

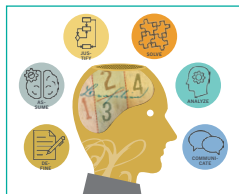
# MathWorks Math Modeling Challenge 2025

## Flint Hill School

Team #17839, Oakton, Virginia

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## M3 Challenge FINALIST—\$5,000 Team Award

### JUDGE COMMENTS

*Specifically for Team #17839—Submitted at the close of triage judging*

**COMMENT 1:** Extremely well-written. Assumptions are clear. Modeling is thorough. Outputs are analyzed.

**COMMENT 2:** Nice job in addressing Q1. Did a good job in citing resources.

**COMMENT 3:** Check that the units reported for the variables and other quantities are compatible and used consistently. Some sections of the report are repeated indicating that a general proofreading is in order. Use consistent rounding in the reporting of numbers.

**COMMENT 4:** You have a well-written and clear report that justifies your choice of models and provides results. You take into account several factors in the development of your models to make them more predictive. Well done! The results of your models are reasonable and show great effort!

# Hot Button Issue: *It's Getting Hot In Here*

March 2, 2025

## 1. EXECUTIVE SUMMARY

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As the years go by, climate change becomes increasingly worrisome. With global temperatures on the rise and with multiple record-breaking temperatures being reached worldwide, heat waves have grown in frequency and intensity. Cities around the world experience power outages due to persistent heat waves. We seek to assess the effects of the increasingly distressing heat waves in Memphis, Tennessee.

We first developed a model to determine the effect of heat waves on individual housing units across Memphis. Using a General Energy Balance Equation model, we estimated the change of the given houses' interior temperatures throughout a day during a heat wave. We took into account factors like heat from conduction, residents to determine internal heat, and air change per hour to determine heat loss and gain from ventilation. Our results from the modified Energy Balance Equation model for Problem 1 were incredibly defensible. For example, Home 4 with more surface area and exposure to sun experience the most drastic temperature changes as well as the highest peak temperature, whilst Home 1 was the least susceptible, barely reaching the peak outdoor temperature around sunset due to retention of heat from conduction and internal heating.

We then developed a model to predict the peak demand that Memphis, Tennessee's power grid will need to accommodate for during increasingly hot summer months in the next 20 years. Using our model from Problem 1, we determined vulnerability for communities experiencing heat waves. We used a thermal equilibrium model to determine power demand during extreme heat events by determining the amount of energy required to keep one house at the desired temperature, 22 C°, then calculating the entire city of Memphis' power consumption based on the demands of different housing types. After fine tuning the baseline demand for power to account for variance, we determined that the peak demand, 32.5% of which came from air conditioning. Accounting for population growth and rising climate change, we determined peak power in the coming years will grow upwards of 4000 MW to 5000 MW; however, legislation and technological advancements in the next decade are predicted to reduce cooling requirements by 40% to 50%. These future changes can be utilized to offset the drastic increase in power consumption from population growth and effects of climate change.

Finally, we developed a third model to determine the vulnerability of certain neighborhoods in Memphis, Tennessee using a Principle Component Analysis (PCA). The PCA determined the weights of each factor by maximizing variance of each factor and basing its weight on how much it effects the output. Using the proportions calculated by this model, we determined the vulnerability scores of each neighborhood, with East Memphis at the most risk, with a score of 100, and Rossville with the lowest score of 9.99.

The 3 models we have developed will be vital to ameliorate possible damages from dangerous heat events worsened by climate change in Memphis, and the world as a whole.

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## 2. PART 1: HOT TO GO

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### 2.1. Problem Statement

Problem 1 asks us to develop a model to predict the indoor temperature of any non-air-conditioned dwelling during a heat wave over a 24-hour period in either Memphis, Tennessee, or Birmingham, United Kingdom. For our model, we are assessing the effect of a heat wave on housing in Memphis, Tennessee. We are asked to explain our choices in determining our model. We are asked to use the M3 dataset as part of our solution.<sup>1</sup>

### 2.2. Assumptions and Justifications

- **Assumption: Function Shade ( $f_{shade}$ ) is defined as the following:**
  - Very shady = 0
  - Not very shady = 0.5
  - Not at all shady = 1
  - Justification: The classification of "very, not very, and not at all shady" within the data set provides us with 3 different levels of shadiness. This allow us to split the three different level as 0, 0.5 and 1, respectively.
- **Assumption:  $\delta(t)$  is equal to 3 C°from 6 AM to 6 PM. At any other point in time,  $\delta(t)$  is equal to 0.**
  - Justification: Due to our research and acquired knowledge about homes in Memphis, Tennessee, we have established 3 C°to be an accurate average change in  $\delta(t)$  from time 6AM to 6PM. As for why it is from 6AM to 6PM, that is when the sun rises and sets. When the sun rises, the sunlight hits the house with nothing in the way of the sun, so the sunlight instantly starts heating the house. Similarly, when the sun sets, the sunlight no longer hits the house all, so the sunlight instantly stops heating the house.
- **Assumption: The height of each floor of a residence is approximately 10 feet.**
  - Justification: In the state of Tennessee, floors of buildings have an average height of 10 ft. <sup>2</sup>
- **Assumption: Older houses have worse insulation.**

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<sup>1</sup>Hot Button Issue, MathWorks Math Modeling Challenge 2025, curated data, <https://m3challenge.siam.org/897bjhb54cgfc/>.

<sup>2</sup><https://www.thempc.org/eagenda/x/mpc/2018/october-9-2018-regular-mpc-meeting/table-2-tn-2-dimensional-standards.pdf>

- Justification: Because of innovation in construction techniques, older houses, particularly ones build before 1960s, do not retain heat as well as modern housing.<sup>3</sup>
- **Homes in Memphis have an air change rate(ACH) of 3 or 0.5 per hour depending on the temperature difference.**
  - Justification: When the temperature outside is cooler then the temperature inside, people will open the windows to cool off the house. This circulation that we are measuring is called Air Change per Hour (ACH). Due to the few house options we have available, we have decided to use an ACH of 3 for our modeling. This is because 3 ACH is a minimum standard that most buildings should be in accordance with when windows are opened. However, the ACH dramatically decreases when the temperature outside is hotter than within, as people will then close the window in an attempt to prevent the heat from building up. This causes our ACH to be reduced to 0.5, as there will still be some circulation from cracks and other areas.<sup>4</sup>
- **The sun has an immediate effect on internal heating when it rises or sets.**
  - Justification: We do not have any addition information on how other external factors (number of obstruction, percentage of sunlight, etc) will effect the amount of solar radiation on heating from the sun. Therefore, the moment that the sun rises/set below the horizon, we will treat it as having full contact with house.
- **All residents of a household are within the residence during the heat wave.**
  - Justification: In order to account for internal heat, we must assume the presence of all residents.
- **A singular person within a home generates 100 Watts of power each day.**
  - Justification: It can be approximated that a human being on average generates 100 Watts just by existing.<sup>5</sup>
- **Assumption: Building thermal capacity (C) is approximately 75,000 J/K.**
  - Justification: This value represents typical residential thermal mass derived from building physics research. Studies of residential buildings show thermal capacities ranging from 40,000-100,000 J/K depending on construction type, with wood-frame homes (common in Memphis) averaging around 75,000 J/K.<sup>6</sup>

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<sup>3</sup><https://www.insulatekansascity.com/insulation-blog/inherited-an-old-house-heres-how-to-check-if-its-under-insulated/#:~:text=According%20to%20experts%20from%20Realtor,built%20with%20insulation%20in%20mind.>

<sup>4</sup><https://www.sanalifewellness.com/blog/air-changes-per-hour-ach>

<sup>5</sup><https://www.fst.com/news-stories/magazine/renewable-energy/human-power-plant/>

<sup>6</sup><https://codes.iccsafe.org/content/IRC2024P2/chapter-11-re-energy-efficiency>

## 2.3. Developing the Model

For Problem 1, we decided to use a General Energy Balance Equation. This equation takes into account sources of thermal energy that affect the house. Our components include capacitance, heat from conduction, heat from internal sources within the house, and heat from the ventilation or airflow of the house. We can use the differential equation to solve for  $\frac{dT_{in}}{dt}$ , which will give us rate of change of  $T_{in}$ , the internal temperature.

$$C \cdot \frac{dT_{in}}{dt} = Q_{conduction} + Q_{internal} + Q_{solar} + Q_{ventilation} \quad (1)$$

Variables of the Energy Balance Equation		
Variables	Definition	Units
$C$	Heat Capacitance	Joules/Kelvin
$Q_{conduction}$	Newton's Law of Cooling	W
$Q_{internal}$	Heat generated from people inside	W
$Q_{solar}$	Heat from solar radiation	W
$Q_{ventilation}$	Heat being circulated	$\frac{J}{h}$

Table 1: Summary of Variables in the General Energy Balance Equation

The General Energy Balance Equation considers  $Q_{solar}$  in order to include heat imposed onto a house due to the sun's radiation; however, we determined that our  $Q_{conduction}$  accounts for the this heat factor due to solar radiation. Therefore, we can safely disregard  $Q_{solar}$ , and we can eschew it from our model.

$Q_{conduction} = UA \cdot (T_{sa} - T_{in})$		
Variables	Definition	Units
$Q_{conduction}$	Newton's Law of Cooling	N/A
$UA$	Constant of heat transfer	W/K
$T_{sa}$	$T_{out} + f_{shade} \cdot \delta t$ ; solar radiation onto the house	C°
$T_{in}$	Temperature in	C°
$T_{out}$	Temperature outside	C°
$f_{shade}$	Shade constant	N/A
$\delta t$	$\delta t = \begin{cases} 6am < x < 6pm & 3 \\ 6pm < x < 6am & 0 \end{cases} \quad (2)$	C°

Table 2: Variables for  $Q_{conduction}$

The equation for  $Q_{conduction}$  uses Newton's Law of Cooling to estimate the heat transfer of the outside temperature to the interior of the residence.<sup>7</sup> Newton's Law of Cooling states

<sup>7</sup><https://mathresearch.utsa.edu>

that the rate at which an object will cool is proportional to the difference in temperatures between the object and its surroundings. In the context of the problem, this means that the rate at which the house gains heat due to conduction is proportional to the temperature outside minus the temperature within the house.

