MathWorks Math Modeling Challenge 2025

Mason High School

Team #18111, Mason, Ohio

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M3 Challenge CHAMPION—$20,000 Team Award

JUDGE COMMENTS

Specifically for Team #18111—Submitted at the close of triage judging

COMMENT 1: You answered the questions and gave details. Well done. You presented good critical and analytical thinking.

COMMENT 2: Executive Summary: Perfect. Problem 1: Good job stating and justifying your differential equation. I couldn't quite see how it was used to get the graphs. Problem 2: What was the specific model you used? I see a lot of variable definitions and explanation, but few formulas. The chart provided contains the coefficients, but it would still help to write out the model so it's clear what you're referring to. Problem 3: Good, if a bit complicated, model with clear explanations and good images and heat maps.

COMMENT 3: Great summary and nice job on all three parts of the challenge. When you did regression, did you check assumptions and residuals?

Hot Button Issue: Staying Cool as the World Heats Up

1 Executive Summary

To the Memphis Office of Emergency Management,

In recent years, the United States experienced rising temperatures and increasingly frequent heatwaves, or extended periods of high temperatures, posing a significant risk to the health of residents [2]. Increases in power demand caused by record-breaking temperatures put major stress on city power grids, furthermore increasing the probability of power outages [31] that then leave even more of the population vulnerable to dangerously high temperatures: a cycle that exacerbates the consequences of heatwaves. Thus, it is imperative for the city of Memphis to examine the impact of heat waves and mitigate their risk.

For households without access to air conditioning, heatwaves pose a particularly severe risk. We first constructed a sinusoidal model for hourly temperatures over a heatwave day and a quadratic model for solar radiation incidence over a heatwave day, then applied both to a model capable of predicting the temperature fluctuations of given residences without air conditioning access during a heatwave day in Memphis. Through a differential equation that determined changes in temperature per hour from incident solar radiation and heat transfer across exterior walls of a dwelling, we determined that homes with less shade coverage were significantly more vulnerable to elevated interior temperatures and that solar radiation exposure was a significantly more important effect than ambient external temperature on interior temperature.

On a larger scale, the integrity of the power grid in Memphis is also particularly susceptible to attack by elevated energy demand during heatwaves. In the second section of our report, we modeled the dependence of both peak hourly load (Pload) and summer peak month total consumption (TC) on the population of Shelby County (which contains Memphis) and maximum annual temperature via a multiple linear regression. We then utilized projected predictors of maximum annual temperature and population to make projections for the peak demand that Memphis' power grid should prepare to handle in 2025 & 2045 respectively. For 2025, Memphis' power grid must be able to support a peak hourly load of 3,433.70 MW, and a TC of 887,513,753 kWh. For 2045, Memphis' power grid must be able to support a peak hourly load between 3527.27 and 3563.07 MW, and a TC between 758,881,008 and 764,797,475 kWh through five comprehensive Shared Socioeconomic Pathways (SSP) greenhouse gas emission scenarios.

In the third section of our report, we assign vulnerability scores (VS) to various neighborhoods in Memphis based on the magnitude of monetarily-quantified impacts they would experience due to heatwaves. 4 predictors (proportion of households with elderly, proportion of households with children, population, and number of residents age 16+ who walk or take public transport to work) were identified through multiple linear regression to be important predictors. These predictors were linearly combined using the coefficients obtained from regression, then normalized and scaled to generate a vulnerability score from 0 to 100. The neighborhoods with zip codes 38028, 38139, 38126, and 38066 had the highest VS (100, 94, 92, and 91, respectively), highlighting them as high-priority targets for resource allocation. Notably, these zip codes had higher proportions of households with elderly and children, suggesting that protecting these vulnerable populations will be essential to minimizing impact of heatwaves in Memphis.

We hope for these results to inform the Office of Emergency Management in adequately preparing for these dire weather crises, as well as deciding best practices for managing them.

Contents

2 Hot to Go

2.1 Defining the Problem:

In the first problem, we are tasked with developing a model to predict the indoor temperature of any non-air-conditioned dwelling during a heat wave over a 24-hour period. We have selected Memphis, Tennessee as our city.

2.2 Assumptions:

1. *During a heatwave, residents will follow all expert advice, which includes closing all windows, blinds, and curtains, and unplugging all electronics.* Experts advise residents to close all windows, blinds, and curtains during a heatwave when outside temperature is hotter than inside temperature [4]. We can assume that all rationally-acting residents will keep their windows closed during the daylight hours of a heatwave. Residents are also advised to unplug all electronics due to them generating a small amount of heat and causing an additional strain on the power grid [5]. We assume residents follow this advice and unplug every device so electrical devices will not contribute to heat gain.
2. *Each story of a home is 10 feet (3.048 meters) tall.* Although story height varies, the average story height is 10 feet tall [6], so we assume every story is 10 feet tall (3.048 meters tall) for simplicity.
3. *Floors and ceilings are fully insulatory, and interior walls are not insulatory.* Thick and rigid insulation is used for ceilings and floors [7], so we assume floors and ceilings are fully insulatory. However, interior walls are frequently left uninsulated in construction [9], so we assume that the house is an open space without interior wall insulation.
4. *Shade is uniformly spread across the dwelling surface area.* It is impossible to know the specific sources and locations of shade sources. For simplicity, the qualitative assessments of shade will comprehensively estimate levels of shade distributed uniformly across the walls and/or ceilings of a dwelling.
5. *Insulation used in a house is independent of year built, and insulation does not deteriorate over time.* Since all the given houses were built in 1953 or later, significantly after the adoption of fiberglass as insulation in Memphis [10], we assume the insulation used in the house (fiberglass) [11] is independent of year. Additionally, because fiberglass insulation does not deteriorate enough to warrant replacement up to a century-long period after initial installation [17], we can assume that insulation quality has remained constant regardless of construction date.
6. *Wind speeds provide a negligible impact on the temperature inside a house.* Since we assume all windows are closed in assumption 2.2.1, wind speeds do not affect the temperature inside a house.
7. *Temperature within a house is uniform at any instant in time, and equal to the temperature on the inside of a wall.* Convection of air promotes even distribution of temperature changes within a house, and this assumption simplifies our model greatly.
8. *A house will be modeled as a rectangular prism with a square base. Each story of a house has equivalent floor surface area.* We do not have floor plans of the given houses, so a rectangular prism with the base being the square footage and height being the number of stories will be used in our model. A square base allows for the most "average" value of total wall surface area.
9. *Furniture has a negligible impact on a dwelling's heat capacity.* The difference in temperature change between an empty room and a fully furnished room of the same size during heating a scenario is less than 0.3 °C [13], so we can assume a dwelling's furniture has no significant impact on its heat capacity.

10. *External walls of all homes are constructed with wood frames.* Wood is a tried-and-true building material. Over 90% of American homes are built with wood frames [22], so we assume that all homes being evaluated are built using wood frames.
11. *Cloud cover during a heatwave is negligible.* During a daytime heatwave, skies are usually clear. [28]
12. *Global horizontal irradiance is an acceptable estimator for the amount of solar irradiation that walls receive.* Although global horizontal irradiance is measured for a horizontal surface, walls experience shortwave radiation reflected from their surroundings as well as more longwave radiation than a flat roof does. Therefore, vertical walls ultimately receive a comparable amount of solar irradiation compared to a horizontal surface. [29]
13. *The outer surface of an external wall will always be 50 degrees hotter than the external atmospheric temperature.* An absorptive roof can be up to 50 degrees hotter than the ambient air temperature due to solar radiation absorption [32]. Because of Assumption 2.2.12, solar radiation incident on an external wall will cause a comparable temperature increase, and temperature will not fluctuate significantly even at night due to the highly insulative properties of exterior walls.

2.3 Variables

Symbol	Definition	Unit	Value
$\frac{dT_{in}}{dt}$	Change of internal temperature	°C/h	
C	Dwelling heat capacity	J/°C	
Q_{rad}	Rate of radiation heat transfer	W	
Q_{flow}	Barrier heat flow	W	
r_s	Shade radiation reduction factor	-	Found below
r_w	Wall radiation reduction factor	-	0.2
$I(\mathrm{t})$	Global Horizontal Irradiance	W/m^2	
A_w	Total surface area of dwelling exterior walls	m^2	
$T_{ext}(\mathrm{t})$	Ambient external temperature	°C	
$T_{int}(\mathrm{t})$	Dwelling internal temperature	°C	
R	Dwelling exterior wall R-value	°C · m^2/W	13 [23]
V	Volume of dwelling	m^3	
c	Specific heat capacity of air	J/g · °C	1.005 [15]
h	Height of one dwelling story	m	3.048 [6]
A_b	Surface area of dwelling floor	m^2	

D	Density of air	kg/m^3	1293 [14]
n	Number of stories in the single dwelling	#	

Table 2.3.1: Variable symbols, definitions, and units used in the model

2.4 The Model

2.4.1 Model Development

To predict the indoor temperature of a dwelling during a heat wave, we develop a model based on the principles of heat transfer. We model the temperature change as a function of time, given by the differential equation:

$$\frac{dT_{in}}{dt} = \frac{1}{C}\left(Q_{rad} + Q_{flux}\right)$$

A differential equation was used due to the several highly time-dependent factors contributing to interior temperature as well as dependence on the instantaneous interior temperature itself due to the physical laws used.

