PREVIEW PAPER: ABOVE AVERAGE

It was noted that the quality of the report was uneven with respect to the three questions. The team did a good job, though, in addressing some basic modeling issues.

With respect to the first question, the results indicated a large discrepancy between the United States and United Kingdom costs. The team did make a note of the difference, though. The team also noted that their model was problematic with respect to the long term cost which will decrease towards zero. On the downside, the formatting and labels of some of their graphs were problematic.

For the second question, the team made use of a linear combination of factors which is reasonable. It was difficult, though, to determine how the coefficients were generated.

For the third question, the final number of towers for each region was rounded up to one for the large majority of the regions which off-hand does not seem like a reasonable result.

The summary was adequate. The summary includes results for the first question but not for the other questions. The summary does not include sufficient insights to provide cues as to what techniques were used to approach the three questions. The team had a number of things that they did well. For example, every section had a list of strengths and weaknesses.

1 Executive Summary

The internet has become an integral part of everyday life across the entire world. However, connection to the Internet varies based on the type of connection and necessary bandwidth required for a certain region. Finding the correct balance between connection cost, connection needs, and cell tower distribution is vital in order to ensure that each person within the region receives the best Internet experience without spending too much money.

The models we built have analyzed the cost per unit of bandwidth in dollars for Mbps over the next 10 years for US and UK consumers. It is necessary to take into account that the dependent variable is not cost, rather, the cost per unit bandwidth. This meant that the raw data given in the sources was not sufficient to produce a model. Ultimately, our model utilized a composite function to propose a solution. In addition, the separation of the UK and US data was done in order to view potential regional differences in network pricings. For the US, the predicted cost was $0.16, while for the UK, the predicted cost was $10.70.

To determine the minimum amount of bandwidth needed for specific households, we created a universal mathematical model that took in several parameters, including the number of activities, Megabits per hour required for a given activity, the input of a given activity, the input of a given age, the number of hours taken for a given activity, the number of people doing a given activity, and number of people of a specific age. The model's efficiency was then tested by three household examples, each with their own unique circumstances. The final data consumption was then calculated over the span of a whole year, which was then processed to reveal the necessary bandwidth to fulfill 90% of the household's needs and 99% of the household's needs. Pre-COVID conditions were assumed in order to make use of data from Table 1, which only counts the first quarter of 2020.

To determine the most efficient way of inputting Mobile broadband nodes for each region, we used information from the previous section and developed a model that would establish the best use of low to high Bandwidth nodes. While a node may have high bandwidth, it must also have a diminished radius, making it necessary to balance the quality of connections against the demand for bandwidth in a given area. It is also necessary to take into account the demographics of each population subset to determine how much bandwidth they would, in theory, require. Examination of the regions will be conducted based on each subregion rather than the entire area as a whole. The final result will report what kinds of towers will be used and in what quantity. It is also necessary to determine the range of each node based on its range.

2 Introduction

Within this section of our report, we will expand on each question whilst also providing fundamental information that was used in the process of reaching conclusions based on the dataset linked to our model. Along with that, we also define features that remained constant or were needed to be assumed to permit our model to remain both valid and accurate within the confines of the overarching scenario.

2.1 Mission & Problem Statement

In our report and through the creation of our models, our task was to:

1. Predict the cost for each unit of bandwidth (Megabits per second) and create a model that indicates the trend of this for users of this service in both the United States and the United Kingdom into the next ten years.
2. Find the minimum amount of bandwidth that would fulfill the needs of three separate households through the usage and creation of a mathematical model that also models their respective internet requirements within the timeframe of a year.
   - Household 1: A family of two adults (one employed) and one child.
   - Household 2: A family of two children and one retired elder.
   - Household 3: A group of three college students working part-time whilst also taking undergraduate college courses.
3. Formulate and test a model which focuses on finding the most optimal location for placing cell phone towers in the scope of three hypothetical regions {A, B, and C}.

2.2 Global Assumptions & Terms:

- Bandwidth is measured in Megabits per second (Mbps) and will be referred to using these units unless specified otherwise or the scope of a year was required by the question.

1. *Broadband service providers apply similar pricing packages or deals.*
   Justification: For calculating costs, companies can afford to make cuts in their pricing packages that they offer to consumers if their base price for providing their services is made cheaper through means of events such as mergers.
2. *Data transfer technology and bandwidth generally follow the same trend.*
   Justification: As the model which is created is based on past data, we must assume that no new breakthroughs in the data transfer industry will occur, as higher-speed connections at cheaper costs are not projected within our model. For instance, the creation of fiber-optic cables which allowed for the transfer of data at extremely high speeds comparative to the previously used telephone network lines.
3. *When collecting data, no virtual private networks (VPNs) were used.*
   Justification: VPNs are known to encrypt data that they transmit so that only the device and VPN service can read the encrypted data. However, the encryption is proven to require further data usage and on average, data usage has been shown to increase data usage by around five to fifteen percent (VPN University et al., 2017).

3 Part 1: The Cost of Connectivity

With the advent of technological advancements in this field, the quality of our internet and mobile connections have begun to increase as well. More and more bandwidth has become available, however, costs of these data plans have begun to change as well.

