MathWorks Math Modeling Challenge 2025

Rye Country Day School

Team #18015, Rye, New York

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M3 Challenge FINALIST—$5,000 Team Award

JUDGE COMMENTS

Specifically for Team #18015—Submitted at the close of triage judging

COMMENT 1: Great work! The executive summary is very well-written, and the solution to problem 1 is nicely thought out. Problem 2 is also done well, considering temperature as a key component to energy expenditure, as opposed to only past values. Problem 3 is nicely done but could have been made stronger with more consideration for vulnerability. Overall, very good work though.

COMMENT 2: This paper models extreme heat impacts in Memphis, focusing on indoor temperatures, peak energy demand, and vulnerability assessment. Strengths include its structured approach using differential equations, regression analysis, and entropy-weighted models to inform emergency planning. However, assumptions of uniform housing conditions and linear energy trends may oversimplify real-world variations. Incorporating policy shifts, adaptive behaviors, and non-linear modeling would improve predictive accuracy and applicability.

COMMENT 3: A) Very nice summary. Everything is summarized very well. B) Nice choice of models in Q1. Everything is presented and explained very well. C) Good use of regression in Q2. D) Reasonable assumptions are used in Q3 and the weights are nicely calculated. Perhaps more detail could be given on how the scores were calculated in the final rankings.

COMMENT 4: The executive summary is very strong, making a compelling argument for your report, as well as giving a good mix of methods and results.

Your answer to question one is very detailed and fairly easy to follow. Basing it on physical principles and calculus makes it easily repeatable, well done! To make all points clear you should specify which data you used for your curve fitting of the outside temperature. Some of the equation simplifications are also a little bit unclear (eq 6-8).

I enjoyed your approach to correlate the rising temperatures and energy consumption as a drawing regression in answer the question two. Once again, a mention of data sources would be of benefit here.

The use of entropy model and answer the question three was an unusual one, but very welcomed! Some additional commentary on how to interpret your results would be of benefit here.

Overall, the report is very well written and well displayed. Citations are easy to follow, well done!

M3 Challenge 2025

Hot Button Issue: *Staying Cool as the World Heats Up*

Team #18015

March 1st, 2025

Executive Summary

To the Director of the Memphis and Shelby County Division of Planning and Development,

The Earth has been steadily warming. In fact, 2024 was named the Earth's warmest year on record. The last time Earth had a colder than average year was 1976: 48 years ago. With increasing global temperatures, heat waves and extreme weather events have been occurring at greater frequencies, disrupting communities around the globe. Power grids, which are vital to a city's ability to provide services for the public, are placed under particular strain during heat waves. This excess strain during heat waves can lead to heat-induced power outages. Certain citizens are disproportionally affected by these heat-induced power outages. Therefore, government resources must be justly allocated to neighborhoods to ensure that help can effectively reach populations in need.

In this paper, we first predicted the interior temperatures of different housing structures in Memphis over a 24-hour period based on the ambient temperature of the surrounding environment with a physical analysis approach. We started by fitting a sinusoidal curve to the ambient temperature data since it is known to oscillate over time, and then used the differential form of Newton's Law of Cooling to calculate the temperature inside of the building with no air conditioning. We analytically solved the differential equation and applied it twice to represent the transfer of heat energy from the surrounding environment into the walls of the building, and the transfer from the walls to the interior of the building. We found that the maximum temperatures for the 4 homes over the 24 hour period were 38.458, 38.441, 38.379, and 38.440°C, respectively.

Next, we built a model to determine the peak demand in Memphis's power grid during the summer months due to extensive energy consumption. We compiled a list of variables with possible correlations to energy consumption and determined peak temperature to have a statistically significant correlation. We created linear regression models for the increase in temperature in Memphis per year due to global warming as well as the anomalies in temperature over time. In the year 2025, we predicted that the temperature will be $33.1°C$ and will correlate to a monthly peak demand of 1.004×10^9 kWh. We predicted that in the year 2045, the predicted temperature will be 33.58°C which correlates to a monthly peak demand of 1.018×10^9 kWh.

Lastly, we created criteria for the vulnerability of certain neighborhoods in Memphis, TN and determined what proportion of total heat wave relief resources should be allocated to each neighborhood. We identified five main factors that affected the vulnerability of the population: the proportion of seniors and children under 18 with respect to the total population, the weighted age of the houses, the amount of open space and forestry in an area, the amount of people that commute to work via public transit as opposed to driving or working from home, and the median income. After defining "scores" for each of these variables, we used an Entropy Weight Model to weight the criteria based on their variability (i.e. parameters with higher variability are considered more important to the final ranking). Then, we multiplied the weights for each neighborhood with the scores for each individual category to get the final scores for each neighborhood, then ranked the neighborhoods by score, with a higher score indicating a higher vulnerability. Finally, we multiplied these final scores with the populations of each neighborhood to determine which neighborhoods require the most resources. We found that Uptown/Pinch District required the most resources out of all the neighborhoods with 13% of all resources, while East Memphis/Colonial Yorkshire required the least with 2% of all resources in the city.

We believe that these results give key insight into the gravity of this issue of heat waves in the city, their effect on non-air-conditioned homes in Memphis, the total possible strain on Memphis power grids during heat waves, and the vulnerabilities of different districts to heat waves. We hope that these results will assist lawmakers to enact policy to support different communities in the city during crises caused by global warming, and to ensure that the city is best prepared to protect its citizens and businesses for similar crises to come.

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Hot Button Issue: *Staying Cool as the World Heats Up*

1 Q1: Hot to Go

1.1 Defining the Problem

The first problem asks us to model the temperature inside of a building of a given size that does not have air conditioning as a function of the **ambient temperature** during a heat wave. We will be utilizing data on heat waves and building sizes from Memphis, TN to create our model.

1.2 Assumptions

1. **Initial building temperature starts at the same temperature as the initial ambient temperature (29.4°C).**
   Justification: With no separate air conditioning, the initial building temperature will initially start at room temperature.
2. **The building temperature does not vary significantly from ambient temperature during a heat wave.**
   Justification: This assumption is necessary to use Newton's Law of Cooling in this model. From the data, the difference between the ambient temperature and room temperature is strictly less than 40°F, which is sufficient for the model to apply as all temperature calculations are done in Kelvin.
3. **Ambient temperature is constant at every point around the building.**
   Justification: This allows us to simplify the model so as to not account for fluctuations in temperature at different points due to wind turbulence, data which is not given for this problem.
4. **Rooms are square in floor plan and of equal size.**
   Justification: This simplifies the surface area calculations, as the dimensions of the houses are otherwise not given and thus the change in temperature could not be calculated.
5. **Ceiling height can be modeled as 2.4 m.**
   Justification: This is the average height for a house in Memphis, TN (Ceiling Heights in Homes and Offices — Zell/Lurie Real Estate Center). This can be used to model the height of a single-story house, which can scale to an n-story building.
6. **Houses modeled are made of wood.**
   Justification: The vast majority of houses in the US are made from wood, and thus such a simplification would be accurate (Semuels, 2021). This simplification allows for the emissivity and specific heat capacity of wood to be used for heat emission.
7. **Heat enters houses through the walls and ceiling only.**
   Justification: We are neglecting the effect of geothermal fluctuations in our model as they are not in the spirit of the problem.
8. **The thickness of the walls and ceiling is 12 in.**
   Justification: This is a standard value for exterior wall thickness in the US (Understanding Wall Thickness: Why It Matters for Effective Insulation, 2024).

1.3 Variables

The below variables are factors we take into account in our model.