We consider three factors when calculating the heat gained by the unit over time due to solar radiation: the net solar radiation on the dwelling, the shade on the dwelling (which we quantify using a shade radiation reduction factor), and the physical barrier provided by walls around the dwelling (which we quantify using a wall radiation reduction factor). It can be calculated as:

$$Q_{rad} = r_s r_w I(t) A_w$$

The barrier heat flow term accounts for the transfer of heat into or from a dwelling's interior due to a temperature difference between the exterior and interior of the dwelling. Traditionally, heat flux (rate of heat flow per square meter area per unit time) is represented by Fourier's law [30]. As all heat transfer between the interior and exterior of a dwelling must occur at exterior walls, heat flow due to the temperature difference may be expressed in terms of R-value [24] of an exterior wall. R-value is a construction-industry standard measure for thermal resistance of an interface per unit area, and accounts for both the wall's thickness and thermal conductivity. Adjusted to be in terms of the ambient external temperature function by Assumption 2.2.13, the equation for barrier heat flow is as follows:

$$Q_{flow} = \frac{(T_{ext}(t) + 50 - T_{int}(t))A_w}{R}$$

Since, by Assumption 2.2.8, we model the base of the dwelling as a square, we can calculate the total surface area of the exterior walls with the equation below:

$$A_w = 4\sqrt{\frac{A_b}{n}}nh$$

The rate of change in thermal energy of the dwelling is obtained by adding the rate of solar radiation heat gain and the external-internal heat flux terms. In order to find the change in temperature of the dwelling over time, the rate of thermal energy change is divided by the overall heat capacity of the given dwelling.

Heat capacity of the given dwelling is calculated by multiplying the volume of the dwelling with the specific heat capacity of air and the density of air:

$$V = A_b h$$

$$C = VcD$$

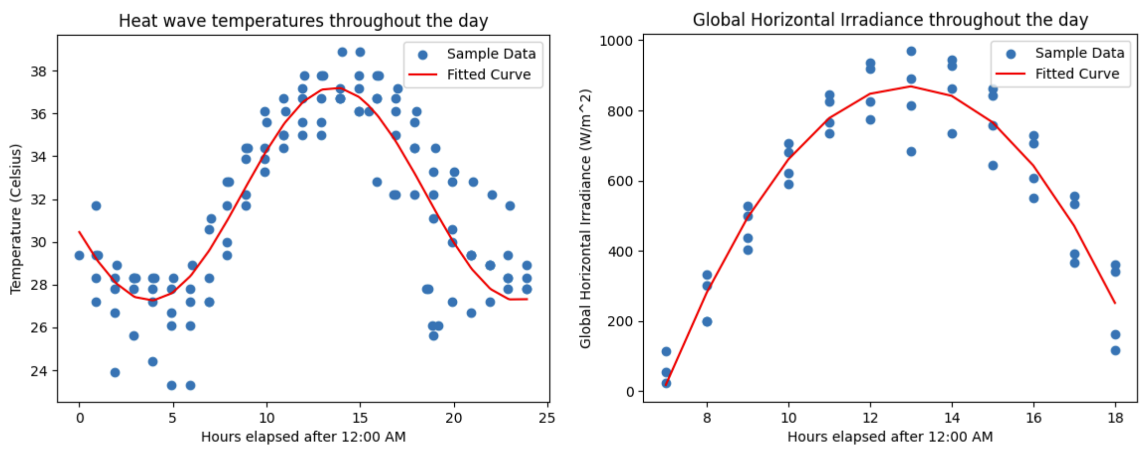
$$C = A_b hcD$$

Combining the heat gained from solar radiation and the heat flux through the walls, the rate of change of indoor temperature is given by:

$$\frac{dT_{in}}{dt} = \frac{1}{A_b hcD}\left(r_s r_w I(t) 4\sqrt{\frac{A_b}{n}} nh + \frac{(T_{ext}(t)+50-T_{int}(t))4\sqrt{\frac{A_b}{n}} nh}{R}\right)$$

2.4.1 Model Execution

To model the ambient external temperature as a function of time during a heatwave, we collected hourly temperature data for heatwave days and plotted them. The resultant graph is a sinusoidal curve, a model frequently used for daily temperature data [21], that represents the average outdoors temperature as a function of hours elapsed after midnight during a heatwave. The Python package SciPy was used to optimize the parameters for this function. Additionally, to determine the solar radiation hitting the homes during heatwaves in Memphis, we collected data and modeled it using a piecewise quadratic model. For times before 7:00 AM and after 6:00 PM, there is 0 horizontal irradiance (due to the Sun being below the horizon). The parameters for this quadratic model were optimized using Python.



Figures 2.4.1 and 2.4.2: Heat wave temperatures and Solar Radiance by hour

We recognized two different forms of heat protection provided by homes: shade from outside sources (i.e a tree) and the wall radiation reduction factor. As shown in Figure 2.4.3, not all the heat prevented by the shade will enter the home, with reductions being shown on both fronts.

The initial temperature (taken at 12:00AM) of the dwelling used in our model was determined from the provided M3 data. Heat transfer occurs due to a tendency towards thermal equilibrium, in which interior and exterior temperatures are equal; it is thus reasonable to adopt the temperature provided as a starting condition for the model.

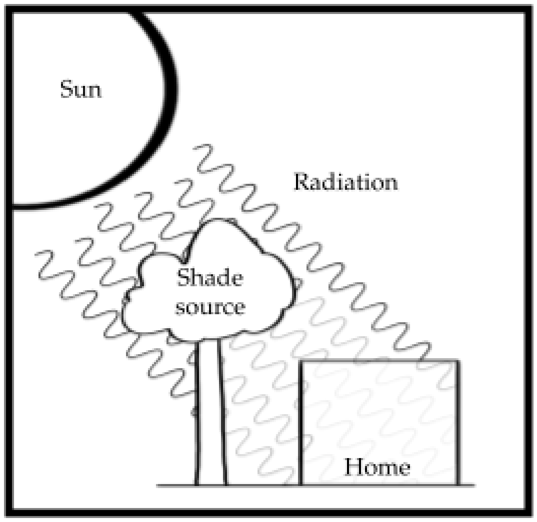


Figure 2.4.3: Visualization of the impact of shade and wall radiation reduction factors

In the Q1 Dwellings tab of the data provided [18], there is categorical data for the shadiness around the dwelling. Based on the approximate amount of solar radiation blocked by varying amounts of shade cover, we assigned each class of shadiness with the corresponding solar radiation reduction rate [25]. By Assumption 2.2.11, cloud cover was ignored in the assignment of shading amount.

Shade Level	r_s
Not at all shady	1
Not very shady	0.7
Somewhat shady	0.4
Very shady	0.1

Table 2.4.1: Values of shade radiation reduction factor corresponding to shade level

The wall radiation reduction factor was determined through taking into account the amount of surface area taken up by windows (approximately 26% of total wall surface area [26]). A standard, room-darkening window curtain blocks approximately 75% of sunlight [27]. By Assumption 2.2.1, all residents will use curtains. Thus, through windows, approximately 20% of incoming solar energy is transmitted into the room, reflected in a multiplier of 0.2 to the solar radiation affecting the dwelling.

The R-value used in the model was determined through the Tennessee International Energy Conservation Code compliance guide. Memphis, located in a Climate Zone 3 region, has a recommended minimum wood-frame wall (by Assumption 2.2.10, all exterior walls are wood-frame) R-value of 13 $°C \cdot m^2/W$ [23].

Python was used to find an analytical solution to our differential equation through solving it as an Initial Value problem and the SciPy.integrate functions. Our initial value is provided above as the average temperature of a dwelling during the summer months without air conditioning (before the heatwave).