3.1 Question-based Assumptions:

1. *Differences in Average Peak Download Speeds from broadband service providers are negligible.*

   Justification: When calculating averages, numerically large differences between the download speeds between broadband service providers would result in average values which would not accurately reflect the dataset or real-values of peak download speeds which consumers would expect.
2. *The U.S. and U.K. have the same traffic and distance from the network centralizer.*

   Justification: Since mobile networks have shared capacities, larger traffic during certain periods, such as when gathering data, could yield significantly lower or even higher speeds than the general average throughout an entire day. Also, the speed of connection depends on one's distance from the broadband network centralizer and the general trend is the further one is from one's network centralizer, the slower their respective data speeds will be (Traficom, 2019). Starlink is a satellite internet constellation project under SpaceX created in 2015, and it can serve as a prime example of innovation within the data transfer field as it is expected to make use of new satellite technologies to provide internet speeds as high as 300Mbps and latency of about 20ms in more remote areas later into 2021 (Michael Sheetz, 2021). The availability of this to the public can prove to pull clients from broadband service providers, marginally changing both the network traffic, service price, and effectively the collected data.
3. *Network providers have the same data peering rules.*

   Justification: Peering is the process in which two Internet networks connect and in doing this they can exchange traffic. Streaming services and service providers generally make use of peering and while it is generally free, there are some exceptions such as Netflix. This is important as the handoff of this data can lift the burden from internet service providers, allowing them to maintain generally higher speeds as their traffic would be lower (Prince, 2018).

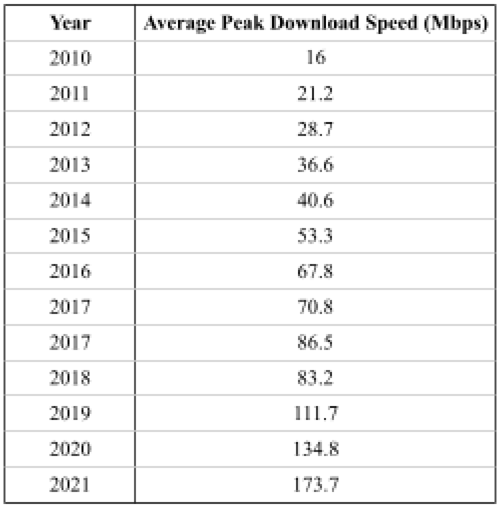
3.2 Defining the Variables

Let t = Time Passed Since 2010 (Years); where $\{ t \in \mathbb{Q}\text{: } 0 \leq t \leq 11\}$
Let b = Average Peak Download Speed (Mbps); where $\{ b \in \mathbb{Q}^{+}\text{: } 16 \leq b \leq 173.7\}$
Let c = Median Yearly Price per Mbps; where $\{ c \in \mathbb{Q}^{+}\text{: } 0.132 \leq c \leq 33.072\}$

There is no data explicitly relating the cost per unit bandwidth as a dependent variable with time in years. As a result, we created a composite function made up of the variables presented in Sheets 1 and 2 (Mathworks). This process will only be performed for the US model because there is sufficient data in Sheet 3 to construct a model for the United Kingdom. By modeling the cost ratio against bandwidth, and then bandwidth against time, a composite function can be constructed that will have the variables of cost ratio vs time.

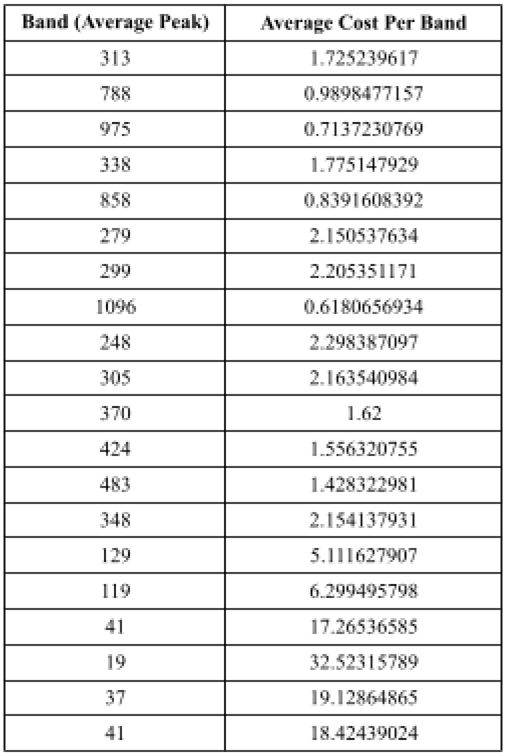
Using Sheet 1, we found the Average Download Speed (bandwidth) vs Time for the U.S.