Table 1: Variable Definitions of Variable Heating Model

Variable	Definition
$T(t)$	Temperature at a Given Time t (K)
T_0	Initial Temperature of House (K)
$T_s(t)$	Surrounding Temperature (K)
ε	Emissivity Coefficient (W m^{-2})
k	Cooling Constant (s^{-1})
σ	Stefan-Boltzmann Constant (W m^{-2} K^{-4})
m	Mass of Building (kg)
c	Specific Heat Capacity of Building (J kg^{-1} K^{-1})
t	Time (h)
A	Surface Area of Building (m^2)
h	Height of Building (m)
F	Floor Area (m^2)
Q	Thermal Energy of the Building (J)

This table shows the variable definitions for Variable Heating Model.

1.4 Model and Solution

1.4.1 Regressing Outside Temperature with Time

One of the things we wanted to model was how the ambient temperature during a heat wave varies with time. Modeling the ambient temperature as non-constant allows us to account for fluctuations in temperature due to heat waves.

We decided to model the ambient temperature as a sinusoidal wave as a function of time because temperature tends to oscillate with time. That is, the ambient temperature T_s can be modeled according to

$$T_s(t) = \alpha \sin(\omega t + \Phi) + C,$$

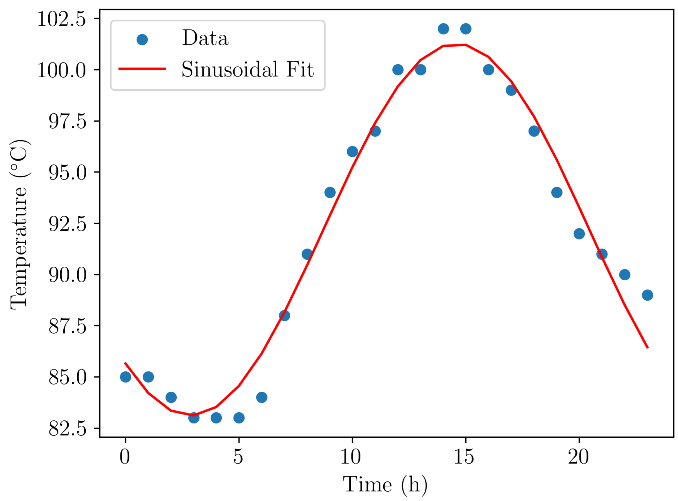
where t represents time (in hours), and ω, Φ, C are constants.

To fit this model to data, we performed sinusoidal regression in Python using the `scipy` library's `curve_fit` function. We found that Equation 1 best fit the data with an R^2 value of 0.9722.

$$T_s(t) = 5.060 \sin(0.268t - 2.333) + 33.427 \tag{1}$$

Figure 1 shows Equation 1 plotted on top of the original data.

Figure 1: Original Data vs Regressed Model



This figure shows Equation 1 plotted on top of the original data. As can be seen, the regressed model closely matches the original data.

An R^2 of 0.9722 indicates a good fit to the data, showing that the heat fluctuations over time can be modeled sinusoidally.

1.4.2 Variable Heating Model

This model is based on Newton's Law of Cooling (Michael, 2012, 5397), which sets up a differential equation for the rate of change of the temperature for an object based on the surrounding temperature, initial conditions, and other factors. We begin with $\frac{\mathrm{d}Q}{\mathrm{d}t}$, the rate of change of thermal energy of the building with respect of time, which is an equation based on the Stefan-Boltzmann Law (Jianjun, 2023, 131).

$$\frac{\mathrm{d}Q}{\mathrm{d}t} = \varepsilon\sigma A\left(T^4 - T_s^4\right) \tag{2}$$

We also know the equation for specific heat capacity (Specific Heat Calculations, 2022), shown in Equation 3.

$$\begin{aligned} q &= mc\Delta T &(3)\\ \implies \frac{\mathrm{d}Q}{\mathrm{d}t} &= mc\frac{\mathrm{d}T}{\mathrm{d}t} &(4)\\ \implies \frac{\mathrm{d}T}{\mathrm{d}t} &= \frac{-\varepsilon\sigma A}{mc}(T^4 - T_s^4) &(5)\end{aligned}$$

We then derive Equation 8 for the rate of change of the interior temperature of a building. Using Equation 2, we find that

$$\begin{aligned} \frac{\mathrm{d}T}{\mathrm{d}t} &= -\frac{\varepsilon\sigma A}{mc}(T - T_s)(T^3 + T^2T_s + TT_s^2 + T_s^3) &(6)\\ &\approx -\frac{4\varepsilon\sigma AT_s^3}{mc}(T - T_s) &(7)\\ &= -k(T - T_s) &(8)\end{aligned}$$

where k, the thermal cooling constant, is equal to $\frac{4\varepsilon\sigma AT_s^3}{mc}$. We chose to simplify T_s as being constant in this case as its variation in in Kelvins, which is what Newton's Law of Differential Cooling uses, is not significant compared to the actual temperature (i.e., a variation of $\sim$10 K over an average of $\sim$300 K, or an $\sim$3% variation). In addition, it also simplified the integration required to solve for an analytical solution. We then expanded Equation 8 to the form of Equation 9.

$$\frac{\mathrm{d}T}{\mathrm{d}t} = -kT + k\alpha\sin(\omega t + \Phi) + kC \tag{9}$$

Since Equation 9 is a linear differential equation, it can be solved as shown below.

$$T(t) = \frac{k_1 \int e^{\int k \ \mathrm{d}t}(k\alpha\sin(\omega t + \Phi) + kC)\ \mathrm{d}t + C_1}{e^{\int k \ \mathrm{d}t}} \tag{10}$$

$$= \frac{1}{k_1 e^{kt}}\left(\alpha\left(\frac{ke^{kt}\sin(\omega t + \Phi)}{\omega^2 + k^2} - \frac{\omega e^{kt}\cos(\omega t + \Phi)}{\omega^2 + k^2}\right) + \frac{C}{k}e^{kt} + C_1\right)k_1 k \tag{11}$$

$$= k\alpha\left(\frac{k\sin(\omega t + \Phi)}{\omega^2 + k^2} - \frac{\omega\cos(\omega t + \Phi)}{\omega^2 + k^2}\right) + C + C_2 e^{-kt} \tag{12}$$

Since $T(t) = T_0$ when $t = 0$, we solved for C_2 in terms of T_0.

$$T_0 = k\alpha\left(\frac{k\sin(\Phi)}{\omega^2 + k^2} - \frac{\omega\cos(\Phi)}{\omega^2 + k^2}\right) + C_2 \tag{13}$$

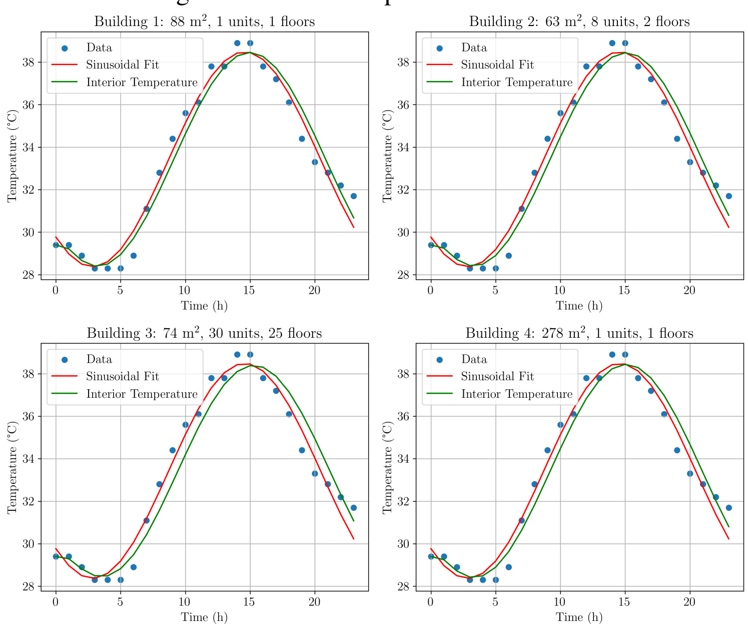
$$C_2 = \frac{k\alpha}{\omega^2 + k^2}\left(\omega\cos(\Phi) - k\sin(\Phi)\right) + T_0 - C \tag{14}$$