2.5 Results

We applied the model to the four homes detailed in the provided data:

	Home 1	**Home 2**	**Home 3**	**Home 4**
A_b	88 m^2	63 m^2	74 m^2	278 m^2
r_s	0.1	0.6	0.95	0.95
n	1	1	1	2
$T_{out}(t)$	$4.979sin(0.320t - 2.775) + 32.236$			
$I(t)$	$-24.1t^2 + 623.69t - 3166.688$			

Table 2.5.1: Input values and model results for given Memphis dwellings

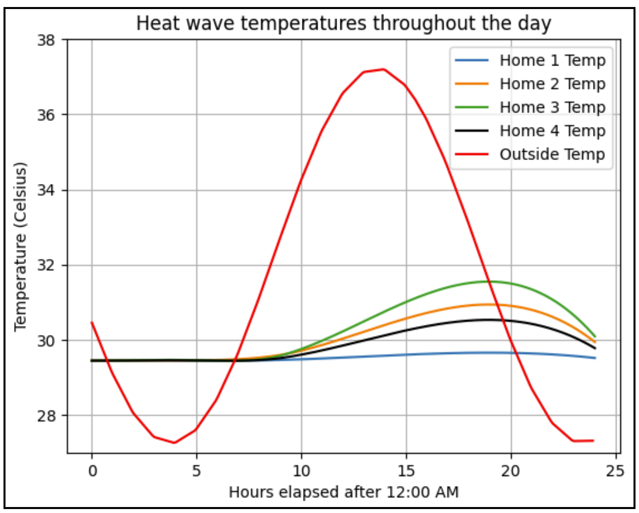


Figure 2.5.2: Heat wave temperatures over 24 hours for the sample data

2.6 Discussion

In summary, our model predicts that homes that are larger and experience more shade coverage will be affected less by the heatwave than smaller homes without shade. For the case of Home 1, our model shows that the heavy shade on the property is significantly reducing the effects of the increased temperature, whereas Home 3 (with a similar area but no shade) is significantly affected by the temperatures. This surprisingly suggests that managing incident solar radiation in a dwelling is significantly more influential in managing interior temperatures–and even for households dependent on the power grid for air conditioning, energy consumption required to maintain comfortable temperatures within the dwelling may be reduced through effective blocking of solar radiation.

Based on this model, a peak dwelling interior temperature occurs at around 18 hours after 12:00 AM, or 6:00 PM, aligning with the intuitive expected location of a temperature peak due to heat-buildup on the interior of the house throughout the day, lending credence to the model.

Additionally, the return of interior temperatures to levels approximately equivalent to the initial temperature at 0 hours demonstrates the cyclical nature of interior temperature levels corresponding to changes in exterior temperatures. This suggests that the initial condition of equal interior and exterior temperatures is valid, because in a day, the temperature ultimately returns to around the outdoor ambient temperature.

2.7 Sensitivity Analysis

To perform a sensitivity analysis on our model, we chose to analyze the impact of variations in the parameters used to determine the Solar Irradiance as a function of time during the heatwave as well as the R parameter representing the wall insulation. The parameters were varied up to 10% and a Monte Carlo simulation for 1000 trials was run:

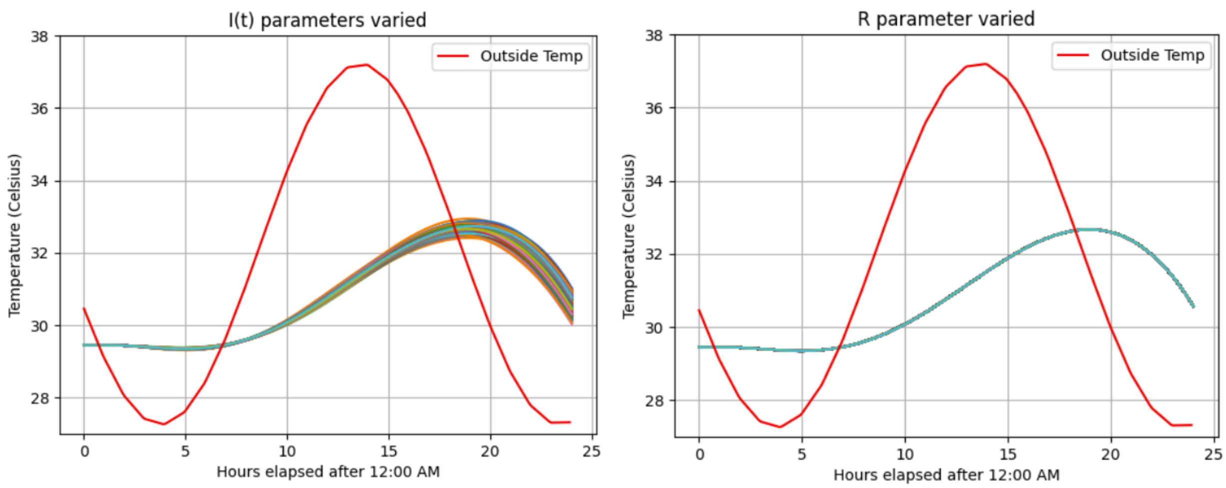


Figure 2.7.1: Model Sensitivity Analysis

By varying the solar irradiance, we found that our model exhibits a slight variation in temperature with either slightly lower or higher irradiations, directly correlating to this variable and maintaining trends regarding the time of day for the peak temperature and general shape of the graph. Varying R, or the thermal resistance of the exterior walls, however, showed very little effect on the temperature change over time, suggesting that solar irradiance is a significantly more impactful contribution to temperature over time.

2.8 Strengths and Weaknesses

The primary strength of our model is its adaptability to different types of dwellings, as it allows for the input of specific parameters such as floor area and shade factor. For individual dwellings, floor area and shade factor are easily obtainable information. Additionally, the model is fairly resistant to changes to both the solar irradiance factor and the thermal resistance of exterior walls, with a maximum difference in temperature of the dwellings 24 hours from the model initiation of about one degree Celsius.

However, the model's strength of adaptability derives from fairly generalized assumptions about the information available about each dwelling. We have limited information about floor plans, construction, and heat absorbed by objects within the house which alter factors like the heat flow through exterior walls. If we wanted to scale this to multiple dwellings or neighborhoods, we would have to have a large amount of information about included residences. We also assumed that heat loss through ventilation is negligible due to closed windows. While this is reasonable during a heat wave, if people choose to open windows or if there are air leaks, this will not hold true. In general, people are not necessarily rational actors in a real-world scenario;

the assumption that residents follow all recommended advice for heat wave mitigation such as closing windows may result in an undershoot of the temperatures that interiors may reach.

2.8.1 Model Refinement

Our model may be improved through more comprehensive consideration of the thermodynamic factors behind heat transfer into the interior of the home. Due to our limited physical sciences knowledge, we were only able to take a basic application of heat flux into account. Our simplifying assumptions, such as the complete insularity of ceilings and floors as well as a fixed exterior wall surface-ambient temperature difference, are intended to reduce the number of thermodynamic relationships we would have to take into account; consequently, they will most likely cause a deviation from actual interior temperatures.

3 Power Hungry

3.1 Defining the Problem:

In this problem, we are asked to develop a model that predicts the peak demand that Memphis should be prepared to handle during the summer months and how it changes 20 years from now.

3.2 Assumptions:

1. *Projected peak global temperature growth is approximately linear over the 78-year period from 2022 to 2100.* Global temperature change is dependent on many unpredictable physical, industry, and political factors. For simplicity, we assume that the temperature in Memphis, Tennessee increases linearly from the historical average according to the Shared Socioeconomic Pathways (SSP) greenhouse gas emission scenarios.
2. *There are no major technological advancements that improve air conditioning technology in the next 20 years*. It is extremely difficult to predict the onset and impact of innovation. Additionally, assuming no advancements would result in a more conservative (and thus safer) estimate for the power demand. Therefore, our model will not account for advancements in air conditioning technology.

3.3 Variables

Symbol	Definition	Unit	Value (2025)	Value (2045)
pop	Annual population of Shelby County	People	897,412 [36]	869,395 [44]
mtemp	Annual maximum recorded temperatures in Memphis	F°	103	Varies
Pload	Peak hourly demand	MW	3,433.70	Varies
TC	Total consumption of peak summer month in Shelby County	kWh	887,513,753	Varies

Table 3.3.1: Variable symbols, definitions, and units used in the model

3.4 The Model

3.4.1 Model Development

Peak demand can be broken into two definitions: 1) the peak hourly demand or 2) the total consumption in the peak summer month. We calculated the total consumption in the peak summer month by multiplying the provided Shelby County (Memphis) annual electricity consumption with the ratio of the provided maximum 2024 monthly electricity consumption for east south central USA (Kentucky, Tennessee, Mississippi, Alabama) in the month of August to the total 2024 electricity consumption for east south central USA.

We first examined various factors to determine correlations with the peak hourly demand and total consumption in the peak summer month, using the provided data for annual maximum recorded temperatures and the annually recorded population of Shelby County from 2012-2022 (which includes the population of Memphis city and explains the total consumption). In our preliminary data analysis, we plot each variable over each year side-by-side.