Table 1: Average Peak Download Speed vs Time in the US



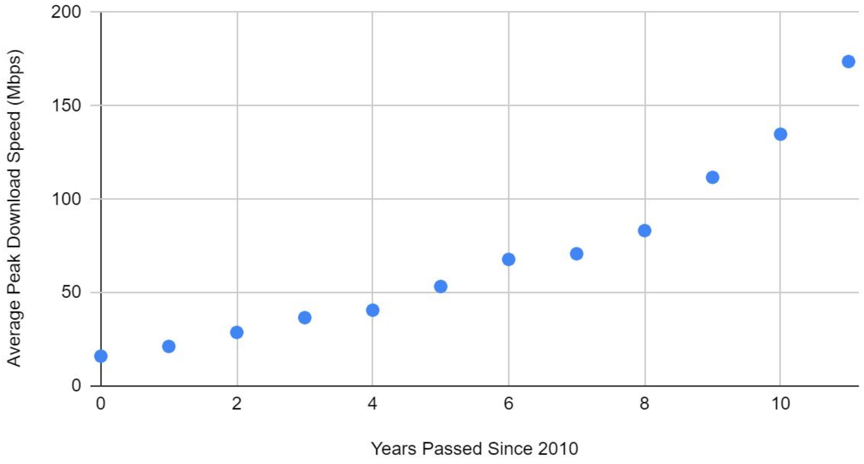
Year	Average Peak Download Speed (Mbps)
2010	16
2011	21.2
2012	28.7
2013	36.6
2014	40.6
2015	53.3
2016	67.8
2017	70.8
2017	86.5
2018	83.2
2019	111.7
2020	134.8
2021	173.7

Table 2 - Average Cost Per Bandwidth vs Bandwidth for the US



Band (Average Peak)	Average Cost Per Band
313	1.725239617
788	0.9898477157
975	0.7137230769
338	1.775147929
858	0.8391608392
279	2.150537634
299	2.205351171
1096	0.6180656934
248	2.298387097
305	2.163540984
370	1.62
424	1.556320755
483	1.428322981
348	2.154137931
129	5.111627907
119	6.299495798
41	17.26536585
19	32.52315789
37	19.12864865
41	18.42439024

<u>Diagram 1: Average Peak Download Speed vs Time in the US</u>



As can be seen from the general shape of the graph, the model exhibits increasing exponential behavior. This means that an exponential regression should be constructed.

$$\text{Exponential Regression: } b(t) = 17.9155(1.22624)^{t}$$

$$r = 0.995$$

$$r^2 = 0.992$$

In context, the possible domain for r-values is all values from -1 to 1 inclusive in which the closer a value is to 1 or -1, the stronger correlation it has whilst 0 indicates no linear correlation whatsoever. Whether the r-value is positive or negative indicates the direction the data takes and whether it is increasing or decreasing as the independent variable increases. An r-value of 0.995 indicates a very strong positive association. In regards to r^2, all values from 0 to 1 are inclusive, and it is generally identified as a percent where the value indicates the percent of variation present within the data that can be explained by the model. The value indicates that the model can explain exactly 99.2% of the variation present within the data. With this done, the results of the model itself should also be explained. The base of the exponential model indicates that every year after 2010, the average peak download speed increases by a factor of 1.23. Furthermore, the constant indicates that in 2010 the expected average peak download speed was about 17.9Mbps. To analyze the validity of this model, we can use percent error. The observed peak download speed in 2010 in the US was 16 Mbps. However, our model projected about 17.9 Mbps.

$$\%\ error = \frac{(projected - observed)}{(observed)} \times 100$$

$$\%\ error = \frac{(17.9-16)}{(16)} \times 100$$

$$\%\ error = (0.11875) \times 100$$

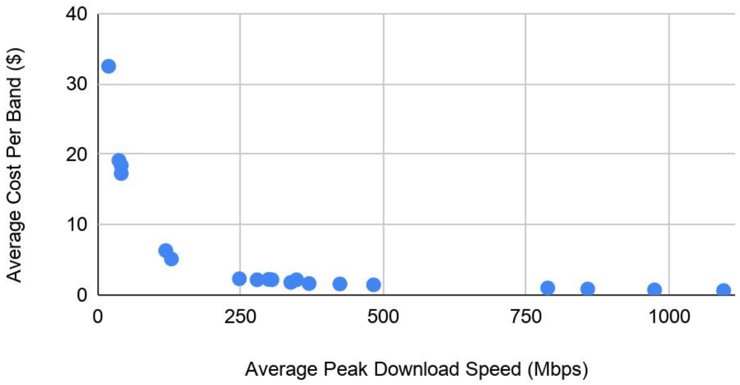
$$\%\ error = 11.875\%$$

$$\%\ error \approx 11.9\%$$

While a percent error of about 11.9% may seem large in the scope of our model, it only correlates to a few Megabits per second, which is somewhat of a negligible value given the amount of data an average consumer uses, alongside the size of modern day data files.

Using Sheet 2, we were able to find the [Median Yearly Price per Unit of Bandwidth] per Unit of Bandwidth. Using the Average Peak Speeds of 2020 and 2012 and calculating the Median Yearly Price in USD for each peak, we were able to create a model based on the median yearly price to a unit of bandwidth ratio and compare it to the unit of bandwidth.