As such, we end up with Equation 15 as the final equation for the temperature as a function of time.

$$T(t) = \frac{k\alpha}{\omega^2 + k^2}\left(k\sin(\omega t + \Phi) - \omega\cos(\omega t + \Phi)\right) + \left(\frac{k\alpha}{\omega^2 + k^2}\left(\omega\cos(\Phi) - k\sin(\Phi)\right) + T_0 - C\right)e^{-kt} + C \tag{15}$$

Applying Equation 15 twice on itself accounts for the heat transfer from the surroundings to the building, and then from the building to the air inside of it. Figure 2 shows the final version of the graph we achieved.

Figure 2: Indoor Temperatures over Time



This figure shows the indoor temperatures for the different housing structures given in the Q1 Dwellings data.

We believed these results to be physically consistent with the actual world, as the temperature of an indoors environment will fluctuate in accordance with the temperature of the surrounding environment with a small delay in time, especially when there is no air conditioning.

The maximum temperature over a 24-hour period for each of the 4 housing structures are shown in Table 2, and the full prediction data is shown in Table 3.

Table 2: Maximum Temperatures by Structure

	Home 1	Home 2	Home 3	Home 4
Temp. (°C)	38.458	38.441	38.379	38.440

This table shows the maximum temperatures of each housing structure over a 24-hour period.

Table 3: Temperature Prediction Data

Time	Home 1	Home 2	Home 3	Home 4
12:00 AM	29.400000	29.400000	29.400000	29.400000
1:00 AM	29.210545	29.244742	29.292694	29.247300
2:00 AM	28.663325	28.713347	28.812856	28.717677
3:00 AM	28.404388	28.429091	28.495996	28.431508
4:00 AM	28.499325	28.489135	28.488773	28.488561
5:00 AM	28.945089	28.899459	28.822861	28.895749
6:00 AM	29.710208	29.632201	29.482348	29.625586
7:00 AM	30.740178	30.635327	30.422245	30.626272
8:00 AM	31.961599	31.837372	31.576080	31.826522
9:00 AM	33.287425	33.152674	32.861752	33.140802
10:00 AM	34.623168	34.487498	34.187667	34.475449
11:00 AM	35.873635	35.746714	35.459340	35.735348
12:00 PM	36.949709	36.840582	36.586146	36.830709
1:00 PM	37.774703	37.691147	37.487780	37.683470
2:00 PM	38.289822	38.237792	38.099988	38.232859
3:00 PM	38.458356	38.441560	38.379139	38.439721
4:00 PM	38.268292	38.287927	38.305338	38.289314
5:00 PM	37.733178	37.787845	37.883847	37.792359
6:00 PM	36.891148	36.976950	37.144702	36.984270
7:00 PM	35.802211	35.913034	36.140580	35.922637
8:00 PM	34.543971	34.671918	34.943041	34.683120
9:00 PM	33.206100	33.342051	33.637430	33.354054
10:00 PM	31.883942	32.018208	32.316793	32.030157
11:00 PM	30.671722	30.794736	31.075248	30.805779

This table shows the full temperature prediction data for all four housing structures for each hour of the day.

1.5 Sensitivity Analysis

To analyze the accuracy of our predictions, we performed a Monte Carlo Error Analysis on our model by running 10,000 trials where we randomly offset each data point by a 5% standard deviation and calculated the percentage uncertainty through the mean and standard deviation of the result. Table 4 displays the uncertainties in the interior temperature calculations for each building.

Table 4: Percent Uncertainty of Each Housing Structure

	Home 1	Home 2	Home 3	Home 4
Percent Uncertainty (%)	5.55	5.53	5.49	5.57

This table shows the uncertainty of each housing structure as calculated through Monte Carlo Error Analysis.

The percent uncertanties for each of the housing structures are all relatively low, so we are relatively confident in our model's accuracy and resistance to random error.

1.6 Discussion

This model was able to take a physics-based approach to model the temperature of the building as a function of time, which we believe is an innovative approach to the mathematical modeling problem. The discovery and use of an analytical solution to the temperature differential equation derived from the physical theory also contributed to the significance of our findings, as while numerical methods can also be accurate, they do not show the full picture of how the temperature inside varies with time. Additionally, the incorporation of an accurate model of the fluctuating ambient temperature helped to increase its validity to match the data provided.

However, since this model primarily relied on Newton's Law of Differential Cooling, it could not take into account the orientation of buildings, the total surface area or orientation of windows on those buildings, or the effects of wind speed on internal heating. For the former two, such information was not given for the houses, though the existence of windows would have a major effect on the heating of the home during a heat wave (due to the absorption of light energy inside the building). In order to simplify our model, we neglected the effect of wind speed on the change in house temperature, which we determined to not have an effect. In the future, we would want to determine the true effect of wind speed on internal heating, and incorporate this into our model. Additionally, the buildings were simplified to be rectangular prisms with square bases, which may not be the case in certain homes. If given data on the dimensions and shape of individual houses, our model would have to be modified slightly to find the surface area of the house for that particular shape.

2 Q2: Power Hungry

2.1 Defining the Problem

This question asks us to develop a model to find the peak demand in Memphis's power grid during the summer months as well as in summer months 20 years from now. We define demand as the average demand for the electrical power in a given month. We then define the peak demand as the highest demand in a given year.

