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M3 Challenge Third Place, $10,000 Team Prize

***Note: This cover sheet has been added by SIAM to identify the winning team after judging was completed. Any identifying information other than team # on an M3 Challenge submission is a rules violation.

***Note: This paper underwent a light edit by SIAM staff prior to posting.

EXECUTIVE SUMMARY

Ever since the invention of the mass produced automobile in the beginning of the 20th century, the automobile has become a vital part of life around the world, especially in America. 95% of American households own at least one car, and 85% of working people in America get to work by car [1]. However, these times are changing. Due to controversy over the environmental footprints of cars and the massive amounts of oil needed to fuel these cars, an increasing percentage of the working class of America are using car-sharing. This relatively new concept is defined by the US Department of State as a process where shared cars owned by private companies parked throughout dense metropolitan areas and university campuses can be rented, based on a variety of factors, by members at any hour of the day [1].

Our job consists of three main parts: one, to determine the percentage of US drivers in varying categories based on the amount of time driven per day and the number of miles driven daily; two, to deduce which of four car-sharing methods would attract the most participation in different cities throughout the country; and three, to then account for the impact of new environmentally friendly automobile technologies and change our model to rerank the aforementioned cities in terms of citizen participation.

To build a model for the first task, we first determined the definitions of low, medium, and high mileage and time spent driving. Using mileage caps from insurance companies, we defined less than 20.548 miles per day to be low, between 20.548 and 41.096 miles to be medium, and greater than 41.096 miles to be high [2]. We then determined the time group bounds by multiplying the previous bounds by the ratio of average time spent driving annually to average miles driven annually to define low as less than 0.71 hours per day, medium being between 0.71 and 1.42 hours, and high as above 1.42 hours. A cumulative probability distribution of miles driven per day was created, and probabilities from that function were multiplied with estimated probabilities of a driver being in the low, medium, or high group given their mileage group.

Our second assignment was to build a model to determine the expected participation of several cities in car-sharing programs like Zipcar. Once we built the basic model, we were then tasked with analyzing three other variants, which were a) a system of jockeys who manually reposition cars after one-way trips, b) a station system where users drop off and pick up cars at stations, and c) a private system, where multiple individuals privately share ownership of a car. We constructed a model, based on factors such as income distribution, population, Cost of car-sharing, average commuting time, and car availability to estimate each city's participation, measured on the car shares index. Our results indicated that the basic structure of a traditional per hour or per day charge would lead to the most participation in the four cities given.

Finally, we adjusted our model to consider the effects of new technological developments such as self-driving cars, and alternative energy sources. After accounting for the change in cost that these new technologies caused, we determined that the traditional hourly- or daily-based system, coupled with new alternative energy sources was the most appealing for consumers.

INTRODUCTION

Background

Centuries have progressed since the introduction of automobiles, and not only has pollution become a threat to human well-being, cities have become concentrated to the point where even ownership of cars can become a burden. In response to these changes, modes of public transportation and conventional car rental methods have been explored, and now car-sharing has emerged to possibly best suit urbanites' needs of a more convenient, environmentally friendly travelling agent. Not only have numerous car-sharing plans been developed for drivers by numerous rising car-sharing enterprises, Zipcar being one of the most prominent in the US, and their well-established auto-manufacturing partners, car-sharing has introduced the idea of automobiles becoming "public" vehicles as accessible as city bikes. The appeal of this innovative model will be evaluated on factors such as cost efficiency and its "fit" with the American population.

Restatement of the Problem

Our task was to do the following:

A) Create a model to determine the current percentage of Americans in each of the brackets and combinations of "low," "medium," and "high" daily mileage and time spent driving per day, the two factors that influence driver's choice of car-sharing program.
B) Build a model to predict future participation in car-sharing programs based on the following scenarios:
    a) Traditional hourly or daily based rental programs.
    b) One-way structure where jockeys manually move cars to where they are needed.
    c) A system of stations where drivers can drop off and pick up cars, similar to New York City's CitiBike system.
    d) Privately centered shared ownership, maintenance, and usage of cars.
C) Analyze the effect of new technologies like alternative energy sources and self-driving cars on the participation of the four cities in car-sharing program.

PROBLEM 1: Who's Driving?

1.1 Assumptions, Justifications, and Simplifications

<u>Assumption 1:</u> The probability of a driver being in the low, medium, or high time group given that the driver is already in the low, medium or high mileage group is equal to the probability that the driver is in the same time group as their mileage group multiplied by ⅓ for every time group that they are "away" from being in the time group corresponding to their mileage group. For example, for a given driver in the low mileage group, let the probability that they are in the low time group be P; then the probability that they are in the medium time group is $\frac{P}{3}$, and the probability that they are in the high time group is $\frac{P}{9}$.

<u>Justification:</u> The distribution of time spent driving per day for a given value of miles driven per day was estimated to be somewhat normal; however, in order to make calculations of given

probabilities, the math is too complex, and thus a simplification is required. The ratio $\frac{1}{3}$was chosen because it is clear that as one moves farther away from their corresponding time group, fewer drivers should be in that range.

Assumption 2: Data from the NHTS 2009 is applicable to today with reasonable accuracy.

Justification: Data collected from years before 2009 did not follow a consisted rate of change with respect to years passed up to 2009. Therefore the data between 2009 and 2015/2016 likely did not change enough to make a large difference.

