MathWorks Math Modeling Challenge
A program of siam

PREVIEW PAPER: ABOVE AVERAGE*

The team's executive summary includes an overview of the problem but lacks specific results and does not disclose the specific questions addressed nor the approaches the team used. The team addressed the first two questions, but their discussion of the third question is incomplete. The team provides references to sources, but their use of citations within the discussion is not consistent.

The team relies on China's experience with e-bikes to address the first question, but a strong argument is not made as to why the trends in the United States follow those of China. The team makes use of a logistic model and includes their reasoning for their choice, and they describe their methodology for estimating the parameters. The team did a good job of describing how their results change if their parameters are changed.

With respect to the second question, the team made use of a Gallup poll, which several other teams in our sample also used. The change in time to questions in different polls is used to determine whether certain questions indicate a willingness for people to purchase e-bikes. The team does a good job of discussing the other aspects they considered and includes supporting data in their discussion.

The team's discussion for the third question is not as complete compared to the first two questions. Less motivation is given for their choices. Their model and results are more difficult to follow than those for the first two questions.

**from among the screened sample of papers examined during pre-triage work.*

E-bikes and their Environmental Impact

Department of Transportation Brief

Department of Transportation Head, Pete Buttigieg,

The United States has been attempting to reduce its carbon footprint for decades, yet we still have not addressed one of our main sources of pollution, cars. A large majority of the United States population lives in urban areas, and as a result much of the population that owns a car lives in the city. Many of these trips are short and could be replaced by trips on e-bikes. We propose that the Department of Transportation pursues the development of e-bike infrastructure, and perhaps even offers tax benefits to citizens for buying e-bikes.

It is imperative that the United States reduces its carbon emissions, and it is better sooner rather than later. Our earth is already feeling the effects of global warming, and to minimize its damage we need to reduce the amount of emissions at the source. One of the primary sources of carbon emissions is transportation. We need to take action on a government level to reduce the environmental impacts of transportation. Delaying any action beyond this only yields negative effects on our children and future generations.

Simply banning diesel-cars could be considered tyrannical, and would be ineffective. Instantly, it would shutdown our economy, however incentivizing other efficient methods of transport, like e-bikes, is an ingenious solution. E-bikes are affordable, cause very few emissions from production, and the charging of the battery also causes very little carbon emissions. E-bikes are already on the rise and E-bike usage increased by 70% in 2021 alone. A major issue with e-biking is the safety of cycling in urban areas, which we have demonstrated in our paper that safety issues are easily preventable with protected biking lanes and looking into offering tax credits to the public for buying e-bikes.

Our paper demonstrates that people feel safer and are more willing to cycle when their roads have Protective Bike Lanes, so it can be extrapolated that the implementation of these Protected Bike Lanes will increase e-bike usage, which in turn will reduce carbon emissions. Additionally, offering tax credits for the purchase of e-bikes would also incentivize purchasing e-bikes, as they would become more affordable. Overall, e-bikes offer a way to propel the U.S. into a future of low carbon emissions.

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Introduction:

I.1 E-Bike Overview

As countries begin urbanizing and form dense population centers, the population increases, and the logistics of transportation become increasingly prevalent. One of the greatest challenges has been finding the right balance in the different forms of transportation.

In recent years a new mode of population has skyrocketed in popularity, the e-bike. The ebike serves as a solution to some of the problems that are often caused by the other forms of transportation. They offer remedies to the problems of parking, traffic congestion and poor public transit planning. They have been continuously growing in popularity, partly caused by a spur in interest as a result of Covid-19. Now the question is how will growth continue in the wake of Covid-19.

I.2 Problem Overview

The first problem was a task to create a model to predict the growth in e-bikes sales in the United States, and use that model to predict e-bike sales two and five years into the future. We then had to identify the causes of the growth. We identified environmental awareness, energy costs, battery innovations as possible causes for the growth of e-bikes. Ultimately, we used the models we created to quantitatively express the benefits of shifting towards e-bikes, deeming carbon emission and safety adequate expressions of future impact.

Assume that all Local Definitions extend to other aspects of the study if applicable however no local assumptions can be justified as global assumptions

Q1: The Road Ahead

1.1 Defining the Problem

This question tasked us to create a mathematical model to predict the future growth of e-bike sales. We then used this model to predict the amount of e-bikes sold two and five years in the future, during the years of 2025 and 2028.

1.2 Local Assumptions

1.2.1 Population Capped Sales

We assumed that at a certain point in time, population would become a limiting factor for e-bike sales. At this point the initial novelty of bike sales would stop, and most people who see themselves benefiting from an e-bike would already have one. Bike sales would be predominantly composed of new e-bikes to replace older e-bikes, and a smaller percentage of people getting their first e-bike. E-bikes sales are also limited by population because there is little interest in most people owning more than one e-bike at a time.

1.2.2 China's E-Bike Sales Have Leveled Off

Since the increase of China's e-bike sales have been leveled off for multiple years, we assumed that China has approached the stagenet stage in e-bike sales. There has only been minor growth in China's e-bike market and we assume that to remain the same going forward.

1.2.3 Infrastructure Improvements

We assume all countries follow similar patterns of improving pedestrian and cycling infrastructure as the country our model is based on, China.

1.2.4 Stability of the Political Environment

We assume that no large-scale cultural or political change will occur that will significantly change the rate of ebike adoption. A drastic change in culture or political environment would not only be unlikely but nearly impossible to accurately predict or incorporate into a model.