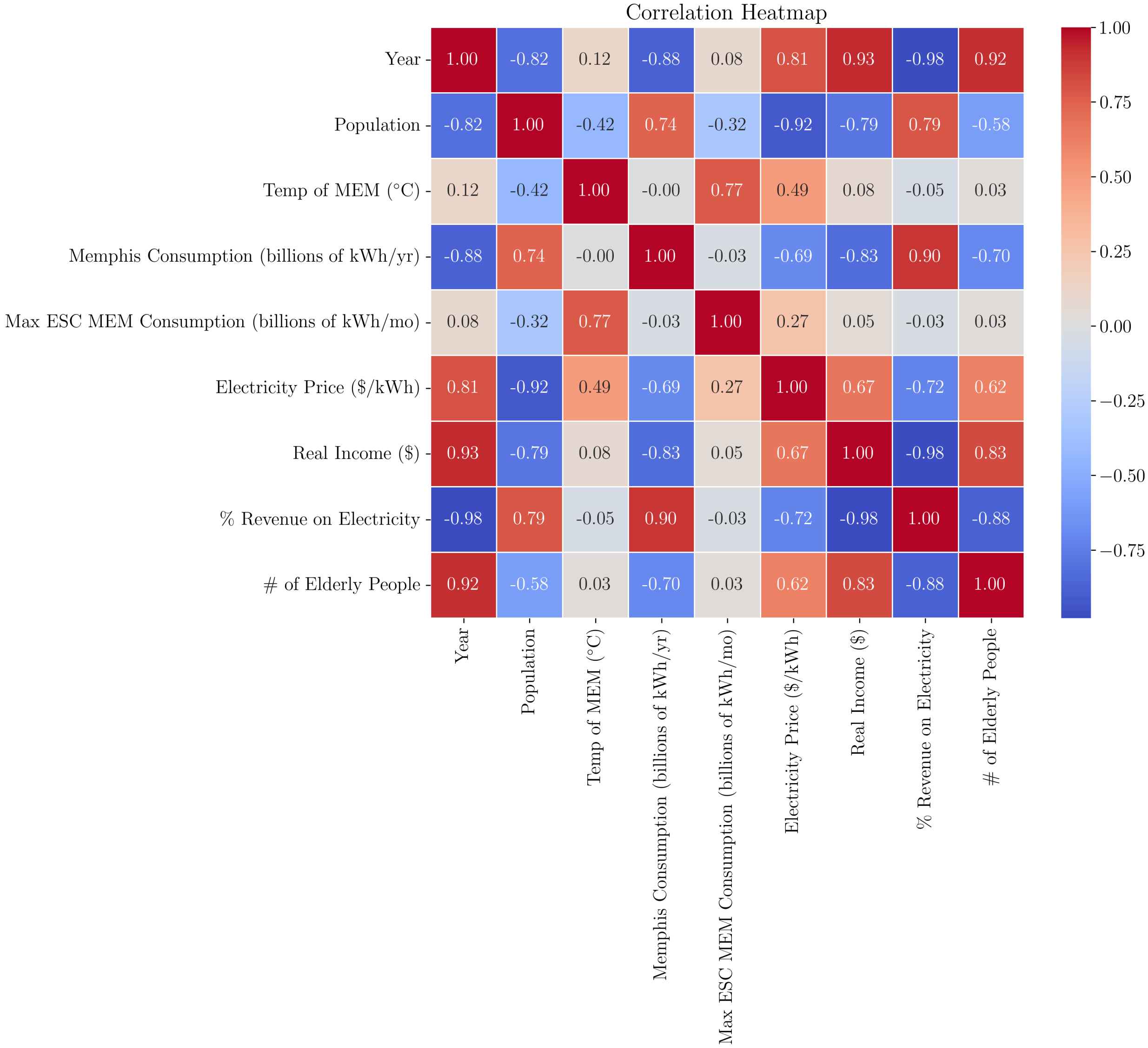
2.2 Assumptions

1. **There will be no drastic changes in the rate of climate change.** Justification: We are using the climate information at our disposal to make the temperature estimations on the hottest days to the best of our ability.
2. **The population of Memphis, TN will follow the population trend from the previous 10 years.** Justification: We are assuming there are no catastrophic events and that population trends in Memphis will continue as documented by the United States Census.
3. **The monthly electricity consumption of for East South Central USA (Kentucky, Tennessee, Mississippi, Alabama) mimics that of Memphis.** Justification: The geographic location of Memphis is reflective of the East South Central USA United States' community make-up and climate, so Memphis will mirror the monthly electricity consumption of Kentucky, Tennessee, Mississippi, and Alabama.
4. **There will be no power disconnects during any given heat crisis.** Justification: Although Tennessee is one of the few states that allow heat crisis disconnects, the most common electrical utility companies in Memphis (Tennessee Valley Authority and Memphis Light, Gas, and Water) do not permit disconnects during heat crises, as stated in their policies (Tennessee 2-1-1, n.d.; Weather-Related Moratorium Policy - Mlgw.com, 2010).

2.3 Model and Solution

We first examined various factors to determine correlations to the peak demand by using the Pearson correlation coefficient, using Python code to analyze a CSV with the necessary data.

Figure 3: Correlation Heatmap



This heatmap shows the Pearson correlation between noted variables. Focusing on the correlations with Max ESC MEM Consumption, only Temp of MEM has a strong correlation.

Upon analyzing the information outputted through the Pearson heat map and the p-values of the regression, we found the only information strongly correlated to the peak demand (titled 'Max ESC MEM Consumption (billions of kWh/mo)') is the peak temperature in Memphis (titled 'Temp of MEM ($°C$).')

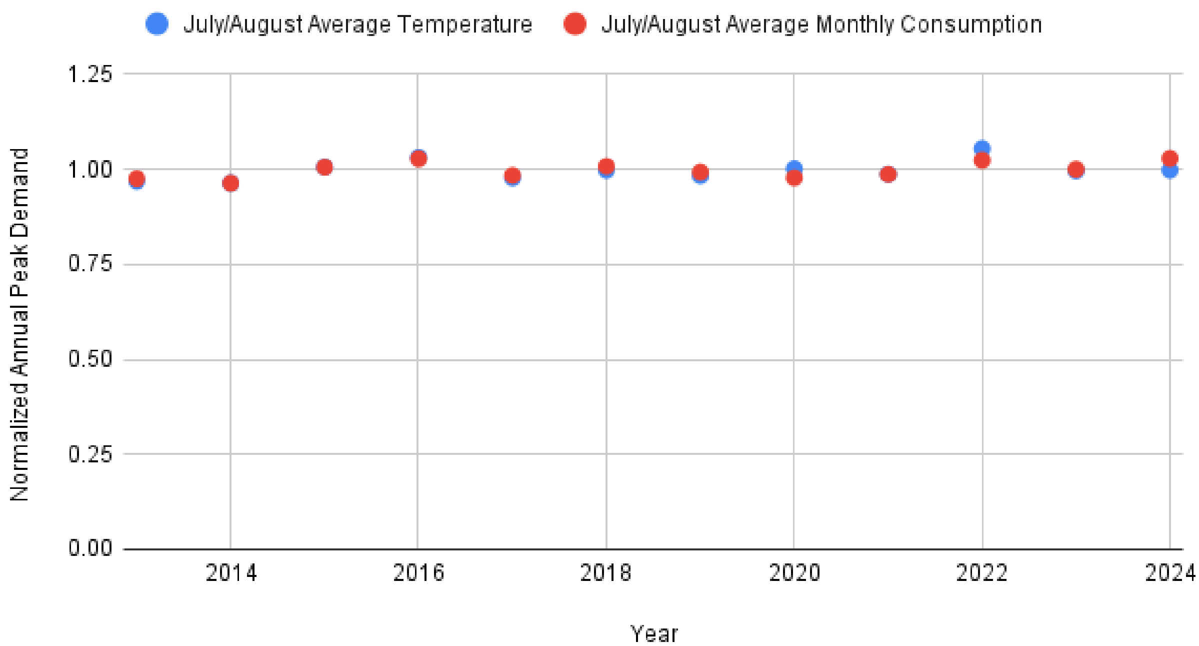
Based on data from the U.S. Energy Information Administration (Total Energy Monthly Data - U.S. Energy Information Administration, n.d.), we found that there was a very consistent and clear peak in electricity sales across the years during July and August. Thus, we took data from the past 20 years and calculated the average of the electricity sales during these two months as the measure of annual peak demand on the power grid. To match this, we also took the average temperature of these two months combined.

Next, we normalized these two time series by dividing each element by the average

$$x_i' = \frac{x_i}{\bar{x}}$$

Plotting the two normalized time series on the same graph, we obtain two nearly overlapping scatter plots, signifying a strong correlation. We attribute the anomalies on 2020 and 2022 to unexpected effects of COVID-19.

Figure 4: Peak Demands and Temperatures over Time



This figure shows the overlapping scatter plots of the annual peak demands and the temperatures in the corresponding months.