1.2 Model Design

Because

$$P(A\&B) = P(A) \times P(B \mid A)$$

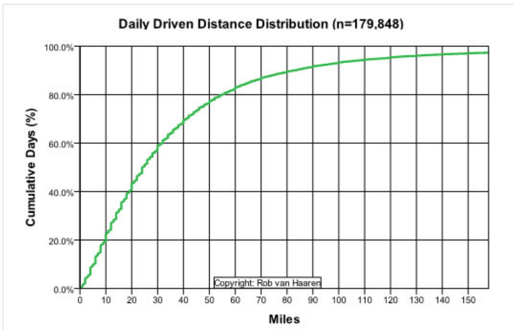
when A and B are not independent events, finding the percentage of drivers in each of the nine car usage characterizations is equivalent to finding the probability that a randomly selected driver is in the low, medium, or high mileages groups, and then multiplying that by the probability that a driver in a given mileage group is in the low, medium, or high time group.

Our first task was to determine the percentage of American drivers in each of the nine categories gotten from all the combinations of low, medium, and high daily miles driven and time spent driving per day using a mathematical model.

We first attacked daily miles driven. The upper bound for the categorical label "low" of miles driven per day was determined to be 7,500 from the "low mileage" cap established by car insurance companies according to Liberty Mutual [2]. Likewise, drivers considered by car insurance companies to have "high mileage" must have exceeded the minimal lower bound of 15,000 miles driven per day, and thus 15,000 was taken as the upper bound for the "medium" category. These values were divided by the number of days in a year, 365, to find the average daily mile upper bounds for the "low" and "medium" categories, being 20.548 and 41.096 miles, respectively. To determine the bounds for the time groups, the mileage bound groups were multiplied by the ratio of the average hours spent driving annually (464.7) to the average miles driven annually (13476), yielding the bounds for the low and medium categories bein 0.71 and 1.42 hours per day, respectively [3][4].

The graph below, taken from Solar Journey USA and originating from the 2009 National Household Travel Survey, shows the cumulative distribution of miles driven per day for a sample of 179,848 Americans meant to approximate the US population.

Figure 1.



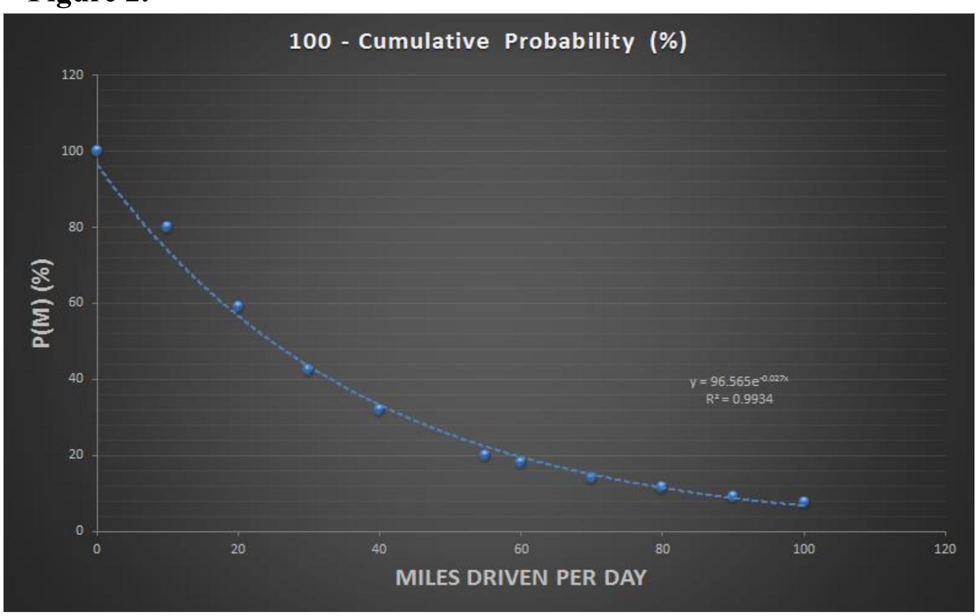
[5]

When estimating data points from Figure 1 in order to build an equation to accurately model it, the regression failed. Therefore, y-values from Figure 1 were taken and subtracted from 100% to obtain a graph which displays the probability that a randomly selected person in the US drives more than a given value of miles driven per day. An exponential regression was run to obtain the function

$$P(m) = 96.565e^{-0.027m},$$

where m is the miles driven, and $P(m)$ is a percentage. The coefficient of determination (r^2) of the regression was 0.9934.

Figure 2.



Based on the manipulated probability equation, the probability of a driver driving more than an average of 41.096 miles per day is calculated as

$$P(High) = P(41.096) = 96.565e^{-0.027(41.096)},$$

which yields that 31.84% of drivers are considered "high" mileage drivers.

The percentage of drivers considered "medium" mileage drivers is equal to the percentage of "high" mileage drivers subtracted from the cumulative percentage of individuals who drive more than the medium lower mileage limit of 20.548,

$$P(Medium) = P(20.548) - P(41.096) = 96.565(e^{-0.027(20.548)} - e^{-0.027(41.096)}),$$

resulting in the probability being 23.61%.

The percentage of drivers considered "low" mileage drivers, because there are only the three categories of "low," "medium," and "high" for daily mileage, is equal to the percentage of American drivers considered "medium" and "high" subtracted from the whole, or 100%,

$$P(Low) = 100 - P(20.548) = 100 - 96.565e^{-0.027(20.548)},$$

leading to a 44.55% chance that a randomly chosen driver is a "low" mileage driver.