1.3 Variables

Symbol	Definition	Units
S	Describes the amount of e-bikes sold per year	In millions of e-bikes
L	Constant that defines maximum sales per year of e-bikes	= 125
k	Logistic constant of proportionality	= -0.28
t	Describes the amount of e-bikes sold after 2011	Years

1.4 The Model

$$S \qquad \frac{125}{1+e^{(-.28)(t-20)}}$$

Because of the initial similarities in growth of E-bikes in the U.S., we predicted bike sales will follow a trend that resembles China's trend, with a slow initial start, then fast growth that eventually levels off. With this in mind, the data for e-bike sales in China we concluded that U.S. e-bike sales closely follow a logistical growth curve for overall sales. China has had a head-start on e-bikes and has been advancing their e-bike system since the 1990's. However they didn't become popular until later on, the data for e-bike sales shows rapid growth near 2006 with more stagnant sales in the 2012- 2022 time period.

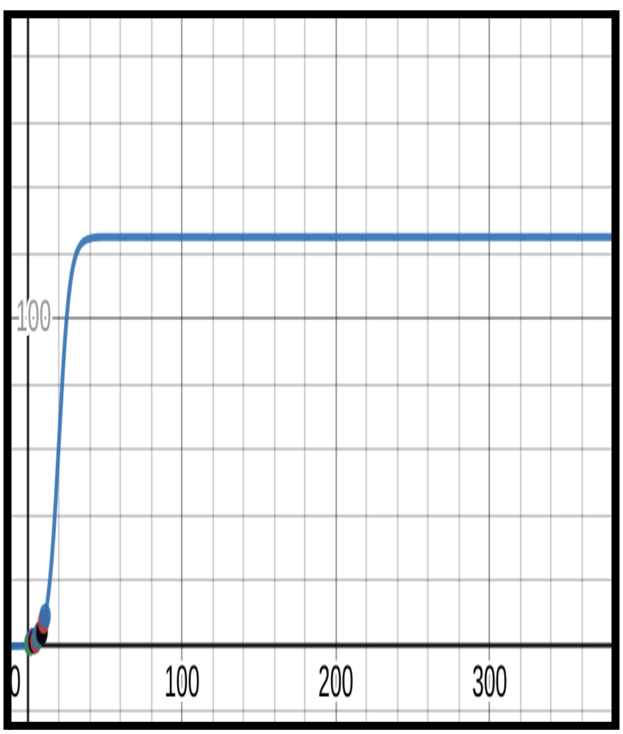
Based on the trends from China's extended time in the e-bike market and the stagnation in sales we concluded that the model for e-bike sales should eventually reach a maximum. Following China's growth, our model shows rapid growth in the beginning that levels off after it approaches a certain number. For these reasons, we chose to model e-bike sales with logistic growth where sales would start off slow, pick up and stop at some value. The model was based off of the base logistic equation. After plugging in the necessary variable values and making adjustments the final model was constructed

$$P(t) = \frac{L}{1 + e^{-kt}}$$

We assumed China's e-bike sales are currently in the stagnant stage of the e-bike sales lifecycle. Taking into account their current sales we began to look at the plausible maximum for e-bike sales in China. Factoring in a little room for the market to still grow, we estimated that the maximum e-bike sales per year for China was 50 million e-bike units per year. Because China has a population of nearly 4 times that of the U.S., and our assumption that e-bike sales would only be determined by population size, we estimated that the maximum number of e-bike sales in the U.S. would be 12.5 million units per year.

Taking this maximum into account, we plotted our logistic growth curve with all the data provided for U.S. e-bike sales per year from 2012 to 2022 and overlaid our logistic growth curve with a maximum value of 125 (with units 100,000 units sold per year). Using the Desmos equation editor we found a value for k that would have the curve follow through the data and translated the graph such that it would go through the data. Now that the logistic equation had been created we plugged in the appropriate times for 2 years from now and 5 years from now and found the following results:

1.5 The Results



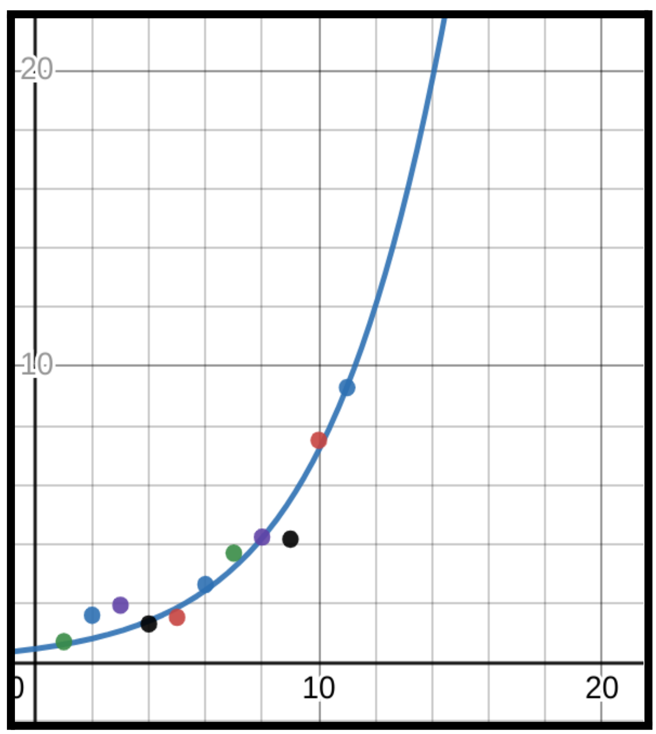


Fig (1.5.1) *Graph of our Model, First Accurate Reporting at 2012 at t = 1*

Year	**Number of E-Bikes Sold**
2025 (*2 Years into the future*)	1.9637 million units sold
2028 (*5 years into the future*)	3.7692 million units sold

1.6 Discussion

Our model reveals that e-bike sales are at the beginning of near exponential growth for the next decade but that e-bikes sales will ultimately become constant around 2040. Ideally, we would have liked to run the data points through a logistic curve algorithm that could determine more exact values for L and K then we could by visually adjusting the values in Desmos.