Diagram 2: Average Cost Per Bandwidth vs Bandwidth for the US



This model also demonstrates exponential behavior because of the steep decrease from 0 to approximately 125 Mbps speeds. Conducting regression analysis, we found the model to be:

$$c(b) = 9.169661579 \cdot 0.9968988226^{b}$$

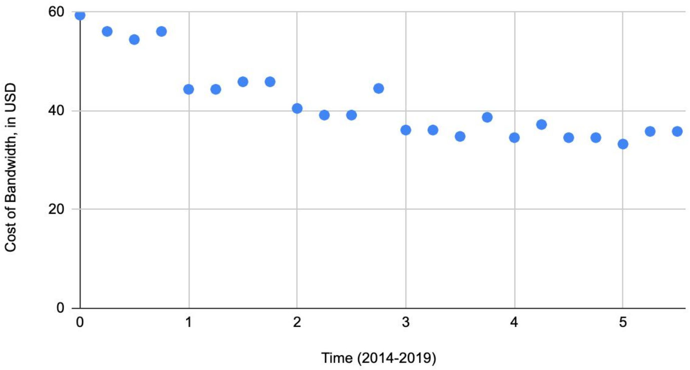
$$r = 0.845$$

$$r^2 = 0.713$$

To explain the magnitude of these results, an r value of 0.845 indicates a moderately strong positive correlation between the two variables whilst the r^2 value indicates that about 71.3% of the variation present in the data can be explained by the model. Yet, the aspects of the exponential regression function also have to be quantified, Within the context of the data, the value of b cannot be equal to zero as an average peak download speed could not have a correlating average cost per band.

Sheet 3 provides data on the monthly cost of "fast connection" and "superfast connection" in the UK from the beginning of 2014 to September of 2019. The peak speeds of each type of connection are displayed, with the peak of fast connection being 30 Mbps and the peak of superfast connection being 300 Mbps. After converting the monthly costs from £ to USD and multiplying the values by 12 to get a predicted yearly cost, the ratio of yearly cost to bandwidth is calculated. This was then graphed against time in years and modeled and extrapolated to 2010.

Diagram 3: Cost per Bandwidth (in USD) vs Years Passed Since 2014



$$c(t) = 53.37554674(0.9097549718)^{t}$$
$$\mathrm{r} = -\ 0.899$$
$$\mathrm{r}^2 = 0.807$$

The r-value of -0.899 unlike the other models indicates a moderately strong negative association between the two variables. This furthers the notion that as the time in years increases, then the cost of bandwidth decreases indicating an inverse relationship. The r^2 value indicates that 80.7% of the variation in the data can be accounted for in the exponential regression model created.

3.3 Solution

We extrapolated the model to 2031 to predict the cost per unit of bandwidth in US dollars:

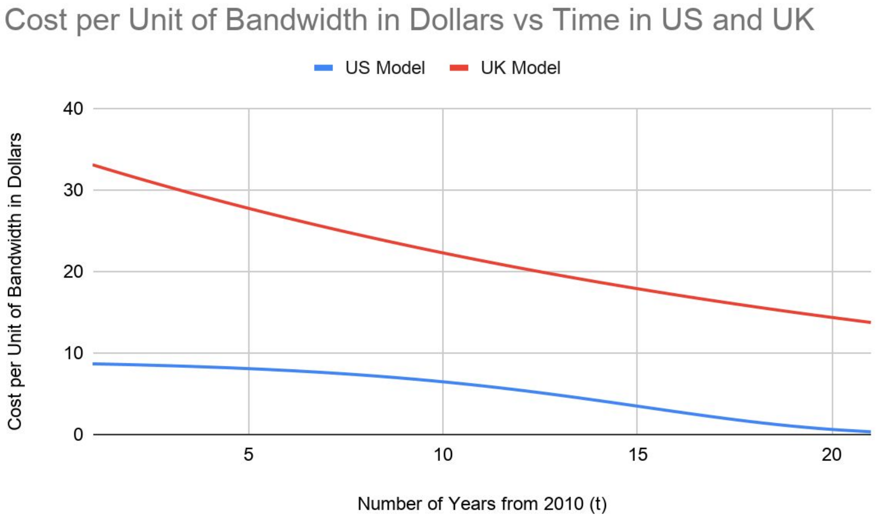
$$c(17) = 53.37554674(0.9097549718)^{17}$$
$$c(17) = 10.69197112/\mathrm{Mbps}$$
$$c(17) \approx 10.7\ \mathrm{USD/Mbps}$$

So in 2031, which is 17 years after 2014, the cost of bandwidth will be about $10.70 dollars per unit of bandwidth {Mbps}

Create the composite function to get Cost/Bandwidth vs Time:

$$c(b(t)) \ = \ c(t) \ = \ 9.169661579(0.9968988226)^{(17.9155\times(1.22624)^{t})}$$
$$c(21) \ = \ 9.169661579(0.9968988226)^{(17.9155\times(1.22624)^{21})}$$
$$c(20) \ = \ 0.1626674683\ \ \mathrm{USD/Mbps}$$
$$c(20) \approx 0.163\ \ \mathrm{USD/Mbps}$$

Diagram 4: Final Models For the US and UK



Conclusively, the composite function highlights that the Cost per Unit of Bandwidth follows a general decreasing trend over time since 2010 in both the US and UK. Comparatively, the UK prices seem to be much higher than the US prices. Because *t* has been defined as years passed since 2010, and ten years from now is 2031, ten years passed would occur at $t = 21$. For the US, the predicted cost to Mbps ratio will be $0.16, while the UK's would be $10.70. This is a remarkable difference. However, this could be explained by examining Sheet 1, where it is shown that the UK, in 2021, had 81.1 Mbps average peak download speeds while the US had speeds of 173.7 Mbps. Following the trend that, as bandwidth increases, the price per Mbps decreases, as shown in Graph 2, the UK price ratios would naturally be higher. The UK model is exponential, however, meaning that, extrapolated further, it should reach similar levels as the US model because both functions asymptotically approach 0.

