MathWorks Math Modeling Challenge 2025

Zionsville High School

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M3 Challenge FINALIST—$5,000 Team Award

JUDGE COMMENTS

Specifically for Team #18231—Submitted at the close of triage judging

COMMENT 1: Very well done executive summary. Paper included 3 solid models that were well explained and analyzed.

COMMENT 2: Model 1 is very thorough, and the sensitivity analysis demonstrates how the different parameters affect the solution. Tables, figures, and overall layout is very well done.

COMMENT 3: Good job explaining the assumptions

COMMENT 4: Extremely well written and organized paper. Good job!

M3 Challenge 2025:
Hot Button Issue: Staying Cool as the World Heats Up

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Team ID: 18231

Contents

1 Executive Summary

To the City Council of Memphis, Tennessee,

In recent years, global warming has only continued, with some of the worst effects yet to be faced. This has been felt particularly in Memphis, with heat waves on the rise [1]. Dangerous weather events, exacerbated by global warming, will especially hurt the elderly, the very young, and those with underlying health conditions [2]. Combating the effects of global warming is difficult given the scope of the issue, and the limited means provided to Memphis's government. However, it is crucial that the government of Memphis does everything it can to help its citizens.

We first predicted the internal temperature of non-air-conditioned dwellings in Memphis over a 24-hour period during a heat wave. We formed a model based around Newton's Law of Cooling and added in heat transfers from other sources such as the sun and internal heat. We found the internal temperature of a home rises much quicker for apartments than for single-family residences. Specifically, apartments are affected 1.5 times as much by external temperature, 1.6 times as much by solar gain, and 1.25 times as much by internal heat generation. Still, both become hotter inside than outside without air conditioning, showing the disastrous effects of heat waves on the citizens of Memphis.

Secondly, we developed a model to predict peak demand on Memphis, Tennessee's electric grid during the months of June, July, and August, and how this peak demand will change in twenty years. For this we used SARIMA time series model with gradient boosting (XGBoost). We showed the peak demand in our projections to be within the range of 3400–3600 kWh per customer, and shows that this peak demand was directly correlated to temperature changes caused by heatwaves. We projected a 2024 peak demand of 1.46 billion kWh, and predicted a 2044 peak demand of 1.755 billion kWh, which is a 20.2% increase in power grid demand over twenty years. This increase in demand will put a sizable strain on the Tennessee and Memphis power grid, and energy infrastructure will need to be updated to meet demand.

We finally standardized a vulnerability score for all of the 27 provided neighborhoods in Memphis. This vulnerability score was found through a principle component analysis (PCA) of over 20 neighborhood characteristics. The vulnerability score predicts risk from a heat wave or power grid failure, with a higher score indicating greater risk of experiencing the worst effects of these events, and a lower score indicating lower risk. This measurement can be used by Memphis's government to prioritize preparation for and response in the case of a heat wave or power grid failure. Our model showed that roughly 49% of variance was made up of by transportation and workforce; 21% by urban density and older housing; 8% by housing diversity; and 7% by recent development and income. By looking at these risk factors, we hope in the long-term Memphis will be able to curb the issues causing these inequities. Our mitigation strategy for Memphis is grid hardening, or preparing the power grid to handle heat and precipitation. This can be accomplished through running underground lines, implementing digital technologies, and upgrading critical equipment. Grid hardening pays off significantly in the long run, with every $1 million spent paying off in a roughly $2.5 million increase in GDP [3]. Mitigation efforts should be allocated depending on specific neighborhoods' vulnerability scores, with the most vulnerable neighborhoods scored at 70–100 prioritized with immediate measures like energy storage and transmission line reinforcement.

We hope these results will assist the city of Memphis in fighting the disastrous effects of heatwaves by providing specific measures of heat waves and power grid failures, as well as specific factors to target through new policies. While the effects of heat waves and power grid failures are significant, they can be curbed with a structured and data-backed response by authorities.

2 Q1: Hot to Go

2.1 Defining the Problem

The first problem asks us to develop a model to predict the indoor temperature of a non-air-conditioned dwelling during a heat wave over a 24-hour period in one of two cities. We chose Memphis, Tennessee. Our model will take into account heat transfer data guidelines by dwelling type.

2.2 Assumptions

1. **Dwellings have no air conditioning**
   Our model assumes that dwellings have no air conditioning so our governed only by conduction, convection, radiation, and internal heat sources.
2. **There will be no major infrastructure changes in the average non-air-conditioned dwelling in Memphis, Tennessee, or its close surroundings**
   We must assume there is no significant change in the infrastructure and material makeup of the average dwelling, as this could greatly shift the data. We further assume that no disaster will remove large amounts of dwellings or displace those within them, and no new housing will be built, as this 24-hour period is to be soon.
3. **Humidity, dew point, and wind speed are not included in the original equation**
   The model uses hourly ambient temperature data from Memphis on July 8, 2022, as the main input. Although other factors such as humidity, dew point, and wind speed are provided, we assume that their primary effect is already captured indirectly by adjusting the heat transfer coefficient k.
4. **A heat wave occurs on any day in which temperature goes above 100 degrees Fahrenheit**
   The Tennessee government defines the heat wave threshold as 100 degrees Fahrenheit [4].
5. **Our analysis will be relegated to single-family residences and apartment homes**
   Single-family residences and apartments are the main home types in our data, so they will be the main ones analyzed in our model.
6. **Our model assumes that heat waves follows Newton's Law of Cooling**
   Our model will utilize a modified version of Newton's Law of Cooling to account for different housing types which have different levels of thermal conductivity [5].
7. **There is a negligible temperature gradient within a dwelling**
   We assume that temperature gradients within the dwelling are negligible, meaning the entire indoor area has one uniform temperature.
8. **Solar Gain is based on ASHRAE standards**
   Both dwelling types are assumed to have windows with standard characteristics as described in the ASHRAE Handbook. Single-family residences are assumed to have moderate shade, whereas apartments are assumed to have greater window exposure [6].
9. **Solar Gain follows a sinusoidal pattern**
   Solar heat gain is a sinusoidal function, representing the daily cycle of the sun. This function peaks at midday and troughs during the night [7].
10. **Internal Heat Generation of a dwelling is assumed constant**
    The internal heat generation factor is assumed constant for each home type, depending on the number of occupants and appliances.

11. **Temperature is steady over each hour**
    The model neglects temporary changes in occupancy, rapid weather changes, or short-term wind gusts. Coefficients are treated as constants over the period of interest.