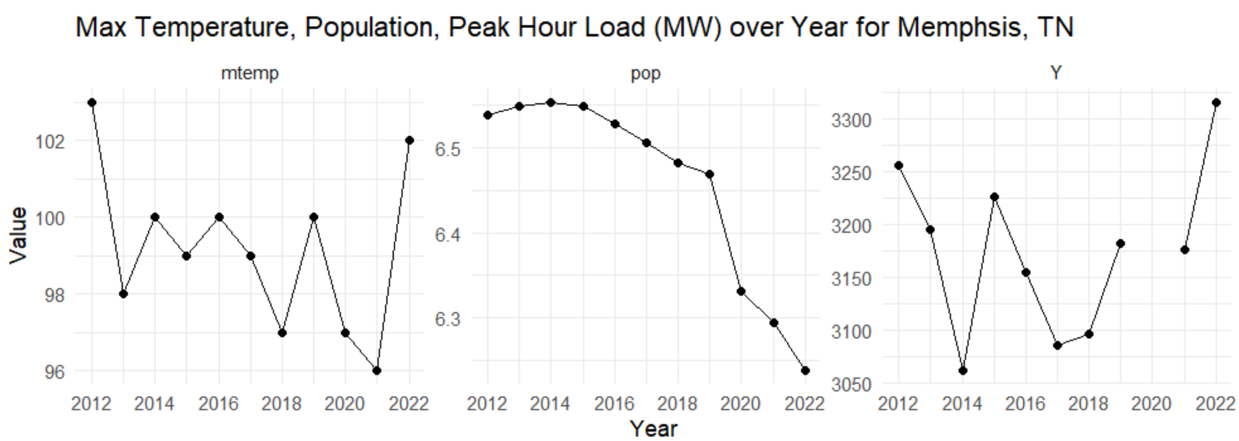


Figure 3.4.2.1: Annual Max Temperature, Population, & Peak Hourly Load for Memphis

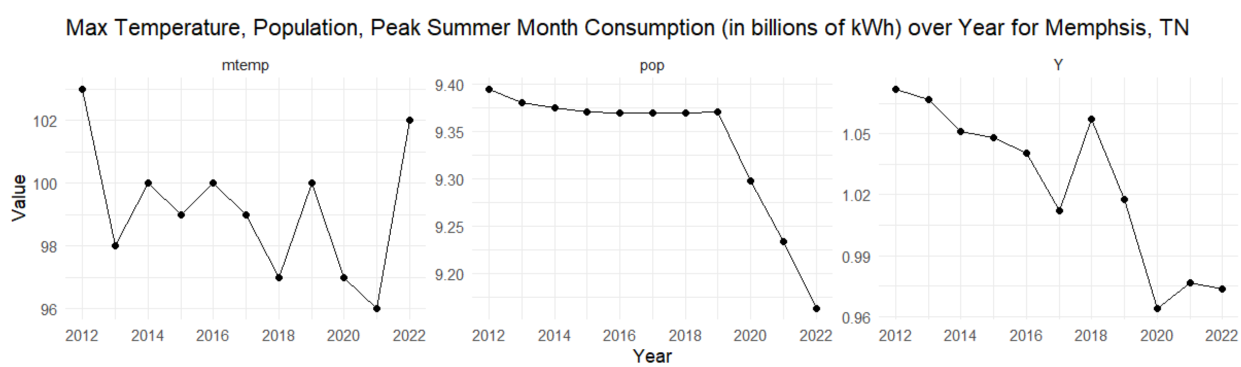


Figure 3.4.3.1: Annual Max Temperature, Population, & Consumption of Peak Summer Months

From our preliminary data analysis, we noticed the population variable is closely correlated with both definitions of peak demand, and thus decided to conduct a multiple linear regression to predict Memphis' peak hourly load and total consumption in the peak summer month. The general form for a multiple linear regression is as given, where y is the dependent variable, x_p is the p^{th} independent variable, β_p is the coefficient of the p^{th} independent variable, and ε is an error term:

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + ... + \beta_p x_p + \varepsilon.$$

Using the provided data for annual maximum recorded temperatures and the annually recorded population of Shelby County, we created multiple linear regressions with either *Pload* or *TC* as the dependent variable and *mtemp* and *pop* as the independent variables.

Peak Hourly Load (MW)		
	Coefficient	p-value
Intercept	6220.61	0.0596
mtemp	18.60	0.1052
pop	-524.02	0.1004
R^2		0.501
Adjusted R^2		0.358

Table 3.4.1.1: Multiple linear regression coefficients for Peak Hourly Load

Total Consumption of Peak Summer Month (kWh)		
	Coefficient	p-value
Intercept	-3.34	0.00966**
mtemp	0.004	0.27616
pop	0.42	0.00307**
R^2		0.7093
Adjusted R^2		0.6366

Table 3.4.1.2: Multiple linear regression coefficients for Consumption of Peak Summer Month

We also fitted various models, including $y \sim mtemp$, $y \sim pop$, $y \sim mtemp + pop + lag(y)$. Despite the resulting p-values being around 0.1, the proposed multiple linear regression model has the best performance in terms of R^2 and adjusted R^2 over the other linear regression models. We chose not to utilize nonlinear models or machine learning models, as the amount of data is limited.

3.4.2 Forecasting 2025 Peak Hourly Demand & Total Consumption of Peak Summer Month

We then project Memphis' 2025 peak hourly load and peak summer month total consumption with our fitted multiple linear regression, using the maximum annual recorded maximum temperature in Memphis from

2012-2022 and the forecasted 2025 population of Shelby County according to World Population Review [36]. Our rationale for using the maximum annual recorded maximum temperature available to us was so that a more conservative estimate for the load could be generated, mitigating the potential effect of year-to-year maximum fluctuations.

3.4.3 Forecasting 2045 Peak Hourly Demand & Total Consumption of Peak Summer Month

To project Memphis' peak hourly load and peak summer month total consumption in 2045, we use the forecasted 2045 population of Shelby County according to the Boyd Center for Business and Economic Research and under varying global temperature projections using Scenario Analysis in Excel. Global temperature projections over the next 20 years vary based on different greenhouse gas emission scenarios. According to the Intergovernmental Panel on Climate Change's (IPCC) Sixth Assessment Report (AR6), global warming is anticipated to reach or exceed 1.5°C above pre-industrial levels by 2040 across all considered scenarios [43].

We computed the 2045 maximum annual temperature as linearly increasing from the average of the 2012-2022 historical maximum annual temperatures by the expected annual change in global temperatures as according to the SSP1-1.9, SSP1-2.6, SSP2-4.5, SSP3-7.0, and SSP5-8.5 greenhouse gas emission scenarios [46]. Each SSP scenario estimates different levels of global greenhouse gas emissions, ranging from optimistic to pessimistic. As seen in Table 3.4.3.1, these directly impact their respective projected global temperature changes.

Shared Socioeconomic Pathways (SSP) Scenario	Optimism Ranking (i)	Projected 2100 Global Change in Temperature (°F)	Projected 2045 Global Change in Temperature (°F)
SSP1-1.9 (Optimistic)	1	2.88	2.88
SSP1-2.6	2	3.24	0.96
SSP2-4.5	3	4.86	1.43
SSP3-7.0	4	6.48	1.91
SSP5-8.5 (Pessimistic)	5	7.92	2.34

Table 3.4.3.1: Shared Socioeconomic Pathways temperature projections

Using the values from Table 3.4.3.1, the 2045 maximum annual temperature values were calculated as follows:

$$mtemp = m_i t + \mu,$$

where m_i is the projected 2045 global change in temperature according for the i^{th} optimism ranking, t is the years since 2022, and μ is the average of the provided annual maximum temperature in Memphis from 2012-2022, 99.18 °F. Our rationale for using the average annual maximum temperature available to us was so that a more robust estimate for the dependent variables could be generated for 20 years into the future.

3.5 Results

We project the coming 2025 summer's peak hourly load and peak month total consumption using a multiple linear regression based on the maximum annual recorded maximum temperature in Memphis and the forecasted 2025 population of Shelby County.

Year	Peak Hourly Load (MW)	Total Consumption in Peak Summer Month (kWh)
2025	3,433.70	887,513,753

Table 3.5.1: Energy demand projections for Memphis in 2025

Similarly, we also project Memphis' 2045 peak hourly load and peak summer month total consumption under varying SSP emission scenarios using the Scenario Analysis in Excel.

