 'realistic' in an idealistic world with no conflict or disease.

Working remotely for this project, in a way, gave us more insight into the idea of remote working, and how the variables may impact the decisions of the employer or the employee. Especially for question 2, where we concluded and decided on the variables almost entirely based on our experience commuting in the morning to meet up or remotely joining and suffering under the torture of unstable internet connection. We suppose this makes the topic somewhat more endearing, and gave us the courage to throw in a little irony in this report.

4. Appendix

4.1 Question 1

4.1.1 Estimate the percentage of workers whose jobs are currently remote-ready.

These tables were formed by the combination of the given data sheets, **d1** and **d3. in** order to minimize the errors occurred by the direct connection of **d1** and **d3**, more detail occupation categories in each region from the internet sites function as mediator.

Seattle	2021 (people)	Remote work (%)	Remote work (people)
Mining, logging, construction	109600	0.01	10.96
Manufacturing	142200	21.53	30615.66
Trade, Transportation, and utilities	332600	3.00	9978
Information	139000	95.96	133384.4
Financial Activities	87600	88.00	77088
Professional and business services	277500	55.40	153735
Education and health services	223500	45.12	100843.2
Leisure and hospitality	133000	30.89	41083.7
Other cervices	59300	0.33	195.69
Government	206700	34.00	70278

Total	1711000		617212

Percentage of workers whose jobs are currently ready-remote = 36.073%

Omaha	2021	Remote work (%)	Remote work (people)
Mining, logging, construction	30700	0.01	3.07
Manufacturing	33500	12.25	4103.75
Trade, Transportation, and utilities	94100	3.00	2823
Information	9800	95.11	9320.78
Financial Activities	44100	88.00	38808
Professional and business services	71900	56.91	40918.29
Education and health services	79600	39.18	31187.28
Leisure and hospitality	47500	29.86	14183.5
Other cervices	18300	0.32	58.56
Government	65200	30.82	20094.64
Total	494700		1661500.87

Percentage of workers whose jobs are currently ready-remote = 32.646%

Scranton	2021	Remote work (%)	Remote work (people)
Mining, logging, construction	10300	0.02	2.06
Manufacturing	27200	9.13	2483.36
Trade, Transportation, and utilities	63900	3.00	1917
Information	2500	91.00	2275
Financial Activities	13000	88.00	11440
Professional and business services	26100	56.12	14647.32
Education and health services	50500	33.85	17094.25

Leisure and hospitality	18200	31.81	5789.42
Other cervices	7700	0.38	29.26
Government	28300	22.85	6466.55
Total	247700		62144.22

Percentage of workers whose jobs are currently ready-remote = 32.646%

Liverpool	2021	Remote work (%)	Remote work (people)
Mining, logging, construction	146240	0.00	0
Manufacturing	103120	29.88	30812
Trade, Transportation, and utilities	146100	3.00	4383
Information	73120	86.56	63293
Financial Activities	25592	88.00	22521
Professional and business services	47528	28.00	13308
Education and health services	23900	24.23	8181
Leisure and hospitality	69700	26.00	18122
Other cervices	73120	0.21	154
Government	26560	6.00	1594
Total	734980		162367

Percentage of workers whose jobs are currently ready-remote = 22.091%

Barry	2021	Remote work (%)	Remote work (people)
Mining, logging, construction	4100	0.29	12
Manufacturing	5700	34.80	1984
Trade, Transportation, and utilities	1400	3.00	42
Information	4000	88.28	3531

Financial Activities	3045	88.00	2680
Professional and business services	5655	28.00	1583
Education and health services	9700	38.02	3688
Leisure and hospitality	9500	26.00	2470
Other cervices	2400	0.24	6
Government	9700	6.00	582
Total	55200		16577