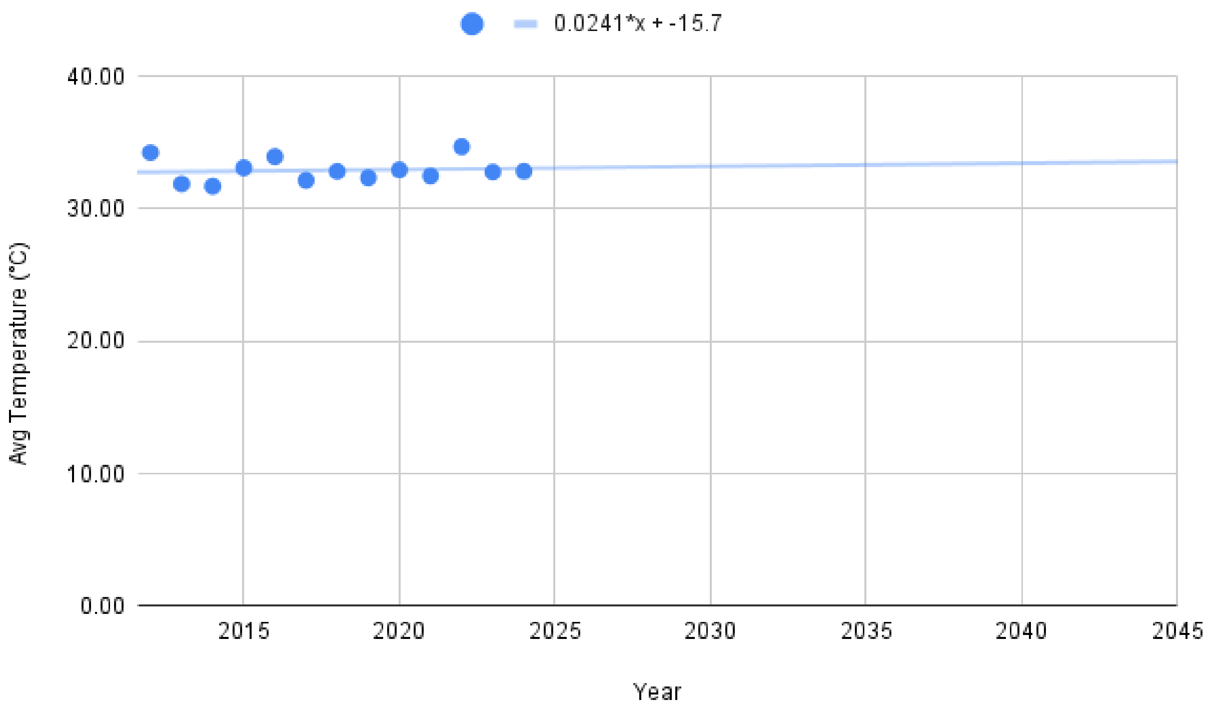
We calculate the pearson correlation R value of the two series using `scipy.stats.pearsonr`, and obtain $R = 0.83$ and $p = 0.0005$, proving its statistical significance. We also calculate the root mean squared of the residues $= 0.015$, which is less than 2% of the overall value.

We can justify this correlation because the higher the temperature, the greater the energy needed by the AC to cool the house down to a comfortable room temperature would be.

Therefore, our task becomes to model and predict the maximum (July/August) temperatures across the years.

Even though there are random fluctuations, we are looking for the overall, underlying trend of the temperatures, so we use linear regression to find the base relationship.

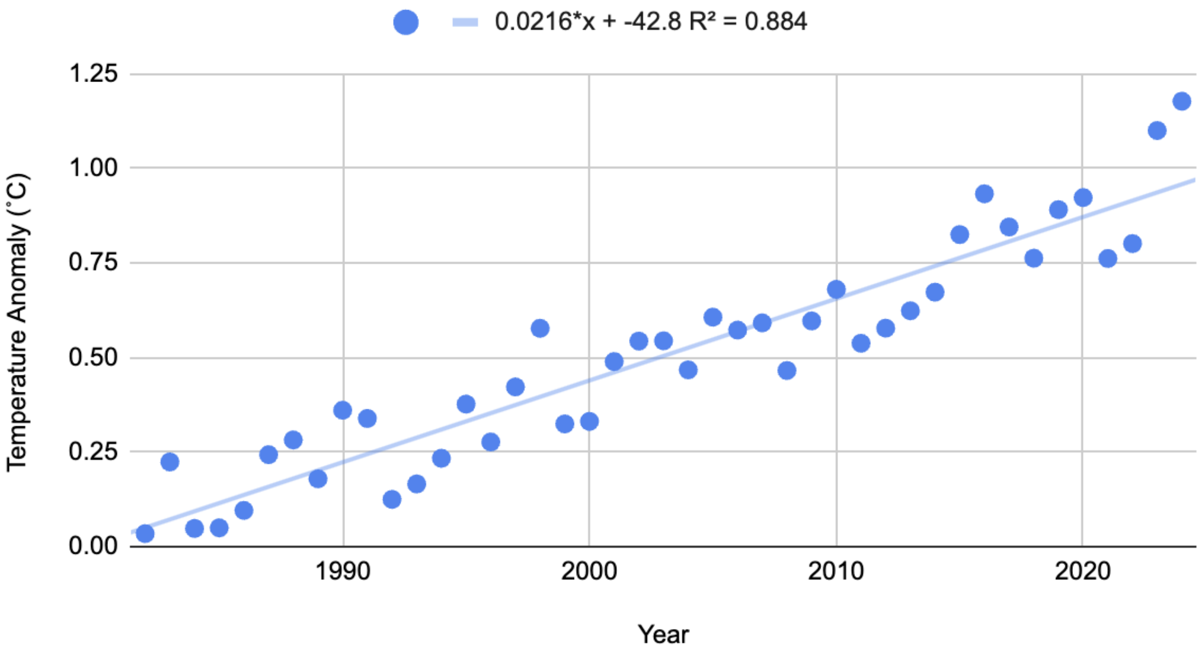
Figure 5: Predicted Temperature



This figure shows the predicted temperature in 20 years based on linear regression.

We note that the slope is $0.0241°C/$yr. We verify this value by comparing it to the rate of warming since 1982 on Climate.gov, which is $0.02°C/$yr (Lindsey, n.d.). This information could be corroborated by the data from Met Office Hadley Centre of the same time period, which is plotted below (Morice, 2022). This is also our justification for using a linear regression model to forecast global temperature trends in the relatively short term (20 years). The slope on this graph is $0.0216°C/$yr. The slight difference between 0.0216 and 0,0241 could be explained by the slight upwards concavity and the fact that this warming rate itself is increasing over time – our slope is extrapolated from the data since 2012 (Rate and Impact of Climate Change Surges Dramatically in 2011-2020, 2023).

Figure 6: Global Temperatures



This figure shows global temperature trends since 1984 modeled with a linear regression.

Following the linear regression model in Figure 6, we find that the predicted temperature in 2045 is $33.58°C$. Using the same normalization function, it becomes 1.02, and thus correlates back to a peak monthly demand of 30.977 billions of kWh in the East South Central region.

Finally, based on our assumptions, we can find the proportionality between the East South Central region and the Memphis region to convert this value to the peak demand in Memphis specifically. We do so by finding the average proportionality constant between these two areas' use of energy which is $= 30.4$.