Given the assumed association between time spent driving per day and daily miles driven for Americans, the percentages of Americans belonging to each of the nine "low," "medium," and "high" combination categories were calculated. Finding the probabilities in the three categories of time associated with "medium" mileage, it is assumed first that the probability of drivers who spend "medium" time driving, given that they fall under "medium" mileage, is *P*. Thus, both the probabilities, given "medium" mileage, of drivers with "low" and "high" times are equal to *P* divided by 3. Therefore, because these three probabilities sum to 1,

$$P + \frac{P}{3} + \frac{P}{3} = 1$$

$$\frac{5P}{3} = 1$$

$$P = \frac{3}{5}, \frac{P}{3} = \frac{1}{5}$$

Because these are the probabilities of drivers given their "medium" mileage, the overall percentages of these three categories must be found after multiplying the probabilities found above with the percentage of any driver having "medium" mileage.

$$P_{m,m} = \frac{3}{5} \times 23.61\% = 14.17\%$$

$$P_{m,h} = P_{m,l} = \frac{1}{5} \times 23.61\% = 4.72\%$$

Next, we must find the probabilities of drivers in each of the three categories with "low" daily mileage. Assuming "low" mileage as a given, P is the probability of a driver spending a "low" amount of time in the car. Continuing this assumption, the probability of a driver spending a "medium" amount of time daily is P divided by 3. However, for "high" times spent per day given "medium" mileage, the probability is P divided by 9 because the "high" category is twice removed from "low" mileage, and so P must be divided by 3 twice. Thus, once again,

$$P + \frac{P}{3} + \frac{P}{9} = 1$$

$$P = \frac{9}{13}, \frac{P}{3} = \frac{3}{13}, \frac{P}{9} = \frac{1}{13}$$

The overall percentages of these three categories must again be found after multiplying the probabilities found above with the percentage of any driver having "low" mileage:

$$P_{l,l} = \frac{9}{13} \times 44.55\% = 30.84\%$$
$$P_{l,m} = \frac{3}{13} \times 44.55\% = 10.28\%$$
$$P_{l,h} = \frac{1}{13} \times 44.55\% = 3.43\%$$

The above procedure is followed precisely to find the three percentages associated with the three "high" mileage categories:

$$P_{h,h} = \frac{9}{13} \times 31.84\% = 22.04\%$$
$$P_{h,m} = \frac{3}{13} \times 31.84\% = 7.35\%$$
$$P_{h,l} = \frac{1}{13} \times 31.84\% = 2.45\%$$

1.3 Sensitivity Analysis

We tested the sensitivity of this model with the ratio of probabilities being $\frac{1}{2}$ and $\frac{1}{4}$.

Figure 3. Sensitivity Analysis with Different Ratios

Ratio $= \frac{1}{2}$	Low Mileage	Medium Mileage	High Mileage
Low Time	25.46%	11.81%	4.55%
Medium Time	12.73%	5.90%	9.10%
High Time	6.36%	5.90%	18.19%

Ratio $= \frac{1}{4}$	Low Mileage	Medium Mileage	High Mileage
Low Time	33.94%	15.74%	1.52%
Medium Time	8.49%	3.94%	6.06%
High Time	2.12%	3.94%	24.26%

Changing the ratio to ½ produced no changes in the values greater than 5%, and changing the ratio to ¼ produced no errors greater than 4%. Because these changes are fairly small, slight errors in the assumed ratio do not significantly affected the final data.

Problem 2: Zippity Do or Don't?

2.1 Assumptions, Justifications, and Simplifications

Assumption 1: Everyone who drives will drive more than 6 times a month.

Justification: This is valid because usually people who drive will tend to drive often since driving is often a necessity for many daily functions like traveling to a workplace, going to the local supermarket, etc. Thus, unless one of these vital functions is accessible by an alternate means of transportation (e.g., public transportation), one needs to drive to reach these places. And the only places where a large enough proportion of these vital locations would be reachable by public transportation is a large city, where the public transportation network is so extensive that one would most likely not drive at all anyway. Thus, it is safe to assume that almost everyone who drives will drive more than 6 days a month.
Assumption 2: The price of gasoline will be held constant.
Justification: The price of gas fluctuates wildly and is affected by factors that are hard to predict, such as the political stability of regions that are rich in oil (e.g., the Middle East).
Assumption 3: Switching from a round-trip model to a one-way model makes little to no difference to the predicted values.
Justification: If a car-sharing company offers only one-way service, customers who use the service to reach a destination are likely to not own their own car. Therefore, they are likely to need to use the service again on their return trip.

2.2 Model Design

We used the car shares index from walkscore.com to measure participation in local car-sharing programs [6]. While developing our model, we determined that the primary factors that affect the participation in a car-sharing program would be the population of the city (*p*), the average amount of time spent commuting to work in minutes (*t*), the daily cost of the local car-sharing program in dollars (*c*), the percentage of households without a car (*h*), the percentage of households with an annual income between $41,000 and $66,000 (*i*), and the percentage of households with an annual income between $66,000 and $106,000 (*j*).

2.2.1: Model for a Round Trip Model for a Car-Sharing Program where Vehicles are Rented by the Hour/Day

$$S = 1258.987 + 47.385p + 30.208t - 16.524c - 9.213h - 80.953i + 59.742j$$

S = car shares index, a measurement of participation in car shares programs

Population (*p*) would affect the participation in the car-sharing program because a higher population would have a larger base to appeal to and naturally lead to more overall participation in the car-sharing program.[1]

An increase in the time spent commuting to work (*t*) would also be expected to lead to an increase in car-sharing participation because as local commuting time increase, so does gas usage and the cost of paying for gas. Car-sharing programs charge only by hours or days used and thus save the driver the hassle of worrying about gas, and therefore they are a natural alternative to

[1] See Appendix A

long commuting times. Furthermore, car-sharing programs are more advantageous than public transportation because they afford the traveler privacy and flexibility, both in timing and in route.