Percentage of workers whose jobs are currently ready-remote = 30.032%

	2024					2027				
	Seattle	Omaha	Cranton	Liverpool	Barry	Seattle	Omaha	Cranton	Liverpool	Barry
Mining, logging,	0.01	0.01	0.02	0.00	0.30	0.01	0.01	0.02	0.00	0.31
Manufacturing	23.91	12.74	9.49	33.18	36.18	24.86	13.19	9.83	34.50	37.46
Trade, transport	3.33	3.12	3.12	3.33	3.12	3.46	3.23	3.23	3.46	3.23
Information	100.00	98.88	94.61	96.11	91.78	100.00	100.00	97.95	99.95	95.02
Financial activiti	97.71	91.49	91.49	97.71	91.49	100.00	94.72	94.72	100.00	94.72
Professional and	61.51	59.17	58.35	31.09	29.11	63.97	61.26	60.40	32.33	30.14
Education and h	50.10	40.73	35.19	38.01	39.53	52.10	42.17	36.43	39.52	40.92
Leisure and hos	34.30	31.04	33.07	28.87	27.03	35.67	32.14	34.24	30.02	27.99
Other services	0.37	0.33	0.40	0.23	0.25	0.38	0.34	0.41	0.24	0.26
Government	37.75	32.04	23.76	6.66	6.24	39.26	33.17	24.59	6.93	6.46

Prediction of Percentage of remote-ready jobs in each sector using the exponential model (we set 100 as the maximum value)

	Seattle		Cranton		Barry		Omaha		Liverpool	
Year	2024	2027	2024	2027	2024	2027	2024	2027	2024	2027
Mining, logging, construction	12	13	3	4	2	0	0	0	11	11
Manufacturing	32720	32576	5056	5019	2178	210	35776	38490	1378	1449
Trade, transportation, and utilities	11171	11956	3034	3237	2008	217	5375	5885	330	351
Information	126647	136911	36324	39769	1666	112	72504	75951	1049	1075
Financial activities	80398	80929	21835	22267	11598	1226	22448	22579	4804	4823
Professional and business services	166930	182537	46572	50775	16298	1768	13264	13557	2382	2502
Education and health services	123190	134487	29052	31622	18860	2035	7880	7466	3558	3761
Leisure and hospitality	51411	54345	13497	14225	6720	711	19294	19597	1351	1535
Other services	234	250	62	66	29	3	178	181	5	5
Government	84882	88479	20897	21718	6603	689	1459	1407	469	486
remote-ready projection	677595	722482	176332	188700	65962	6972	178179	185113	15336	15999
labour force projection	1745201	1792901	506206	520785	248798	25409	745518	748749	56048	57121
percentage of remote ready	38.8%	40.3%	34.8%	36.2%	26.5%	27.4%	23.9%	24.7%	27.4%	28.0%

4.2 Question 2

4.2.1 Model I

Seattle, Washington	2019	2020	2024	2027
T(min)	31.6	31.284	30.05128558	29.15873234
E	4.7885	4.7885	4.7885	4.7885
H($)	112.35	114.70935	121.7303629	128.4228343
A(year)	37.1	38.5	39.0	39.4

Omaha, Nebraska	2019	2020	2024	2027
T(min)	21.1	20.889	20.06589005	19.46991305
E	4.5225	4.5225	4.5225	4.5225
H($)	93.6	95.5656	101.4148818	106.9904521
A(year)	36.1	34.5	39.0	39.4

Scranton, Pennsylvania	2019	2020	2024	2027
T(min)	23.7	23.463	22.53846418	21.86904926
E	3.92	3.92	3.92	3.92
H($)	81.275	82.981775	88.06083884	92.90223285
A(year)	42.4	38.5	39.0	39.4

Liverpool, England	2019	2020	2024	2027
T(min)	29	28.71	27.57871145	26.75959614
E	3.657	3.657	3.657	3.657
H($)	63.995	61.70549353	75.5795742	88.75683382
A(year)	38	40.5	41.1	41.6