This proportionality could be cross-verified by calculating the population ratio of the two regions. The East South Central region population is 19759744 (Resident Population in the East South Central Census Division, n.d.). The Memphis region population is 618639. (U.S. Census Bureau QuickFacts: Memphis City, Tennessee, n.d.). The proportionality $= 31.9$. The difference between these two ratios implies that people in Memphis use more energy than an average person living in the East South Central region. This is caused by the fact that people living in cities (Memphis) usually use more electricity than people living in more rural areas.

Ultimately, we predict that the peak monthly demand would be 1.018×10^9 kWh in Memphis in 2045.

2.4 Discussion

In the construction of this model, we thoroughly analyzed the different factors that could impact the peak monthly demand using a heat map. Thus, we were able to systematically rule out unrelated variables and identify correlated ones that serve as accurate reflections of our target time series based on quantitative evidence. Moreover, we cross-verified all of the important constants and proportions that we calculated to ensure and check that they accurately correspond to and reflect related real-world trends and phenomena as we would expect.

On the other hand, in the future, we would like to consider potential policy changes in the future due to climate change and global warming. Another area for improvement is to analyze the random fluctuations in temperature above and below the average trend more closely. We hope to determine and mathematically prove whether these changes are actually random or if we could mathematically predict them and incorporate this aspect into our prediction.

3 Q3: Beat the Heat

3.1 Definition

This question asks us to create a set of criteria to quantify the vulnerability of neighborhoods in Memphis, TN to heat waves and power grid failure to properly allocate resources to different communities.

3.2 Assumptions

- **Building shape and size have an effect on heat vulnerability** Justification: The shape and size of the building affect its surface area, which affects the amount of heat that can enter or leave the building.
- **Building material has an effect on heat vulnerability** Justification: The cooling factor k is inversely proportional to the mass and the specific heat capacity of the building, which depend on the building material.
- **Separation between buildings has an effect on heat vulnerability** Justification: Closely packed houses lead to one house's heat loss being another house's heat gain. This sustains high temperatures in homes long after the heatwaves stop.
- **Forestry/parks in a community has an effect on heat vulnerability** Justification: Foliage provides shade in an area, lessening the burden of heatwaves.
- **Socioeconomic conditions have an effect on heat vulnerability** Justification: Many socioeconomically vulnerable populations have less sophisticated cooling systems and rely more on public transportation, increasing their vulnerability to heatwaves.
- **Elderly people (65+) and minors (0-18) are more vulnerable than working-age adults** Justification: Elderly people and minors are more susceptible to disease and are less able to take care of themselves, rendering them more vulnerable than adults ages 18-65 (Schröder-Butterfill, 2006, 10).
- **People who take public transportation/walk to work are more vulnerable than those who commute by car.** Justification: People who walk to work are more expose to the heatwaves, leaving them more vulnerable. People who take public transportation are outside more often as they often have to walk to get picked up by their transportation and walk to their destination after being dropped off.

3.3 Model and Solution

To rank a particular neighborhood in its vulnerability to heat waves, we must first understand the specifics of that house. The main criteria we will be ranking are as follows:

- Is the neighborhood in an urban, suburban, or exurban area? What is the density of houses in that area per square kilometer?
- How old are the houses in this this neighborhood?
- What is the socioeconomic level of the area?
- Is the neighborhood in a well-forested area?
- Are there trends in the population of the neighborhood for higher populations of elderly people or children?
- Do many people commute via public transportation or walking to their workplace in this neighborhood?

These factors can be summarized in five key variables: **Age**, **Housing**, **Open Space**, **Income**, and **Transportation**. We created individual rankings for each of these categories using a variety of different methods, and used an Entropy Weighted Model (EWM) to apply weights to the scores of each of these categories. An EWM weights individual criteria by analyzing the variability in the sample distribution of each criterion (Zhu et al., 2020). The weights for an EWM are calculated by first normalizing the data:

$$p_{ij} = \frac{x_{ij}}{\sum_{j=1}^{n} x_{ij}}$$

Where n represents the total number of "samples" measured (the number of neighborhoods), j represents an individual neighborhood, i represents an individual criterion, and x represents the value for an individual criterion for a certain neighborhood. The "entropy", or differentiation in the sample values for a certain criterion, is defined as:

$$E_i = \frac{\sum_{j=1}^{n} p_{ij} \cdot \ln(p_{ij})}{\ln n}$$

From this entropy value (which ranges from 0 to 1, where a higher value represents more variability and thus a higher weight), we can derive the final weights:

$$w_i = \frac{1 - E_i}{\sum_{i=1}^{m} 1 - E_i}$$

First, we find the scores for the age category. We want to model the proportion of seniors and minors in a given neighborhood; the greater this proportion, the more vulnerable this community is. Thus, the score for this category is:

$$A_j = \frac{P_{old} + P_{young}}{P}$$

where P represents the population of the neighborhood, P_{old} represents the population of elderly people in that community, and P_{young} represents the population of children under 18 in that community.

In general, older houses are more susceptible to overheating during heat waves than newer ones (Bean Falls, n.d.). Using this, we can rank the houses in a given neighborhood by age and weight them accordingly (1 for houses built after 2010, 2 for houses built from 1990-2010, etc.), then dividing this number by the number of houses. In other words, the category is scored as follows:

$$H_j = \frac{\sum_{i=1}^{5} iN_i}{N}$$

where N represents the total number of houses in the neighborhood and N_i is the number of houses built in the ith housing age block (2010+, 1990-2009, 1970-1989, 1950-1969, and before 1950).

The scoring for open space can be represented as follows:

$$O_j = O$$

where O represents the amount of open space (in m^2) for a given neighborhood, and O_{avg} is the average amount of open space over all neighborhoods in Memphis.

The scoring for income is:

$$I_j = I$$

where I represents the median income in neighborhood j and I_{avg} represents the average median income over all neighborhoods in Memphis.

People who take public transportation are more vulnerable to heat waves due to an increased time being outside. In particular, those who walk are affected the most, with those taking other forms of public transport following, and car-commuters being affected the least. We "zero" this model around work-at-home individuals, i.e. people who do not commute for work are not affected by heat waves. Thus, using the AHP discussed earlier, we can weight the model as follows:

$$T_j = \frac{T_{car} + 2T_{public}}{W_{tot}}$$

where T_{car} represents the number of car commuters in neighborhood j, T_{public} is the number of commuters on public transit, and W_{tot} represents the size of the workforce in that community.

Next, we normalize the individual values by dividing them by the average across all cities. Generally, the more open space there is in an area, the lower the vulnerability to heat waves because of increased foliage. Furthermore, the higher the income/socioeconomic status of a neighborhood, the greater resistance they will have to heat waves due to build quality of the house. Thus, there are negative relationships between income and heat vulnerability and open space with heat vulnerability. Therefore, in normalizing these two scores, we multiply them by -1.

Using the previously described weighting method (EWM), we can derive the weights for each of these parameters:

Table 5: Weighting Processes by Variable

Variable	Entropy	Weights
Ages	0.996	0.0325
Transport	0.999	0.0063
Income	0.961	0.334
Housing	0.986	0.116
Open Spacing	0.940	0.511

This table shows how each variable progresses through the Entropy Weighted Method.