The cost of the local car-sharing program (c) should have a negative relationship with participation because the law of demand illustrates that as price increases, demand, which is represented by participation in the car-sharing program, will decrease.

An increase in percentage of carless households (*h*) is expected to have a negative relationship with participation because people in carless households already do not use cars and most likely do not have a reason to drive. Thus, they are not potential customers and will decrease the proportion of the population that would consider joining a car-sharing program.

The percentage of people who have an annual salary between $41,000 and $66,000 (*i*) should have a negative correlation with participation because people in this income bracket are unlikely to be able to afford the costs of participating in a car-sharing program, and thus their proportion of the population detracts from the possible consumer base.

The percentage of people who have an annual salary between $66,000 and $106,000 (*j*) should have a positive correlation with participation because people in this income bracket are upper middle class citizens who frequently commute and are able to afford a car-sharing program.

2.2.2: Model for a "Floating" Car-Sharing Program with Jockeys

For this part of the model, the primary adjustment is that we need to adjust the cost of the program to account for the additional cost of paying the jockeys.

<u>Assumption:</u> Jockeys will be paid minimum wage.

<u>Justification:</u> Jockeys are a low-skilled occupation that has a large supply of potential workers from the population of unskilled and untrained workers.

<u>Assumption:</u> Each jockey can move 2 cars per hour.

<u>Justification:</u> We reached this conclusion by determining that a jockey could move one car in 10 minutes, find the location of another car in 10 minutes, and get to the location of the other car in 10 minutes:

$$S = 1258.987 + 47.385p + 30.208t - 16.524(c + P) - 9.213h - 80.953i + 59.742j$$

where *P* = the additional cost for the consumer due to the increased cost of production due to the jockeys' salary:

$$P = 0.5M$$

where *M* is the minimum wage of the state that the town is located in.

Since each jockey can move two cars per hour, it is reasonable to determine that each participant in the car-sharing program will be charged one half of the jockey's hourly salary. Since the coefficient of the cost is negative, by increasing the variable associated with that coefficient, *S* will always decrease.

Figure 4.

City	Minimum Wage ($/hour)	Increase in Cost per Person ($)
Poughkeepsie	$9.00	$4.50
Richmond	$7.75	$3.87
Riverside	$10.00	$5.00
Knoxville	$7.75	$3.87

[7]

2.2.3: Model for a Car-Sharing Station Program

For this plan, the major difference from the base model of round trip car-sharing is that participants must spend extra time every day to walk to and from the car-sharing station. This lost time leads to an opportunity cost of lost wages during said time period.

Assumption: People will not be willing to walk more than 15 minutes from a car-sharing station to reach their workplace or home.
Justification: The average time traveled to work ranges from 21.7 minutes to 29.2 minutes in the four cities we are studying [8]. Thus, any walk longer than 15 minutes will more than double the travel time and severely inconvenience the traveler.

We can model the system of car-sharing stations as a field of tangent circular regions of radius *r*, where *r* equals the distance that an average person could walk 15 minutes:

$$\bar{T} = \frac{\int_0^{0.775\ mi} \frac{r}{3.1\ mi/h}}{0.775\ mi}$$

We find the average time to be 0.125 hours. We multiply this number by 4 to account for the fact that on each trip, the traveler must make 4 separate walking trips: 2 to the car-sharing station from work and home and 2 from the car-sharing station to work and home.

Thus, we get the total time lost due to this plan to be 0.5 h / day. We can calculate the cost of this extra travel time by multiplying the average lost time by the median income in each of these cities, which yields the following.

Figure 5.

City	Median Income ($/hour)	Time Lost to Extra Walking (hours)	Lost Income ($)
Poughkeepsie	29.26	0.5	12.63
Richmond	18.06	0.5	9.03

Riverside	24.96	0.5	12.48
Knoxville	15.02	0.5	7.51

[8]

Thus,

$$S = 1258.987 + 47.385p + 30.208t - 16.52(c+l) - 9.213h - 80.953i + 59.742j$$

where l is the amount of wages lost due to the time spend at the car-sharing stations, calculated above.

2.2.4: Model for Fractional Ownership of a Private Car

Assumption: No more than two people will share a car.
Justification: If more than 2 people share a car, their time needs for the car will overlap too much, leading to significant conflict.
Assumption: The real value for car shares is equivalent to 30% of the output of our model.
Justification: Many people view a car as a symbol of status and are unwilling to share a car. In addition, sharing car inconveniences both users. According to a German industrial study, 70% of people are unwilling to even consider sharing a car [9]. Thus, we can assume that 70% of our car shares will not actually be realized because of "soft" factors like pride and inconvenience.

We can adjust our model for this scenario by redefining the cost variable to be the equivalent of the daily cost owning and maintaining a car shared by two people:

$$D = \frac{A}{E\,yr. * 365\,days/yr} + \frac{M}{365*2}$$

where D is the average daily cost per person of operating and owning a car shared by 2 people; A is average car cost, which is \$33,560; E is the average lifetime of a car, which is 8 years; and M is the average cost of yearly maintenance of the car, which is \$8,968 [10][11][12].
Thus, after adjustment, we now have