2.3 Model

Newton's Law of Cooling provides a versatile foundation that can be built upon according to a set of factors instrumental in modeling the transfer of heat. Considering the absence of a comprehensive AC system, the development of the model is carried out using the physical features of the home. By adjusting the heat transfer coefficient, the amplitude of the solar heat gain, and the internal heat generation, a thorough framework has been implemented to model the twenty-four-hour effect of a heat wave on the indoor temperature of both an apartment and a single-family residence.

2.3.1 Model Development

The Newton's Law of Cooling states: $\frac{dT}{dt} = k[T_{\text{ambient}} - T]$ [5]. This equation shows that the rate of change of an object's temperature is in proportion to the difference between the object's temperature and the ambient temperature, or the temperature of the surrounding area. We chose this as our base model as it is the real-world equation used in classical physics for heat transfers. We can expand this to include other factors which could transfer heat to a dwelling. Solar radiation and internal gains will also have a considerable effect on the indoor temperature. To incorporate these effects, we will add two more terms.

First, we add Solar Heat Gain: $A\sin(\frac{\pi t}{24})$. The sine function measures the sun's daily pattern, with a peak at midday and a trough at night [7]. The constant A is chosen based on key characteristics of the window. The ASHRAE Fundamentals Handbook shows solar heat gain coefficients for building openings, which supports our A values. The ASHRAE Handbook provides a set of guidelines and numbers developed by the American Society of Heating, Refrigeration, and Air-Conditioning Engineers that ensures buildings are designed to be energy efficient. The constants utilized through the remainder of this question will be taken from the ASHRAE Handbook based on standard housing materials for a given air-conditioner-less domicile [6].

Second, we add Internal Heat Generation: Q. This term represents a constant contribution from internal sources such as occupants and appliances. ASHRAE guidelines and building energy codes provide typical values for internal loads, which allow us to estimate Q [6].

Consequently, the fully integrated and adjusted model becomes:

$$\frac{dT}{dt} = k[T_{\text{ambient}} - T] + A\sin(\frac{\pi t}{24})\ + Q$$

Table 1: Newton's Law of Cooling Model Variables

Constant	Description	Units
k (Heat Transfer Coefficient)	rate at which heat travels from the environment into a house	hrs^{-1}
A (Solar Heat Gain Amplitude)	predicts how much heat is added to a home from the sun	$\frac{C}{hr}$
Q (Internal Heat Generation)	rate at which appliances and occupants produce heat	$\frac{C}{hr}$
T (Internal Temperature)	Temperature inside a dwelling at a certain time of day	C

Each constant in this equation is tied to measurable or standard values: k is influenced by the building's makeup. As described in the ASHRAE Handbook (Section 27.2 and Table 5), overall U-factors for building components are derived from averages of conduction, convection, and radiation through walls, windows, and roofs. The U-factor represents the overall heat transfer coefficient of a building component A is determined from solar gain properties, including the glazing solar heat gain coefficient, which is provided in ASHRAE literature. Glazing solar heat gain is the thermal energy transmitted by the sun through glass into a building. Q is estimated from typical internal loads reported in residential energy studies and ASHRAE design guidelines [6].

2.3.2 Determination of k (Heat Transfer Coefficient):

For a single-family residence, an effective R-value of R = 5 can be utilized. The R-value measures how well a material resists heat flow and a higher R-value is an indication of better insulation [8]. Given this, the corresponding U-factor value can be determined using $U \approx \frac{1}{R}$. Since U-factor is found using the inverse of the R-value, a higher U-factor indicates worse insulation and greater heat transfer [9]. The U-factor is analogous with the k factor so we can determine that $k_{sf} \approx \frac{1}{5} \approx 0.20\ \text{hr}^{-1}$. The same can be done for the apartment utilizing an effective R-value of approximately 3. $k_{apt} \approx \frac{1}{3} \approx 0.33\ \text{hr}^{-1}$. k_{apt} can be rounded to 0.30 for simplicity as well as to stay consistent with ASHRAE's guidelines on fenestration [6].

2.3.3 Determination of A (Solar Heat Gain Amplitude):

For the single-family residence, due to partial shading (roof overhangs, vegetation), we assume a moderate solar gain: $A_{\text{sf}} = 0.25\,^{\circ}\text{C/hr.}$. For the apartment, with larger or more exposed windows and less shading, we assign a higher value: $A_{\text{apt}} = 0.40\,^{\circ}\text{C/hr.}$. These values are supported by typical solar heat gain coefficients found in ASHRAE literature [6].

2.3.4 Determination of Q (Internal Heat Generation):

For the single-family residence, estimating an internal load of approximately 400 W distributed over the dwelling's thermal mass, we obtain: $Q_{\text{sf}} = 0.80\,^{\circ}\text{C/hr}$. For the apartment, with higher occupant density and smaller volume, the effective internal heat generation is higher: $Q_{\text{sf}} = 1.00\,^{\circ}\text{C/hr}$. These estimates are in line with residential energy load studies and ASHRAE guidelines [6].

2.3.5 Results

Combining the coefficients and other factors above, we arrive at the following equations for modeling the proposed situation. The differential equations are later numerically integrated in order to predict the indoor temperature over the 24-hr period and qualitatively compare it using a graph to the ambient temperature from M3's provided data over the same time frame [10].

The equation for the single-family residence was determined to be:

$$\frac{dT_{\text{sf}}}{dt} = 0.20[T_{\text{ambient}} - T_{\text{sf}}(t)] + 0.25\sin(\frac{\pi t}{24}) + 0.80$$

The equation for the apartment was determined to be:

$$\frac{dT_{\text{apt}}}{dt} = 0.30[T_{\text{ambient}} - T_{\text{apt}}(t)] + 0.40\sin(\frac{\pi t}{24}) + 1.00$$