Table 6: Final Rankings

Rank	Neighborhood	Score	Percent Allocated
1	Uptown / Pinch District	-1075	13.16
2	Rossville	-1488	10.04
3	South Forum / Washington Heights	-1552	9.7
4	Downtown / South Main Arts District / South Bluffs	-3923	5.0
5	South Memphis	-4265	4.75
6	Oakland	-5102	4.29
7	Hickory Withe	-5304	4.2
8	North Memphis / Snowden / New Chicago	-7516	3.53
9	Hollywood / Hyde Park / Nutbush	-7799	3.47
10	Midtown / Evergreen / Overton Square	-11925	2.93
11	Frayser	-12807	2.86
12	Coro Lake / White Haven	-13075	2.84
13	East Midtown / Central Gardens / Cooper Young	-13329	2.82
14	Bartlett, Zipcode 1	-16676	2.64
15	Egypt / Raleigh	-23470	2.43
16	Lakeland / Arlington / Brunswick	-24104	2.42
17	Germantown, Zipcode 2	-25034	2.4
18	Cordova, Zipcode 1	-28899	2.34
19	Bartlett, Zipcode 2	-28964	2.34
20	Cordova, Zipcode 2	-31085	2.31
21	Bartlett, Zipcode 3	-31424	2.3
22	Windyke / Southwind	-33796	2.28
23	Germantown, Zipcode 1	-35037	2.26
24	Collierville / Piperton	-40449	2.22
25	East Memphis	-41016	2.21
26	East Memphis – Colonial Yorkshire	-54754	2.14

This table shows the final results and rankings of the Entropy Weighted Method.

3.4 Discussion

This model was able to quantitatively rank different communities in the Memphis area by their vulnerability to heat waves using data on socioeconomic status. The use of the Entropy Weighted Model to determine the weights for the model allowed for a data-driven approach to weighting rather than a subjective one, which increased the validity of our predictions. In addition, our score ranking of each community was able to directly correlate to the proportion of resource allocation the City of Memphis should follow for each community in the case of a heat wave, which we believe provides extra support for policy makers in the city to properly plan in the case of a similar crisis in the future.

However, our model does not account for house density in a given area, which might increase the heat retention of the homes even after the heat wave recedes. We found that other factors correlated more directly to the heat vulnerability of individual communities during a heat wave, but this model could be expanded to include house density to model the aftereffects of a heat wave in these communities. Since data was not given on building materials and house shape in different neighborhoods in the city, we were unable to incorporate the heat retention of specific materials and specific house dimensions on the necessity of aid (e.g. large houses made out of brick might need more resources than small houses made of wood). With the aforementioned data, we would hope to create a more all-encompassing model that takes the true needs of each community best into account.

4 Conclusion

We created a model to predict the indoor temperature in four different dwellings over a 24-hour period in Memphis, TN. We modeled the ambient temperature of the environment over time and then created a differential equation to predict the change in temperature of a room based on the surrounding temperature and initial conditions. We applied the equation twice to account for the heat transfer from the surroundings into the building and found the maximum temperature each of the four homes reached throughout the course of the day. The model predicted that Home 1, Home 2, Home 3, and Home 4, would have a maximum temperature of 38.458, 38.441, 38.379, and 38.440°C respectively.

We then created a model to determine the peak demand in Memphis's power grid during the summer months and the predicted peak demand in 20 years from now. We compiled a list of possible variables which could have an effect on the peak electricity consumption and performed a heat map. After an analysis of our heat map, we found that the peak temperature in Memphis had a strong correlation to the maximum electricity consumption. After normalizing the peak demand and corresponding temperature, we found that the correlation was statistically significant. Using a linear regression, we predicted the increase in temperature in Memphis per year due to global warming and the temperature anomaly over time. Following our linear regression models, we estimate that the predicted temperature in 2045 will be 33.58°C and the ultimately correlates to a monthly peak demand of 1.018×10^9 kWh. In the year 2025, we predicted that the temperature will be $33.1°C$ and will correlate to a monthly peak demand of 1.004×10^9 kWh.

Finally, we ranked the vulnerability of different communities in Memphis, TN based on five main factors which include Age, Housing, Open Space, Income, and Transportation. We created an entropy weighted model to calculate the weight of each of these categories. We found that Ages, Transport, Income, Housing, and Open Spacing had an entropy of 0.996, 0.996, 0.961, 0.986, and 0.940 with a weight of 0.0325, 0.0063, 0.334, 0.116, and 0.511 respectively. We found that Uptown and Pinch District were the most vulnerable neighborhoods in Memphis, and we created a way to allocate a percentage of Memphis's resources equitably.

Our findings in predicting the temperature of unconditioned homes, electricity consumption over time, and determining the most vulnerable neighborhoods in Memphis will be helpful to local authorities when compiling aid and emergency protocol for their citizens.