1.6.1 Strengths

We based our model on a country that has been involved with e-bikes for a very lengthy amount of time and has data on what e-bikes sales can be expected to look like in the U.S.

1.6.2 Weaknesses

The political and cultural differences between China and the U.S. may result in a different trend in the United States than what China experienced which we assumed to be the same. Our model also does not account for the fact that the population of America is not exactly one fourth that of China. We also did not account for differences in population growth and population demographics in our assumptions of around maximum e-bike sales. Our model also does not account for future developments of technology.

1.7 Sensitivity Analysis

Most of our assumptions deal with what sales will look like after extrapolating a great number of years. In particular, our assumption that the maximum number of e-bike units sold per year would be one quarter of the maximum e-bikes sold in China and that China was already near its maximum both influence the L variable in our equation. However, because both L and K were chosen to make the curve fall through the data points, changes in L from a different maximum requires adjusting K as well. For this reason, our predictions are not thrown off extremely by small changes in L because K helps to compensate for the differences. For example reducing L by 5% to 118.75 and readjusting K to 0.278 the prediction at 2 years in the future is changed to 1.8845 million units a 4.0% difference and at 5 years the new value is 3.5957 million units a 4.6% difference. If L is increased by 5% to 131.25 and K is readjusted to 0.289 the prediction at 2 years in the future is changed to 1.9698 million units a 0.3% difference and at 5 years the new value is 3.8834 million a 3% difference. These findings suggest that our model reduces the effects of changes based off assumption and our model is more favorable if L is higher.

Q2: Shifting Gears

2.1 Defining the Problem

The second problem was to determine what factors contribute to the increase in e-bike usage, to prove whether they are significant or not, and to use mathematical modeling to represent our findings.

2.2 Assumptions

2.2.1 E-bike Types

We assume that the most sales of e-bikes will be of Class 1, 2, and 3 e-bikes, as they are the most common varieties of e-bike sold. They are the most commonly available off-the-shelf commercial e-bike variety, with limited government oversight and regulation. They are designed for low powered transportation, and are best suited for shorter distances at lower speeds, none legally operating at a speed greater than 28 mph.

2.2.2 Negligible Electricity Cost

Despite increasing energy prices, we assume that the cost to charge the battery will not be a major factor in the adoption of e-bikes. Since the most common e-bike only consumes 500-800 watts of energy per charge and assuming the average electricity cost of 10.6 cents per kWh of electricity, a typical e-bike will only cost 7 cents to fully charge. This number is not significant enough to be a barrier for sales or adoption. [A]

2.2.3 Adequate Storage

We assume that storage and e-bike parking are not a significant inhabitant of e-bike adoption. E-bikes are small enough and can be stored in most environments and can be stored in most spare spaces such as a living room, bike rack, or tied up against a pole. If additional bike parking is necessary its cost and space required to implement are negligible, especially when compared to other modes of transportation.

2.3 Variables

Symbol	Definition	Units
x_{1i}	An individual's response to the question from the earlier date (our case represents 2003)	Scale from 1 to 5
x_{2i}	An individual's response to the question from the later date (our case represents 2017)	Perception score(1-5)
$\bar{X}_2$	The sample mean score from the later date (our case represents 2017)	Perception score(1-5)
$\bar{X}_1$	The sample mean score from the later date (our case represents 2003)	Perception score(1-5)
$P(x_i)$	the probability of a survey respondent giving a particular response of x_i to the question calculated by dividing the amount of equivalent responses by the total number of responses that year	Perception score(1-5)
n	$n_1 = n_2 = n$ = the sample size of the survey each year	Number of people
S_x	the sample standard deviation (how much an individual response typically differs from the sample mean)	Perception score(1-5)
σ_X	the sampling distribution standard deviation (how much a sample difference in mean score typically varies from the true difference in mean score)	Perception score(1-5)
t^*	the standard t-statistic used with 1016 (n - 1) degrees of freedom and Confidence Level of 95%	N/A

2.4 The Model

2.4.1 Modeling of Environmental Perception

Throughout the 21st century, significant reforms have come about in hopes of fixing the damage humans have caused to the environment. Numerous laws and regulations have been passed in hopes of reducing the negative impact of daily life on the Earth. In recent years e-bikes have come to the forefront of emission-reducing technologies, among the likes of EVs, and renewable forms of energy. Because they do not run on fossil fuels, electric vehicles are a cleaner alternative to the typical cars, buses, and other forms of transportation.

To find any correlation between the two variables, concern for the environment must be represented as a quantifiable value to allow for proper analysis. The data presented in the Gallup Poll provides opinions of U.S. citizens about the environment. Due to the qualitative nature of this data, the responses were converted to a 1 to 5 scale, with one representing "not at all" and five corresponding to a "great deal." These values are arbitrary, serving solely to differentiate the different responses while providing them with a numerical value, allowing us to use inferential statistics.