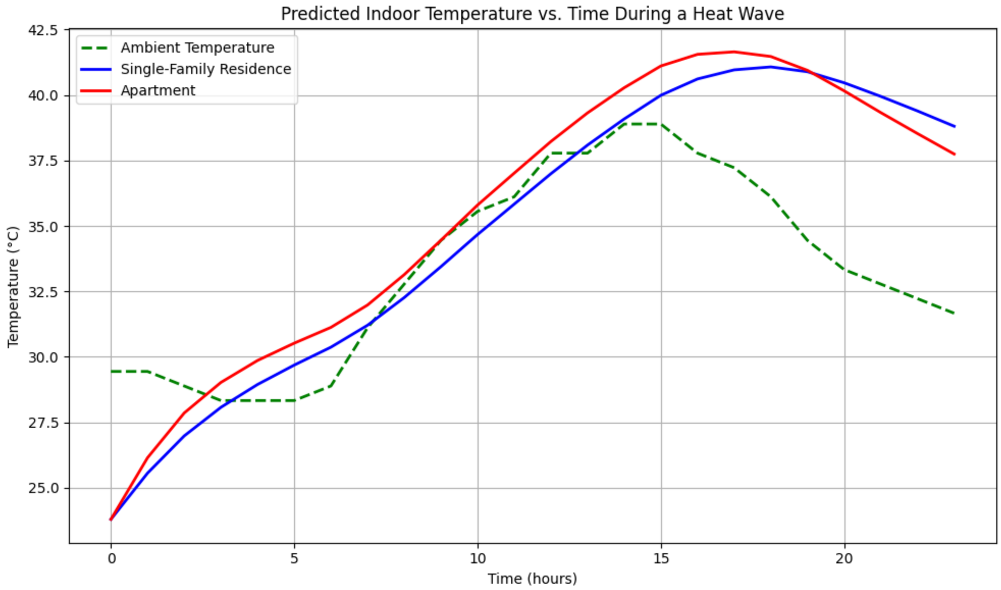


Figure 1: Predicted Indoor Temperatures during a Heat Wave

2.4 Discussion

The model reflects external conditions through Newton's Law of Cooling, as well as solar gain and internal heat generation.

2.4.1 The effects of k

The k value for the apartment model ($k = 0.30$) is higher than that of the single-family residence ($k = 0.20$), indicating that heat will transfer faster into apartments than single-family residences. This is reasonable as apartments are less dense and insulated than single-family residences [11].

2.4.2 The effects of A

The A value shows the effects of solar gain on indoor temperature, and is higher for apartments ($A = 0.40$) than single-family residences ($A = 0.25$), showing they will gain heat faster from the sun's cycle. This correlates with data from the ASHRAE handbook [6].

2.4.3 The effects of Q

The higher Q for apartments ($Q = 1.00$) than for single-family residences ($A = 0.80$) shows that the effect of internal heat generation is amplified in smaller spaces, such as those of an apartment. This is in line with data from the ASHRAE handbook [6].

2.4.4 Single-Family Residences vs Apartments

All our constants (k, A, Q) were higher for apartments than single-family residences, showing that apartments gain heat much faster during a heat wave than single-family residences, which correlates with real-life results.

It's imperative that policy-makers understand the dire need to work with industrial companies to improve the insulation in single-family homes, and especially, apartments. Areas with high concentrations of lower-income housing (apartments) are likely to be severely affected by heat waves and abnormally high temperatures and targeted cooling strategies are necessary to alleviate this. Apartment buildings reaching up to 41 C during a heat wave is a clear sign that intervention from government officials is needed.

It is worth noting that both our models predict a higher indoor temperature than the outdoor temperature during a heat wave, which correlates with what we see in real-world non-air-conditioned dwellings. This can be seen in findings from researchers from the Arizona Institute for Resilience who state, "When it's 110 F (43.3 C) outside, the 1950s house will likely feel at least 10 F (5.6 C) warmer inside, even with the same air temperature [12]."

2.5 Sensitivity Analysis

A sensitivity analysis has been performed for the indoor temperature model for both an apartment and a detached single-family home. In the analysis, three ambient temperature scenarios are considered and the differential equation is solved for each ambient temperature case.

We vary the ambient temperature input by considering three scenarios:

1. **Cool Scenario**: The ambient temperature is decreased by 2°C.
2. **Baseline Scenario**: The ambient temperature is used as measured.
3. **Hot Scenario**: The ambient temperature is increased by 2°C.

Let $T_{\text{ambient, cool}}(t) = T_{\text{ambient}}(t) - 2$ and let $T_{\text{ambient, hot}}(t) = T_{\text{ambient}}(t) + 2$.

Beginning with the apartment, the following baseline parameters are established: $k = 0.30\,\text{hr}^{-1}, \quad A = 0.40\,°\text{C/hr}, \quad Q = 1.00\,°\text{C/hr}$.. The differential equation for the apartment in this scenario is $\frac{dT_{\text{apt}}}{dt} = 0.30[T_{\text{ambient, scenario}} - T_{\text{apt}}(t)] + 0.40\sin(\frac{\pi t}{24}) + 1.00$. The following Table 2 assumes an initial indoor temperature of $T(0) = 23.8\,°\text{C}$.

Table 2: Indoor Temperature Change in Ambient Scenarios in an Apartment

Ambient Scenario	Indoor Temperature at t=15 (°C)
Cool (-2°C)	39.127175
Baseline	41.104957
Hot (+2°C)	43.082739

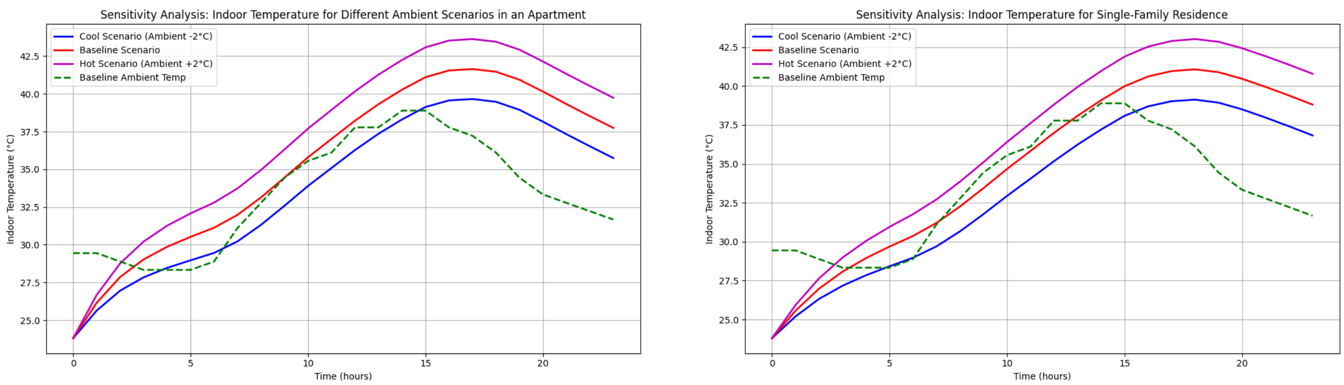
A similar sensitivity analysis can be done for the single-family house model. The ambient temperature input will be accordingly adjusted in the same manner as above and both a graph and table is presented to summarize the indoor temperature at a key time (t = 15 hours). The following baseline parameters are established: $k = 0.20\,\text{hr}^{-1}, \quad A = 0.25\,°\text{C/hr}, \quad Q = 0.80\,°\text{C/hr}$. The differential equation for the single-family home in this scenario is $\frac{dT_{\text{sf}}}{dt} = 0.20[T_{\text{ambient, scenario}} - T_{\text{sf}}(t)] + 0.25\sin(\frac{\pi t}{24}) + 0.80$. The following Table 3 assumes an initial indoor temperature of $T(0) = 23.8\,°\text{C}$.