$$S = 1258.987 + 47.385p + 30.208t - 16.524D - 9.213h - 80.953i + 59.742j$$

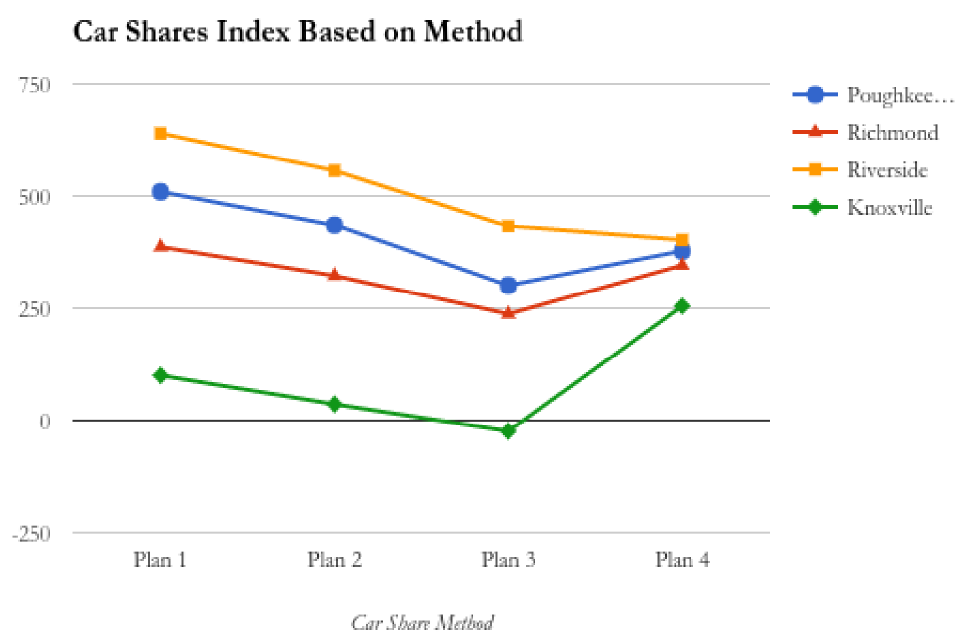
2.3 Results[2]

Figure 6.

City	Plan 1	Plan 2	Plan 3	Plan 4
Poughkeepsie, NY	509.649	435.291	300.951	377.654
Richmond, VA	386.473	322.442	237.261	345.659

[2] See Appendix B

Riverside, CA	639.336	556.716	433.116	401.689
Knoxville, TN	100.311	36.281	-23.784	254.853



2.4 Validity Analysis

Overall, our model was very strong, with an R^2 value of 0.9769, which indicates that 97.69 % of the variation in the car shares values is accounted for by our model. Furthermore, all of our key variables have strong t-stats and p-values.

Figure 7.

Variable	*Coefficients*	*t Stat*	*P-value*
Intercept	1258.986684	2.952978	0.059881
Population (millions)	47.3852675	3.417936	0.041906
Time travel to work (min)	30.20802014	3.59064	0.037006
Cost per day (USD)	-16.5239141	-3.11875	0.052523
% Household without a car	-9.21348248	-4.13949	0.025595
% Income between 66k and 106k	59.74181808	7.076596	0.005803
% Income between 41k and 66k	-80.9529699	-6.57786	0.007148

In addition, the residual plots of all of our variables display a random distribution, which indicates that our choice of a linear model was accurate. However, there are still several trends and unexplained phenomena in the graph that must be explained for a complete and correct interpretation that goes beyond simple visual conclusions.

Plan 2 results in a decrease in car shares index in comparison to Plan 1 due to being quite different in methodology. The addition of a jockey to reposition the cars after they are used increases the amount each driver has to spend, so the participation will have a decrease as more people will participate the less the service costs. The jockeys are paid minimum wage and are expected to adjust two cars an hour, so each user will have to increase the amount they pay by half the value of the state's minimum wage.

Plan 3 results in a decrease of car shares index, no matter what the demographics of the city are. This can only be explained with an in depth analysis of this method. Plan 3, which is a one way car-sharing station model, is a model where customers pick up and drop off the cars at pre-designated, existing stations. This model causes a sharp decrease in the index of every town because, in this model, customers must first transport themselves, usually by walking, to a station to pick up their car. Then, after getting to their destination, they must find a car-sharing station and walk back to their destination. This causes massive inconvenience, which we quantified in the amount of wages that would be theoretically lost in the time wasted walking to and from stations. Since there is a negative correlation between cost and car shares index, and since a loss of wages is synonymous to an increase in cost, a loss of wages causes a decrease in participation in each city, thus decreasing the car shares index of each city.

Plan 4 is a method that causes the indices of all cities to come close to the average car shares index. This is because in Plan 4, the average amount of a car is the same all around the country so the cost per day in each of the four cities is equal. Cost per day is a large factor in calculating the participation, so the values get relatively close for this plan.

A strange phenomenon that occurs in this graph is that Riverside has a different minimum from all the other cities', which occurs because every other method of car-sharing is feasible in the town. This is due to the fact that Riverside has large values for variables that have a strong positive correlation with our regression model, and small values for variables that have a strong negative correlation with our regression model. Specifically, the California town has the largest population and commute time of all cities, both of which have large positive coefficients, and the smallest cost per day and percentage of households without a car of all cities, both of which have large fairly large negative coefficients. In Plan 4, Riverside's minimum, the cities' car shares indices become much closer together due to the fact that the cost per day of buying a shared car is the same in every city. This will always cause indices of high quantity to decrease to a middle range of values, and indices of low quantity to increase to a middle range of values.

A very interesting occurrence that must be interpreted in this graph is the presence of a negative car shares index for Plan 3 in Knoxville. Plan 3, which is a one way car-sharing station model, is a model where customers pick up and drop off the cars at predesignated, existing stations. This, as explained before, is a massive inconvenience to those using the car-sharing program due to the fact that stations must be dispersed throughout a city, and there will undoubtedly be a certain level of walking required to travel from a station to a destination. In Knoxville's case, this inconvenience, quantified in the loss of wages that could occur based on median income in the city, is too great to justify using a car-sharing station model in the city. In other words, a positive car shares index calculated from our regression proves that significant participation will be present in a city; a negative car shares index leads us to believe that participation of a specific method of car-sharing in a city will be minimal, almost insignificant. Thus, it would be unfeasible for a car-sharing company who wishes to establish a business in that environment to do so.