Barry, Wales	2019	2020	2024	2027
T(min)	25.4	25.146	24.15514727	23.43771524
E	3.2545	3.2545	3.2545	3.2545
H($)	63.995	61.70549353	75.5795742	88.75683382
A(year)	38	40.5	41.1	41.6

2021	seattle	Omaha, Nebraska	scranton	liverpool	barry
overall percentage	6.5%	4.9%	5.00%	11.62%	14.41%

	Seattle, Washington	Omaha, Nebraska	Scranton, Pennsylvania	Liverpool, England	Barry, Wales
	0.180055402	0.150306748	0.199203187	0.525791855	0.480333333
k	0.000392935	0.000607504	0.00111859	0.002943937	0.003450343
w2024	0.176488849	0.143358315	0.223151261	0.545996677	0.498791302
w2027	0.17882761	0.145258044	0.226108375	0.614669176	0.561526565

4.2.2 Model II codes

```
import numpy as np

def seattle(x,y):
    count = 0
    for i in range(0,10000):
        commute = np.random.choice([2.5,7.5,12,17,22,27,32,37,42,52,74.5,100], p =
[0.015,0.056,0.101,0.142,0.165,0.081,0.181,0.041,0.053,0.099,0.051,0.015] )
        income = np.random.choice([5,15,25,35,45,55,65,75,85,95,105,115,125,135,145,155,165,175,185,195,205], p =
[0.0887,0.0937,0.12,0.125,0.105,0.0816,0.0771,0.0534,0.0481,0.0331,0.0373,0.0187,0.023,0.0139,0.0105,0.0136,0.0077
,0.00615,0.00527,0.00299,0.03519])
        age = np.random.choice([16,18.5,20,21,23,27,32,37,42,47,52,57,60.5,63,65.5,68,72,77,82,90], p =
[0.0413,0.0267,0.0133,0.0461,0.1007,0.0995,0.0934,0.0801,0.0777,0.0752,0.0777,0.0303,0.0437,0.0255,0.0340,0.0485,
0.0316,0.0194,0.0206,0.0147])
        if x * commute + y * income -  age >= 250:
            count += 1
        i += 1
    return(6500 - count)
```

seattle.py

```
import numpy as np

def scranton(x,y):
    count = 0

    for i in range(0,10000):
        commute = np.random.choice([2.5,7.5,12,17,22,27,32,37,42,52,74.5,100], p =
[0.03,0.145,0.244,0.216,0.145,0.039,0.079,0.015,0.021,0.025,0.025,0.016] )
        age = np.random.choice([16,18.5,20,21,23,27,32,37,42,47,52,57,60.5,63,65.5,68,72,77,82,90],p =
[0.0433,0.0325,0.0156,0.0144,0.0455,0.0806,0.0794,0.0746,0.0686,0.0722,0.0782,0.0842,0.0349,0.0505,0.0301,0.0409,
0.0566,0.0409,0.0265, 0.0305])
        income = np.random.choice([5,15,25,35,45,55,65,75,85,95,105,115,125,135,145,155,165,175,185,195,205], p =
[0.11,0.113,0.134,0.154,0.129,0.1,0.0732,0.0507,0.0359,0.0212,0.0198,0.0115,0.0122,0.0056,0.00333,0.00468,0.00175,
0.00243,0.00206,0.000894,0.014756])
        if x * commute + y * income - age >= 250:
            count += 1
        i += 1
    return(5000 - count)
```