As presented above in both the apartment and single-family home scenarios, a decrease of 2 degrees Celsius in ambient temperature yields as significantly lower indoor temperature for the entirety of the day. An increase of 2 degrees Celsius in outside temperature also results in a significantly higher indoor temperature in both instances - especially during peak solar hours. In both cases, an increase or decrease in ambient temperature of 2 degrees Celsius yields a corresponding 1.9 degrees Celsius increase or decrease in indoor temperature.

This analysis depicts the importance of accurate ambient temperature data in order to predict indoor temperature conditions. Seemingly small changes in input data yield a fair yet not abnormal difference in the output. This noticeable effect shows why data analysts and policymakers must be scrupulous in the collection and analysis of temperature data.

Table 3: Indoor Temperature Change in Ambient Scenarios in a Single-Family Residence

Ambient Scenario	Indoor Temperature at t=15 (°C)
Cool (-2°C)	38.085783
Baseline	39.986210
Hot (+2°C)	41.886637



(a) Apartment Sensitivity **(b) Single-Family Residence Sensitivity**

Figure 2: Comparison of Sensitivity for Apartment and Single-Family Residences

2.6 Strengths and Weaknesses

The model is straightforward and provides a first-order approximation of indoor temperature dynamics. By incorporating solar heat gain and internal heat generation, the model captures all major aspects of a dwelling's temperature change. The model stratifies by different dwelling types, helping to ensure each is analyzed properly.

The model assumes constant internal heat generation and a constant sinusoidal pattern for solar gain, which may not reflect short-term fluctuations, such as those caused by clouds. The assumption of a uniform indoor temperature ignores temperature gradients, such as those in larger buildings. Although wind speed and humidity data are inputted, their effects are only indirectly included through adjustments in k, rather than inputted explicitly Without extensive measured indoor temperature data, the constants k, A, and Q are estimated from literature and typical values, but further calibration to the specific buildings of Memphis would likely yield better results.

3 Q2: Power Hungry

3.1 Defining the Problem

The second problem asks us to develop a model which predicts the peak demand on a city's power grid during the summer months, and asks us to predict any changes in this maximum demand in 20 years. We chose the city of Memphis, Tennessee, and our model will take into account previous power grid demand from 2008–2024.

3.2 Assumptions

1. **There will not be any major changes in the setup of Memphis's power grid**
   In predicting the future demand, we must assume that there will be no major changes, either towards more efficiency or less, of the power grid system within Memphis, Tennessee.
2. **The summer months are June, July, and August**
   We assume the common definition that the "summer months" are inclusive of June, July, and August. Thus, our analysis will include data from these months.
3. **The effect of climate change on demand from Memphis's power grid will experience no major shift over the next 20 years**
   We assume there will no major and unpredicted changes in the rate of global warming within Memphis. This allows us to ensure it continues to follow the same curve as before.
4. **Peak demand will be the max aggregate demand of all nodes on the power grid**
   Peak demand will add up the demand of all sources taking up electricity, including businesses, homes, and public facilities.
5. **Our model assumes that the ratio of Memphis's average demand to the national average demand is constant**
   We assume that the ratio of Memphis's average in kWh to the national average demand in kWh is constant. We are finding this ratio with annual demands and assuming it applies the same during the summer months.
6. **Peak summer demand pertains to the demand of the summer month with the greatest value**
   This assumption allows us determine response variables for our modeling.
7. **The total electric customers in Memphis will increase every year at 0.6%**
   We have to use the 0.6% average population growth of the Memphis metropolitan area to estimate the total number of customers up to 2044. This is reasonable, since electric demand is largely derived from this area, even with outside rural and suburbs. We can account for supply-chain relationships this way.

3.3 Model

3.3.1 Development

To project peak demand, we first defined historical peak demand as the demand of the summer month (June, July, August) with the highest value. Using this definition, we extracted data from the U.S. Energy Information Administration for the peak demand per customer averaged across the whole U.S from 2008–2024 [13]. The M3 Challenge data provides that the Memphis average annual consumption is 15172 kWh while the national average is 10791 kWh, creating a ratio of approximately 1.4 [10]. We then estimated Memphis's historical peak demand per customer by multiplying 1.4 to the national average.

Our explanatory variable was the set Memphis heatwave temperatures provided [10]. We could not find high-resolution data for future projections of heatwave temperature, but we noticed that the historical data followed a sinusoidal, seasonal pattern. Taking advantage of this, we fitted a SARIMA (Seasonal Auto-Regressive Integrated Moving Average) time series on temperature to project 20 years ahead to 2044. Figure 3 shows our SARIMA results, in which we observed that sinusoidal seasonality was captured accurately.