Not only does Knoxville have a negative car shares index in a certain method but, overall, it has the lowest average car shares index. This is because in Knoxville, only 14.6% of the population has a household income of $66,000 to $106,000, which would be considered upper middle class [13]. Therefore, there are not as many people to participate in the car share program because they cannot afford to rent a car everyday in order to commute to work.

2.5: Sensitivity Analysis

For the "floating" one-way system that uses jockeys:

Key Assumption: One jockey can move 2 cars per hour.

Now Assume If: (A) One jockey can move 1.5 cars per hour (-25% assumption)

(B) One jockey can move 2.5 cars every hour (+25% assumption)

For Assumption (A):

We still use

$$S = 1258.987 + 47.385p + 30.208t - 16.524(c+P) - 9.213h - 80.953i + 59.742j$$

where P = the additional cost for the consumer due to the increased cost of production due to the jockeys' salary.

Only now,

$$P = 0.667M$$

For Assumption (B):

$$P = 0.4M$$

Now we get the following.

Figure 8. Sensitivity Analysis of Cost Increase Due to Jockeys

City	Minimum Wage ($/hour)	Increase in Cost per Person ($)	Increase in Cost per Person - A ($)	Increase in Cost per Person - B ($)
Poughkeepsie	$9.00	$4.50	$6.00	$3.60

Richmond	$7.75	$3.87	$5.16	$3.10
Riverside	$10.00	$5.00	$6.67	$4.00
Knoxville	$7.75	$3.87	$5.16	$3.10

[7]

These changes in cost increase per day lead to the following changes in the car shares index for Assumption (A).

Figure 9. Sensitivity Analysis of Car Shares for Lower Bound Test

City	Original Car Shares	New Car Shares	Percent Error
Poughkeepsie	435.291	410.505	5.7 %
Richmond	322.44	301.209	6.5 %
Riverside	566.716	530.277	6.3 %
Knoxville	36.28084449	15.048	57.99 %

These changes in cost increase per day lead to the following changes in the car shares index for Assumption (B).

Figure 10. Sensitivity Analysis of Car Shares for Upper Bound Test

City	Original Car Shares	New Car Shares	Percent Error
Poughkeepsie	435.291	450.163	3.44 %
Richmond	322.44	335.248	4.03 %
Riverside	566.716	573.240	1.24 %
Knoxville	36.280	49.087	35.3 %

Overall, the model shows strong resistance to changes in the assumption of the jockey's production rate. The 25% change in the jockey's efficiency resulted in a percent error of less than 7 % for all cities except Knoxville. However, the alarming figure for Knoxville can be attributed to its position on the extrema of the distribution of car shares; because its car shares index is so low, its percent error is magnified and exaggerated.

Problem 3: Road Map to the Future

$$G = \frac{q*u}{y}$$

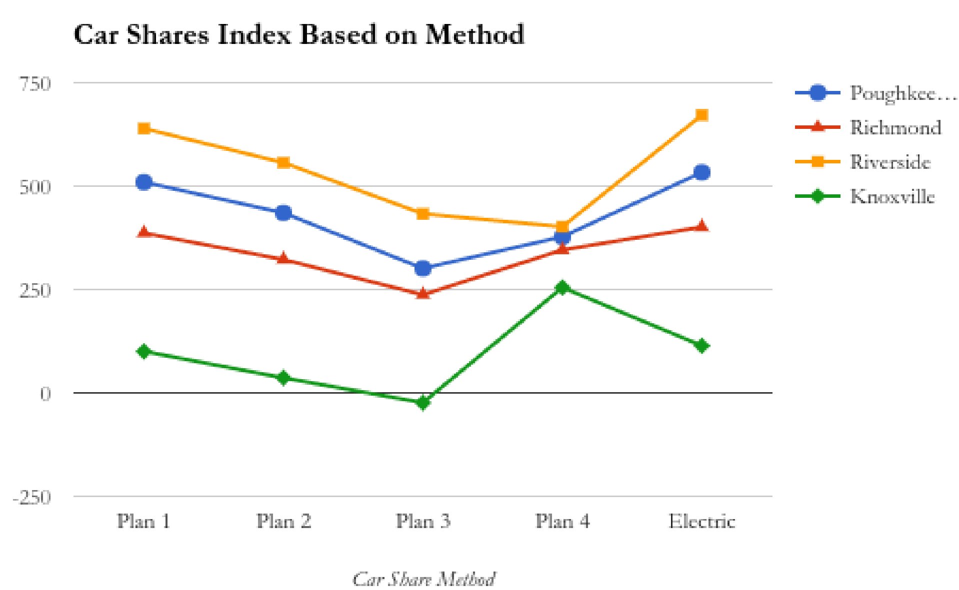
where *G* is the average price spent by one person per day for gas, q is the average gas price of the town, u is the average miles driven per person per day, which is 36.9, and y is the average gas mileage of cars in the United States, which is 25.5 [4][14].

If electricity costs $0.11 per kWh, and the average vehicle consumes 34 kWh to travel 100 miles, the cost per mile is about $0.04 [15]. If we assume that the average miles driven per person per day is 36.9, then the cost to charge an electric car for a day is $1.38 [4].

Figure 11.