3.4 Evaluating the Model

3.4.1 Implications

Our model states that the cost per unit of bandwidth will decrease whether in the US or in the UK. The model also revealed that there is a difference in the overall prices per unit of bandwidth between the US and the UK.

3.4.2 Strengths

Within our model, we used a composite function in order to derive the correct trend for the cost of bandwidth in the US, showing a higher level of mathematical complexity. Also, we were able to incorporate data that was not specifically related to the situation in order to end up with a more refined model of the data.

3.4.3 Weaknesses

We did not account for inflation rates within our model so the extrapolation is not exactly precise as the years add up. In addition, the overall trend for the UK is slightly higher than it probably should be because it seems illogical that there would be such vast differences in prices in bandwidth between the US and the UK.

4 Part 2: Bit by Bit

The amount of bandwidth required for each person will typically differ based on their age and the how people of those ages use the Internet. There are a myriad of factors that influence the data consumption of every household, meaning that this model's goal is to properly reflect their bandwidth usage.

4.1 Question-based Assumptions:

1. *Background internet consumption for each device is relatively the same and also negligible.*
   Justification: Background uploads and downloads can occur on any device and they can drastically impact one's required bandwidth. Some examples of how they can be transmitted are anti-virus or operating system updates and while these can be turned off, they can inhibit one's direct control over their device and the device's usability itself.

Malware can also increase background uploads and downloads as it can make use of one's device as a file uploader or receiver having the same paralleled effect as operating system updates, however, these devices generally are continuous as opposed to system updates (Murmson, 2016).

2. *The number of devices per household will be the same.*
   Justification: To calculate the expected bandwidth of each household a standard has to be set in regards to traffic and even the per-device usage as having multiple devices could involve processes such as data transfer. Thus, in our models and in the scope of this data the number of devices will be proportional to the number of members within the household in a 1:1 ratio.
3. *The same or similar workflow.*
   Justification: While workflow cannot be actively quantified, in our analytical models and in the scope of the question we are forced to assume that all the members of each household work from home. For example, the teacher would fulfill her job requirements over video conferencing given that their household has to take care of their child and classes have a consistent yearly and weekly schedule in regards to time spent on video conferences. The husband is looking for a job. In the case of the three part-time working m3 challenge participants, we will also have to assume that they follow the same or a similar schedule and work from home using video conferencing as a medium for communication.
4. *The software used for each device will remain mostly the same in regards to bandwidth.*
   Justification: Software can play a large role in regards to the bandwidth required as programs, despite having the same general outcome or purpose, may have different levels of optimization or even different databases which require more data to complete or sort through (Opera, 2019). With this in mind, we will assume that households will most notably use the same remote desktop protocols to access their respective work online portals and they will also use the same internet search engine.

4.2 Defining the Variables

Assuming that all of the data reflects Pre-COVID conditions

Flexible Mathematical Model:

Let a = the number of activities

Let b = Megabits per hour required for a given activity

Let c = the input of a given activity

Let g = the input of a given age

Let h = the number of hours taken for a given activity

Let n = the number of people doing a given activity

Let t = number of people of a specific age

$C = f(\mathrm{a}, b, t, h, n, c, g)$

$C = 52 \times ([\mathrm{t}_1(\mathrm{h}(\mathrm{g}_1) \times \mathrm{b}(\mathrm{c})) \ldots + \mathrm{t}_\mathrm{n}(\mathrm{h}(\mathrm{g}_\mathrm{n}) \times \mathrm{b}(\mathrm{c}))]_1 \ldots + [\mathrm{t}_1(\mathrm{h}(\mathrm{g}_1) \times \mathrm{b}(\mathrm{c})) \ldots + \mathrm{t}_\mathrm{n}(\mathrm{h}(\mathrm{g}_\mathrm{n}) \times \mathrm{b}(\mathrm{c}))]_\mathrm{a})$

The mathematical model *C* estimates the amount of bandwidth in Megabits in a year that would cover the total amount of bandwidth required to cover the need of the internet. The model is calculated by adding up the total number of Megabits per hour, which is determined by the number of activities and type of activity. These two variables are dependent on the age of the individual, and the number of individuals that age. The data reflects information and extrapolation based on Pre-Covid conditions.