For each year provided, an average score for importance of the environment was calculated using the values assigned to the different responses. To demonstrate a positive correlation between care about the environment and the sale of e-bikes, it must be shown that care about the environment is increasing. This is because if there is any meaningful correlation between the two a change in the dependent variable would not occur without a change in the explanatory variable. Testing this meant constructing a 95% confidence interval for the difference between the concern for the environment in the 2017 and the 2003.

Justification of the use of t procedures to create a confidence interval:

Initial statement from the surveying body (Gallup): Results for this Gallup poll are based on telephone interviews conducted March 1-18, 2022, with a random sample of 1,017 adults, aged 18 and older, living in all 50 U.S. states and the District of Columbia.[B]

Sample Size Accuracy

Gallup states that they conducted a random survey. Randomness gives us the ability to generalize the results of surveys to all Americans and make further conclusions about the American consensus on environmental perception.

The 10% Condition

Because the sample size of 1,017 adults is less than 10% of the total population of adults aged 18 and older, living in all 50 U.S. states and the District of Columbia, the 10% condition is satisfied.The fulfillment of this condition is important because it ensures that a sample can be drawn without replacement while the independence of each individual is still guaranteed. This condition essentially allows us to assume that $P(x)$ remains constant so we can proceed with our standard deviation (σ) formula, and further to our standard deviation of x bar ($\sigma_{\bar{x}}$) formula.

$$S_x = \sqrt{\Sigma[(x_i - \bar{x})^2 \cdot P(x_i)]}$$

$$\sigma_{\bar{x}} = \frac{S_x}{\sqrt{n}} \qquad \sigma^2_{\bar{x}} = \frac{S_x^{\,2}}{n}$$

$$\text{ME(2 sample interval)} = t^* \sqrt{\sigma^2_{x_1} + \sigma^2_{x_2}}$$

The Central Limit Theorem

Because the sample size is greater than or equal to 30 (1,017 ≥ 30) the central limit theorem applies. This theorem guarantees an approximately normal sampling distribution of $\bar{x}$ (being the mean environmental perception score for a given year). This is important because it allows the use of t procedures which we used to calculate our margin of error and therefore the confidence interval.

$$(\bar{x}_1 - \bar{x}_2) \pm t^* \sqrt{\frac{\Sigma[(x_{1i} - \bar{x}_1)^2 \cdot P(x_{2i})]}{n_1} + \frac{\Sigma[(x_{2i} - \bar{x}_2)^2 \cdot P(x_{2i})]}{n_2}}$$

The interval

A two sample t interval for $\mu_{\bar{x}1 - \bar{x}2}$ (the true mean difference in Environmental Perception score across two years) was constructed at the 95% confidence level using the data from the table of responses The confidence interval produced: (-0.638, 0.019)

Environmental Perceptions

US Gallup poll: percentage of respondants in the United States for each answer choice**

Year	"How much do you personally worry about the quality of the environment?"				
	Great deal	Fair amount	Only a little	Not at all	No opinion
2001	42	35	17	5	1
2002	35	31	27	6	1
2003	34	34	21	10	1
2004	35	27	31	7	0
2005	35	30	28	6	1
2006	40	37	18	5	0
2007	43	33	18	6	0
2008	40	34	19	7	1
2010	34	34	24	7	1
2011	34	34	24	7	0
2012	37	36	19	7	1
2013	36	33	23	8	0
2014	31	35	24	10	0
2015	34	34	22	10	0
2016	42	31	19	7	0
2017	47	30	16	7	1
2018	42	30	20	8	0
2019	47	27	18	8	0
2020	43	26	22	9	0
2021	46	29	15	9	0
2022	44	27	18	10	0

***Note that due to rounding errors, percentages do not sum to exactly 100 for all years.*

[B]

2.4.2 Modeling of Energy Costs

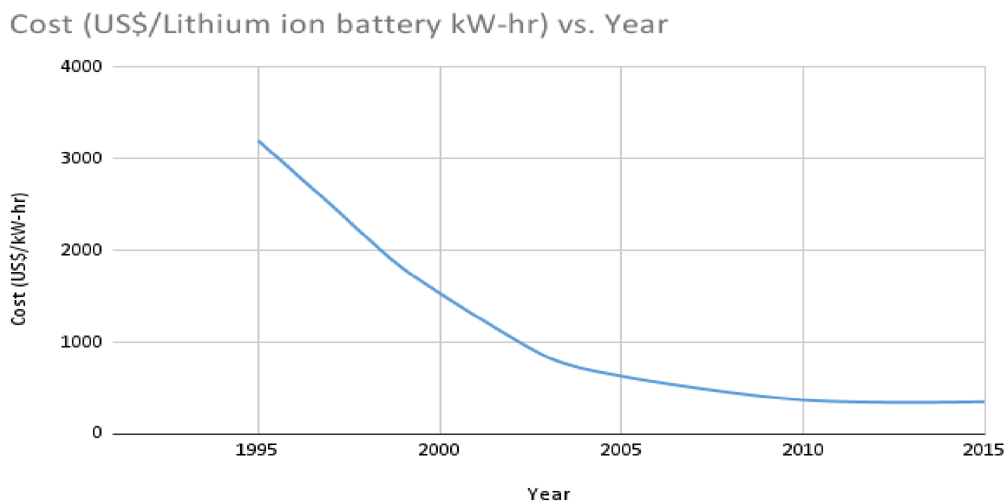


Fig (2.4.2.1) [C]

Prices of batteries have been decreasing, at a rate of ~20% annually. The fall is expected to continue at a rate of ~10% annually over the next five years. [D]. Because batteries are the most expensive part of the EBike [E] it is an important thing to note that their prices have been going down.