City	Plan 1	Plan 2	Plan 3	Plan 4	Electric
Poughkeepsie, NY	509.649	435.291	300.951	377.654	533.279
Richmond, VA	386.473	322.442	237.261	345.659	401.179
Riverside, CA	639.336	556.716	433.116	401.689	671.723
Knoxville, TN	100.311	36.281	-23.784	254.853	113.861

Figure 12.



With the alternative option that comes with the introduction of self-driving and electric cars, the plans from before are slightly altered. When the cars being used are switched to electrical power, the cost per day no longer includes the price of gas. Electrical power is $0.11 per kWh, which amounts to a daily total of $1.38, compared to the daily total one person would spend on gas, which averages to $2.66 in the four cities [16]. By lowering the overall cost, the participation increases in Poughkeepsie, Richmond, and Riverside. There is a decrease in participation in Knoxville as their gas prices are already low enough that the amount a person saves on gas is insignificant compared to how much they would be paying per day to share a car with someone else.

The self-driving car was not taken into account in the calculations because it does not change the numbers, but instead just eliminates plan 2, where the cars are to be repositioned by jockeys. If self-driving cars are used, then this position is no longer needed, as the car will relocate itself to its next desired location. This would correlate to the pricing of plan 3, the one-way car-sharing station model, because the cars will just be dropped off at any spot and can return to the station by themselves. There will be no extra charges applied by using these self-driving cars. This was not included in question 3 as question 3 was based off of our core model, not the reiterations of the models used in smaller parts of question 2.

Conclusion

The results of our model describing the nine characterizations of drivers led to the least number of drivers being in the medium mileage group when compared to the other two mileage groups. Out of the nine characterizations, the least number of drivers were in the high mileage, low time group, while the greatest number of drivers were in the low mileage, low time group. This fits with the distribution of drivers by miles per day being skewed right, causing there to be more drivers with low mileage and time spent driving.

From our model, we concluded that for most cities, the traditional hourly- or daily-based rent plan was the most effective at raising participation in car-sharing programs. We based our model on factors such as income distribution, commuting time, car availability, cost of car sharing programs, and metropolitan population. However, for Knoxville, the private, shared ownership plan was more effective at raising participation, which is probably caused by Knoxville's low starting car shares index. Because of this, even though 70% of people would refuse to consider sharing a privately owned car, the relief gained by the decrease in costs overcomes this hostile attitude. Overall, new technology such as new energy sources and self-driving cars further reduces the cost and will enhance the effectiveness of car-sharing programs in the four cities.

Appendix A: Data of 10 Cities Used for Regression

City	Population (millions)	Time Travel to Work (min)	Cost per Day (USD)	% Household without a Car	% Income between 41k and 65k	% Income between 66k and 106k
NYC	8.175	39.4	84	55.97	17.2	17.8
San Francisco	0.805	31	79	30.4	13.7	17.3
Chicago	2.696	33.7	74	27.3	18.3	17.8
Portland	0.583	24.7	74	15	18.7	21.4

Washington D.C.	0.601	28.4	69	37.9	16	17.4
Seattle	0.608	26	73	16.2	18.1	19.9
San Diego	1.307	22.9	77	7.6	19.7	20.3
Austin, TX	0.79	23.2	74	6.9	20.2	19.9
Miami	0.399	26.6	67	21.5	15.6	11.8
Boston	0.617	29.4	78	35.8	18	17.6

[17][18][19][20][21][22][23]

Appendix B: Data of 4 Cities Analysis

Plan 1: Round Trip Model for a Car-Sharing Program where Vehicles are Rented by the Hour/Day

City	Population (millions)	Time Travel to Work (min)	Cost per Day (USD)	% Households without a Car	% Income Between 41k and 5k	% Income Between 66k and 106k
Poughkeepsie, NY	**0.032736**	**23.9**	**63**	**7**	**17.46**	**17.51**
Richmond, VA	**0.204214**	**21.7**	**64**	**17.2**	**20.8**	**22.8**
Riverside, CA	**0.303871**	**29.2**	**60**	**6.3**	**22**	**22**
Knoxville, TN	**0.178874**	**23**	**63**	**9.8**	**19.8**	**14.6**

Plan 2: Model for a “Floating” Car-Sharing Program with Jockeys

City	Population (millions)	Time Travel to Work (min)	Cost per Day (USD)	% Households without a Car	% Income Between 41k and 5k	% Income Between 66k and 106k
Poughkeepsie, NY	**0.032736**	**23.9**	**67.5**	**7**	**17.46**	**17.51**
Richmond, VA	**0.204214**	**21.7**	**67.875**	**17.2**	**20.8**	**22.8**
Riverside, CA	**0.303871**	**29.2**	**65**	**6.3**	**22**	**22**
Knoxville, TN	**0.178874**	**23**	**66.875**	**9.8**	**19.8**	**14.6**

Plan 3: Model for a Car-Sharing Station Program

City	Population (millions)	Time Travel to Work (min)	Cost per Day (USD)	% Households without a Car	% Income Between 41k and 5k	% Income Between 66k and 106k

Poughkeepsie, NY	**0.032736**	**23.9**	**75.63**	**7**	**17.46**	**17.51**
Richmond, VA	**0.204214**	**21.7**	**73.03**	**17.2**	**20.8**	**22.8**
Riverside, CA	**0.303871**	**29.2**	**72.48**	**6.3**	**22**	**22**
Knoxville, TN	**0.178874**	**23**	**70.51**	**9.8**	**19.8**	**14.6**