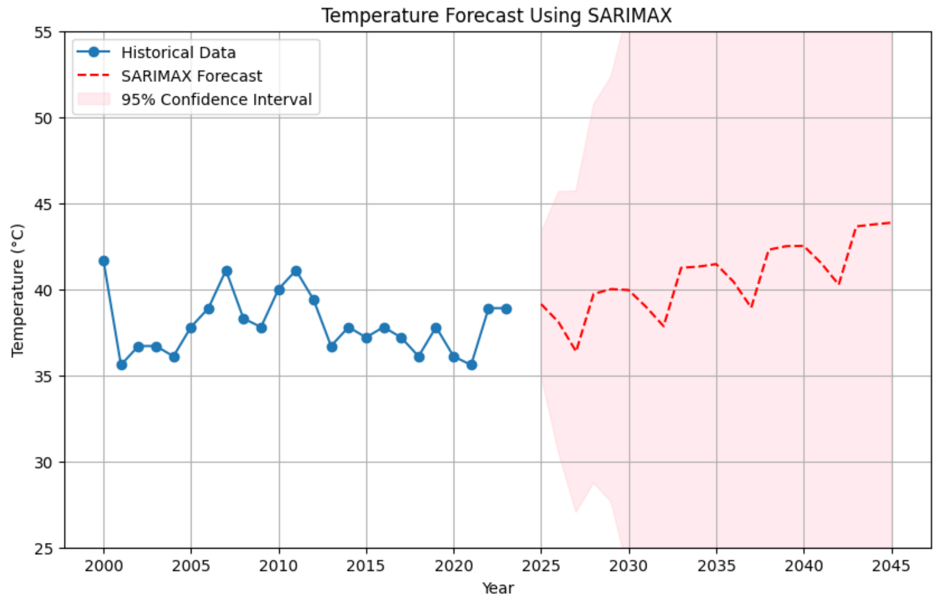
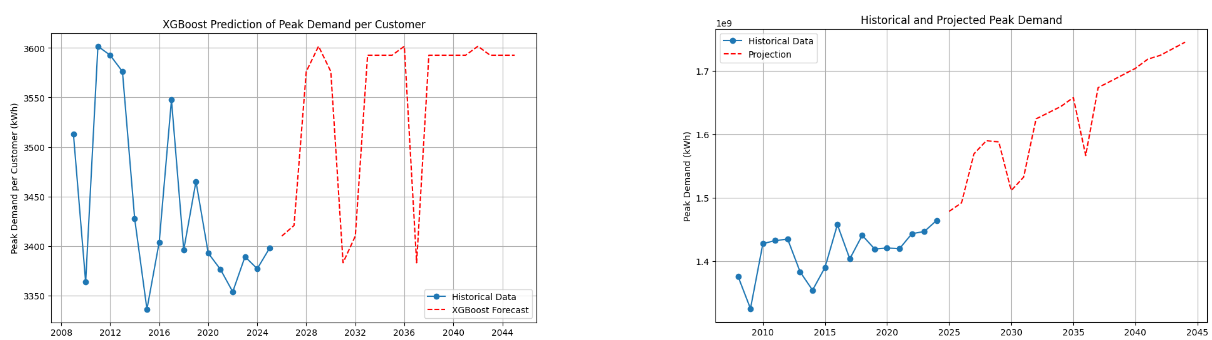


Figure 3: Forecasted Heatwave Temperatures

After preparing the explanatory variable, we were ready to project Memphis's peak demand per customer based on heatwave temperatures. We opted for a gradient boosting algorithm, XGBoost, for its suitability with non-linear and strongly seasonal data.

3.3.2 Results

In our projections, we found that the peak demand per customer generally ranged from 3400–3600 kWh and was directly correlated to heatwave temperature fluctuations (Figure 4a. However, these projections do not reflect the maximum demand the entire Memphis grid handles, as they are only per-customer values. We multiplied the total electric customers in Memphis to these projections to calculate the total grid demand. In 2024, the total electric customers was around 431,000, and we estimated the future number of customers based on the Memphis metropolitan area's 0.6% average annual population growth [14]. Figure 4b shows the final projected peak demand for the whole grid after multiplying per-customer values by our estimated electric customers.



(a) Peak Demand per Customer **(b) Total Grid Peak Demand**

Figure 4: Peak Demand Projections

3.4 Discussion

Examining Figure 4, we found a steady increase in peak demand due to the population rise of electric customers over the next two decades [13]. In 2044, we projected a peak demand of 1.755 billion kWh, which is a clear 20.2% increase from 2024's peak demand of 1.46 billion kWh. Due to the projected rising of summer peak demand, the Memphis power grid will likely face load issues, supply-demand deficits, and outages if the capacity of energy infrastructure is not upgraded.

3.5 Strengths and Weaknesses

Our model pipeline is logical: we first extend the explanatory temperatures through SARIMA, then project Memphis peak demand per customer using XGBoost, and finally calculate total grid demand with our electric customer population multipliers. Additionally, both our SARIMA and gradient boosting models were trained to capture the seasonality of heatwave temperatures well and outputted reasonable projections. However, without high-resolution data for established temperature projections and specific city-level electric consumption, our methodology is built upon assumptions. Our extrapolation of heatwave temperatures may be problematic due to long-term climate change, although we do see a gradual incline in projections in Figure 3. Additionally, we gathered data for the whole U.S., then calculated Memphis values based on an assumed ratio, so this could be a factor of instability.

4 Q3: Beat the Heat

4.1 Defining the Problem

The goal for the third problem is to develop a vulnerability score for the 27 neighborhoods throughout Memphis to help allocate resources equitably in the case of a heat wave or power grid failure. We are also tasked with recommending an approach on incorporating vulnerability scores into mitigating heat wave effects.

4.2 Assumptions

1. **There will not be any major changes in the setup of Memphis's power grid within the scope of our prediction**
   In predicting future vulnerability in particular areas, we must assume that there will be no major changes of the power grid setup within Memphis, Tennessee.
2. **The implementation of the developed scores will require allocating more resources to at risk areas**
   Our model is based on the idea that a higher score means an area is more at risk and thus will need more help in our final implementation.
3. **Neighborhood characteristics can be reduced or grouped into components**
   Multicollinear variables can be grouped together to simplify our analysis to less variables. While variables may not be perfectly correlated, if they are closely correlated, they can be grouped to make analysis possible.

4.3 Model

4.3.1 Development

To develop a vulnerability metric, we examined the neighborhood characteristics and variables (around 20+ variables including number of households, population, etc.). All variables are measured in counts and can be found in the M3 Challenge's provided data (not included since there are too many to put in this paper) [10]. We found prominent multicollinearity between some variables (Figure 5) due to overlapping metrics like household or transportation data. To clarify our analysis, we chose to utilize Principle Component Analysis (PCA), an unsupervised learning technique to reduce dimensionality while preserving the key "principle components" that explain the variability between the neighborhood characteristics.