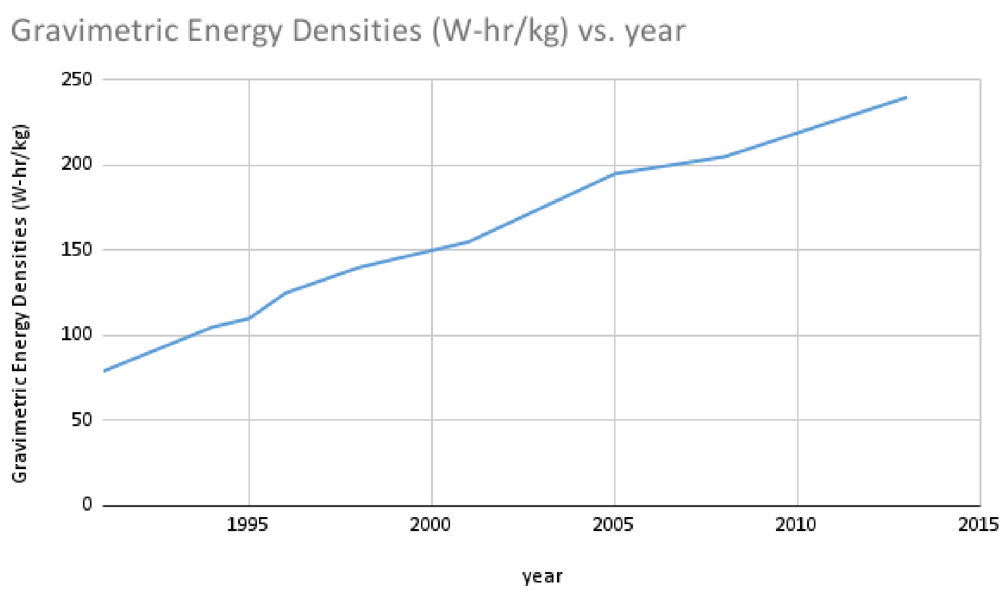


Fig (2.4.2.2) [C]

Gravimetric energy density is effectively the efficiency of the battery(or rather how much the battery can store). Because batteries are progressively able to store more energy it is becoming easier to integrate them into modern technology such as e-bikes. You can have a smaller lighter battery that still carries enough charge to warrant the implementation of it.

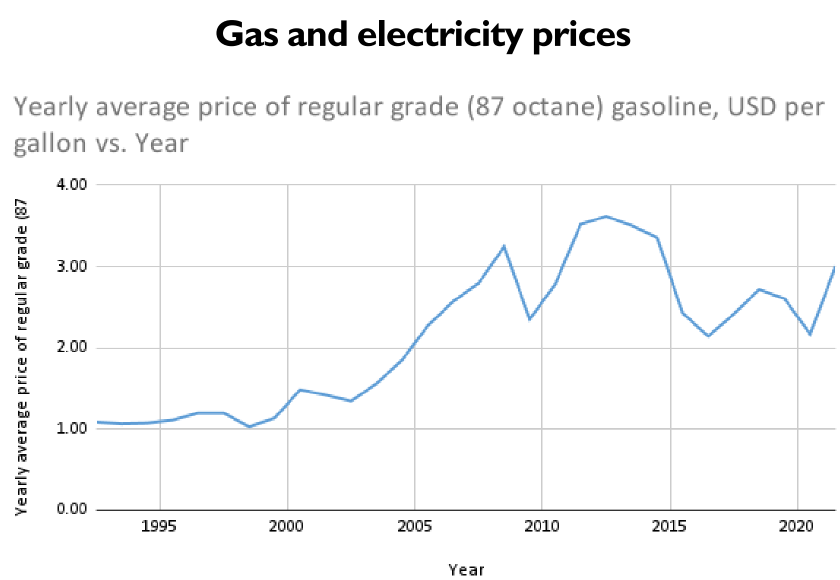


Fig (2.4.2.3) [F]

Over the 30 years, gas prices in the United States have been steadily rising from a little over 1$/gallon in 1990 to 3.95$/gallon in 2022 as shown in the graph. Electricity prices, however, have not seen as drastic increases going from 0.08$/kWh 1990 to 0.15$/kWh. Over the same span of time, gas prices have quadrupled while electricity prices have only doubled.