This equation utilizes UA, temperature due to solar radiation on the house and temperature inside the house. UA is a constant of insulation that determines how well a building retains heat. We considered the building type, with apartments having a better heat transfer capacity, and houses having a worse one. This means apartment-style buildings will gain or lose heat at a much faster rate than house-style buildings do. In addition to building type, we took into account building age, in which the older the building is, the worse the heat transfer capacity will be, as stated by one of our assumptions.  $T_{sa}$  is the sol-air, which is composed of the outside temperature plus the temperature gained from solar radiation projected onto the house.<sup>8</sup> Our model also takes into account the surface area, and larger surface areas will result in a higher  $T_{sa}$ . The level of shade a house has is determined by the given data on Memphis, TN, which qualifies a houses' shadiness on a level of "Very Shady" to "Not Shady at All." We can roughly use the shadiness and constant of insulation to estimate the amount of heat added to the internal temperature of a house due to this direct solar radiation.

$Q_{internal} = p \cdot \frac{100}{V_h}$		
Variables	Definition	Units
$p$	Number of people inside	People
100	Heat generated per person	W
$V_h$	Volume of the house	$m^3$

Table 3: Variables for  $Q_{internal}$ 

The equation for  $Q_{internal}$  is undoubtedly refreshing after seeing the monstrosities of our previous variables. Internal heat is the heat generated within the residence. For our purposes, we can simply calculate internal heat by considering the number of people within a residence. The average human generates 100 watts of energy at rest<sup>9</sup>. However,  $Q_{internal}$  has a small overall impact on the overall temperature of the house because it mostly just impacts the temperature that the people within experience, which we are not considering.

$Q_{ventilation} = ACH \cdot 0.9 \cdot A \cdot (T_{out} - T_{in})$		
Variables	Definition	Units
$ACH$	Air change per hour	N/A
0.9	Ventilation coefficient	$W \frac{m^2}{K}$
$A$	Area of floor	$m^2$
$T_{out} - T_{in}$	Change in temperature	$^{\circ}C$

Table 4: Variables for  $Q_{ventilation}$ 

Finally,  $Q_{ventilation}$  is the heat due to the circulation of air. This component determines

<sup>8</sup><https://www.sciencedirect.com/science/article/abs/pii/S0306261977900174>

<sup>9</sup><https://www.fst.com/news-stories/magazine/renewable-energy/human-power-plant/>

the amount of heat gained or lost from ventilation. The  $ACH$  is the amount of air changes per hour, which tells us at what rate the indoor air is replaced.<sup>10</sup> This factor is important to consider in order to assess how the circulation of air within a home affects its internal temperature. For most homes, the  $ACH$  is 3 air changes per hour<sup>11</sup>.

$$C \cdot \frac{dT_{in}}{dt} = UA \cdot (T_{sa} - T_{in}) + p \cdot 100 + ACH \cdot 0.9 \cdot A \cdot (T_{out} - T_{in}) \quad (3)$$

Alas, here is the final equation for our model. Each component has been described above. We can utilize this equation to determine the effect of a heat wave on internal temperatures in Memphis, Tennessee.

## 2.4. Analyzing the Results

Using the given data from the Memphis, Tennessee data set, our model predicted the heat flow throughout the 4 given homes. Figure 3.1 shows the change in heat, in Watts, throughout a day. All 4 houses experienced a rapid influx of heat around 5 to 6 AM in the morning, and a rapid decrease in heat around 6 to 7 pm. This occurrence is due to the sun. Because our model greatly depends on temperature outside, it is reasonable to see these two time-frames as points of drastic changes due to the sun rising and setting.

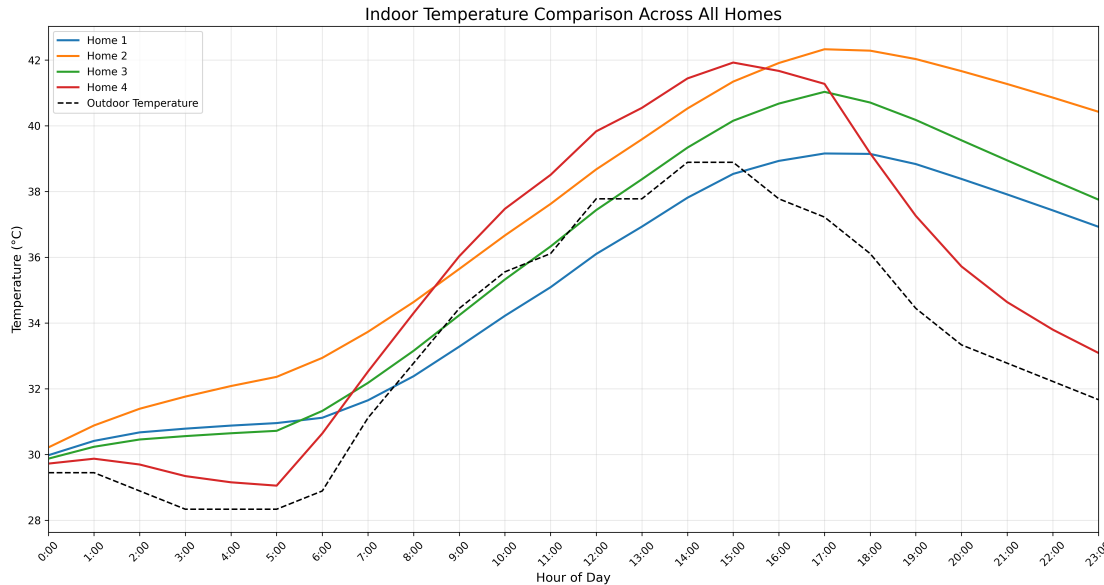


Figure 2.1: Indoor Temperature Comparison Across All Homes

Figure 3.2 displays the internal temperature throughout a day. All four homes get increasingly warm throughout the day and then slowly cool off after sunset. Homes 3 and 4 eventually exceed the highest outdoor temperature, reaching over 40 C°. Homes 1 and 2 stay below the highest outdoor temperature throughout the day, although they do retain heat, even hours after sunset.

<sup>10</sup><https://cleanair.camfil.us/2021/10/22/how-to-calculate-air-changes-per-hour>

<sup>11</sup><https://www.sanalifewellness.com/blog/air-changes-per-hour-ach>.



Home 1		
Time	Out (°C)	In (°C)
00:00	29.4	30.0
01:00	29.4	30.4
02:00	28.9	30.7
03:00	28.3	30.8
04:00	28.3	30.9
05:00	28.3	31.0
06:00	28.9	31.1
07:00	31.1	31.7
08:00	32.8	32.4
09:00	34.4	33.3
10:00	35.6	34.2
11:00	36.1	35.1
12:00	37.8	36.1
13:00	37.8	36.9
14:00	38.9	37.8
15:00	38.9	38.5
16:00	37.8	38.9
17:00	37.2	39.2
18:00	36.1	39.1
19:00	34.4	38.8
20:00	33.3	38.4
21:00	32.8	37.9
22:00	32.2	37.4
23:00	31.7	36.9

Figure 2.2: Home 1 Temperatures

Home 2		
Time	Out (°C)	In (°C)
00:00	29.4	30.2
01:00	29.4	30.9
02:00	28.9	31.4
03:00	28.3	31.8
04:00	28.3	32.1
05:00	28.3	32.4
06:00	28.9	32.9
07:00	31.1	33.7
08:00	32.8	34.6
09:00	34.4	35.6
10:00	35.6	36.7
11:00	36.1	37.6
12:00	37.8	38.7
13:00	37.8	39.6
14:00	38.9	40.5
15:00	38.9	41.3
16:00	37.8	41.9
17:00	37.2	42.3
18:00	36.1	42.3
19:00	34.4	42.0
20:00	33.3	41.7
21:00	32.8	41.3
22:00	32.2	40.9
23:00	31.7	40.4

Figure 2.3: Home 2 Temperatures

Home 3		
Time	Out (°C)	In (°C)
00:00	29.4	29.9
01:00	29.4	30.2
02:00	28.9	30.5
03:00	28.3	30.6
04:00	28.3	30.6
05:00	28.3	30.7
06:00	28.9	31.3
07:00	31.1	32.2
08:00	32.8	33.2
09:00	34.4	34.2
10:00	35.6	35.3
11:00	36.1	36.3
12:00	37.8	37.4
13:00	37.8	38.4
14:00	38.9	39.3
15:00	38.9	40.2
16:00	37.8	40.7
17:00	37.2	41.0
18:00	36.1	40.7
19:00	34.4	40.2
20:00	33.3	39.6
21:00	32.8	39.0
22:00	32.2	38.4
23:00	31.7	37.8

Figure 2.4: Home 3 Temperatures

Home 4		
Time	Out (°C)	In (°C)
00:00	29.4	29.7
01:00	29.4	29.9
02:00	28.9	29.7
03:00	28.3	29.3
04:00	28.3	29.2
05:00	28.3	29.1
06:00	28.9	30.6
07:00	31.1	32.5
08:00	32.8	34.3
09:00	34.4	36.0
10:00	35.6	37.5
11:00	36.1	38.5
12:00	37.8	39.8
13:00	37.8	40.5
14:00	38.9	41.4
15:00	38.9	41.9
16:00	37.8	41.7
17:00	37.2	41.3
18:00	36.1	39.2
19:00	34.4	37.3
20:00	33.3	35.7
21:00	32.8	34.6
22:00	32.2	33.8
23:00	31.7	33.1

Figure 2.5: Home 4 Temperatures

According to our model, the home most susceptible to overheating due to heat waves is Home 4. Home 4 is a single-family residence made in 1990, and it is not located in a shaded area. It is also the largest of the four homes, with a square-meterage of  $278 \text{ m}^2$ , and 6 people living in it. The house that is least susceptible is Home 1, which is a 3 person, single-family residence made in 1953, located in shade with a square-meterage of  $88 \text{ m}^2$ . However, all houses reach or surpass the max temperature of the day,  $39 \text{ C}^\circ$ .

There are 3 glaring differences between Home 1 and Home 4. Firstly, the size of the homes are different. Despite the residences both being single-family homes, Home 4 is drastically bigger than Home 1, being over 3 times the size of Home 1. Secondly, the shadiness of the homes proved to be important. Houses 3 and 4, the most susceptible homes, were both classified as "Not at all in shade," and House 2 was classified as "Not very shady," whilst House 1 was "Very shady." The major factors that made Home 1 the least susceptible was the square-meterage, with it being  $88 \text{ m}^2$ , as well as the fact it had only 3 residents.

An important observation is the rapid rate at which the internal temperature of House 4 decreases after sunset. This is due to the fact it is incredibly large, making its Q conduction very, very large. Thus, when the sun goes down, the major source of heat practically vanishes and allows for rapid cooling. The Homes 2 and 3, however, have 2 to 3 people living in significantly smaller, unshaded homes, leading high retention of heat gained during the day, even after sunset.