Table 3: Hours per Week Spent by 1 Person In Need of Bandwidth

	Age Category					
Activity	**2-11**	**12-17**	**18-34**	**35-49**	**50-64**	**65+**
Watching Traditional Television	12.825	8.185	12.375	24.935	40.425	50.7
TV Connected Game Console	2.82	4.13	3.68	1.675	0.46	0.16
TV Connected Internet Device	6.635	3.975	6.14	5.91	4.175	2.685
Internet on a Computer (not including video)	N/A	N/A	3.835	4.51	4.715	2.97
Video on a Computer	N/A	N/A	1.3	0.955	0.785	0.365
Total App/Web on a Smartphone	N/A	N/A	27.075	27.57	22.38	16.245
Video Focused App/Web on a Smartphone	N/A	N/A	2.765	1.95	1.18	0.775
Streaming Audio on a Smartphone	N/A	N/A	1.16	0.71	0.485	0.31
Total App/Web on a Tablet	N/A	N/A	4.72	6.58	6.695	7.545
Video Focused App/Web on a Tablet	N/A	N/A	1.015	0.995	0.715	0.625
Streaming Audio on a Tablet	N/A	N/A	0.15	0.185	0.135	0.09

Table 4: Required Bandwidth for Differing Activities

Activity	**Required Bandwidth**
General web surfing, email, social media	1 Mbps
Online gaming	1-3 Mbps
Video conferencing	1-4 Mbps
Standard definition video streaming	3-4 Mbps
High definition video streaming	5-8 Mbps
Frequent large file downloads	50+ Mbps

4.3 Solution

Household 1. A couple in their early 30's (one is looking for work and the other is a teacher) with a 3-year-old child.

$$52 \times ([t_1(h(g_1) \times b(c)) \ldots + t_n(h(g_n) \times b(c))]_1 \ldots + [t_1(h(g_1) \times b(c)) \ldots + t_n(h(g_n) \times b(c))]_a) \times .9$$

$$\begin{gathered} 52 \times ([1((12.8 \times 12600) + (2.8 \times 7200) + (6.6 \times 23400)) + 2((24.9 \times 12600) + (1.7 \times 7200) \\ + (5.9 \times 23400) + (4.5 \times 3600) + (1.0 \times 12600) + (27.6 \times 3600 + (2.0 \times 23400) + (0.7 \times 23400) \\ + (6.6 \times 3600) + (1.0 \times 23400) + (0.2 \times 23400)]) \times .9 \end{gathered}$$

The final value received is 70,989,048 Megabits per year. This means that covering the household's internet needs ninety percent of the time, about 70.9 million Megabits will be used over a year.

$$52 \times ([t_1(h(g_1) \times b(c)) \ldots + t_n(h(g_n) \times b(c))]_1 \ldots + [t_1(h(g_1) \times b(c)) \ldots + t_n(h(g_n) \times b(c))]_a) \times .99$$

$$\begin{gathered} 52 \times ([1((12.8 \times 12600)+(2.8 \times 7200) + (6.6 \times 23400)) + 2((24.9 \times 12600) + (1.7 \times 7200) \\ + (5.9 \times 23400) + (4.5 \times 3600) + (1.0 \times 12600) + (27.6 \times 3600) + (2.0 \times 23400) \\ + (0.7 \times 23400) + (6.6 \times 3600) + (1.0 \times 23400) + (0.2 \times 23400)]) \times .99 \end{gathered}$$

After simplification of the procedure above, the final attained value would be 78,087,952.8 Megabits per year. Such a value indicates that covering the needs of the household ninety-nine percent of the time will require about 78.1 million Megabits for a year.

Household 2. A retired woman in her 70's who cares for two school-aged grandchildren twice a week.

$$52 \times ([t_1(h(g_1) \times b(c)) \ldots + t_n(h(g_n) \times b(c))]_1 \ldots + [t_1(h(g_1) \times b(c)) \ldots + t_n(h(g_n) \times b(c))]_a) \times .90$$

$$\begin{gathered} 52 \times ([2(2/7)((8.2 \times 12600) + (4.1 \times 7200) + (4.0 \times 23400)) + 1((50.7 \times 12600) + (0.2 \times 7200) \\ + (2.7 \times 23400) + (3.0 \times 3600) + (0.4 \times 12600) + (16.2 \times 3600) + (0.8 \times 23400) + (0.3 \times 23400) + \\ (7.5 \times 3600) + (0.6 \times 23400) + (0.1 \times 23400)]) \times .9 \end{gathered}$$

Following the same pattern as before, the simplification yields 70,989,048 Megabits per year at a ninety percent coverage. So, covering ninety percent of the family's needs requires about 71.0 Megabits, and as previously calculated this is also over a year.

$$52 \times ([t_1(h(g_1) \times b(c)) \ldots + t_n(h(g_n) \times b(c))]_1 \ldots + [t_1(h(g_1) \times b(c)) \ldots + t_n(h(g_n) \times b(c))]_a) \times .99$$

$$52 \times ([2(2/7)((8.2 \times 12600) + (4.1 \times 7200) + (4.0 \times 23400)) + 1((50.7 \times 12600) + (0.2 \times 7200)$$

$$+ (2.7 \times 23400) + (3.0 \times 3600) + (0.4 \times 12600) + (16.2 \times 3600) + (0.8 \times 23400) + (0.3 \times 23400) + (7.5 \times 3600) + (0.6 \times 23400) + (0.1 \times 23400)]) \times .99$$

After substitution and simplification, the value returned is 78,087,952.8 Megabits per year which covers ninety-nine percent of the family's needs. Furthermore, this means that about 78.1 million Megabits are required for the data-related tasks of the retired woman and her two grandchildren.