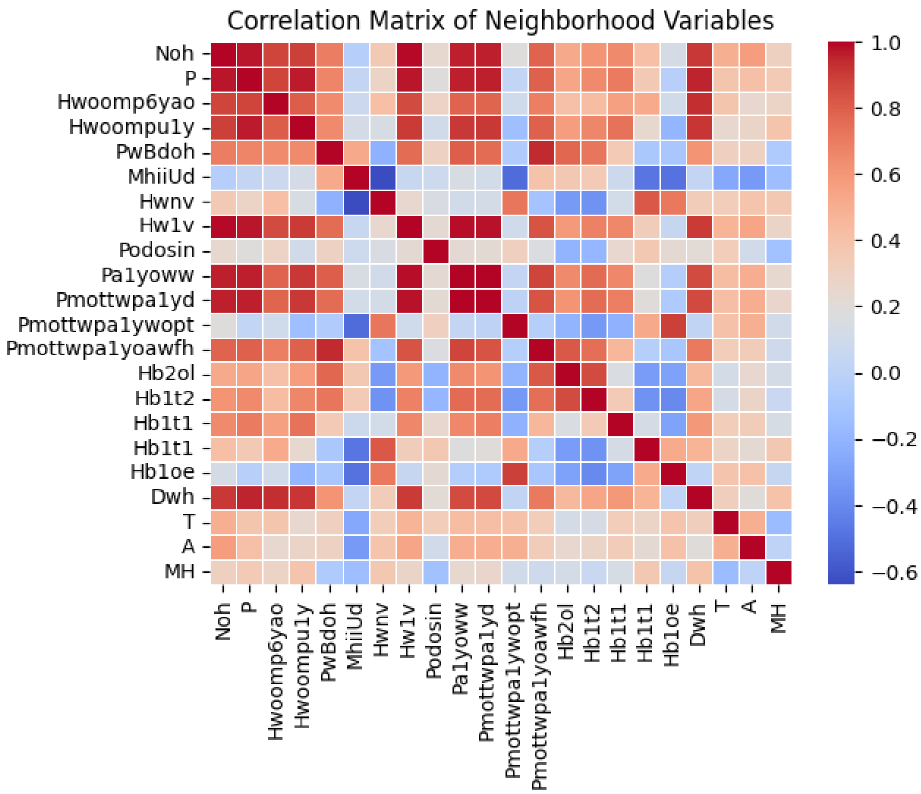


Figure 5: Correlation Matrix

We then scaled the neighborhood characteristics to balance their weight, and calculated a covariance matrix to quantify the similarity between each variable. From the direction of maximum variance within the data, we determined four eigenvectors (principle components v) and corresponding eigenvalues λ. The explained variance of each principle component (PC) v_j was calculated with the ratio of λ_j over the summed eigenvalues of all PCs. We also used the loading factors of each PC to determine the most important neighborhood characteristics and define PCs into categories like infrastructure, transportation, etc.

$$\text{Explained Variance} = \frac{\lambda_j}{\sum_{k=1}^{p} \lambda_k}$$

4.3.2 Results

From our four main components, we examined their key neighborhood characteristics and divided PC1–PC4 into the categories of transportation/workforce, urban density/older housing, housing diversity and recent development/income. Results are shown in Table 4.

Table 4: Principal Component Vulnerability Analysis

PC	Category	Top Variables (Loading Factor)	Explained Variance
PC1	Transportation/Workforce	Households w/ 1+ vehicles (0.30), Population aged 16+ working (0.30), Primary mode of transportation to work is driving (0.30)	0.49
PC2	Urban Density/Older Housing	Primary mode of transportation to work is walking/public transit (0.38), Homes built before 1950 (0.38), Homes built 1950–1969 (0.37)	0.21
PC3	Housing Diversity	Mobile homes/other (0.47), Townhouse (0.35), Apartments (0.33)	0.08
PC4	Recent Development/Income	Proportion of developed/open space (0.65), Homes built after 2010 (0.32), Median household income (0.30)	0.07

Our vulnerability score was calculated based on these four principle components for each neighborhood, weighted by their proportion of explained variance, then normalized on a 0–100 scale. However, PCA does not automatically correlate with heatwave or outage vulnerability. We negated the contributions of PC1 and PC4 due to their inverse relationships with vulnerability. For example, in PC1, higher transportation and workforce numbers indicate economically strong, car-dependent areas, and therefore less vulnerability. In PC4, more development and income also correspond with less vulnerability, an inverse effect. Our PCs combined explained around 85% of total variance, which shows the strength of our model. Our vulnerability calculation is shown below with the coefficients representing each PC's explained variance.

$$\text{Vulnerability Metric} = (-PC1 \times 0.49) + (PC2 \times 0.21) + (PC3 \times 0.08) + (-PC4 \times 0.07)$$

4.4 Discussion

Our model showed transportation of people in the workforce as the most correlated variable with increased vulnerability, followed by urban density/older housing, then housing diversity, and finally recent development/income. It is worth noting that these variables in total do not explain all the variance, yet offer a significant portion of it, at 85%. This shows our model is quite strong.

Figure 6 depicts a heat map displaying the most at-risk areas. West Memphis consists of the most areas which are considerably vulnerable to heat waves and the ensuing power outages. The most at-risk areas residing in West Memphis include Uptown/Pinch District, South Forum/Washington Heights, and Rossville. The neighborhoods that have the highest chance of withstanding heatwaves and persisting without power outages are East Memphis neighborhoods including Arlington, Cordova, and Collierville. Given this, it's crucial that policymakers pay special attention to West Memphis neighborhoods whose citizens are most vulnerable to power outages. Our vulnerability score is distributed from 0–100, so we recommend dividing the neighborhoods into 0–30 (least