From our results, it can be determined that the 3 major factors to overheating in homes are the square-meterage, the number of people inside, as well as the degree of shade. These results are logical given the nature of our model and the factors it takes into account.

## 2.5. Strengths and Weaknesses

### 2.5.1 Strengths

- We chose a differential equations model, which is effective in capturing the rate of change. This is useful in the context of determining inside temperature because the inside temperature can be determined from the rate of change of the initial temperature. The rate of cooling depends on the instantaneous temperature, so we have to use a differential equation because it takes into account the instantaneous temperature.
- This model is not overly complicated, every part have a logical explanation and the math doesn't rely on neural networks or simulation.

### 2.5.2 Weaknesses

- Our model fails to take into account wind into our internal temperature. Since the wind speed can affect the air changes per hour (ACH), this lack of the wind speed can lead our predicted temperature to vary from the actual value depending on the wind speed.
- Our model has variable constants which vary from house to house. Some factors including the function for shade and capacitance of the house vary from house to

house. We did not account for each individual house and instead generalized to houses in Memphis, Tennessee.

- Our model always assumes that it is sunny outside, leading to inaccurate predictions on days in which it is cloudy. Although this effect is drastically reduces as during a heatwave it is almost always sunny.
- We solved this differential equation using Euler's method, which leads our data to not be continuous and is only close approximate to the actual solution. However, in order for us to solve Euler's method, we had to take small steps, seconds instead of hours, which led us to interpolate our given temperature in hours to temperature in seconds. This additional interpolation could potentially make our model less accurate.

### 3. PART 2: POWER HUNGRY

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#### 3.1. Problem Statement

Problem 2 asks us to develop a model that predicts the peak demand that Memphis, Tennessee's power grid should be prepared to handle during the summer months. We are asked if we foresee any changes in the maximum demand 20 years from now.

#### 3.2. Assumptions and Justifications

- **Assumption: The average temperature that people in Memphis keep their house at is between 70-73 degrees Fahrenheit or 22 degrees Celsius**
  - Justification: This is the average that an American keeps their home temperature at. <sup>12</sup>
- **Assumption: Older homes have reduced AC capacity and efficiency.**
  - Justification: We implemented capacity factors (0.7-1.0) based on home age. This reflects real-world conditions where older AC systems experience reduced capacity due to wear, outdated technology, and improper sizing. <sup>13</sup>
- **Assumption: Buildings can be represented by a single thermal zone with uniform temperature.**
  - Justification: While real buildings have temperature variations between rooms, the single-zone approach is standard in building energy modeling for city-scale simulations. For predicting overall energy consumption patterns, this simplification introduces minimal error while greatly reducing complexity.
- **Assumption: Memphis housing stock can be reasonably represented by four archetypal building models.**

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<sup>12</sup><https://www.adt.com/resources/average-room-temperature>: :text=That%20being%20said%2C%20the%20average,68%20

<sup>13</sup><https://www.aceee.org/research-report/a2001>

- Justification: While there may be other type of building models, considering all possible models increases complexity while have four types does not drastically increase the error.

### 3.3. Developing the Model

We chose a thermal equilibrium model based on our previous model from Problem 1 to determine the power demand during extreme heat events. This approach allowed us to measure the heat exchange between indoor and outdoor environments. A physics-based approach is more appropriate than others because power demand during extreme heat events are largely driven by cooling needs by integrating the thermodynamics of buildings to create an accurate representation of citywide power demand. We considered other approaches, such as statistical regression based on historical patterns of temperature and power consumption, but those models often fail to capture non-linear relationships between temperature and power consumption. They also cannot account for building-specific factors such as the insulation, population, shade, and AC-efficiency of each house.

We first solved the Energy Balance Equation 3 in Question 1 for the amount of energy required to keep one house at the desired temperature, giving us:

$$P_{AC} = \min \left( \frac{UA(T_{sa} - T_{setpoint}) + P \times H_p + ACH \times 0.9 \times A \times (T_{out} - T_{setpoint})}{COP}, P_{rated} \cdot CF \right) \quad (4)$$

Energy Balance Equation Solved for Energy Required

Symbol	Variables	Unit
$UA$	Building envelope heat transfer coefficient	$\frac{W}{K}$
$T_{sa}$	Sol-air temperature	K
$T_{in}$	Indoor temperature	K
$T_{out}$	Outdoor temperature	K
$PP$	Number of persons	people
$H_p$	Heat gain per person	W
$ACH$	Air changes per hour	$\frac{1}{hr}$
$AA$	Floor area	$m^2$
$P_{AC}$	AC power consumption	W
$COP$	Coefficient of Performance	N/A
$CF$	Capacity factor	N/A
$Q_{required}$	Required cooling power	W

Table 5: Parameter definitions and typical values.

In order to calculate the entire city of Memphis' power consumption during an extreme heat event, we first summed up the various categories of residence types: detached houses,

townhouses, apartments, and mobile or other types. We then calculate the proportion of each housing type. In order to factor in the age of the houses, we created a distribution for homes, breaking down the housing stock into categories. By combining the style-type percentages with these age brackets, we mapped the data provided—being the number of dwellings—to four housing model categories, similar to the four provided example dwelling types. In order to determine the baseline of power that the city of Memphis requires outside of an extreme heating event, we first calculated the region’s average power demand. In order to do so, we divided the annual consumption into hourly watts. We then adjusted the baseline to reflect seasonal variations, such as increased appliance and lighting demands to accommodate for increased demand for A/c during summer. In order to account for systemic factors that are not residential heating, such as commercial or industrial electrical consumption, we then further refined the baseline.

To align the results with realistic peak demand scenarios, further refinements are applied to account for systemic factors not explicitly modeled, such as commercial or industrial consumption. These adjustments ensure that the baseline serves as a foundational reference point, allowing cooling demands—calculated separately through thermodynamic simulations—to be superimposed while maintaining plausible total grid demand levels. The approach emphasizes practical alignment with established utility benchmarks. We chose a proportionality constant of 1.5 for the baseline to help align it better with real data from Memphis’ power grid.

We then summed the power required to cool all the different housing categories:

$$P_{total} = P_{baseline} + \sum_{i=1}^4 P_{AC,i} \cdot N_i \cdot SF \quad (5)$$

Equation: Peak Energy Consumption Required

Symbol	Variables	Unit
$P_{total}$	Total power consumption	W
$P_{baseline}$	Baseline power consumption	W
$P_{AC,i}$	Air conditioning power consumption	W
$N_i$	Number of dwellings in housing category $i$	(count)
$SF$	Scaling factor	N/A

Table 6: Parameter definitions for the peak energy consumption equation.

The total peak power demand for Memphis is calculated by adding the baseline power consumption of the city with the aggregated air conditioning of the different housing categories. For each of the housing models( $i$ ), we multiply the air conditioning power consumption ( $P_{AC,i}$ ) by the number of households in that category ( $N_i$ ), and then apply a scaling factor ( $SF$ ) of 70 to account for system-wide effects that are not captured in the individual building simulations.

### 3.4. Analyzing the Results

Using the model we created, we predict that at any random heatwave, the peak outdoor temperature is going to be 38.89 C°. This peak will occur at around 2 PM, as it follows the pattern of most heat waves. From this information, we determined that the Peak Power Demanded is 2850.51 MW, which 32.5% of comes from air conditioning. This results in an AC contribution of 926.96 MW at the peak of the heatwave. By 2045, Memphis's electrical consumptions patterns will likely have changed drastically due to climate change, population growth, and technological evolution. Increasing global temperatures will bring hotter average temperatures and harsher extreme heat events, such as heat waves.<sup>14</sup> These changes in consumer population and frequency of climate events will increase the demands for power as many more people will want A/C to cool their houses in a hotter climate. Even with a moderate population growth of the city, the peak power required by the city could grow upwards of 4000 MW: a substantial increase from current usage in 2024. Additionally, commercial and industrial sectors will increase in their power usage during potential heat events as they expand their businesses.

Building codes implemented in the 2020s and 2030s will result in approximately 30% of Memphis housing stock being constructed to much higher efficiency standards by 2044, reducing cooling requirements by 40-50% in these newer developments.<sup>15</sup> Retrofitting programs for existing buildings could improve another 25% of the housing stock, yielding 15-25% efficiency gains in these structures.<sup>16</sup> AC technology improvements should increase average system efficiency by 30-40%.<sup>17</sup> These technological and zoning changes could help offset the gain in power consumption from projected population growth and effects from climate change.

### 3.5. Sensitivity Analysis

We tested the sensitivity of our power demand model by systematically varying its value while holding all other inputs constant at their baseline values. This allowed us to isolate the impacts that each variable had on the power demand. We strategically selected four critical parameters: AC setpoint temperature (ranging from 19-25°C), AC efficiency (COP from 1.9-2.7), AC load scaling factor (50-90), and capacity factor scenarios (representing different distributions of AC system efficiencies across housing stock).

The analysis revealed that each 1°C increase in thermostat settings reduces peak power demand by approximately 39 MW (1.4%), with higher temperatures yielding progressively smaller AC contributions. It also showed that each 1°C increase in thermostat settings reduces peak power demand by approximately 39 MW (1.4%), with higher temperatures yielding progressively smaller AC contributions. The scalefactor highlights the substantial impact of the scaling factor, which accounts for commercial buildings and other effects not directly modeled, with each 10-point increase adding approximately 132 MW (4.6%) to peak demand. The capacity factor highlights the substantial impact of the scaling

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<sup>14</sup><https://www.vox.com/a/weather-climate-change-us-cities-global-warming>

<sup>15</sup><https://www.tn.gov/commerce.html>

<sup>16</sup><https://www.energy.gov/eere/wap/weatherization-assistance-program>

<sup>17</sup><https://www.energy.gov/eere/buildings/appliance-and-equipment-standards-program>

AC Setpoint Temperature (°C)	Peak Power (MW)	AC Contribution (%)
19	2928.24	34.3
20	2889.38	33.4
21	2850.51	32.5
22	2811.65	31.6
23	2772.79	30.6
24	2733.93	29.6
25	2695.06	28.6

Table 7: Effect of AC setpoint temperature on peak power demand.

AC Efficiency (COP)	Peak Power (MW)	AC Contribution (%)
1.9	3045.66	36.8
2.1	2938.80	34.5
2.3	2850.51	32.5
2.5	2776.36	30.7
2.7	2713.19	29.1

Table 8: Effect of AC efficiency (COP) on peak power demand.