Scenario Summary	Projected 2045 Emission Scenarios				
	SSP1-1.9	SSP1-2.6	SSP2-4.5	SSP3-7.0	SSP5-8.5
Projected Change in Temp (°F)	2.88	0.96	1.43	1.91	2.34
Peak Hourly Load (MW)	3,563.07	3,527.27	3,536.15	3,545.04	3,552.94
Summer Peak Month Total Consumption (kWh)	764,797,475	756,927,721	758,881,008	760,834,287	762,570,543

Table 3.5.2: Energy demand projections for Memphis in 2045

3.6 Discussion, Strengths, and Weaknesses

Our model shows a promising fit based on the given data, and our projected peak load and summer peak month total consumption would provide insightful guidelines for future planning. However, our results are reliant on inputted projections for temperature change and population level, and those projections could change significantly over time. In this case, we would have to adjust our model accordingly to follow new projected predictions. Given the annual data available, we were limited to linear models, but if we had access to high frequency, hourly data across several years, then we may be able to use more complex models such as SARIMA.

3.7 Sensitivity Analysis

We conducted sensitivity analysis on the 2045 projections using the different temperature changes projected by the SSP's varying greenhouse gas emissions standards. The percent difference between the maximum and minimum peak hourly loads was 1.009%, and the percent difference between the maximum and minimum summer peak month total consumption was 0.78%. These low percent differences, in spite of a fairly significant difference in projected temperature changes, indicate the resilience of our model against slightly altered predictor variables.

4 Beat the Heat

4.1 Defining the Problem:

In the third problem, we are tasked with developing a vulnerability score for various neighborhoods to help them allocate resources to minimize the effects of a heat wave or power grid failure. Additionally, we are asked to propose a single approach for how Memphis can incorporate these scores into their management of heat waves.

4.2 Assumptions:

1. *The effects of a heatwave on a population can be quantified using Expected Annual Loss.* Expected Annual Loss is the average economic loss calculated using a multiplicative equation that includes exposure, annualized frequency, and historic loss ratio risk factors for natural hazards (heatwaves in our case) [34].
2. *Fatalities and other health complications can be monetarily quantified.* Using a value of statistical life (VSL) approach, one fatality or 10 injuries can be treated as $11.6 million lost [34]. By quantifying the population Expected Annual Loss in terms of dollars, we can ensure a common unit of measurement across each type of loss.

4.3 Variables

Symbol	Definition	Unit	Value
L_H	Monetary loss from population health	dollars	
L_A	Monetary loss from agriculture	dollars	
L_B	Monetary loss from buildings	dollars	
L_{total}	Total monetary loss from a heat wave	dollars	Appendix 7.3.1
x_e	Proportion of households with one or more people 65 years and over		Appendix 7.3.1
x_c	Proportion of households with one or more people 18 years and under		Appendix 7.3.1
x_p	Population of neighborhood	10,000 people	Appendix 7.3.1
x_t	Number of persons aged 16+ whose primary mode of transportation to work is walking or public transit	people	Appendix 7.3.1
VS	Vulnerability score		Table 4.5.1

Table 4.3.1: Variable symbols, definitions, and units used in the model

4.4 The Model

We define our vulnerability score as a measure of a population's susceptibility to harmful effects from a heatwave.

In order to evaluate the extent of these harmful effects within a neighborhood, we examined 3 main impacts of heatwaves: health decline, agriculture strain, and property damage. Our data regarding these effects comes from the National Risk Index (NRI) Census Tracts, where each census tract has their expected annual loss due to heat waves for our three consequence types: population health (L_H), agriculture (L_A), and building value (L_B). For all the loss values, we use annualized numbers using National Risk Index data from 2005 to 2021 [34]. We define total loss:

$$L_{total} = L_H + L_A + L_B$$

Since our data only had census tract data, we used zip codes to draw a sum of loss within the census tracts within each neighborhood [35].

We then identified predictors that may potentially increase the susceptibility of a population to the heatwave consequences, such as age, income, and proportion of open space (a full list of evaluated predictors can be found below). In order to determine the most important predictors, we ran a multiple linear regression model between the selected predictors and the heatwave expected total annual loss per capita. Independent predictor variable data values used were also scaled to be comparable for ease of interpretation. A level of significance of $\alpha < 0.05$ indicated that a given predictor had a significant impact on the neighborhood's heatwave susceptibility. The general form for a multiple linear regression is as given, where y is the dependent variable, x_p is the pth independent variable, β_p is the coefficient of the pth independent variable, and ε is an error term:

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + ... + \beta_p x_p + \varepsilon.$$

Python was used to create a multiple linear regressions model and determine our p-values and coefficients for use as our vulnerability score weights. Data was first imported from the xlsx file as a Dataframe and SkLearn was used to develop our model. The code for this model can be found in our appendix.

After running the multiple linear regression model, we determined that there were 4 variables that could be determined as significant. The fitted coefficients of two models we used are given below.

	Coefficient	p-value
Proportion of households with elderly	78.9062	0.083
Proportion of households with children	96.2070	0.044

Population (10,000 people)	-7.1583	0.018
Primary mode of transportation to work (persons aged 16 years+): walking or public transit	0.0707	0.039
Proportion of developed, open space in neighborhood	-44.1021	0.212
Median household income (in 100,000s of US dollars)	12.1282	0.307
Mean number of homes built 1950 or earlier (in 1000s)	-3.6117	0.406

Table 4.4.1: Regression coefficients and p-value between loss per capita and various predictors

Based on these values, we determined that the proportion of developed open space in a neighborhood, the median household income, and the mean number of homes built before 1950 were not significant when considering the vulnerability ratings of the neighborhoods around Memphis, allowing us to remove them and create a simpler model as our final model with coefficients below.

	Coefficient (w)	p-value
Proportion of households with elderly	84.9225	0.026
Proportion of households with children	105.4934	0.023
Population (10,000 people)	-7.3073	0.014
Number of residents aged 16+ who walk or take public transport to work.	0.0336	0.025

Table 4.4.2: Final regression coefficients and p-value between loss per capita and final set of predictors

We propose to calculate our vulnerability scores (VS) for each given neighborhood using a weighted sum (linear combination of predictors), where the above coefficients are the "weights" (w) for each predictor being considered, modeled below:

$$VS = w_e x_e + w_c x_c + w_p x_p + w_t x_t$$

Vulnerability scores were then normalized using min-max normalization, multiplied by 100, and rounded to the nearest integer for simplicity.

$$VS_{final} = \frac{VS - min(VS)}{max(VS) - min(VS)} \times 100$$

4.5 Results

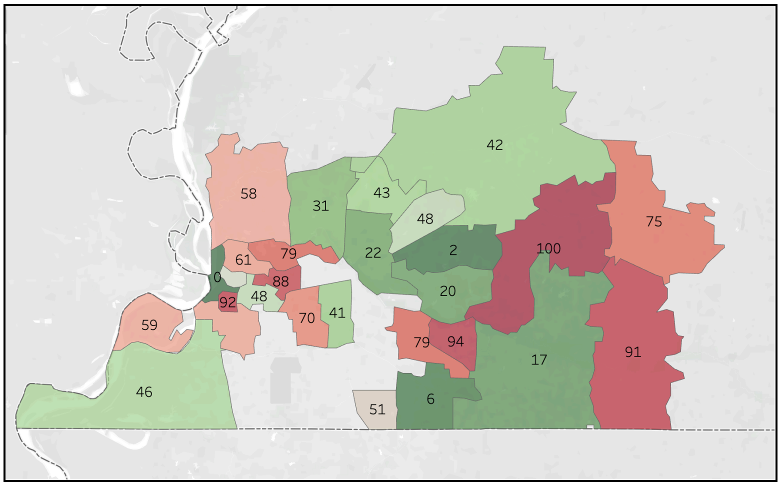


Figure 4.5.1: Heat map of vulnerability scores (0-100) for Memphis neighborhoods by ZIP code

Figure 4.5.1, generated from Tableau [45], geographically depicts the vulnerability scores with a red to green gradient with darker red shades indicating higher vulnerability and darker green shades indicating lower vulnerability. The final numerical vulnerability scores are also shown on the map. The map identifies high-risk areas, such as the east and west corners of Memphis, guiding resource allocation.

Neighborhood ZIP Code	**Vulnerability Score**	38109	46
38103	0	38111	70
38002	42	38112	88
38017	17	38117	41
38016	2	38125	6
38018	20	38126	92
38028	100	38127	58
38060	75	38128	31
38066	91	38133	48
38104	48	38134	22
38105	50	38135	43
38106	59	38138	79
38107	61	38139	94
38108	79	38141	51

Table 4.5.1: Vulnerability scores (0-100) for Memphis neighborhoods by ZIP code

Table 4.5.1 gives the final vulnerability scores scaled from 0 to 100 with provided ZIP codes. The top third of VS is shown in red, the middle third of VS is shown in yellow, and the lower third of VS is shown in green.

4.6 Discussion

From our calculated VS, we recommend that Memphis should first target areas with higher vulnerability scores such as ZIP codes 38028 (100), 38139 (94), 38126 (92), and 38066 (91). For example, the highest risk was associated with ZIP code 38028, Hickory Withe, with a loss per capita of $98.672 million. Hickory Withe has a proportion of households with at least one person 65 or over of 39.4%, a proportion of households with at least one child of 36%, a total population of 76,990, and 9 residents aged 16+ who walk or use public transit as their primary mode of transportation.