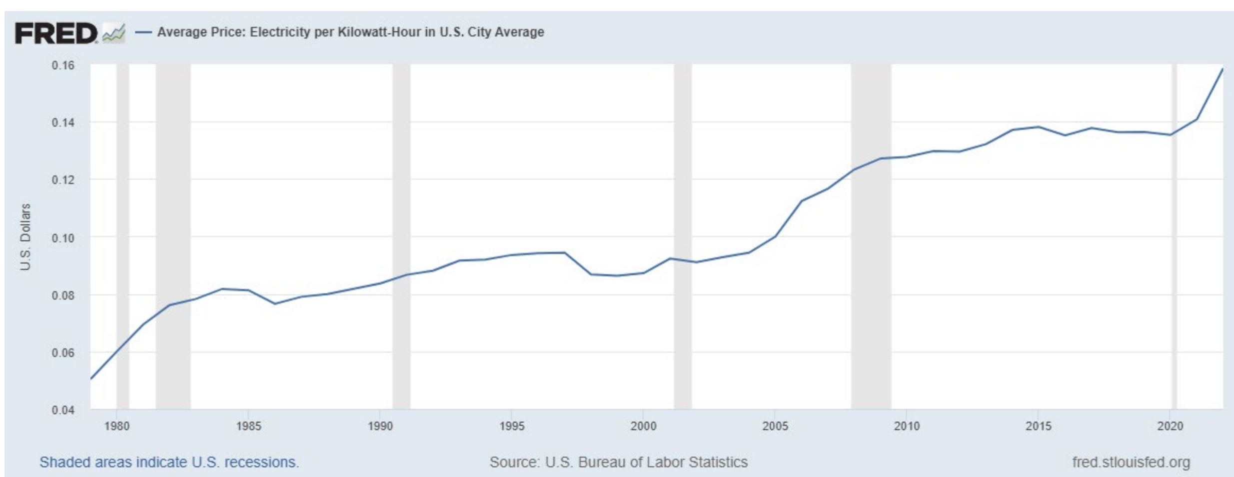
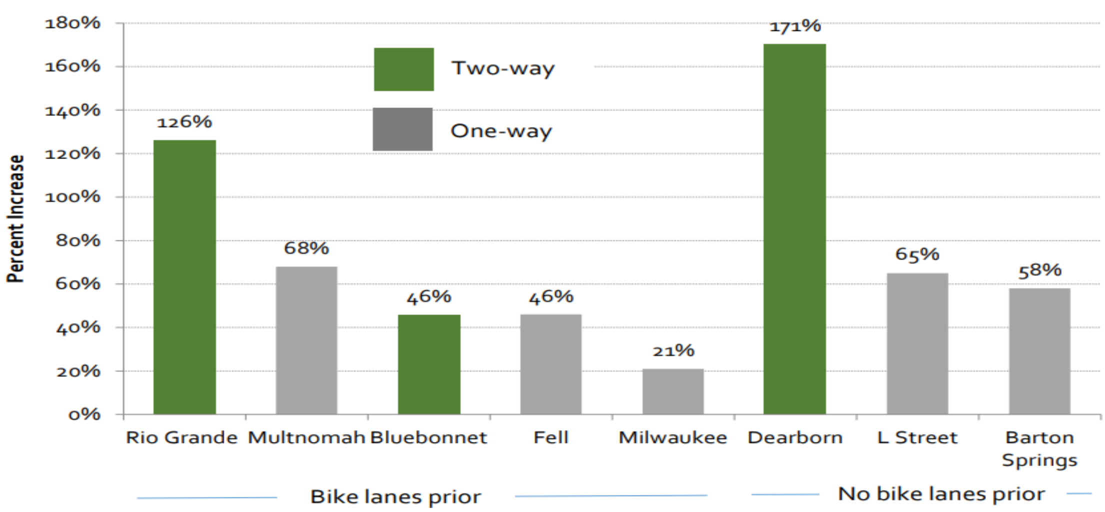


Fig (2.4.2.4)[G]

Gas is a non-renewable resource that will continually be depleted. As the supply of petroleum (the key ingredient of gasoline) dwindles, prices will rise even further. The U.S. in particular faces even more uncertainty in its supply of gasoline as most ingredients necessary to make gas are imported from countries that the U.S.has poor political relationships with. Meanwhile, electricity can be produced completely within the states and is not dependent on foreign countries. Electricity generation, while currently dominated by coal and other non-renewable sources, can be continually improved with scientific developments. As clean and renewable energy sources are discovered and optimized, U.S. energy prices can be reduced or at least held constant at some point.

2.4.3 Modeling the Impact of Bike Safety (Infrastructure)

Increases in Amount of Biking in Streets After Protected Bike Lane Installation



(Fig 2.4.3.1)Source: [H]

A concern of many people when biking is their safety, and without any cyclist infrastructure they are forced to ride in the same lane as cars. Whereas cars have dozens of safety features implemented in them, such as airbags, seat-belts, and even automatic braking in some cars, bicycles have virtually no safety features. Protected biking lanes are an ideal remedy for this ailment. Government investments into these types of cyclist infrastructure has increased the use of e-bikes by urban residents [H].Installing protected bike lanes increases usage of the installed bike line by at least 20% in its first year alone. At most it will increase by 171% in its first year alone. [H] These increases in biking on streets with Protected Bike Lanes demonstrate that people feel safer and confident enough to bike in urban areas with these additions.

2.5 Solution

Throughout our process we referenced the following data on electric bike sales from different regions ranging from 2006-2022. It is important to note that the data given had holes which were partially filled by supplementing the table with data from Statista, which will be included in the citation.