AC Load Scale Factor	Peak Power (MW)	AC Contribution (%)
50	2585.67	25.6
60	2718.09	29.2
70	2850.51	32.5
80	2982.94	35.5
90	3115.36	38.3

Table 9: Effect of AC load scaling factor on peak power demand.

Capacity Factor Scenario	Peak Power (MW)	AC Contribution (%)
All high efficiency	2850.51	32.5
Baseline	2850.51	32.5
All low efficiency	2850.51	32.5
High variation	2850.51	32.5

Table 10: Effect of capacity factor scenarios on peak power demand.

factor, which accounts for commercial buildings and other effects not directly modeled, with each 10-point increase adding approximately 132 MW (4.6%) to peak demand.

## 3.6. Strengths and Weaknesses

### 3.6.1 Strengths

- We opted for a thermal equilibrium model, which effectively captures the essential physics of heat transfer and building thermodynamics. This approach allows for explicit modeling of conduction, ventilation, and cooling processes, setting it apart from simpler statistical models.
- The model accurately reflects the non-linear relationship between outdoor temperature and power consumption. It takes into account the rate of temperature change, which is crucial for understanding how buildings react to heat.
- This model is tailored to the region of Memphis, Tennessee, as it incorporates housing distribution data and electricity consumption patterns specific to the city. This tailoring ensures that local building characteristics and climate conditions are considered.
- By integrating Newton's Cooling Law and varying capacity factors for different types of homes, the model mirrors real-world scenarios where older buildings find it challenging to maintain comfort during heat waves, resulting in more realistic power demand forecasts.

### 3.6.2 Weaknesses

- Our model simplifies the diverse housing stock in Memphis into just four categories of buildings. This feature likely under-represents the true variety in building performance, especially for unique housing types or those with unusual characteristics.
- We apply a scaling factor of 70.0 to extend our residential model to citywide demand. This introduces potential inaccuracies by assuming proportional scaling relationships across different sectors and does not explicitly account for commercial and industrial buildings.
- The model assumes uniform thermostat settings and occupant behaviors across all households. In reality, preferences and behaviors can vary significantly, which can greatly influence power consumption.
- Additionally, our model lacks thorough validation against Memphis-specific power consumption data during similar heat events, relying instead on general patterns observed in other cities.

## 4. PART 3: BEAT THE HEAT

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### 4.1. Problem Statement

We are asked to develop a vulnerability score for various neighborhoods in Memphis to help them equitably allocate resources for minimizing the effects of a heat wave or a power grid failure. We are asked to justify all factors we choose to include in our vulnerability scores.



In addition, we must propose a single approach for how Memphis officials can incorporate our vulnerability scores into their management of heat waves.

## 4.2. Assumptions and Justifications

- **Assumption: All houses created in each decade was made out of the same material**
  - Justification: The majority of houses created in the same decade were constructed using the same material, which each have different properties that affect the neighborhoods vulnerability to heat.
- **Assumption: The results of our model reveal accurate vulnerabilities in Memphis, Tennessee.**
  - Justification: In order for our solutions and analysis to address Problem 3, we must assume, dare I say safely, that our model from Problem 1 accurately reveals flaws in certain housing types.

## 4.3. Developing the Model

To create our model, we first determined the factors relating to the vulnerability score. We have compiled these factors into the following table.

Symbol	Variable	Unit
$N_H$	Number of households	#
$P_N$	Proportion of developed, open space in neighborhood	N/A
$H_{2010}$	Homes built 2010 or later	#
$H_{1990}$	Homes built 1990 to 2009	#
$H_{1970}$	Homes built 1970 to 1989	#
$H_{1950}$	Homes built 1950 to 1969	#
$H_{<1950}$	Homes built 1950 or earlier	#
$D_H$	Detached whole house	#
$T_H$	Townhouse	#
$P_H$	Apartments	#
$M_H$	Mobile Homes/Other	#

Table 11: Individual factors which determine the vulnerability score

We choose the number of households as a factor because the more households there are the more demand on the power grid, and the more buildings are at risk of high levels of heat. The proportion of developed, open space in neighborhood is important because it helps determine the density of houses, and the amount of free space in the neighborhood. The more free space, the less vulnerable homes are to high levels of heat or the effects of power outages. The type of house and date of house construction is important because each type of house is differently effected by heat, and the date of house construction helps us

determine the material the house was created. This is important as different materials have different constants for heat absorption.

After we compiled all of these factors, we needed to decide the weights of each factor. To do this, we wanted to avoid subjectivity, and we opted to do a Principle Component Analysis or PCA. PCA is used to reduce variable data to its Principle Components. Yet even though this is helpful, this was not the main reason we decided to use this statistical method. PCA is a method which helps us determine the weights of each factor in a non-biased way. It does this by maximizing the variance of each of the factors, and basing its weight on how much that effects the output. And by normalizing our data, we ensure that our weights are not skewed towards the greater numbers. To perform a PCA, we first had to create a covariance matrix using the normalized values (z-scores) for each value in our data table. After this, we computed the eigenvalues and vectors. This is shown below.

$N_H$	$P_N$	$H_{2010}$	$H_{1999}$	$H_{1970}$	$H_{1950}$	$H_{<1950}$	$D_H$	$T_H$	$P_H \& M_H^*$
5.2920	1.5067	2.7740	3.1703	3.5748	2.0020	1.1228	4.2394	3.500544	4.128
0.9387	4.9542	7.4997	8.1032	1.7743	7.2830	8.3998	1.7124	3.5100	2.579
0.5348	1.1817	0.5243	0.2659	1.0218	1.0562	0.7529	1.4103	0.2509	1.494
0.8717	2.0741	1.4610	0.5200	2.9344	1.9662	2.1205	1.8057	0.9839	0.877
0.1816	1.9229	1.0498	0.5600	1.2783	1.0134	0.6253	0.1933	1.6470	0.688
0.1065	1.4253	0.3648	0.2003	0.4644	0.0927	0.0402	0.1031	1.8432	0.491
0.2632	0.0621	0.6004	0.2952	0.6125	1.1442	1.3542	0.3687	0.4984	0.219
0.0644	0.0560	1.3271	1.2179	0.7021	0.1166	0.3918	0.1455	0.1602	0.113
0.0296	0.0014	0.0036	0.0077	0.0062	0.0048	0.0044	0.0135	0.0003	0.003

Table 12: Absolute value of the vulnerability eigenvectors scaled by the eigenvalues,  $P_H$  and  $M_H$  are combined as they have the same eigenvalues

Then we multiplied each of the vectors by its corresponding eigenvalue and summed each of the rows. The eigenvalues represent the importance of each of the eigenvalues. By taking the proportion of each weight (Initial weight divided by total weights) we have the percentage values of each weight. All the components in the eigenvectors represent a factor that affects the vulnerability.

After this we determined the proportions of each factor by dividing the initial value given above by the total. Below is our final equation for the vulnerability score.

$S_i = \frac{v_i}{v_{max}} * 50 + 50$  where  $S_i$  is the vulnerability score for the  $i$ th neighborhood and  $v_i$  is the value for the  $i$ th neighborhood and  $v_{max}$  is the highest value out of the neighborhoods. A higher vulnerability score means that the neighborhood is more vulnerable.

After creating our equation we wanted to create a resource score, which can help the city determine which neighborhood needs the most resources diverted to it. We accomplished this by using the same method as described above, except adding factors relating to demographics in the respective neighborhoods.

After computing the PCA values we have the following equation for resource score:

$S_i = \frac{v_i}{v_{max}} * 50 + 50$  where  $S_i$  is the resource score for the  $i$ th neighborhood and  $v_i$  is the value for the  $i$ th neighborhood and  $v_{max}$  is the highest value out of the neighborhoods. A higher resource score means that the neighborhood has more resources, so it needs less

Variables	Eigenvectors multiplied by Eigenvalues summed up	Proportional weights
$N_H$	8.282963	0.060527
$P_N$	13.18483	0.096347
$H_{2010}$	15.60522	0.114034
$H_{1990}$	14.34078	0.104794
$H_{1970}$	12.36929	0.090388
$H_{1950}$	14.67964	0.107271
$H_{<1950}$	14.81246	0.108241
$D_H$	9.992278	0.073018
$T_H$	12.39486	0.090575
$P_H$	10.592	0.077400
$M_H$	10.592	0.077400

Table 13: Summation of scaled eigenvectors their proportional rate

Neighborhood	Resource Score
Downtown/South Main Arts District/South Bluffs	28.2831
Lakeland/Arlington/Brunswick	74.6408
Collierville/Piperton	100
$\vdots$	$\vdots$
Germantown, Zipcode 1	65.6563
Germantown, Zipcode 2	48.9859
South Riverdale	32.9456

Table 14: Resource scores for the neighborhoods

support.

#### 4.4. Analyzing the Results

Based on our results for our vulnerability score, we have determined that East Memphis, Tennessee, is the at the most risk with a vulnerability score of 100. Contrary to this, Rossville has a vulnerability score of 9.99853204. These results are consistent with the data, as Rossville is a small town in Memphis which has a low number of buildings in proportion to open space and most of its homes built more recently. East Memphis, on the other hand, is much larger with a much higher proportion of developed, open space in neighborhood.

Based on the results for our resource score we can see that South Forum / Washington Heights needs the most resources diverted to it with a resource score of 14.406 and Collierville / Piperton needs the least with a resource score of 100. This is consistent with our results and data, along with our previous vulnerability score.

During heat waves, the city should prioritize resource allocation based on our given resource score, with a higher resource score meaning that the city has more resources and

needs less resources. This would include deploying cooling centers and mobile cooling units to the most vulnerable areas, targeting public health messaging and outreach to vulnerable populations in those areas, prioritizing power restoration in those areas in case of outages, and increasing the number of emergency services in those areas.

## 4.5. Strengths and Weaknesses

### 4.5.1 Strengths

- Principal component analysis allows us to find the importance of the various factors without needing previous data of vulnerability scores and resource scores. Without PCA, we wouldn't have been able to determine the importance factors since we were unable to run a regression.
- The PCA relies on data to determine vulnerability, making the assessment more objective and less prone to bias.

### 4.5.2 Weaknesses

- PCA does not give the most accurate values as it is a way to manipulate data and reduce data dimensions. Using PCA to determine feature importance isn't the most effective because the principal components in PCA are linear combinations of the original factors and not the factors themselves, which means we have to extract the factor importance from the eigenvectors.
- While the report provides vulnerability scores, it could offer a more in-depth interpretation of the principal components themselves. Understanding which factors contribute most to each component would provide valuable insights.