Plan 4: Model for a Car-Sharing Station Program

City	Population (millions)	Time Travel to Work (min)	Cost per Day (USD)	% Households without a Car	% Income Between 41k and 5k	% Income Between 66k and 106k
Poughkeepsie, NY	**0.032736**	**23.9**	**17.66**	**7**	**17.46**	**17.51**
Richmond, VA	**0.204214**	**21.7**	**17.66**	**17.2**	**20.8**	**22.8**
Riverside, CA	**0.303871**	**29.2**	**17.66**	**6.3**	**22**	**22**
Knoxville, TN	**0.178874**	**23**	**17.66**	**9.8**	**19.8**	**14.6**

[24][25][26][27][28][29]

References:

1. Chase, Robin. "Does Everyone in America Own a Car?" *YOU ASKED* (n.d.): n. pag. *BUREAU OF INTERNATIONAL INFORMATION PROGRAMS*. US Department of State. Web. 27 Feb. 2016.
2. "Discounts Offered for Auto Insurance from Liberty Mutual." *Car Insurance Comparison*. Car Insurance Comparison, n.d. Web. 27 Feb. 2016.
3. Santos, A., N. McGuckin, H.Y. Nakamoto, D. Gray, and S. Liss. "Summary of Trends: 2009 National Household Travel Survey." *National Household Travel Survey*. US Department of Transportation, June 2011. Web. 27 Feb. 2016.
4. "Average Annual Miles per Driver by Age Group." *Federal Highway Administration*. US Department of Transportation, 20 Feb. 2015. Web. 27 Feb. 2016.
5. "Theory/How Far Do We Drive?/7. Results." *Solar Journey USA*. SolarJourneyUSA.com, n.d. Web. 27 Feb. 2016.
6. Cedar, Jocelyn Milici. "Car Share Infographic: Top 10 U.S. Car Sharing Cities." *Walk Score*. Walk Score, 13 Feb. 2013. Web. 27 Feb. 2016.
7. "What's the Minimum Wage in Your State?" *What's the Minimum Wage in Your State?* N.p., n.d. Web. 27 Feb. 2016.
8. "QuickFacts United States." *United States Census Bureau*. United States Department of Commerce, n.d. Web. 27 Feb. 2016.

9. Dr. Andres Cornet. N.p.: Dr. Andres Cornet, n.d. *Mobility of the Future*. McKinsey and Company, Feb. 2012. Web. 27 Feb. 2016.
10. "Kelley Blue Book - New and Used Car Price Values, Expert Car Reviews."*Kbb.com*. N.p., n.d. Web. 27 Feb. 2016.
11. "What's the Life Expectancy of My Car?" *Msnbc.com*. N.p., 28 Mar. 2006. Web. 27 Feb. 2016.
12. Lee, Jeanne. "What Is the Total Cost of Owning a Car?" *Nerdwallet*. Nerdwallet, Inc., 9 Oct. 2015. Web. 27 Feb. 2016.
13. Berube, Alan, and Alec Friedhoff. "How Big (or Small) Is Your City's Middle Class?" *Brookings*. The Brookings Institution, 9 June 2015. Web. 27 Feb. 2016.
14. Sivak, Michael. "Average U.S. Mpg Edges up to 25.5 in May." *Automotive News*. Crain Communications Inc, 04 June 2015. Web. 27 Feb. 2016.
15. "Charging Plug-In Electric Vehicles at Home." *Alternative Fuels Data Center:*. USA.gov, 5 Jan. 2015. Web. 27 Feb. 2016.
16. "US Average Gas Prices by State - GasBuddy.com." *US Average Gas Prices by State - GasBuddy.com*. GasBuddy, n.d. Web. 27 Feb. 2016.
17. "2010 Census Interactive Population Search." *United States Census 2010*. US Census Bureau, n.d. Web. 27 Feb. 2016.
18. "Land Area for U.S. Cities Map." *Governing the States and Localities*. E.Republic, n.d. Web. 27 Feb. 2016.
19. "Median Household Income in the Top 20 Most Populated U.S. Cities in 2014." *Statista*. Statista, n.d. Web. 27 Feb. 2016.
20. "Miami-Fort Lauderdale-Pompano Beach Florida Household Income." *Department of Numbers*. Department of Numbers, n.d. Web. 27 Feb. 2016.
21. "Portland-Vancouver-Hillsboro Oregon Household Income." *Department of Numbers*. Department of Numbers, n.d. Web. 27 Feb. 2016.
22. "Boston-Cambridge-Quincy Massachusetts Household Income." *Department of Numbers*. Department of Numbers, n.d. Web. 27 Feb. 2016.
23. "Rates and Plans." *Zipcar*. Zipcar, Inc., n.d. Web. 27 Feb. 2016.
24. "Population Demographics for Poughkeepsie, New York in 2016 and 2015." *Suburban Stats*. Suburban Stats, Inc, n.d. Web. 27 Feb. 2016.
25. "Population Demographics for Richmond, Virginia in 2016 and 2015." *Suburban Stats*. Suburban Stats, Inc, n.d. Web. 27 Feb. 2016.

26. "Poughkeepsie, New York (NY) Income Map, Earnings Map, and Wages Data." *City-Data.com*. Advameg, Inc., n.d. Web. 27 Feb. 2016.
27. "California Ranks No. 5 for Annual Pay." *Sacramento Business Journal*. Sacramento Business Journal, 16 Apr. 2012. Web. 27 Feb. 2016.
28. "May 2014 State Occupational Employment and Wage Estimates Tennessee." *Bureau of Labor Statistics*. Bureau of Labor Statistics, 25 Mar. 2015. Web. 27 Feb. 2016.
29. "Commuting." *ETindex*. ET Index, n.d. Web. 27 Feb. 2016.