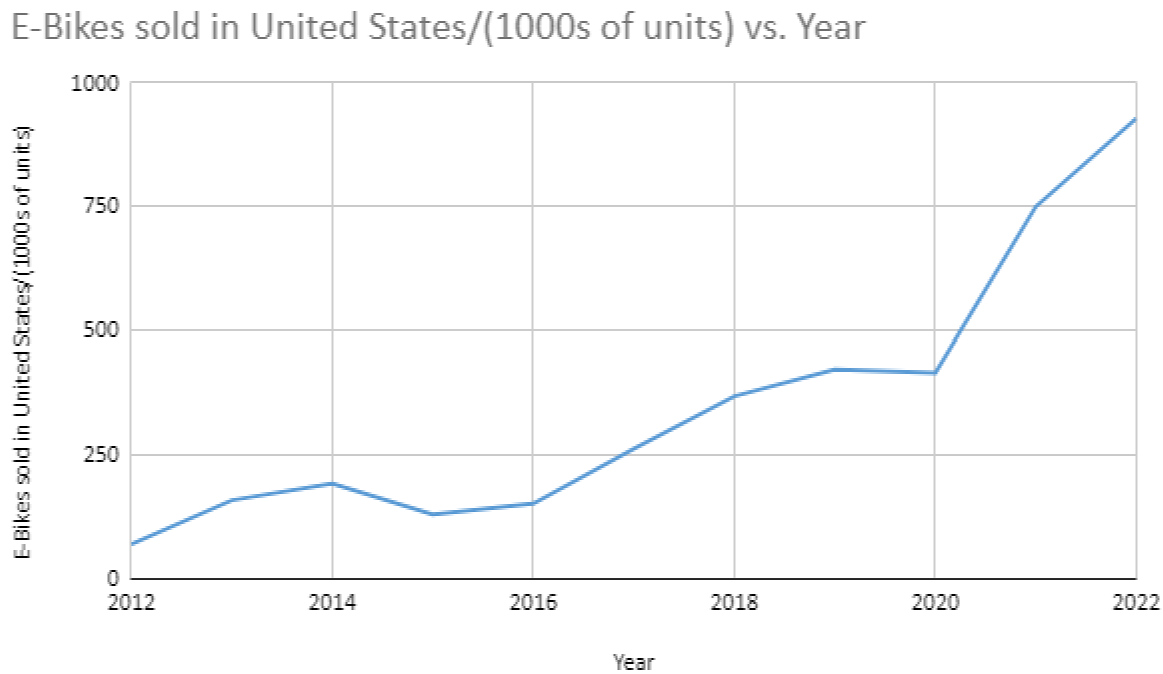
Year	E-Bikes sold in United States (1000s of units)	E-Bikes sold in Europe (1000s of units)	E-Bikes sold in France (1000s of units)	Electric two-wheelers* sold in China (1000s of units)	Electric two-wheelers* sold in India (1000s of units)	E-Bikes sold in Japan (1000s of units)
2006	--	98	--	19500	--	--
2007	--	173	--	21380	--	--
2008	--	279	15	21880	--	--
2009	--	422	24	22200	--	--
2010	--	588	38	29540	--	--
2011	--	716	37		--	409
2012	70	854	46	34500	--	392
2013	159	907	57	36000	--	446
2014	193	1139	78	34400	--	474
2015	130	1364	102	31800	--	468
2016	152	1637	134	31400	20	540
2017	263	2074	278	30500	23	616
2018	369	2767	338	32200	55	666
2019	423	3397	388	36800	126	698
2020	416	4537	--	47600	152	738
2021	750	5057	--	41000	144	--
2022	928	--	--	44200	--	--

[I][J][K][L][M][N][O][P]
Fig (2.5.1)

These trends in pricing likely stimulated the growth of e-bike sales. Despite common assumptions, the effects of environmental concerns on e-bike sales were low. When faced with the growing gas prices, Americans likely moved away from cars and towards electric transportation methods. The main factor boosting the sale of e-bikes is their cost efficiency when compared to other more common means of transportation.

2.6 Discussion

Our data shows that batteries are becoming cheaper to make as well as being able to store more energy over the years. The result of this is an incentive for e-bike manufacturers to decrease their prices which in turn makes the choice of an e-bike more appealing to the average American. It is a safe assumption to make that the advancement in battery technology has directly contributed to the growth of the e-bike market. Based off of the historic gas and battery data it can be concluded that price fluctuations of energy as well as innovations in battery construction had an effect on the total sales of e-bikes as seen in the graph:



Moving on to the result of the confidence interval. Because the interval contains 0 as a plausible value for the difference between the years, there is reason to believe that care did not change from year to year.Despite the static opinion on the environment, the interest and sale of e-bikes grew irregardless, suggesting that increases in environmental concerns were not leading to an increase in sales, or at the very least were not influencing the boom in popularity seen in the last five years. As such, the constructed model does not take the environmental concerns of consumers into account, being that no data suggests it has a meaningful impact on sales.

2.6.1 Strengths

The confidence interval that was calculated was done so with extreme mathematical rigor so the numerical data produced is very trustworthy. Data was able to be filled into gaps for sales data in multiple sectors which acts to strengthen the overall trends we were able to draw from the information itself.

2.6.2 Weaknesses

Causation could not be proven due to the time restricted nature of this challenge as well as the limited access to data (most of which was blocked by paywalls). So it is informed speculation that was used to reach the conclusions for Q2. We would need immense resources to conduct many controlled experiments to be able to prove a true cause and effect relationship but our current procedure is a plausible correlation.

There are also possible sources of bias from the data used for the confidence interval although this is a gallup survey the data cannot be overstretched. Some possible concerns include possible nonresponse bias because the survey was conducted by phone. It is possible that the people who were prone to answer a phone survey hold a specific opinion about the environment that people who wouldn't answer the same survey dont hold. Another possible shortfall stems from possible response bias in that people may intentionally lie about how they care about the environment so as to not seem like an unfavorable person by the surveyor. These two forms of bias could possibly affect the results but were taken to be negligible to the test.

Q3: Off the Chain

3.1 Defining the Problem

As more and more e-bikes are purchased and subsequently used, it becomes important to determine the positives brought about. One of the main purposes of electric cars is to reduce the carbon waste produced while traveling. This problem requires a way for determining the reduction in carbon emissions caused by e-bikes. This can be understood to mean finding the amount of carbon that is not being produced due to the usage of e-bikes over other forms of transportation.

3.2 Assumptions

3.2.1 E-bikes for Utility

We assume that the majority of E-Bikes will be primarily intended for transportation rather than recreation. This means that every mile traveled by an e-bike is a mile that would have normally been traveled by a car.