## 5. CONCLUSIONS

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### 5.1. Further Studies

We have focused our model on Memphis as discussed in the initial problem. Future studies could expand this by developing similar models for other cities with varying climate and building stock characteristics, allowing for comparative analyses and broader applicability. We could also add more granular and niche data related to specific neighborhoods, which would increase the accuracy of the model. In addition to this, future research could assist us in incorporating dynamic occupancy models that account for variations in household schedules, thermostat preferences, and adaptive behaviors during heat waves. This would lead to more realistic power demand projections.

### 5.2. Summary

This report explores how heat waves are impacting households in Memphis, Tennessee. It employs various analyses to examine indoor temperatures, forecast power demand, and

pinpoint neighborhoods that are most at risk. The use of a General Energy Balance Equation highlights how building features impact indoor heat levels during heat waves, with some designs exacerbating the situation. Additionally, the report employs a thermal equilibrium model to project a notable rise in future electricity demand due to heat waves. This underscores the necessity for new technologies and policies to address the anticipated increase in power requirements. Moreover, a Principal Component Analysis is used to identify neighborhoods that face the greatest risk during heat waves. This allows for targeted interventions, such as improving building designs or implementing community programs. Together, these models provide a comprehensive understanding of heat wave impacts and suggest ways to provide resources to impacted community. The report stresses the need for enhancing building efficiency, modernizing the power grid, and reducing neighborhood vulnerability to strengthen urban resilience against extreme heat.

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## 7. APPENDIX: CODE LISTED FOR TECHNICAL COMPUTING CONSIDERATION

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### 7.1. Question I

#### Question I

```

1 import pandas as pd
2 import matplotlib.pyplot as plt
3 import numpy as np
4 from matplotlib.dates import DateFormatter
5 import datetime
6
7 # Load data from Excel files
8 accommodation_data = pd.read_excel("Accomadation_Memphis.xlsx", sheet_name="Sheet1")
9 weather_data = pd.read_excel("weather.xlsx", sheet_name="Sheet1")
10
11 # Process accommodation data
12 homes = []
13 for i in range(1, 5): # Columns B to E correspond to Home 1 to Home 4
14     home = {
15         'name': f'Home {i}',
16         'type': accommodation_data.iloc[1, i], # Row 1: Accommodation type
17         'year_built': accommodation_data.iloc[7, i], # Row 7: Year structure built
18         'shade': accommodation_data.iloc[9, i], # Row 9: Shade assessment
19         'size_sq_ft': accommodation_data.iloc[5, i], # Row 5: Size of unit (square feet)
20         'persons': accommodation_data.iloc[8, i], # Row 8: Persons living in unit
21     }
22     homes.append(home)

```

```

23
24 # Shading factor mapping
25 shade_map = {
26     'Very shady': 0,
27     'Not very shady': 0.67,
28     'Not at all shady': 1,
29 }
30
31 # Calculate additional parameters for each home
32 for home in homes:
33     home['size_sq_m'] = home['size_sq_ft'] / 10.764 # Convert square feet to square ↵
34     meters
35     if 'Single-family' in home['type']:
36         if home['year_built'] < 1970:
37             a = 1
38         elif 1970 <= home['year_built'] <= 2000:
39             a = 0.7
40         else:
41             a = 0.25
42     else: # Apartment
43         if home['year_built'] < 1970:
44             a = 0.5
45         elif 1970 <= home['year_built'] <= 2000:
46             a = 0.35
47         else:
48             a = 0.25
49     home['UA'] = a * home['size_sq_m']
50     home['f_shade'] = shade_map[home['shade']]
51     home['C'] = 75000
52     home['Q_internal'] = home['persons'] * 100 # Internal heat gains in watts (W)
53
54 # Process weather data
55 weather_data['Temp_C'] = (weather_data['Temperature ( F )'] - 32) * 5/9
56 weather_data['Temp_K'] = weather_data['Temp_C'] + 273.15 # Convert to Kelvin
57
58 # Extract hour from the Time column
59 if isinstance(weather_data['Time'].iloc[0], str):
60     weather_data['hour'] = weather_data['Time'].apply(lambda x: int(x.split(':')[0]) if ':'↵
61     ' in x else int(x))
62 else:
63     weather_data['hour'] = weather_data['Time'].apply(lambda x: x.hour if hasattr(x, 'hour'↵
64     ') else int(x))
65
66 weather_data['delta_t'] = weather_data['hour'].apply(lambda x: 3 if 6 <= x < 18 else 0)
67
68 # Simulate indoor temperature for each home (per-second steps)
69 for home in homes:
70     UA = home['UA']
71     f_shade = home['f_shade']
72     C = home['C']
73     Q_internal = home['Q_internal'] # In watts (W)
74     A = home['size_sq_m']
75
76     T_in = weather_data['Temp_K'].iloc[0] # Initial indoor temperature in Kelvin
77     results = [] # Store all relevant values for output
78
79     print(home)
80     for hour_idx in range(len(weather_data)):
81         current_weather = weather_data.iloc[hour_idx]
82         T_out = current_weather['Temp_K']
83         delta_t = current_weather['delta_t']
84         T_sa = T_out + f_shade * delta_t # Sol-air temperature in Kelvin
85
86     # Simulate 3600 seconds (1 hour)
87     for _ in range(3600):
88         ACH = 2 if T_out < T_in else 0.3 # Air changes per hour (ACH)
89         ACH_per_second = ACH / 3600 # Convert ACH to per-second

```

```

88         # Heat flows (W)
89         Q_conduction = UA * (T_sa - T_in)
90         Q_ventilation = ACH_per_second * 0.9 * A * (T_out - T_in)
91
92
93         # Temperature change (K)
94         dT = (Q_conduction + Q_internal / (home['size_sq_m'] * 3) + Q_ventilation) * 1 / C ←
95             # t = 1 second
96         T_in += dT # Update indoor temperature
97
98     # Append hourly results
99     results.append({
100         "Time": current_weather["Time"],
101         "Hour": hour_idx, # Add numeric hour index for plotting
102         "Outdoor_Temp_C": T_out - 273.15,
103         "Sol_Air_Temp_C": T_sa - 273.15,
104         "Q_Conduction_W": Q_conduction,
105         "Q_Ventilation_W": Q_ventilation,
106         "Q_Internal_W": Q_internal,
107         "ACH": ACH,
108         "Indoor_Temp_C": T_in - 273.15
109     })
110
111     home["results"] = results # Store results in each home
112
113 # Output results
114 for home in homes:
115     print(f"\n{home['name']} - Temperature Simulation Results:")
116     print(f"{'Time':<10} {'Outdoor ( C )':<15} {'Sol-Air ( C )':<15} {'Q_Cond (W)':<15} {'Q_Vent (W)':<15} {'Q_Int (W)':<15} {'ACH':<10} {'Indoor ( C )':<15}")
117     print("-" * 120)
118
119     for entry in home["results"]:
120         print(f"{'Time':<10} {'Outdoor_Temp_C':<15.2f} {'Sol_Air_Temp_C':<15.2f} {'Q_Conduction_W':<15.2f} {'Q_Ventilation_W':<15.2f} {'Q_Internal_W':<15.2f} {'ACH':<10.2f} {'Indoor_Temp_C':<15.2f}")
121
122 def generate_temperature_graphs(homes):
123     """
124     Generate and save temperature graphs for each home and a combined comparison graph.
125
126     Args:
127         homes (list): List of home dictionaries containing simulation results
128     """
129     # Use the default style and set white backgrounds for figures and axes
130     plt.style.use('default')
131     plt.rcParams['figure.facecolor'] = 'white'
132     plt.rcParams['axes.facecolor'] = 'white'
133
134     # Create a figure for all homes comparison with white background
135     plt.figure(figsize=(15, 8), facecolor='white')
136
137     # Colors for each home
138     colors = ['#1f77b4', '#ff7f0e', '#2ca02c', '#d62728']
139
140     # Use numeric hour indices for x-axis
141     x_values = [entry['Hour'] for entry in homes[0]['results']]
142     hour_labels = [f"{i}:00" for i in range(len(x_values))]
143
144     # Plot indoor temperature for each home on the combined graph
145     for i, home in enumerate(homes):
146         indoor_temps = [entry['Indoor_Temp_C'] for entry in home['results']]
147         plt.plot(x_values, indoor_temps, label=f"{home['name']}", color=colors[i], ←
148                 linewidth=2)
149
150     # Create individual graph for each home with white background

```



```

150     plt.figure(figsize=(12, 6), facecolor='white')
151
152     indoor_temps = [entry['Indoor_Temp_C'] for entry in home['results']]
153     outdoor_temps = [entry['Outdoor_Temp_C'] for entry in home['results']]
154     sol_air_temps = [entry['Sol_Air_Temp_C'] for entry in home['results']]
155
156     plt.plot(x_values, indoor_temps, label='Indoor Temperature', linewidth=2.5)
157     plt.plot(x_values, outdoor_temps, label='Outdoor Temperature', linewidth=1.5, ←
158             linestyle='—')
159     plt.plot(x_values, sol_air_temps, label='Sol-Air Temperature', linewidth=1.5, ←
160             linestyle=':')
161
162     # Add home details as text annotation
163     details = (f"Type: {home['type']}\n"
164               f"Year Built: {home['year_built']}\n"
165               f"Size: {home['size_sq_ft']} sq ft\n"
166               f"Shade: {home['shade']}\n"
167               f"Occupants: {home['persons']}")
168
169     plt.annotate(details, xy=(0.02, 0.02), xycoords='axes fraction',
170                 bbox=dict(boxstyle="round,pad=0.5", fc="white", alpha=0.8),
171                 fontsize=9, verticalalignment='bottom')
172
173     plt.title(f"Temperature Profile for {home['name']}", fontsize=16)
174     plt.xlabel('Hour of Day', fontsize=12)
175     plt.ylabel('Temperature ( C )', fontsize=12)
176     plt.legend(loc='best')
177     plt.grid(True, alpha=0.3)
178
179     plt.xticks(x_values, hour_labels, rotation=45)
180     plt.xlim(0, len(x_values)-1)
181
182     plt.figtext(0.5, 0.01, f"UA: {home['UA']:.2f}, Shade Factor: {home['f_shade']}",
183                 ha='center', fontsize=10, bbox=dict(boxstyle="round,pad=0.3", fc="white"←
184                 ", alpha=0.8))
185
186     plt.tight_layout()
187     plt.savefig(f"{home['name'].replace(' ', '_')}_temperature.png", dpi=300)
188     plt.close()
189
190     # Return to the combined graph
191     plt.figure(1)
192     plt.title('Indoor Temperature Comparison Across All Homes', fontsize=16)
193     plt.xlabel('Hour of Day', fontsize=12)
194     plt.ylabel('Temperature ( C )', fontsize=12)
195     plt.legend(loc='best')
196     plt.grid(True, alpha=0.3)
197
198     plt.xticks(x_values, hour_labels, rotation=45)
199     plt.xlim(0, len(x_values)-1)
200
201     # Add outdoor temperature to combined graph
202     outdoor_temps = [entry['Outdoor_Temp_C'] for entry in homes[0]['results']]
203     plt.plot(x_values, outdoor_temps, label='Outdoor Temperature', color='black', ←
204             linewidth=1.5, linestyle='—')
205
206     plt.legend(loc='best')
207     plt.tight_layout()
208     plt.savefig("All_Homes_Temperature_Comparison.png", dpi=300)
209     plt.close()
210
211     # Create a heat gain comparison chart with white background
212     plt.figure(figsize=(15, 8), facecolor='white')
213
214     for i, home in enumerate(homes):
215         conduction = [entry['Q_Conduction_W'] for entry in home['results']]
216         ventilation = [entry['Q_Ventilation_W'] for entry in home['results']]
217         internal = [entry['Q_Internal_W'] for entry in home['results']]