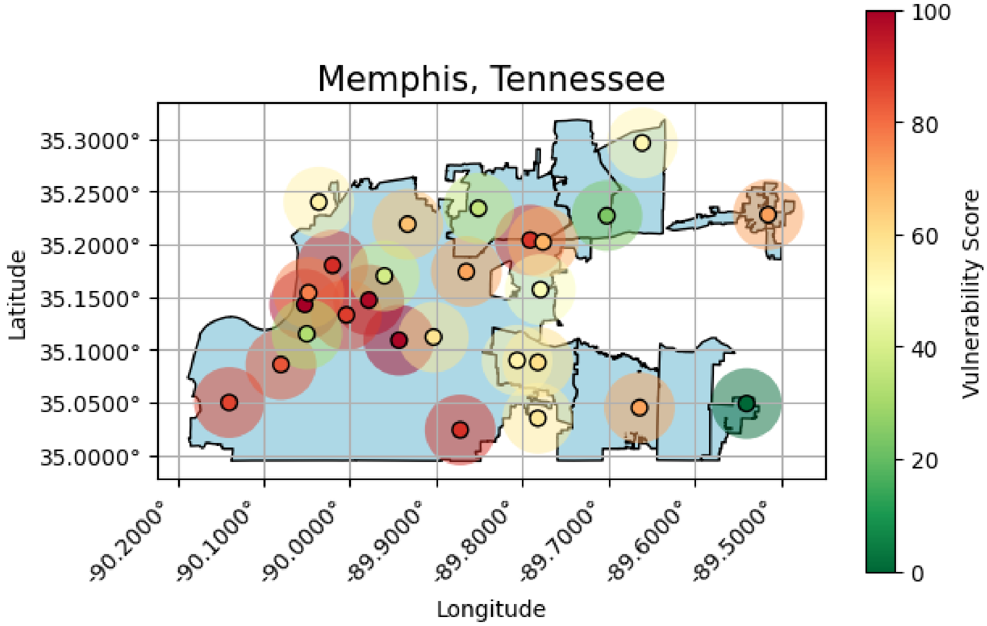


Figure 6: Heatmap of Vulnerability

vulnerable), 30–70 (moderately vulnerable), and 70–100 (most vulnerable). The city should be actively engaging in mitigation strategies with the most vulnerable neighborhoods, utilizing energy storage, transmission line upgrades, or emergency measures. Of course, neighborhoods labeled 0–30, are much less impacted by heatwaves or outages, and the government should implement passive measures like energy usage awareness. By looking at the variables most correlated with vulnerability, policymakers can not only provide more for highly at-risk areas, which will have a higher vulnerability score, but also work to fix underlying issues leading certain areas to be vulnerable to heat waves and power grid failures.

4.4.1 Recommendations

One key policy that we recommend to support the most at-risk areas is climate-resilient grid hardening. Hardening the power grid allows it to withstand weather events such as heat waves much better and leads to significantly less power grid failures [15]. From 2003 to 2012, weather-related power outages cost the U.S. economy an inflation-adjusted yearly average of between $18 billion and $33 billion. As of 2022, the annual cost to GDP is around $150 billion [3]. To combat these losses, grid hardening brings substantial economic benefits by preventing outages and eventually raising GDP. According to the Federal Energy Regulatory Commission (FERC), every $1 million in direct spending on grid modernization and hardening generates some $2.5 million in GDP growth, mainly due to avoided or reduced outages [3].

Grid hardening can take many forms and should be tailored to specific areas as needed. Crucial approaches include implementing underground power lines, using digital technologies, and upgrading critical equipment. Other methods include rapid response preparation by utilities and government, as well as installing advanced protective devices into the power grid. The point of all these forms is to weatherproof critical infrastructure within power grids from inclement weather events, thus mitigating the effects of heat waves and power grid failures in turn [3] [15] [16].

The government of Memphis should implement these grid hardening technologies within high vulnerability score areas first, then go through and implement these technologies in lower and lower vulnerability scores. By focusing on first helping the most at-risk areas, we can ensure that those who are most threatened by heat waves and power grid failures are aided first.

4.5 Sensitivity Analysis

To evaluate the robustness of our vulnerability metric, we examined how small changes in the weighting coefficients affect the final scores. Our vulnerability metric is defined as

$$V = -w_1\,\mathrm{PC}_1 + w_2\,\mathrm{PC}_2 + w_3\,\mathrm{PC}_3 - w_4\,\mathrm{PC}_4,$$

with baseline weights

$$w_1 = 0.49, \quad w_2 = 0.21, \quad w_3 = 0.08, \quad w_4 = 0.07.$$

To understand the sensitivity of our model, we varied each weight by $\pm 10\%$. For example, w_1 varies as

$$w_1 \in \{0.9 \times 0.49,\, 0.49,\, 1.1 \times 0.49\} = \{0.441,\, 0.49,\, 0.539\}.$$

We performed similar variations for w_2, w_3, and w_4 while keeping the other weights fixed at their baseline values. For each case, we recalculated the vulnerability metric V for all 27 neighborhoods. Figure 7 shows sensitivity plots for each weight, where the vulnerability scores (before 0–100 normalization) for all neighborhoods are plotted under three scenarios: the baseline weight, -10% variation, and $+10\%$ variation. These plots clearly demonstrate that even a small change in the weights leads to noticeable shifts in the vulnerability scores. Table 5 below summarizes the vulnerability scores for the first five neighborhoods when w_1 is varied by $\pm 10\%$ while the other weights remain at their baseline values. These values indicate that reducing w_1 by 10% increases

Table 5: Summary of w_1 Variations

Neighborhood	V_{baseline}	$V_{w_1=0.441}$	$V_{w_1=0.539}$
1	V_1^{baseline}	$V_1^{0.441}$	$V_1^{0.539}$
2	V_2^{baseline}	$V_2^{0.441}$	$V_2^{0.539}$
3	V_3^{baseline}	$V_3^{0.441}$	$V_3^{0.539}$
4	V_4^{baseline}	$V_4^{0.441}$	$V_4^{0.539}$
5	V_5^{baseline}	$V_5^{0.441}$	$V_5^{0.539}$

the vulnerability score, while increasing w_1 by 10% decreases it Similar trends are observed for the other weights.

Our sensitivity analysis shows that the vulnerability metric is moderately sensitive to small changes in the weighting coefficients. Components with higher baseline weights, such as w_1, have a greater impact on the overall score, causing variations of up to 8.2% from the baseline. We found that the sensitivity variations proportionally followed the explained variances of the principle components. Although the four principal components collectively explain about 85% of the total variance, precise calibration of these weights is essential for accurately identifying vulnerable