The primary contributors to these areas' high vulnerability scores are a higher proportion of households with elderly people and children, a lower population, and more people that walk to work. Intuitively, this result makes sense: for example, elderly and child populations are significantly more susceptible to heat-related illnesses [42]. In order to mitigate this effect, Memphis should focus on creating more cooling centers and increasing emergency supplies, specific for the elderly and children, in these areas. Additionally, in the long term, Memphis should invest in long term infrastructure improvements such as improving the accessibility and integrity of public transportation to reduce exposure to extreme heat for those who walk or use public transit as their primary mode of transportation to work.

We can also see a trend in areas with high vulnerability scores in the generated map. Neighborhoods in the southeast area and west of Memphis tend to have higher vulnerability scores. Thus, since city funds are limited and we can assume the government cannot establish cooling centers in every Memphis neighborhood, we recommend Memphis institutes cooling centers in the east and west corners of the city for maximum accessibility.

4.7 Sensitivity Analysis

To determine the accuracy of our prediction, we randomly offset each coefficient by up to 5% and then ran the model again to get new vulnerability scores. Then we calculated the percent difference between the original prediction and new prediction. We repeated this process five times and averaged the percent differences across all the neighborhood vulnerability scores. The average jittered variation of vulnerability score was found to be 3.47%. Since this jittered variation is relatively low, we are relatively confident in our model's resilience to random error.

4.8 Strengths and Weaknesses

The primary strength of our model is that by using Expected Annual Loss values, we can account for multiple impacts of heatwaves in our vulnerability score. Instead of only using impacts on health, we also consider impacts on agriculture and buildings, making our vulnerability scores more reflective of a population's susceptibility to harmful effects from a heat wave. We were able to do so by monetarily quantifying losses due to fatalities or other health complications, and measuring all losses in terms of expected dollars lost per year.

In addition, we generated a heat map of Memphis neighborhood vulnerability scores by ZIP code. This visualization is easier for the city planners to digest and provides clearer insights to help them decide where resources should be allocated. For instance, a neighborhood may seem in dire need of resources based on a high vulnerability score, but in reality make up a tiny amount of Memphis' total population. In this case, it would likely be better to allocate resources to a less-vulnerable, but larger neighborhood.

A major limitation of our model was our inability to incorporate additional predictors into the vulnerability score, such as the heat island effect (dependent on vegetation cover of a neighborhood) due to insufficient available data even after exhaustive searching online. If, for example, the heat island effect were a significant variable, our recommendations would likely have included investing in green spaces.

4.8.1 Model Refinement

Finding more data on relevant predictors could greatly expand the number of predictors found to be significant contributors to heatwave impact, improving the accuracy of the vulnerability score as a measure of overall impact on a neighborhood. In order to more definitively allocate resources, the budget of the city of Memphis could also be taken into consideration; to further inform policy decisions that aim to mitigate heat wave impacts, spatial correlation could be applied to identify optimal locations for our recommended cooling centers as an extra measure to accompany vulnerability scores.

5 Conclusion

5.1 Further Studies

To enhance the applicability of our models, there are many ideas that warrant further studies. For our indoor temperature prediction model, future investigation could incorporate dynamic weather patterns to better capture localized temperature variations. Additionally, exploring the impact of building spaces, colors (white vs. black building) and ventilation strategies could provide more accurate predictions for different types of dwellings. For our power grid demand forecasting model, we could attempt to integrate variables such as heating or cooling degree days to improve long-term prediction. Furthermore, understanding the SSP scenarios used and accounting for global trends that come with climate change could help mitigate peak energy loads. Finally, for our vulnerability scoring model, incorporating real-time data on population movement, behavioral responses during heat waves, finding data on the current Memphis data and getting clearer data about the health risks (deaths directly caused by heat vs. deaths caused by conditions exacerbated by heat) for each neighborhood would allow us to create a more comprehensive resource allocation plan. We could also consider adding spatial correlation to understand the optimal locations for cooling centers.

5.2 Summary

In this paper, we address the growing threat of heat waves in Memphis, Tennessee, by developing models to predict indoor temperatures in non-air-conditioned homes, forecast future energy demands on the power grid, and assign vulnerability scores to neighborhoods based on heat wave impacts.

Our indoor temperature model revealed that homes with less shade coverage are significantly more vulnerable to elevated interior temperatures, emphasizing the importance of shading and insulation in mitigating heat wave risks. For our second model, we projected that Memphis must prepare for a peak hourly load of 3,433.70

MW in 2025 and between 3,527.27 and 3,563.07 MW in 2045, alongside significant increases in summer peak month total consumption. These findings demonstrate the need for investments in grid resilience and energy efficiency to handle rising energy demands during extreme heat events. The reduction in efficiency over the years also brings to attention the importance of increasing efficiency of AC.

Finally, our vulnerability scoring model identified neighborhoods with the highest risk, particularly those with large proportions of elderly and children, such as ZIP codes 38028, 38139, 38126, and 38066. These areas should be prioritized for resource allocation, including cooling centers and emergency supplies. Together, these models provide actionable insights for the Memphis Office of Emergency Management to better prepare for and manage the impacts of heat waves, ensuring the safety and resilience of the city's residents in the face of a warming climate.