scraton.py

```
import numpy as np

l1 = 1 -
sum([0.124,0.117,0.129,0.13,0.11,0.0929,0.069,0.0515,0.0399,0.0271,0.0263,0.0134,0.0137,0.00783,0.00558,0.00665,0.
00477,0.00295,0.00334,0.00199])
l2 = 1 -
sum([0.0491,0.0352,0.0189,0.0176,0.0516,0.0855,0.083,0.0855,0.073,0.0692,0.0717,0.078,0.0327,0.0478,0.0264,0.0377
,0.0516,0.034,0.0252])

def ohama(x,y):
```

```
    count = 0
    for i in range(0,10000):
        commute = np.random.choice([2.5,7.5,12,17,22,27,32,37,42,52,74.5,100], p = 
[0.027,0.118,0.189,0.224,0.203,0.074,0.1,0.01,0.013,0.021,0.013,0.008] )
        income = np.random.choice([5,15,25,35,45,55,65,75,85,95,105,115,125,135,145,155,165,175,185,195,205], p = 
[0.124,0.117,0.129,0.13,0.11,0.0929,0.069,0.0515,0.0399,0.0271,0.0263,0.0134,0.0137,0.00783,0.00558,0.00665,0.0047
7,0.00295,0.00334,0.00199,l1])
        age = np.random.choice([16,18.5,20,21,23,27,32,37,42,47,52,57,60.5,63,65.5,68,72,77,82,90], p = 
[0.0491,0.0352,0.0189,0.0176,0.0516,0.0855,0.083,0.0855,0.073,0.0692,0.0717,0.078,0.0327,0.0478,0.0264,0.0377,0.05
16,0.034,0.0252,l2])
        if x * commute + y * income - age >= 250:
            count += 1
        i += 1
    return(4900 - count)
```

ohama.py

```
from seattle import seattle
from ohama import ohama
from scranton import scranton
import matplotlib.pyplot as plt
from mpl_toolkits.mplot3d import Axes3D


X = []
Y = []
Z = []


for i in range (30,50):
    x = i * 0.1 + 1
    for i in range (30, 50):
        y = i * 0.1 + 1
        X.append(x)
        Y.append(y)
        Z.append(abs(seattle(x,y)) + abs(ohama(x,y)) + abs(scranton(x,y)))


fig = plt.figure()
ax = Axes3D(fig)
ax.set_xlabel("X")
ax.set_ylabel("Y")
ax.set_zlabel("Z")
ax.plot(X,Y,Z,marker="o",linestyle='None')
plt.show()
```

define.py

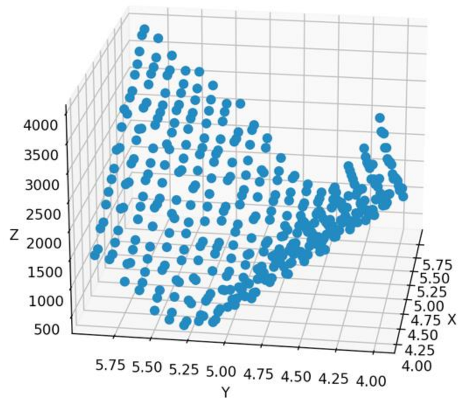

Figure 1

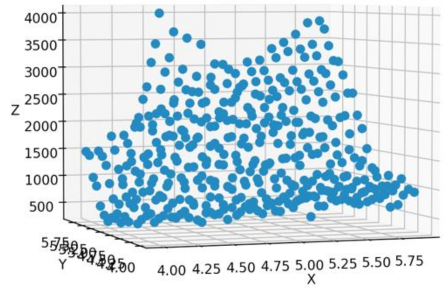

Figure 2

4.3 Question 3

	seattle		omaha		scranton		liverpool		barry	
year	2024	2027	2024	2027	2024	2027	2024	2027	2024	2027
remote availability	38.8%	40.3%	34.8%	36.2%	26.5%	27.4%	23.9%	24.7%	27.4%	28.0%
remote supporter	17.6%	17.9%	14.3%	14.5%	22.3%	22.6%	54.6%	61.4%	49.9%	56.2%
work force	1.7E+06	1.8E+06	5.1E+05	5.2E+05	2.5E+05	2.5E+04	7.5E+05	7.5E+05	5.6E+04	5.7E+04
overall percentage	6.9%	7.2%	5.0%	5.3%	5.9%	6.2%	13.0%	15.2%	13.6%	15.7%

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