Household 3. Three former M3 Challenge participants sharing an off-campus apartment while they complete their undergraduate degrees full-time and work part-time.

$$52 \times ([t_1(h(g_1) \times b(c)) \ldots + t_n(h(g_n) \times b(c))]_1 \ldots + [t_1(h(g_1) \times b(c)) \ldots + t_n(h(g_n) \times b(c))]_a) \times .9$$

$$52 \times ([3((12.4 \times 12600) + (3.7 \times 7200) + (6.1 \times 23400) + (3.85 \times 3600) + (1.3 \times 12600) + (27.1 \times 3600) + (2.8 \times 23400) + (1.2 \times 23400) + (4.7 \times 3600) + (1.0 \times 23400) + (0.2 \times 23400)]) \times .9$$

The work above returns a value of 82,815,080.4 Megabits per year which covers ninety percent of the needs of the family. And to summarize in a year, about 82.8 million Megabits are needed to account for ninety percent of the family's data usage.

$$52 \times ([t_1(h(g_1) \times b(c)) \ldots + t_n(h(g_n) \times b(c))]_1 \ldots + [t_1(h(g_1) \times b(c)) \ldots + t_n(h(g_n) \times b(c))]_a) \times .99$$

$$52 \times ([3((12.4 \times 12600) + (3.7 \times 7200) + (6.1 \times 23400) + (3.85 \times 3600) + (1.3 \times 12600) + (27.1 \times 3600) + (2.8 \times 23400) + (1.2 \times 23400) + (4.7 \times 3600) + (1.0 \times 23400) + (0.2 \times 23400)]) \times .99$$

As a concluding element of this question, 91,096,588.4 Megabits per year is the value returned from the simplification of the work above. Returning to the original scenario, this value indicates that about 91.1 million Megabits account for ninety-nine percent of the three past participants' data usage over the timeframe of a year.

4.4 Evaluating the Model

4.4.1 Implications

Our model states that based on the number of people within the house and the age of those people, a certain amount of annual bandwidth needed can be calculated. The model also takes into account the certain types of activities that people of a specific range group perform.

4.4.2 Strengths

The algebra for this model remained consistent for each of the given households and the outputs seemed logical for each situation. Also, the model incorporates specific bandwidth requirements based on certain online activities, meaning that the results are specific towards each age group.

4.4.3 Weaknesses

We did not account for global shifts in online education and the changing patterns in telecommuting. In addition, the model above does not consider any usage during the COVID-19 pandemic so the results from this model don't actually reflect what the results would be for a given household during the pandemic.

5 Part 3: Mobilizing Mobile

The distribution of cell towers is also key because depending on the population density of a certain region, a certain type of cell tower is used. These cell-towers can range from low, medium, and high bandwidth cell towers.

5.1 Question-based Assumptions:

1. *There has to be at least one cellular node per subregion.*
   Justification: In order for consumers to be pleased with the broadband service providers and in order to decrease the number of dropped calls, disconnects, missing texts, and poor reception the providers will respectively have at least one tower for ear subregion.
2. *The cost of placing a cellular node within various subregions is the same*
   Justification: Varying costs between the placement of cellular nodes could motivate cellular providers to employ more cost-effective methods such as placing a cellular node with a large range in a different region and making use of the overlap into the other subregion as opposed to placing an entire cellular node within that region.
3. *Man-Made Terrain does not interfere with cellular node range or strength.*
   Justification: Since radio waves go in straight lines from devices to cellular and data nodes, large topographical structures such as hills can inhibit or block data transmission from reaching either end (Buckler, 2019). This notion also holds true in cities where buildings apply the same rules. Concrete, steel, and other materials that are also used in building construction block the radio waves. Yet, on upper levels of buildings signals will return as the waves will leave from windows and experience diffraction.

5.2 Defining the Variables

Let A_r = the area of the subregion in square miles
Let A_t = the area of the chosen tower in square miles
Let a = median age of the subregion
Let b = Megabits per hour required for a given activity
Let c = the input of a given activity
Let d = overall bandwidth demand for the subregion
Let h = the number of hours taken for a given activity
Let m = Mbps usage of one person of the given age using the equation from Q2
Let n = number of towers in a given subregion
Let p = population of the subregion

5.3 Model Development

$$m = \{[h(a) \times b(c)]_1 \ldots + [h(a) \times b(c)]_a\} \times \frac{1}{604800}$$

m is converted from Megabits per week to Megabits per seconds using $\frac{1}{604800}$

$$m\left(\frac{1\ week}{7\ days}\right)\left(\frac{1\ day}{24\ hours}\right)\left(\frac{1\ hour}{60\ minutes}\right)\left(\frac{1\ minute}{60\ seconds}\right)$$

$$d = m \times p$$

$$d = \{[h(a) \times b(c)]_1 \ldots + [h(a) \times b(c)]_a\} \times \frac{1}{604800} \times p$$