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7 Code Appendix

7.1 Hot To Go

```
# Code for Figure 2.4.1
import numpy as np
import matplotlib.pyplot as plt
from scipy.optimize import curve_fit
import pandas as pd

# Data import goes here, we are linking to a spreadsheet that contains data we found for 5
different Heatwave days
df =
pd.read_excel('https://docs.google.com/spreadsheets/d/1gQrosZuqWKW6CsYGKLHJ49BeoSHT1T_l/exp
ort?format=xlsx', sheet_name='LOTS OF HEATWAVE TEMPS', engine='openpyxl')

#Removes the null rows at the top and renames the first row to be the titles
df = df.drop(0)
df.columns = df.iloc[0]
df = df[1:]
df.head()

# Convert the datetime.time objects to numerical representations (the hours elapsed since
12:00 AM) so that we can graph the various days on top of each other
x = (pd.to_datetime(df.iloc[:,1].astype(str)).dt.hour * 3600 + pd.to_datetime(df.iloc[:,
1].astype(str)).dt.minute * 60 + pd.to_datetime(df.iloc[:, 1].astype(str)).dt.second)/3600
y = df.iloc[:,3]

# Definition of the curve that we are going to use scipy.optimize to find the parameters
for.
def sine_function(x, A, B, C, D):
    return A * np.sin(B * x + C) + D

# Perform the curve fitting using time normalized x-values and provided y-value
temperatures. Additionally, we provide a "guess" for the parameters.
params, covariance = curve_fit(sine_function, x, y, p0=[1, 2*np.pi/(24*3600), 0,
np.mean(y)])

# Extract the fitted parameters using the curve fit optimization built into SciPy
A_fit, B_fit, C_fit, D_fit = params

# Displaying said parameters
```

```
print(f"Fitted parameters: A={A_fit}, B={B_fit}, C={C_fit}, D={D_fit}")

# Sort normalized x-values for plotting so similar time data points are located next to
each other (original data is sorted by day)
x_sorted = np.sort(x)
y_fit_sorted = sine_function(x_sorted, A_fit, B_fit, C_fit, D_fit)

# Using MatPlotLib to plot the scatter plot of the data combined with the optimized curve
plt.scatter(x, y, label='Sample Data')
plt.plot(x_sorted, y_fit_sorted, label='Fitted Curve', color='red')
plt.xlabel('Hours elapsed after 12:00 AM')
plt.ylabel('Temperature (Celsius)')
plt.title('Heat wave temperatures throughout the day')
plt.legend()
plt.show()
```

```
# Code for Figure 2.4.2
import numpy as np
import matplotlib.pyplot as plt
from scipy.optimize import curve_fit
import pandas as pd

# Data import goes here, we are linking to a spreadsheet that contains data we found for 5
different Heatwave days
df =
pd.read_excel('https://docs.google.com/spreadsheets/d/1RaHFbtKV04xqiwwVMWwfupebBhJqOre5cCYb
UhLKPu0/export?format=xlsx', sheet_name='SOLAR IRRADIANCE', engine='openpyxl')

# Convert the datetime.time objects to numerical representations (the hours elapsed since
12:00 AM) so that we can graph the various days on top of each other
x = (pd.to_datetime(df.iloc[:,2].astype(str)).dt.hour * 3600 + pd.to_datetime(df.iloc[:,
2].astype(str)).dt.minute * 60 + pd.to_datetime(df.iloc[:, 2].astype(str)).dt.second)/3600
y = df.iloc[:,3]

# Definition of the curve that we are going to use scipy.optimize to find the parameters
for.
def quadratic(x, A, B, C):
    return A * x**2 + B * x + C

# Perform the curve fitting using time normalized x-values and provided y-value
temperatures. Additionally, we provide a "guess" for the parameters.
```

```
params, covariance = curve_fit(quadratic, x, y, p0=[1, 2*np.pi/(24*3600), 0])

# Extract the fitted parameters using the curve fit optimization built into SciPy
A_fit, B_fit, C_fit = params

# Displaying said parameters
print(f"Fitted parameters: A={A_fit}, B={B_fit}, C={C_fit}")

# Sort normalized x-values for plotting so similar time data points are located next to
each other (original data is sorted by day)
x_sorted = np.sort(x)
y_fit_sorted = quadratic(x_sorted, A_fit, B_fit, C_fit)

# Using MatPlotLib to plot the scatter plot of the data combined with the optimized curve
plt.scatter(x, y, label='Sample Data')
plt.plot(x_sorted, y_fit_sorted, label='Fitted Curve', color='red')
plt.xlabel('Hours elapsed after 12:00 AM')
plt.ylabel('Global Horizontal Irradiance (W/m^2)')
plt.title('Global Horizontal Irradiance throughout the day')
plt.legend()
plt.show()
```

```
# Code for visualizing and solving the Model discussed in 2.5 and for Figure 2.5.2.
import numpy as np
import matplotlib.pyplot as plt
from scipy.integrate import solve_ivp
import math

# This function defines the temperature model we discussed in Section 2.4 and has inputs
with all the predefined variables.
def temperature_model(A_b,h,c,D,r_s,r_w,R,n):
  # Equation based on the rectangular prism assumption we made before
  A_w = 4 * math.sqrt(A_b/n) * n * h

  # Temperature as a function of time as modeled in Figure 2.4.1
  def T_out(t):
      return 4.9796776692867555 * np.sin(0.31958467920431916 * t + -2.7753696625091497) +
32.23606417137194

  # Global Horizontal Irradiance as a function of time as modeled in Figure 2.4.2
  def I(t):
```

```
      return np.piecewise(t, [t < 7, (t >= 7)], [0,-24.099845890995997 * t**2 +
623.6904007623635 * t + -3166.6881716727808])

  # Define the differential equation incorporating T_out(t) and R(t)
  def dT_dt(t, T_in):
      return (1 / (A_b * h * c * D)) * (r_s * r_w * I(t) * A_w +
(((T_out(t)+50-T_in)*A_w)/R))

  # Time span and initial conditions, t_span is the 24 hour period for modeling and t_in_0
is the initial temperature.
  t_span = (0, 24)
  T_in_0 = 29.444

  # Solving the Initial Value Problem using SciPy.integrate libraries
  solution = solve_ivp(dT_dt, t_span, [T_in_0], t_eval=np.linspace(0, 24, 500))
  return solution.t, solution.y[0]

# Calling the function described above with the parameters discussed and listed in Section
2.5
home1_x, home1_y = temperature_model(88,3.048,1.005,1293,0.1,0.2,13,1)
home2_x, home2_y = temperature_model(63,3.048,1.005,1293,0.6,0.2,13,1)
home3_x, home3_y = temperature_model(74,3.048,1.005,1293,0.95,0.2,13,1)
home4_x, home4_y = temperature_model(278,3.048,1.005,1293,0.95,0.2,13,1)

# Graphing all the plots together, as well as the data for the outside temperatures. The
data for outside temperatures can be found above in the Appendix.
plt.plot(home1_x, home1_y, label="Home 1 Temp")
plt.plot(home2_x, home2_y, label="Home 2 Temp")
plt.plot(home3_x, home3_y, label="Home 3 Temp")
plt.plot(home4_x, home4_y, label="Home 4 Temp", color='black')
plt.xlabel("Time (hours)")
plt.ylabel("Temperature (K)")
plt.ylim(27,38)
plt.title("Temperature Evolution Over Time")
plt.legend()
plt.grid()
plt.plot(x_sorted, y_fit_sorted, label='Outside Temp', color='red')
plt.xlabel('Hours elapsed after 12:00 AM')
plt.ylabel('Temperature (Celsius)')
plt.title('Heat wave temperatures throughout the day')
plt.legend()
```

```
plt.show()
```

```
# Code for the Sensitivity Analysis
import numpy as np
import matplotlib.pyplot as plt
from scipy.integrate import solve_ivp
import math

# Using the parameters for Home 1 to perform this sensitivity analysis
A_b = 88
h = 3.048
c = .1005
D = 1293
r_s = 0.15
r_w = 0.9
R = 13
n = 1

# This function defines the temperature model we discussed in Section 2.4 and has inputs
with all the predefined variables.
def temperature_model(A_b,h,c,D,r_s,r_w,R,n):
  # Equation based on the rectangular prism assumption we made before
  A_w = 4 * math.sqrt(A_b/n) * n * h

  # Temperature as a function of time as modeled in Figure 2.4.1
  def T_out(t):
      return 4.9796776692867555 * np.sin(0.31958467920431916 * t + -2.7753696625091497) +
32.23606417137194

  # Global Horizontal Irradiance as a function of time as modeled in Figure 2.4.2 with
jittered parameters, up to 10% different.
  def I_jittered(t, jitter_amount=0.1):
    coeff1 = -24.099845890995997 + np.random.normal(scale=jitter_amount)
    coeff2 = 623.6904007623635 + np.random.normal(scale=jitter_amount)
    coeff3 = -3166.6881716727808 + np.random.normal(scale=jitter_amount)
    return np.piecewise(t, [t < 7, (t >= 7)], [0, coeff1 * t**2 + coeff2 * t + coeff3])

  # Define the differential equation incorporating T_out(t) and R(t)
  def dT_dt(t, T_in):
      return (1 / (A_b * h * c * D)) * (r_s * r_w * I_jittered(t) * A_w +
(((T_out(t)+50-T_in)*A_w)/R))
```

```
  # Time span and initial conditions, t_span is the 24 hour period for modeling and t_in_0
is the initial temperature.
  t_span = (0, 24)
  T_in_0 = 29.444
  # Solving the Initial Value Problem using SciPy.integrate libraries
  solution = solve_ivp(dT_dt, t_span, [T_in_0], t_eval=np.linspace(0, 24, 500))
  return solution.t, solution.y[0]

# Running a   simulation using 1000 iterations of the jittered I(t) parameters modeling the
Solar Irradiance.
num_runs = 1000
for _ in range(num_runs):
    home_x, home_y = temperature_model(A_b, h, c, D, r_s, r_w, R, n)
    plt.plot(home_x, home_y)
# Graphing all the plots together, as well as the data for the outside temperatures. The
data for outside temperatures can be found above in the Appendix.
plt.xlabel("Time (hours)")
plt.ylabel("Temperature (K)")
plt.ylim(27,38)
plt.title("Temperature Evolution Over Time")
plt.legend()
plt.grid()
plt.plot(x_sorted, y_fit_sorted, label='Outside Temp', color='red')
plt.xlabel('Hours elapsed after 12:00 AM')
plt.ylabel('Temperature (Celsius)')
plt.title('Model Sensitivity Analysis')
plt.legend()
plt.show()
```

7.2 Power Hungry

```
## The following code is written in R for Model Two##

#### M3 Challenge, 03/03/2025, Team #######
#### Q2, Memphis, TN, ####
#### TC (KW): Annual Consumption ####
#### Pload (MW): maximum hourly electricity demand in megawatts (MW) in Memphis,Tennessee,
Siemens' reports augmented by Fact sheets ###
#### mtemp: Maximum Temperature ####
#### pop: Population of Shelby County (including Memphis) ####

Mdata <- read.csv(file = "Mdata.csv", header=T)
```

```
attach(Mdata)
Mdata$Year <- as.integer(Mdata$Year)
## multiply TC by the factor of percentage of peak-month load converted from the monthly
country data
Mdata$TC <- Mdata$TC*0.099672319 /10^9
Mdata$pop <- Mdata$pop/10^5
Mtrain <- Mdata[1:11,]
attach(Mtrain)
data <- Mtrain

### Uncomment whichever Y to forecast

#Y <- Pload #peak hour load (summer)
Y <- TC #total consumption of peak summer month

## plot the data ##
# Load the ggplot2, tidyverse packages
library(ggplot2)
library(tidyverse)
library(mgcv)

data <- data.frame(
  Year = Year,
  Y = Y,
  mtemp = mtemp,
  pop = pop
)

# Fit linear regression model for mtemp over time
mtemp_model <- lm(mtemp ~ Year, data = Mtrain)

# Predict mtemp for 2025 with confidence interval
new_data <- data.frame(Year = 2025)
prediction <- predict(mtemp_model, newdata = new_data, interval = "confidence", level =
0.95)

# Print prediction and confidence interval
print(prediction)

# Reshape the data into long format
data_long <- data %>%
```

```
  pivot_longer(cols = c(Y, mtemp, pop), names_to = "Variable", values_to = "Value")

# Create a trellis plot
ggplot(data_long, aes(x = Year, y = Value)) +
  geom_line() +         # Add lines for each variable
  geom_point() +        # Add points for each year
  facet_wrap(~ Variable, scales = "free_y") +  # Create separate panels for each variable

#  labs(title = "Max Temperature, Population, Peak Hour Load (MW) over Year for Memphsis,
TN",
  labs(title = "Max Temperature, Population, Peak Summer Month Consumption (in billions of
kWh) over Year for Memphsis, TN",
       y = "Value") +
  scale_x_continuous(breaks = c(2012, 2014, 2016, 2018, 2020, 2022)) +
  theme_minimal()

## Build linear regression models with PeakCh/TC as dependent variable#

model1 <- lm(Y ~ mtemp + pop, data = Mtrain)
summary(model1) # best adj-R2
model2 <- lm(Y ~ mtemp, data = Mtrain)
summary(model2)
model3 <- lm(Y ~ mtemp + pop + lag(Y), data = Mtrain)
summary(model3)
```