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A Code Appendix

A.1 Part 1: Hot to Go

```
import matplotlib.pyplot as plt
import numpy as np
import pandas as pd
from scipy.optimize import curve_fit

plt.rcParams["font.family"] = "serif"
plt.rcParams["font.serif"] = "Computer Modern"
plt.rcParams["text.usetex"] = True
plt.rcParams["text.latex.preamble"] = r"\usepackage{amsmath}"
plt.rcParams["font.size"] = 14
plt.rcParams["figure.dpi"] = 300

df = pd.read_csv("./Q1tempdata.csv")
df["Hours"] = df["Time"].apply(lambda x: int(x.split(":")[0]) \
                + (12 if "PM" in x and x != "12:00 PM" else 0) \
                - (12 if x == "12:00 AM" else 0))
df.head()

plt.plot(df['Hours'], df['Temperature (C)'])
plt.show()

def sinusoidal_model(t: int, A: float, B: float, C: float, D: float) -> float:
    return A * np.sin(B * t + C) + D

time = np.array(df['Hours'])
temperature = np.array(df['Temperature (C)'])

params, _ = curve_fit(sinusoidal_model, time, temperature, p0=[10, 0.1, 0, 85])

print(params)

temperature_pred = sinusoidal_model(time, *params)

ss_total = np.sum((temperature - np.mean(temperature))**2)
ss_residual = np.sum((temperature - temperature_pred)**2)
r_squared = 1 - (ss_residual / ss_total)

print(f"R^2: {r_squared:.4f}")

plt.scatter(time, temperature, label="Data")
plt.plot(time, temperature_pred, color='red', label=f"Sinusoidal Fit")
plt.xlabel("Time (h)")
plt.ylabel("Temperature ($^\circ$C)")
plt.legend()
plt.show()

def surface_area(sqm: int, num_units: int, floors: int) -> float:
    units_per_floor = num_units / floors
    total_base_area = units_per_floor * sqm
    perimeter = 2 * (np.sqrt(total_base_area) + np.sqrt(total_base_area))
```

```
    wall_area = perimeter * (2.4 * floors)
    roof_area = total_base_area if floors > 1 else 0
    return total_base_area + wall_area + roof_area

def k1(sqm: int, num_units: int, floors: int) -> float:
    return (4 * 0.80 * (5.67 * 10 ** -8) * ((30 + 273) ** 3) \
            / (454.5 * surface_area(sqm, num_units, floors) * 2300)) \
        * surface_area(sqm, num_units, floors)

def k2(sqm: int, num_units: int, floors: int) -> float:
    return (4 * 0.16 * (5.67 * 10 ** -8) * ((30 + 273) ** 3)) \
            / (1.2 * sqm * num_units / floors * 2.4 * floors * 1000) \
        * surface_area(sqm, num_units, floors)

def interior_temperature(t: int, A: float, omega: float, phi: float, \
                         T0: float, C: float, k: float) -> float:
    return ((k * A) / (omega ** 2 + k ** 2)) * (k * np.sin(omega * t + phi) \
            - omega * np.cos(omega * t + phi)) \
            + np.exp(-k * t) \
            * ((k * A / (omega ** 2 + k ** 2)) * (omega * np.cos(phi) - k * np.sin(phi)) + T0 - C) \
            + C

# Parameters found from curve_fit regression
A = 5.0595291
omega = 0.26775697
phi = -2.33298053
C = 33.42658701

# Housing unit details; formatted as (size of unit in m^2, # units, # floors)
units = [
    (88, 1, 1),    # Home 1
    (63, 8, 2),    # Home 2
    (74, 30, 25), # Home 3
    (278, 1, 1)    # Home 4
]

# Graph the results for each of the 4 homes
fig, axes = plt.subplots(2, 2, figsize=(12, 10))
axes = axes.flatten()

interior_temp_preds = []

for i, (sqm, num_units, floors) in enumerate(units):
    SA = surface_area(sqm, num_units, floors)
    k_2 = k2(sqm, num_units, floors) * 3600
    k_1 = k1(sqm, num_units, floors) * 3600
    print(k_1, k_2)
    T0 = temperature[0]

    interior_temp_pred = interior_temperature(time, A, omega, phi, \
                         interior_temperature(time, A, omega, phi, T0, C, k_1), \
                         C, k_2)
    interior_temp_preds.append(interior_temp_pred)
    print(max(interior_temp_pred))
```

```
    axes[i].scatter(time, temperature, label="Data")
    axes[i].plot(time, temperature_pred, color='red', label=f"Sinusoidal Fit")
    axes[i].plot(time, interior_temp_pred, label=f"Interior Temperature", color='green')
    axes[i].set_title(f"Building {i+1}: {sqm} m$^2$, {num_units} units, {floors} floors")
    axes[i].set_xlabel("Time (h)")
    axes[i].set_ylabel("Temperature (°C)")
    axes[i].legend()
    axes[i].grid()

plt.tight_layout()
plt.show()

# Number of Monte Carlo Simulations
N = 10000

# Define standard deviations for parameter uncertainty as 5% offset
sigma_A = 0.05 * A
sigma_omega = 0.05 * omega
sigma_phi = 0.05 * abs(phi)
sigma_C = 0.05 * C
sigma_T0 = 0.05 * abs(T0)

monte_carlo_results = []
overall_relative_uncertainties = []

overall_relative_uncertainties = []

for i, (sqm, num_units, floors) in enumerate(units):
    monte_carlo_results = np.zeros((N, len(time)))

    for sim in range(N):
        # Randomly vary input parameters
        A_mc = np.random.normal(A, 0.05 * A)
        omega_mc = np.random.normal(omega, 0.05 * omega)
        phi_mc = np.random.normal(phi, abs(0.05 * phi))
        C_mc = np.random.normal(C, 0.05 * C)

        k_1_mc = k1(sqm, num_units, floors) * 3600
        k_2_mc = k2(sqm, num_units, floors) * 3600
        T0_mc = np.random.normal(33, 2)

        temp_pred = interior_temperature(time, A_mc, omega_mc, phi_mc,
                    interior_temperature(time, A_mc, omega_mc, phi_mc, T0_mc, C_mc, k_1_mc), \
                    C_mc, k_2_mc)

        monte_carlo_results[sim] = temp_pred  # Store results for this simulation

    mean_temp = np.mean(monte_carlo_results, axis=0)
    std_temp = np.std(monte_carlo_results, axis=0)
    relative_uncertainty = np.mean((std_temp / mean_temp) * 100)

    overall_relative_uncertainties.append(relative_uncertainty)

    print(f"Building {i+1} ({sqm}m², {num_units} units, {floors} floors) - \
```

```
            Overall Relative Uncertainty: {relative_uncertainty:.2f}%")
```

A.2 Part 2: Power Hungry

```python
import scipy
import pandas as pd

df = pd.read_csv("./Q2data.csv")
scipy.stats.pearsonr(df['NormTemp'], df['NormConsumption'])
```

A.3 Part 3: Beat the heat

```python
from matplotlib import pyplot as plt
import scipy
import numpy as np
import pandas as pd
from google.colab import files

uploaded = files.upload()

def p_weights(arr): #normalizes data
  p_arr = np.zeros((len(arr), len(arr[0]))) #create dummy array
  for i in range(len(arr)): #iterate over criteria
    sumAvg = sum(arr[i, :]) #sum sample values
    for j in range(len(arr[0])): #iterate over samples
      p_arr[i][j] = arr[i][j]/sumAvg

  print(p_arr)
  return p_arr

def entropy(p_arr): #Entropy of given p array
  E = np.zeros(len(p_arr)) #dummy array
  for i in range(len(p_arr)): #iterate over criteria
    sumPlnP = 0
    for j in range(len(p_arr[0])): #iterate over samples
      sumPlnP += p_arr[i][j] * np.log(p_arr[i][j])
    E[i] = -1*sumPlnP / np.log(len(p_arr[0]))

  return E

def weights(E_arr): #weights based on entropy
  sumAvg = len(E_arr) - sum(E_arr)
  weights = np.zeros(len(E_arr))
  for i in range(len(weights)):
    weights[i] = (1-E_arr[i])/sumAvg

  return weights

df = pd.read_csv("for_all - Memphis.csv") #read from csv
data = df.values
ages = []
transport = []
income = []
```

```
housing = []
open_space = []

for i in range(3,29): #get data on given criteria
  ages.append((float(data[i][5])+float(data[i][4]))/float(data[i][3]))
  income.append(float(data[i][7]))
  open_space.append(float(data[i][10]))
  transport.append((float(data[i+30][3])+2*float(data[i+30][4]))/float(data[i+30][2]))
  housing.append((float(data[i+60][2]) + 2*float(data[i+60][3]) + 3*float(data[i+60][4]) +
  4*float(data[i+60][5]) + 5*float(data[i+60][6]))/(float(data[i][2])))

#Normalize data
ages = np.array(ages)/(sum(ages)/(len(ages)))
transport = np.array(transport)/(sum(transport)/(len(transport)))
income = -1 * np.array(income)/(sum(income)/(len(income)))
housing = np.array(housing)/(sum(housing)/(len(housing)))
open_space = -1 * np.array(open_space)/(sum(open_space)/(len(open_space)))

print("INITIAL VALUES")
criteria_matrix = np.array([ages, transport, income, housing, open_space])
print(criteria_matrix)

p_arr = p_weights(criteria_matrix)
print("P-Values")
print(p_arr)
ent = entropy(p_arr)

print("ENTROPY")
print(ent)
weights_matrix = weights(ent)

print("WEIGHTS")
print(weights_matrix)


def value_of(weights, x, criteria_matrix): #Applies weights to scores for all neighborhoods
  val = 0
  for i in range(len(weights)):
    val += (weights[i]*criteria_matrix[i][x])
  return val

def all_values(weights, criteria_matrix, data):
  vals = []
  neighborhoods = []
  for i in range(3,29):
    vals.append(value_of(weights, i-3, criteria_matrix) *
    float(data[i][3]))
    neighborhoods.append(data[i][0])
  return neighborhoods, vals

n, v = all_values(weights_matrix, criteria_matrix, data)

ordered_array = [elem[0] for elem in sorted(zip(n, v), key=lambda x: x[1])]
```

```
val_ord = [elem[0] for elem in sorted(zip(v, v), key=lambda x: x[1])]

props = np.array(val_ord)/sum(val_ord)

for i in range (1, 27, 1):
  print(i, "& ", ordered_array[26-i], "& ", int(val_ord[26-i]), "& ",
  int(props[26-i] * 10000)/100, "\\\\")

props = np.array(val_ord)/sum(val_ord)
```