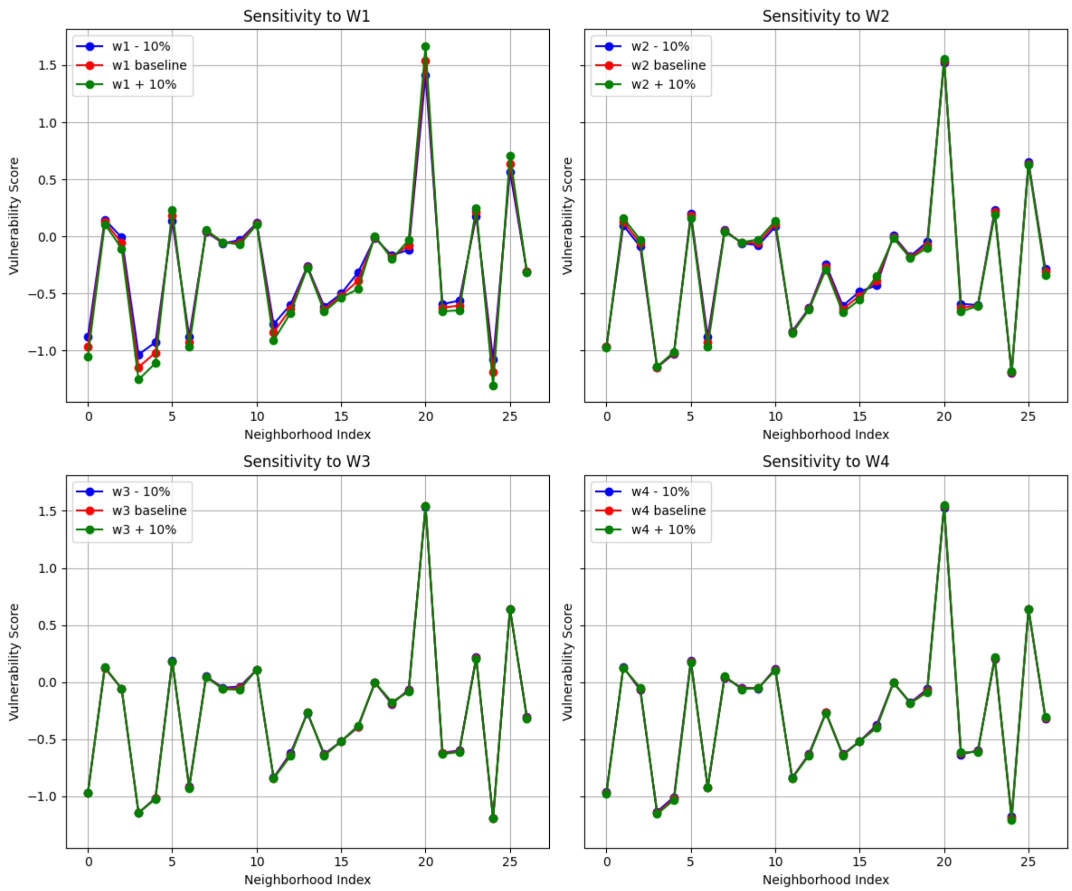


Figure 7: Sensitivity Plots for w_1, w_2, w_3, and w_4

neighborhoods. This insight is crucial for resource allocation and effective mitigation of heat wave effects.

Overall, even minor adjustments to the weighting coefficients result in significant changes in the vulnerability scores. This highlights the importance of careful calibration of the weights to ensure that our metric accurately reflects the true vulnerability of each neighborhood.

4.6 Strengths and Weaknesses

Our model is quite strong, with its components explaining 85% of variance in vulnerability. This is because it combined several different variables and reduced dimensionality while preserving the key "principle components" which explain variability. Our PCA modeling identified key areas to target: for example, policymakers can focus on the top variables calculated for PC1, which pertain to transportation and workforce metrics. We were also able to distribute high-resolution vulnerabilities to specific neighborhoods, structuring future mitigation strategies.

The weakness of our model is revealed in the extra 15% of variance in vulnerability not explained by our data. We could try to rectify this by finding more explanatory data which we may have not included. However, 85% is usually a sufficient start for PCA modeling. The primary weakness is that we do not have any response variables, like actual outages by neighborhood or heat-related hospital visits, so our vulnerability score calculations are partially based on educated assumptions.

5 Conclusion

In this study, we addressed three critical questions related to the challenges posed by extreme heat waves in Memphis, Tennessee. The answers, and most importantly, implementations in reaction to these given questions can have significant benefits to the well-being of Memphis, Tennessee. The application of similar modeling work to other major southern cities that suffer from heat waves and failing electric grids can have outsized effects on society at large.

For Q1, we developed a predictive model for indoor temperatures in non–air-conditioned dwellings during a heat wave using a modified form of Newton's Law of Cooling. Our results indicate that apartments, due to higher heat transfer coefficients and internal loads, experience faster and higher indoor temperature increases compared to single–family residences. This is of immense importance for policymakers to consider when regulating housing in the Memphis area. A well-insulated apartment or single-family home can be the difference between a life of good health and prosperity and that of sickness. The effects that an indoor temperature of 41.1 C and 40.0 C in apartments and single-family homes respectively can have on a family has repercussions for both their community, the legal system, and the city's economy. Significant improvements in insulation for non-AC homes are necessary to ensure the sustenance of thousands of families who depend on low-income housing. The social implications of lower-quality insulation and the economic effects of higher-quality insulation must be weighed when constructing and legislating non-AC homes.

For Q2, we projected the peak power demand on Memphis's electric grid during the summer months and forecasted changes over the next 20 years using a SARIMA model integrated with XGBoost features. Our time series analysis predicted an increase in peak demand over the next two decades, suggesting that grid efficiency improvements and modernization are absolutely necessary to offset the rising ambient temperatures and increased energy needs due to heat waves. Our projections indicated to us that the average peak demand during the summer months per customer (individual and business) in Memphis is around 3400 to 3600 kWh. Across the total Memphis grid, demand is expected to increase from 1.46 billion kWh in 2024 during the highest month in the summer months to 1.76 billion kWh by 2044. This is a massive 20 percent increase over only two decades which can have catastrophic consequences on Memphis's electric grid and thereby its citizens without the proper precautions and structural improvements. Our findings are both eye-opening and alarming and point to an urgent need for policymakers to collaborate with the electrical sector to improve grid capacity at a fundamental level and avoid widespread outages that have often irrecoverable damages.

Finally, for Q3, we developed a vulnerability score for Memphis neighborhoods using principal component analysis (PCA) on over 20 neighborhood socioeconomic and infrastructure variables. Our four computed principle components, each covering their own aspects of neighborhoods like housing diversity and income, explained a substantial 85% of the data. The vulnerability metric was calculated on a 0–100 normalization, and we recommended grid hardening, along with other subsidiary measures, to be allocated accordingly based on each neighborhood's vulnerability rank.

With sensitivity analysis, we confirmed the responsiveness of PCA to variable shocks, with ±10% variations in PC1 weighting introducing vulnerability score variations of up to 8.2%. The adaptability of our vulnerability score will allow calculations to remain effective in a variety of circumstances in the future and can actively used to diagnose new neighborhoods in addition to our original 27 neighborhoods. This enables policymakers to identify and prioritize at–risk neighborhoods for interventions such as grid hardening and resource allocation.

In summary, our findings suggest that during heat waves, indoor temperatures in non–air-conditioned dwellings can rise significantly—more so in apartments than in single–family homes—while future power demand may decline due to grid improvements. Moreover, our neighborhood vulnerability score offers a robust, data–driven tool for targeted mitigation strategies. These results provide valuable insights for policymakers tasked with ensuring equitable resource allocation and enhancing the resilience of Memphis's power grid and public infrastructure.

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