7.3 Beat the Heat

```
import pandas as pd
import numpy as np
from sklearn.linear_model import LinearRegression
from sklearn.model_selection import train_test_split
from sklearn.preprocessing import OneHotEncoder
from sklearn.compose import ColumnTransformer
import statsmodels.api as sm

# Loading the data from our sheet into a Pandas Dataframe
data =
pd.read_excel("https://docs.google.com/spreadsheets/d/1RaHFbtKV04xqiwwVMWwfupebBhJqOre5cCYb
UhLKPu0/export?format=xlsx", sheet_name="Sheet21", engine='openpyxl')
df = pd.DataFrame(data)
#df.head()

# Seperating the Independant and Dependant Variables, our Dependant variable is the loss.
X = df.iloc[:,1:8]
y = df.iloc[:,0]

# Handle missing values in both X and y, ensuring they have the same index to avoid errors
X = X.replace([np.inf, -np.inf], np.nan)
# Aligning indices of X and y before dropping NaN values to avoid errors
X, y = X.align(y, axis=0, join='inner')
X = X.dropna()
y = y[X.index] # Ensuring y corresponds to X after dropping NaNs


# Training the multiple linear regression model using statsmodels
model = sm.OLS(y, X).fit()

# Print the model summary to analyze
print(model.summary())

# Print p-values for each x variable
print("\nP-values for x variables:")
print(model.pvalues[1:]) # Exclude the intercept (constant)

# Print coefficients and intercept
print('Coefficients:', model.coef_)
print('Intercept:', model.intercept_)
```

```
# Code for the sensitivity analysis for Model 3

data =
pd.read_excel("https://docs.google.com/spreadsheets/d/1RaHFbtKV04xqiwwVMWwfupebBhJqOre5cCYb
UhLKPu0/export?format=xlsx", sheet_name="vscoredata", engine='openpyxl')
df = pd.DataFrame(data)
data.head()
# A, B, C, and D are parameters that were found by our Multiple Regressions line. We are
using the coefficients determined by that model.
A = 84.9225
B = 105.4934
C = -7.3073
D = 0.0336

# Jittering the parameters by a random value of up to 5% to test the sensitivity of the
model.
A_jit = A + np.random.normal(scale=0.05)
B_jit = B + np.random.normal(scale=0.05)
C_jit = C + np.random.normal(scale=0.05)
D_jit = D + np.random.normal(scale=0.05)

# Convert the relevant columns to numeric type to avoid any Type errors while calculating
the vulnerability scores.
for col in ["Proportion of households with elderly", "Proportion of households with
children", "Population (10,000 people)", "Primary Transportation"]:
    df[col] = pd.to_numeric(df[col], errors='coerce') # 'coerce' will replace non-numeric
values with NaN

# Our Vulnerability scores as determined by our base model
df["Vulnerability Normal"] = df["Proportion of households with elderly"]*A+df["Proportion
of households with children"]*B+df["Population (10,000 people)"]*C + df["Primary
Transportation"]*D
min_normal = df["Vulnerability Normal"].min()
max_normal = df["Vulnerability Normal"].max()
df["Min-max normalized Normal"] = ((df["Vulnerability
Normal"]-min_normal)/(max_normal-min_normal))*100

# New vulnerability scores as determined by the jittered model
df["Vulnerability"] = df["Proportion of households with elderly"]*A_jit+df["Proportion of
households with children"]*B_jit+df["Population (10,000 people)"]*C_jit + df["Primary
Transportation"]*D_jit
```

```
min = df["Vulnerability"].min()
max = df["Vulnerability"].max()
df["Min-max normalized"] = ((df["Vulnerability"]-min)/(max-min))*100

# Comparing the percent differences between the data
print(df["Min-max normalized"])
print(df["Min-max normalized Normal"])
df["Percent Differece"] = abs(df["Min-max normalized"]-df["Min-max normalized
Normal"])/df["Min-max normalized Normal"]*100
# Dropping the infinite values that may be caused when one of the values being divided by
for the percent difference is 0, otherwise we will not be able to get an accurate mean
value.
df = df.replace([np.inf, -np.inf], np.nan)
df.dropna()
average = df["Percent Differece"].mean()
print(average)
```

7.2.1 Power Hungry Data

Year	pop	pop (10,000s of people)	mtemp	Pload (MW)	TC (billion kWh)
2012	939421	9.39421	103	3256	1.07187532
2013	938069	9.38069	98	3195	1.067037226
2014	937441	9.37441	100	3062	1.050957091
2015	937020	9.3702	99	3226	1.048039781
2016	936961	9.36961	100	3155	1.040242715
2017	936961	9.36961	99	3086	1.012139307
2018	936961	9.36961	97	3097	1.05699823
2019	937070	9.3707	100	3182	1.01752221
2020	929744	9.29744	97		0.964066949
2021	923382	9.23382	96	3177	0.976826102
2022	916357	9.16357	102	3316	0.973628714

7.3.1 Beat the Heat Data

ZIP code	Loss per capita	Proportion of households with elderly	Proportion of households with children	Population (10,000 people)	Primary Transportation	Open Space	Median Income (in 100k)	Built before 1950 (1000s)
38103	45.195	0.115	0.079	1.182	307	0.0451	0.758	1.463
38002	49.041	0.302	0.412	4.369	65	0.0596	1.155	0.33
38017	36.149	0.298	0.391	5.623	65	0.0888	1.359	0.365
38016	37.526	0.249	0.261	4.427	131	0.1602	0.757	0.047
38018	36.638	0.243	0.313	3.8	99	0.1981	0.896	0.076
38028	98.672	0.394	0.36	0.77	9	0.0566	1.508	0.095
38060	30.205	0.347	0.308	1.236	42	0.0578	0.843	0.121
38066	73.416	0.475	0.228	0.371	3	0.0238	1.053	0.044
38104	42.971	0.215	0.139	2.212	755	0.1998	0.565	7.367
38105	72.03	0.223	0.131	0.496	411	0.1004	0.293	0.893
38106	43.619	0.364	0.195	2.17	347	0.1047	0.298	4.181
38107	58.322	0.245	0.219	1.4	427	0.2102	0.364	3.643
38108	39.46	0.336	0.33	1.843	186	0.1728	0.354	2.483
38109	29.092	0.407	0.28	4.364	268	0.1171	0.369	2.406
38111	35.098	0.27	0.217	4.206	1102	0.4575	0.528	5.787
38112	65.318	0.301	0.249	1.511	585	0.2764	0.526	3.688
38117	53.601	0.326	0.266	2.626	67	0.4829	0.937	0.64
38125	32.916	0.189	0.329	4.273	89	0.1969	0.832	0.288
38126	70.382	0.256	0.368	0.546	173	0.1165	0.308	0.514
38127	31.029	0.281	0.379	3.94	350	0.1277	0.378	1.55
38128	31.006	0.233	0.376	4.37	191	0.1855	0.432	0.939
38133	40.547	0.221	0.36	2.09	16	0.2037	0.825	0.196
38134	41.587	0.242	0.331	3.885	90	0.2278	0.612	0.351
38135	10.132	0.309	0.31	3.028	90	0.2763	0.925	0.324
38138	52.834	0.462	0.311	2.517	78	0.3539	1.301	0.129
38139	26.961	0.396	0.396	1.63	0	0.3305	1.741	0.061
38141	46.37	0.186	0.411	2.377	48	0.1302	0.641	0.074