```

```

214     total_heat = [c + v + i for c, v, i in zip(conduction, ventilation, internal)]
215
216
217     plt.plot(x_values, total_heat, label=f"{home['name']} Total Heat Flow", color=↵
        colors[i], linewidth=2)
218
219     plt.title('Total Heat Flow Comparison Across All Homes', fontsize=16)
220     plt.xlabel('Hour of Day', fontsize=12)
221     plt.ylabel('Heat Flow (W)', fontsize=12)
222     plt.legend(loc='best')
223     plt.grid(True, alpha=0.3)
224
225     plt.xticks(x_values, hour_labels, rotation=45)
226     plt.xlim(0, len(x_values)-1)
227
228     plt.axhline(y=0, color='black', linestyle='-', alpha=0.3)
229     plt.tight_layout()
230     plt.savefig("All_Homes_Heat_Flow_Comparison.png", dpi=300)
231
232     print("All graphs have been generated successfully!")
233
234 # Generate the graphs after running the simulation
235 generate_temperature_graphs(homes)

```

## 7.2. Question II

### Question II

```

1  import pandas as pd
2  import numpy as np
3  import matplotlib.pyplot as plt
4  from copy import deepcopy
5
6  # Load data from Excel files
7  try:
8      accommodation_data = pd.read_excel("Accomadation_Memphis.xlsx", sheet_name="Sheet1")
9      weather_data = pd.read_excel("weather.xlsx", sheet_name="Sheet1")
10     print("Successfully loaded Excel files")
11 except Exception as e:
12     print(f"Error loading Excel files: {e}")
13     print("Using fallback data instead")
14     # Use fallback data if Excel files can't be loaded
15     weather_data = pd.DataFrame({
16         'Time': [f"{hour:02d}:00:00" for hour in range(24)],
17         'Temperature ( F )': [85, 85, 84, 83, 83, 83, 83, 84, 88, 91, 94, 96, 97,
18                               100, 100, 102, 102, 100, 99, 97, 94, 92, 91, 90, 89],
19     })
20
21 # Process homes data
22 homes = []
23 try:
24     for i in range(1, 5): # Columns B to E correspond to Home 1 to Home 4
25         home = {
26             'name': f'Home {i}',
27             'type': accommodation_data.iloc[1, i], # Row 1: Accommodation type
28             'year_built': accommodation_data.iloc[7, i], # Row 7: Year structure built
29             'shade': accommodation_data.iloc[9, i], # Row 9: Shade assessment
30             'size_sq_ft': accommodation_data.iloc[5, i], # Row 5: Size of unit (square ↵
                feet)
31             'persons': accommodation_data.iloc[8, i], # Row 8: Persons living in unit
32         }
33         homes.append(home)
34 except Exception as e:
35     print(f"Error processing home data: {e}")

```

```

36     print("Using fallback home data instead")
37     # Use fallback data if accommodation data can't be processed
38     homes = [
39         {'name': 'Home 1', 'type': 'Single-family home', 'year_built': 2010, 'shade': '←
40         Very shady', 'size_sq_ft': 88.26, 'persons': 3},
41         {'name': 'Home 2', 'type': 'Single-family home', 'year_built': 1985, 'shade': 'Not←
42         very shady', 'size_sq_ft': 185.70, 'persons': 4},
43         {'name': 'Home 3', 'type': 'Apartment', 'year_built': 2015, 'shade': 'Not at all ←
44         shady', 'size_sq_ft': 88.26, 'persons': 2},
45         {'name': 'Home 4', 'type': 'Apartment', 'year_built': 1960, 'shade': 'Not at all ←
46         shady', 'size_sq_ft': 391.05, 'persons': 2}
47     ]
48
49     # Shading factor mapping
50     shade_map = {
51         'Very shady': 0,
52         'Not very shady': 0.67,
53         'Not at all shady': 1,
54     }
55
56     # Calculate additional parameters for each home
57     for home in homes:
58         home['size_sq_m'] = home['size_sq_ft'] / 10.764 # Convert square feet to square ←
59         meters
60         if 'Single-family' in home['type']:
61             if home['year_built'] < 1970:
62                 a = 1
63             elif 1970 <= home['year_built'] <= 2000:
64                 a = 0.7
65             else:
66                 a = 0.25
67         else: # Apartment
68             if home['year_built'] < 1970:
69                 a = 0.5
70             elif 1970 <= home['year_built'] <= 2000:
71                 a = 0.35
72             else:
73                 a = 0.25
74         home['UA'] = a * home['size_sq_m']
75         home['f_shade'] = shade_map[home['shade']]
76         home['C'] = 75000
77         home['Q_internal'] = home['persons'] * 100 # Internal heat gains in watts (W)
78
79     # Process weather data
80     try:
81         weather_data['Temp_C'] = (weather_data['Temperature ( F )'] - 32) * 5/9
82         weather_data['Temp_K'] = weather_data['Temp_C'] + 273.15 # Convert to Kelvin
83
84         # Extract hour from the Time column
85         if isinstance(weather_data['Time'].iloc[0], str):
86             weather_data['hour'] = weather_data['Time'].apply(lambda x: int(x.split(':')[0]) ←
87                 if ':' in x else int(x))
88         else:
89             weather_data['hour'] = weather_data['Time'].apply(lambda x: x.hour if hasattr(x, '←
90                 hour') else int(x))
91
92         weather_data['delta_t'] = weather_data['hour'].apply(lambda x: 5 if 6 <= x < 18 else ←
93             0)
94     except Exception as e:
95         print(f"Error processing weather data: {e}")
96         print("Recreating weather data with proper structure")
97         # Create properly structured weather data if processing fails
98         weather_data = pd.DataFrame({
99             'Time': [f"{hour:02d}:00:00" for hour in range(24)],
100             'Temp_C': [29.4, 29.4, 28.9, 28.3, 28.3, 28.3, 28.9, 31.1, 32.8, 34.4, 35.6, 36.1,
101                 37.8, 37.8, 38.9, 38.9, 37.8, 37.2, 36.1, 34.4, 33.3, 32.8, 32.2, ←
102                 31.7],
103             'Temp_K': [t + 273.15 for t in [29.4, 29.4, 28.9, 28.3, 28.3, 28.3, 28.9, 31.1, ←

```

```

95         32.8, 34.4, 35.6, 36.1,
96         37.8, 37.8, 38.9, 38.9, 37.8, 37.2, 36.1, 34.4, ←
97         33.3, 32.8, 32.2, 31.7]],
98     'hour': list(range(24)),
99     'delta_t': [0, 0, 0, 0, 0, 0, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 0, 0, 0, 0, 0, ←
100                0]
101 })
102
103 # Memphis demographic data
104 MEMPHIS_POPULATION = 652236 # Population of Memphis
105 AVERAGE_HOUSEHOLD_SIZE = 2.5 # Average number of people per household
106 MEMPHIS_HOUSEHOLDS = MEMPHIS_POPULATION / AVERAGE_HOUSEHOLD_SIZE
107
108 # Housing distribution based on Memphis data
109 def calculate_housing_distribution():
110     """Calculate housing distribution based on Memphis housing data"""
111     # Distribution by housing type and age from data
112     return [
113         (0.321, 0), # 32.1% mapped to Home 1
114         (0.415, 1), # 41.5% mapped to Home 2
115         (0.110, 2), # 11.0% mapped to Home 3
116         (0.154, 3), # 15.4% mapped to Home 4
117     ]
118
119 HOUSING_DISTRIBUTION = calculate_housing_distribution()
120
121 def calculate_baseline_power():
122     """Calculate baseline power consumption for Memphis excluding AC"""
123     # From the data:
124     # Memphis annual usage per household: 15,172 kWh
125     annual_usage_per_household = 15172 # kWh
126
127     # Total number of households in Memphis/Shelby County
128     total_households = MEMPHIS_HOUSEHOLDS
129
130     # Calculate total annual residential consumption
131     annual_residential_kwh = annual_usage_per_household * total_households
132
133     # From the data, Shelby County annual consumption (2022): 9,768,296,000 kWh
134     total_annual_consumption = 9768296000 # kWh
135
136     # Calculate average power in watts
137     hours_in_year = 365 * 24
138     avg_power_w = (total_annual_consumption * 1000) / hours_in_year
139
140     # Summer baseline is higher than annual average
141     summer_non_ac_factor = 1.15 # 15% higher baseline during summer
142
143     # Adjust baseline to align with target power values
144     summer_baseline_w = avg_power_w * summer_non_ac_factor
145
146     return summer_baseline_w * 1.5 # Scaling factor to align with expected city-wide ←
147         impact
148
149 def run_sensitivity_analysis(base_params, homes, weather_data, housing_distribution, ←
150                             memphis_households):
151     """
152     Run sensitivity analysis on key model parameters
153
154     Args:
155         base_params: Dictionary with baseline parameters
156         homes: List of home dictionaries with thermal properties
157         weather_data: DataFrame with weather information
158         housing_distribution: List of (percentage, home_index) tuples
159         memphis_households: Number of households in Memphis
160
161     Returns:
162         Dictionary with sensitivity analysis results
163     """