3.2.2 Surface Level Emissions

We assume that surface level emissions are primarily caused by transportation. Surface level emissions will greatly affect the quality of air in metropolitan areas. Because factories and power plants are generally located further away from metropolitan areas, they affect surface level pollution less. Thus, we can assume tailpipe emissions have the greatest impact on local air quality, whereas factory and power plant emissions have a negligible impact on local air quality.

3.2.4 Bicycle Lifespan

We assume that the e-bikes reach the end of their life cycle and need to be replaced every 10 years. We calculate this based on the average date of motor failure. We assume that all bicycles are bought on January 1st of each year, meaning the data of Carbon emissions saved, is calculated for a full year of 3-bike replacements.

3.3 Variables

Symbol	Definition	Units
P(t)	The reduction in CO_2 caused by e-bikes in the given year (t) by e-bikes	Grams of CO_2

L_{km}	The expected number of kilometers travel per year by an e-bike $L = \frac{total(miles)}{lifespan(years)} = \frac{10000\ miles}{10\ years} * \frac{1.60934\ km}{1\ year} = 1609.34\ km/year$	Kilometers/Year
S	the difference in grams of CO_2 pollution per kilometer traveled between a car(C_{out}) and an e-bike(E_{out}) $C_{out} - E_{out} = 229g/km - 1.5g/km = 227.5g/km$	Grams of CO_2/km
b(t)	The total number of e-bikes sold in the given year (t)	E-bikes

3.4 The Model

$$P(t) = L \cdot S \cdot \sum_{t-9}^{t} b(t)$$

In order to find the amount of CO_2 e-bikes will save, the difference in CO_2 produced by an e-bike and an average car over one kilometer must be calculated. The average e-bike produced 1.5g of CO_2 per kilometer and the average car produced 229g of CO_2 per kilometer[Q]. This means that an e-bike uses S = 227.5g of CO_2 less than a car per kilometer. Based on the assumed mile lifespan of an e-bike and the assumed time(years) lifespan of an e-bike, a value for L(km traveled/year)This equates to 1609.34 kilometers per year. This means that each e-bike produces (L*S)g of CO_2 less per year. This would mean that the total reduction in CO_2 output is (L*S)g multiplied by the total number of e-bikes still functioning. Because we assume that e-bikes last for 10 years based on the average lifespan of its parts, the total number of bikes still working by the end of the year is equal to the sum of bikes sold in year t-9 to year t. We assume that bike sales can all be condensed to January 1st, because bikes will continually be sold and expire over the course of the year in similar patterns to every other year we introduce and remove all bikes at one time. This allows us to deal with the fact that the data given is discrete as it is only given on a yearly basis. For these reasons all bikes sold ten years before a desired year can be considered expired and all bikes sold in a given year will contribute to the carbon savings of that year.

3.5 The Results

The produced model indicates that each e-bike produces significantly less CO_2 than a car would. E-bikes have thus proven to be a much more efficient and environmentally friendly method of travel than a normal car. With our models predicting rapid growth in the number of e-bikes sold in the coming years, the carbon reduction from e-bikes will grow as well, saving countless tons of CO_2 pollution from entering the atmosphere.

3.6 Discussion

The model we created for carbon emissions unfortunately assumes a constant lifespan and emissions for all e-bikes. This prevents the consideration of any variability in the quality of e-bikes which may make the model less accurate. If possible, it would be beneficial to do more research on the life expectancy of an e-bike and determine if e-bikes are producing less emissions as the technology becomes better.

3.6.1 Strengths

Our model for predicting pollution rests on fairly safe assumptions surrounding the life expectancy of e-bikes.

3.6.2 Weaknesses

Our model does not take into account the carbon emissions of Electric Vehicles, like Teslas. We also do not take into account that as technologies improve the carbon emissions of the e-bikes may be reduced, further increasing the carbon savings from e-bikes. We also do not take into account the carbon emissions from producing the large number of e-bikes and from recycling the e-bikes at the end of their lives.

Conclusion

We projected the total amount of E-bike sales in 2 and 5 years. Our model uses a logistical growth curve to predict the number of e-bike sales based on China's growth of e-bike usage. We approximated the equation (specifically the values for L and k) for the growth curve by trial and error, and then adjusted our value of L to fit the population of the U.S.

Then, we explored the causes of the exponential growth pattern of e-bike sales and determined multiple factors that affected (or did not affect) the growth in sales of e-bikes. To do this we created a Confidence Interval for the amount of people who care about the environment at various levels, a line graph of the decreasing price of e-bike batteries, and one of the increasing efficiency of e-bike batteries. We also made a line graph of increasing gas prices, and one of increasing electricity prices. We also referenced a histogram to determine that increasing the safety of urban biking increases the popularity of biking on streets with protected biking lanes.

Finally, we created a linear series equation modeling the reduction of carbon emissions through the increase of e-bike usage. We found the difference in carbon emissions from cars and e-bikes, and multiplied it by the numbers of e-bikes in the U.S. This quantifies the amount of carbon-emissions saved through the use of e-bikes using the amount of e-bikes predicted to be in use from Q1.

Global pollution has slowly but surely become more and more prevalent in our current day. In order to reduce our carbon footprint, it is imperative that we increase the use of e-bikes throughout the nation, along with other methods of reducing our dependence on fossil-fuels. By dramatically reducing our carbon emissions, we can help guarantee a future to our children where a shrinking coastline or global warming wouldn't be an issue.

Sources(Independent of page count)

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