Based on the demand of the subregion, the type of towers that need to be used is determined. If the demand of a subregion exceeds the maximum bandwidth of a tower, the next band is used. Our methodology is detailed by the following pseudocode:

```
if $d < 250$ Mbps:
    Use lowBand tower
else if $900 > d > 250$ Mbps:
    Use midBand tower
else:
    Use highBand tower
```

To determine the number of tower, we used the equation:

$$n = \frac{A_r}{A_t}$$

Table 5: Conversion of Weekly Time Spent To Mbps per Person

	Age Category					
Activity	**2-11**	**12-17**	**18-34**	**35-49**	**50-64**	**65+**
Watching Traditional Television	161595	103131	155925	314181	509355	638820
TV Connected Game Console	20304	29736	26496	12060	3312	1152
TV Connected Internet Device	155259	93015	143676	138294	97695	62829
Internet on a Computer (not video)	--	--	13806	16236	16974	10692
Video on a Computer	--	--	16380	12033	9891	4599
Total App/Web on a Smartphone	--	--	97470	99252	80568	58482
Video Focused App/Web on a Smartphone	--	--	64701	45630	27612	18135
Streaming Audio on a Smartphone	--	--	27144	16614	11349	7254
Total App/Web on a Tablet	--	--	16992	23688	24102	27162
Video Focused App/Web on a Tablet	--	--	23751	23283	16731	14625
Streaming Audio on a Tablet	--	--	3510	4329	3159	2106
Total (Mbpw)	337158	225882	589851	705600	800748	845856
Total (Mbps)	0.5575	0.3734	0.9753	1.1667	1.3240	1.3986

Table 6: Number of Cell Towers Based on Bandwidth Demand for Region A

Subregion	Bandwidth Demand	Band of Tower	Number of Towers	Rounded
A_1	672.9450893	Mid	0.04279499581	1
A_2	1386.852054	High	0.000962887406	1
A_3	1520.166667	High	0.0005331690595	1
A_4	368.0686904	Mid	0.05835681247	1
A_5	1450.166667	High	0.0002864788977	1
A_6	1356.618289	High	0.001702957892	1

Table 7: Number of Cell Towers Based on Bandwidth Demand for Region B

Subregion	Bandwidth Demand	Band of Tower	Number of Towers	Rounded
B_1	4518.500001	High	1.107718404	2
B_2	2466.333334	High	1.384648005	2
B_3	1658.957083	High	1.476957872	2
B_4	1317.166667	High	0.7384789359	1
B_5	3300.702321	High	2.400056542	3
B_6	2797.666667	High	1.200028271	1
B_7	2375.234642	High	2.400056542	3

Table 8: Number of Cell Towers Based on Bandwidth Demand for Region C

Subregion	Bandwidth Demand	Band of Tower	Number of Towers	Rounded
C_1	1712.666667	High	0.1209577567	1
C_2	1583.859167	High	0.04456338407	1
C_3	1180.666667	High	0.03183098862	1
C_4	1510.833334	High	0.07639437268	1
C_5	1276.645104	High	0.0413802852	1
C_6	1176	High	0.09867606472	1
C_7	2087.166667	High	0.1082253613	1

The results show that no subregion had a demand low enough for a low-band cell tower to be used. The vast majority of the subregions required the use of a high-band cell tower, with some having low enough populations for a medium-band cell tower to suffice. Using the data from Sheet 4 and 5, the weekly hours spent could be multiplied by the megabit usage per week for each different activity to determine the bandwidth usage of an individual. Multiplying this by the population results in the overall bandwidth necessary in a specific region. The type of tower needed is directly tied to the demand for bandwidth in the subregion, meaning that high bandwidths would have been necessary for areas of high population regardless of their area. Also, all cell tower values must be rounded up, as rounding down leads to a lack of bandwidth.

5.4 Evaluating the Model

5.4.1 Implications

This model suggests that, more often than not, the high-band tower is necessary in areas that are highly dense in population. However, even in areas that are larger and have less people, a medium-band tower is typically used. Most regions only require one of each tower to meet the bandwidth demands as well, which are typically in the 1000-2000 Mbps range.

5.4.2 Strengths

Through examination of specific subregions, this model disregards the overlap of the regions and ensures that the entire region is covered by nodes. This also ensures that the bandwidth demand is always met because many of the high-band nodes have large enough coverage to span more than one subregion.

5.4.3 Weaknesses

The core assumption that each subregion must have one node, however, is a limiting factor in using the model. Especially in areas in which the bandwidth demand is just barely over the 900 Mbps necessary, there would be a lot of unused bandwidth. In reality, this would be incredibly uneconomical. Furthermore, the small areas of each subregion served to exacerbate the problem, especially in Region C, where each subregion area was below 1. This issue can only be resolved through an examination of the overall area of the region.

6 Conclusion

Our model for Part 1 implies that over the next 10 years, the cost per unit of bandwidth will decrease. This trend applies to both the US as well as the UK. Our model for Part 2 shows that by taking into account the number of people in a given household, the age of the people within the household, and the types of Internet activities that someone of that age will typically perform, an annual required amount of bandwidth can be calculated. Our model for Part 3 demonstrates that based on the same calculations from model 2 along with the population size of a certain region and its area, the number and type of cell towers can be determined for that region.

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