```

```

158     """
159     print("Running sensitivity analysis...")
160
161     # Define parameter ranges to test
162     param_ranges = {
163         "AC_SETPPOINT_TEMP_C": [19, 20, 21, 22, 23, 24, 25],
164         "AC_POWER_RATING": [6000, 7000, 8000, 9000, 10000],
165         "AC_EFFICIENCY": [1.9, 2.1, 2.3, 2.5, 2.7],
166         "AC_LOAD_SCALE_FACTOR": [50, 60, 70, 80, 90]
167     }
168
169     # Capacity factor scenarios
170     capacity_factor_scenarios = [
171         {
172             "name": "All high efficiency",
173             "factors": {
174                 'Home 1': 1.0,
175                 'Home 2': 0.95,
176                 'Home 3': 0.9,
177                 'Home 4': 0.85
178             }
179         },
180         {
181             "name": "Baseline",
182             "factors": base_params['AC_CAPACITY_FACTOR']
183         },
184         {
185             "name": "All low efficiency",
186             "factors": {
187                 'Home 1': 0.9,
188                 'Home 2': 0.8,
189                 'Home 3': 0.7,
190                 'Home 4': 0.6
191             }
192         },
193         {
194             "name": "High variation",
195             "factors": {
196                 'Home 1': 1.0,
197                 'Home 2': 0.9,
198                 'Home 3': 0.7,
199                 'Home 4': 0.5
200             }
201         }
202     ]
203
204     # Initialize results containers
205     sensitivity_results = {}
206
207     # Run baseline model first
208     baseline_results = simulate_power_model(
209         base_params,
210         homes,
211         weather_data,
212         housing_distribution,
213         memphis_households
214     )
215
216     print(f"\nBaseline results:")
217     print(f"    Peak Power: {baseline_results['peak_power_mw']:.2f} MW")
218     print(f"    AC Contribution: {baseline_results['ac_contribution_mw']:.2f} MW ({←
        baseline_results['ac_percentage']:.1f}%)")
219
220     # Single parameter variations
221     for param_name, param_values in param_ranges.items():
222         results = []
223         print(f"\nTesting {param_name}...")
224 
```

```

225     for value in param_values:
226         # Update just this parameter
227         params = base_params.copy()
228         params[param_name] = value
229
230         # Run model
231         model_results = simulate_power_model(
232             params,
233             homes,
234             weather_data,
235             housing_distribution,
236             memphis_households
237         )
238
239         # Store results
240         results.append({
241             "param_value": value,
242             "peak_power_mw": model_results["peak_power_mw"],
243             "ac_percentage": model_results["ac_percentage"]
244         })
245
246         print(f"    {param_name} = {value}: Peak Power = {model_results['peak_power_mw']:.2 f} MW, AC = {model_results['ac_percentage']:.1 f}%")
247
248         sensitivity_results[param_name] = results
249
250     # Capacity factor scenarios
251     capacity_results = []
252     print("\nTesting capacity factor scenarios...")
253
254     for scenario in capacity_factor_scenarios:
255         # Update capacity factors
256         params = base_params.copy()
257         params["AC_CAPACITY_FACTOR"] = scenario["factors"]
258
259         # Run model
260         model_results = simulate_power_model(
261             params,
262             homes,
263             weather_data,
264             housing_distribution,
265             memphis_households
266         )
267
268         # Store results
269         capacity_results.append({
270             "scenario": scenario["name"],
271             "peak_power_mw": model_results["peak_power_mw"],
272             "ac_percentage": model_results["ac_percentage"]
273         })
274
275         print(f"    {scenario['name']}: Peak Power = {model_results['peak_power_mw']:.2 f} MW, AC = {model_results['ac_percentage']:.1 f}%")
276
277         sensitivity_results["capacity_factor"] = capacity_results
278
279     # Plot results
280     plot_sensitivity_results(sensitivity_results, baseline_results)
281
282     return sensitivity_results
283
284 def simulate_power_model(params, original_homes, original_weather, housing_distribution, ←
    memphis_households, temp_increase=0):
285     """
286     Run the Memphis power model with specific parameters
287
288     Args:
289         params: Dictionary of model parameters

```

```

290     original_homes: List of home dictionaries
291     original_weather: DataFrame with weather information
292     housing_distribution: List of (percentage, home_index) tuples
293     memphis_households: Number of households in Memphis
294     temp_increase: Optional temperature increase in C
295
296 Returns:
297     Dictionary with simulation results
298 """
299 # Get parameters
300 ac_power_rating = params.get('AC_POWER_RATING')
301 ac_setpoint_temp_c = params.get('AC_SETPOINT_TEMP_C')
302 ac_efficiency = params.get('AC_EFFICIENCY')
303 ac_capacity_factor = params.get('AC_CAPACITY_FACTOR')
304 ac_load_scale_factor = params.get('AC_LOAD_SCALE_FACTOR')
305
306 # Make deep copies to avoid modifying originals
307 homes = deepcopy(original_homes)
308 weather_data = original_weather.copy()
309
310 # Apply temperature increase if specified
311 if temp_increase > 0:
312     weather_data = weather_data.copy()
313     weather_data["Temp_C"] += temp_increase
314     weather_data["Temp_K"] = weather_data["Temp_C"] + 273.15
315
316 # Simulate indoor temperature for each home
317 for home in homes:
318     UA = home['UA']
319     f_shade = home['f_shade']
320     C = home['C']
321     Q_internal = home['Q_internal']
322     A = home['size_sq_m']
323
324     T_in = ac_setpoint_temp_c + 273.15
325     results = []
326
327     # Get capacity factor for this home type
328     capacity_factor = ac_capacity_factor[home['name']]
329
330     for hour_idx in range(len(weather_data)):
331         current_weather = weather_data.iloc[hour_idx]
332         T_out = current_weather['Temp_K']
333         delta_t = current_weather['delta_t']
334         T_sa = T_out + f_shade * delta_t
335
336         # AC parameters
337         AC_setpoint_K = ac_setpoint_temp_c + 273.15
338
339         # Calculate heat gains
340         Q_conduction_gain = UA * (T_sa - AC_setpoint_K)
341         Q_internal_gain = Q_internal / (home['size_sq_m'] * 3)
342         ACH = 0.3
343         ACH_per_second = ACH / 3600
344         Q_ventilation_gain = ACH_per_second * 0.9 * A * (T_out - AC_setpoint_K)
345
346         # Total heat gain that AC must remove
347         Q_total_gain = Q_conduction_gain + Q_internal_gain + Q_ventilation_gain
348
349         # Required cooling power
350         required_cooling_power = max(0, Q_total_gain)
351
352         # Max cooling capacity
353         max_cooling_capacity = ac_power_rating * ac_efficiency * capacity_factor
354
355         # Required electrical power
356         required_electrical_power = required_cooling_power / ac_efficiency
357

```

```

358     # Actual AC power usage
359     ac_power_usage = min(required_electrical_power, ac_power_rating * ←
        capacity_factor)
360     actual_cooling_power = ac_power_usage * ac_efficiency
361
362     # Can AC maintain setpoint?
363     can_maintain_setpoint = actual_cooling_power >= required_cooling_power
364
365     # Calculate indoor temperature
366     actual_T_in = T_in
367
368     if not can_maintain_setpoint and required_cooling_power > 0:
369         # Calculate equilibrium temperature
370         a = UA + (ACH_per_second * 0.9 * A)
371         b = UA * T_sa + (ACH_per_second * 0.9 * A * T_out) + Q_internal_gain - ←
            actual_cooling_power
372
373         equilibrium_T_in = b / a
374
375         # Using Newton's cooling for temperature transition
376         cooling_rate = 1/C
377         temp_change = (equilibrium_T_in - T_in) * (1 - np.exp(-cooling_rate * ←
            3600))
378
379         actual_T_in = T_in + temp_change
380     else:
381         actual_T_in = AC_setpoint_K
382
383     # Update temperature for next hour
384     T_in = actual_T_in
385
386     # Store results
387     results.append({
388         "Time": current_weather["Time"],
389         "Hour": hour_idx,
390         "Outdoor_Temp_C": T_out - 273.15,
391         "Indoor_Temp_C": T_in - 273.15,
392         "AC_Power_W": ac_power_usage
393     })
394
395     home["results"] = results
396
397     # Calculate baseline power
398     baseline_power = calculate_baseline_power()
399
400     # Calculate hourly power consumption
401     hourly_power = []
402
403     for hour in range(len(weather_data)):
404         # Calculate total AC power for this hour
405         total_ac_power = 0
406         for dist_pct, home_idx in housing_distribution:
407             home = homes[home_idx]
408             ac_power = home["results"][hour]["AC_Power_W"]
409             households_of_type = memphis_households * dist_pct
410             total_ac_power += ac_power * households_of_type
411
412         # Apply scaling factor
413         total_ac_power *= ac_load_scale_factor
414
415         # Total power
416         total_power = baseline_power + total_ac_power
417
418         hourly_power.append({
419             "Hour": hour,
420             "Baseline_Power_MW": baseline_power / 1e6,
421             "AC_Power_MW": total_ac_power / 1e6,
422             "Total_Power_MW": total_power / 1e6

```



```

423     })
424
425     # Find peak power
426     power_df = pd.DataFrame(hourly_power)
427     peak_power = power_df["Total_Power_MW"].max()
428     peak_hour_idx = power_df["Total_Power_MW"].idxmax()
429     peak_hour_data = power_df.iloc[peak_hour_idx]
430
431     # Calculate AC percentage
432     ac_percentage = peak_hour_data["AC_Power_MW"] / peak_hour_data["Total_Power_MW"] * 100
433
434     # Return key results
435     return {
436         "peak_power_mw": peak_power,
437         "peak_hour": peak_hour_data["Hour"],
438         "ac_contribution_mw": peak_hour_data["AC_Power_MW"],
439         "baseline_power_mw": peak_hour_data["Baseline_Power_MW"],
440         "ac_percentage": ac_percentage
441     }
442
443 def plot_sensitivity_results(results, baseline):
444     """
445     Create visualizations of sensitivity analysis results
446     """
447     # Set up the figure
448     plt.figure(figsize=(20, 15))
449     plt.suptitle("Sensitivity Analysis of Memphis Power Model", fontsize=16)
450
451     # Plot continuous parameters
452     continuous_params = ["AC_SETPOINT_TEMP_C", "AC_POWER_RATING", "AC EFFICIENCY", "↔
453         AC_LOAD_SCALE_FACTOR"]
454
455     for i, param in enumerate(continuous_params):
456         # Extract results for this parameter
457         param_results = results[param]
458         param_values = [r["param_value"] for r in param_results]
459         peak_powers = [r["peak_power_mw"] for r in param_results]
460         ac_percentages = [r["ac_percentage"] for r in param_results]
461
462         # Plot peak power
463         plt.subplot(3, 2, i+1)
464         plt.plot(param_values, peak_powers, 'b-o', linewidth=2)
465         plt.axhline(y=baseline["peak_power_mw"], color='r', linestyle='—', label='↔
466             Baseline')
467
468         # Format labels based on parameter
469         if param == "AC_SETPOINT_TEMP_C":
470             plt.xlabel("AC Setpoint Temperature ( C )", fontsize=12)
471         elif param == "AC POWER_RATING":
472             plt.xlabel("AC Power Rating (W)", fontsize=12)
473         elif param == "AC EFFICIENCY":
474             plt.xlabel("AC Coefficient of Performance", fontsize=12)
475         elif param == "AC_LOAD_SCALE_FACTOR":
476             plt.xlabel("AC Load Scaling Factor", fontsize=12)
477
478         plt.ylabel("Peak Power Demand (MW)", fontsize=12)
479         plt.title(f"Effect of {param} on Peak Power", fontsize=14)
480         plt.grid(True, alpha=0.3)
481         plt.legend()
482
483         # Plot AC percentage
484         plt.subplot(3, 2, i+3)
485         plt.plot(param_values, ac_percentages, 'g-o', linewidth=2)
486         plt.axhline(y=baseline["ac_percentage"], color='r', linestyle='—', label='↔
487             Baseline')
488
489         # Same x-label
490         if param == "AC_SETPOINT_TEMP_C":

```

```

488         plt.xlabel("AC Setpoint Temperature ( C )", fontsize=12)
489     elif param == "AC_POWER_RATING":
490         plt.xlabel("AC Power Rating (W)", fontsize=12)
491     elif param == "AC EFFICIENCY":
492         plt.xlabel("AC Coefficient of Performance", fontsize=12)
493     elif param == "AC_LOAD_SCALE_FACTOR":
494         plt.xlabel("AC Load Scaling Factor", fontsize=12)
495
496     plt.ylabel("AC Contribution (%)", fontsize=12)
497     plt.title(f"Effect of {param} on AC Percentage", fontsize=14)
498     plt.grid(True, alpha=0.3)
499     plt.legend()
500
501     # Plot capacity factor scenarios
502     capacity_results = results["capacity_factor"]
503     scenarios = [r["scenario"] for r in capacity_results]
504     peak_powers = [r["peak_power_mw"] for r in capacity_results]
505     ac_percentages = [r["ac_percentage"] for r in capacity_results]
506
507     plt.subplot(3, 1, 3)
508     x = np.arange(len(scenarios))
509     width = 0.35
510
511     ax1 = plt.gca()
512     bars1 = ax1.bar(x - width/2, peak_powers, width, label='Peak Power (MW)', color='b')
513     ax1.set_ylabel('Peak Power (MW)', fontsize=12)
514     ax1.set_xticks(x)
515     ax1.set_xticklabels(scenarios, rotation=45, ha='right')
516
517     ax2 = ax1.twinx()
518     bars2 = ax2.bar(x + width/2, ac_percentages, width, label='AC Percentage (%)', color='g')
519     ax2.set_ylabel('AC Contribution (%)', fontsize=12)
520
521     ax1.set_title('Effect of Capacity Factor Scenarios', fontsize=14)
522     ax1.legend(loc='upper left')
523     ax2.legend(loc='upper right')
524
525     plt.tight_layout()
526     plt.savefig('memphis_power_sensitivity.png', dpi=300)
527     plt.show()
528
529 def project_2044_power_demand(base_params, homes, original_weather, housing_distribution, ←
    memphis_households):
530     """
531     Project power demand for 2044 under different climate and technology scenarios
532
533     Args:
534         base_params: Dictionary with baseline parameters
535         homes: List of home dictionaries
536         original_weather: DataFrame with weather information
537         housing_distribution: List of (percentage, home_index) tuples
538         memphis_households: Number of households in Memphis
539
540     Returns:
541         List of dictionaries with projection results
542     """
543     # Define scenarios for 2044
544     scenarios = [
545         {
546             "name": "Current Conditions (Baseline)",
547             "params": base_params.copy(),
548             "temp_increase": 0 # No change
549         },
550         {
551             "name": "Climate Change Only",
552             "params": {
553                 **base_params,

```

```

554         'AC_LOAD_SCALE_FACTOR': base_params['AC_LOAD_SCALE_FACTOR'] * 1.3 # 30% ↔
555         increase in cooling demand
556     },
557     "temp_increase": 2.0 # 2 C warmer
558 },
559 {
560     "name": "Tech Improvement Only",
561     "params": {
562         **base_params,
563         'AC_POWER_RATING': base_params['AC_POWER_RATING'] * 0.9, # Smaller units ↔
564         needed
565         'AC_EFFICIENCY': base_params['AC_EFFICIENCY'] * 1.4, # 40% more efficient
566         'AC_CAPACITY_FACTOR': { # Better capacity across all homes
567             'Home 1': 1.0,
568             'Home 2': 0.98,
569             'Home 3': 0.95,
570             'Home 4': 0.9
571         },
572         'AC_LOAD_SCALE_FACTOR': base_params['AC_LOAD_SCALE_FACTOR'] * 0.7 # ↔
573         Reduced by 30% due to efficiency
574     },
575     "temp_increase": 0 # No change
576 },
577 {
578     "name": "Climate Change + Tech Improvement",
579     "params": {
580         **base_params,
581         'AC_POWER_RATING': base_params['AC_POWER_RATING'] * 0.9,
582         'AC_EFFICIENCY': base_params['AC_EFFICIENCY'] * 1.4,
583         'AC_CAPACITY_FACTOR': {
584             'Home 1': 1.0,
585             'Home 2': 0.98,
586             'Home 3': 0.95,
587             'Home 4': 0.9
588         },
589         'AC_LOAD_SCALE_FACTOR': base_params['AC_LOAD_SCALE_FACTOR'] # Combined ↔
590         effects roughly cancel out
591     },
592     "temp_increase": 2.0 # 2 C warmer
593 }
594 ]
595
596 results = []
597 print("\nProjecting 2044 Power Demand Scenarios:")
598
599 for scenario in scenarios:
600     # Run model with scenario parameters and temperature increase
601     model_results = simulate_power_model(
602         scenario["params"],
603         homes,
604         original_weather,
605         housing_distribution,
606         memphis_households,
607         scenario["temp_increase"]
608     )
609
610     # Store results
611     results.append({
612         "scenario": scenario["name"],
613         "peak_power_mw": model_results["peak_power_mw"],
614         "ac_contribution_mw": model_results["ac_contribution_mw"],
615         "ac_percentage": model_results["ac_percentage"],
616         "temp_increase": scenario["temp_increase"]
617     })
618
619 print(f" {scenario['name']}:")
620 print(f" - Peak Power: {model_results['peak_power_mw']:.2f} MW")
621 print(f" - AC Contribution: {model_results['ac_contribution_mw']:.2f} MW (↔

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        model_results['ac_percentage']:.1f}%")
618
619 # Plot results
620 plt.figure(figsize=(12, 8))
621
622 # Extract data for plotting
623 scenario_names = [r["scenario"] for r in results]
624 peak_powers = [r["peak_power_mw"] for r in results]
625 ac_contributions = [r["ac_contribution_mw"] for r in results]
626 temp_increases = [r["temp_increase"] for r in results]
627
628 # Create bar plot
629 x = np.arange(len(scenario_names))
630 width = 0.35
631
632 fig, ax1 = plt.subplots(figsize=(14, 8))
633 ax1.bar(x, peak_powers, width, label='Total Peak Power', color='blue', alpha=0.7)
634 ax1.bar(x, ac_contributions, width, label='AC Contribution', color='red', alpha=0.7)
635
636 # Add temperature increase indicators
637 for i, temp in enumerate(temp_increases):
638     if temp > 0:
639         ax1.annotate(f"+{temp} C ", xy=(i, peak_powers[i] + 50), ha='center', va='↵
        bottom',
        bbox=dict(boxstyle="round,pad=0.3", fc="yellow", alpha=0.7))
640
641 # Customize plot
642 ax1.set_ylabel('Power (MW)', fontsize=14)
643 ax1.set_title('Projected Memphis Power Demand in 2044', fontsize=16)
644 ax1.set_xticks(x)
645 ax1.set_xticklabels(scenario_names, rotation=45, ha='right', fontsize=12)
646 ax1.legend(loc='upper left', fontsize=12)
647 ax1.grid(axis='y', alpha=0.3)
648
649 # Add baseline line
650 ax1.axhline(y=results[0]["peak_power_mw"], color='k', linestyle='—', alpha=0.5, label='↵
    'Current Baseline')
651
652 # Add percentage labels on bars
653 for i, v in enumerate(peak_powers):
654     ac_pct = (ac_contributions[i] / v) * 100
655     ax1.text(i, v/2, f"{ac_pct:.1f}%", color='white', fontweight='bold', ha='center')
656
657 plt.tight_layout()
658 plt.savefig('memphis_power_2044_projection.png', dpi=300)
659 plt.show()
660
661 return results
662
663 # Create a parameters dictionary
664 base_params = {
665     'AC_POWER_RATING': 8000,
666     'AC_SETPOINT_TEMP_C': 21,
667     'AC EFFICIENCY': 2.3,
668     'AC_CAPACITY_FACTOR': {
669         'Home 1': 1.0,
670         'Home 2': 0.9,
671         'Home 3': 0.8,
672         'Home 4': 0.7
673     },
674     'AC_LOAD_SCALE_FACTOR': 70.0
675 }
676
677 # Run the sensitivity analysis
678 sensitivity_results = run_sensitivity_analysis(
679     base_params,
680     homes,
681     weather_data,
682 
```

```
683     HOUSING_DISTRIBUTION ,
684     MEMPHIS_HOUSEHOLDS
685 )
686
687 # Project power demand for 2044
688 projection_2044 = project_2044_power_demand(
689     base_params ,
690     homes ,
691     weather_data ,
692     HOUSING_DISTRIBUTION ,
693     MEMPHIS_HOUSEHOLDS
694 )
```