PREVIEW PAPER: AVERAGE

It was noted that there is no one part of the paper that stands out, but the team did manage to provide an answer for each of the three questions. Moreover, their approaches to each question were relatively straightforward.

With respect to the first question, the team's approach to determining the coefficients of their exponential model was simplistic, but they did find a way to make a determination. Little motivation or insight for their choice of an exponential decay model is provided.

For the second question, their assumption that every demographic group has similar video streaming habits drew some concern.

For the third question, the team provided a number of nodes that seemed somewhat reasonable, but it was difficult to interpret how they arrived at their answer.

The summary was seen as being problematic. The results for the second and third questions are not well presented in the summary, and the summary was more than one page. Within the report, the team did not provide a good critique of their model, and there were no strengths and weaknesses for any of the three questions.

Defeating the Digital Divide

Executive Summary

The use of high-speed internet has become more of a need than a want these days. It no longer is used solely for entertainment. Without sufficient access to the internet, many will suffer, including those who are currently learning remotely, working at home, accessing healthcare in a protected way, participating in civic duty, and many more. The goal is to present which form of internet most optimally helps people(especially those in low-income or rural locations) with connectivity issues.

Over the past twenty years, the average price of mbps has significantly decreased. In 2000, the average price of mbps was $28.13. In 2020, its average price fell to $0.64 [1]. With the demand of the internet rising, the cost of bandwidth will continue to decrease. However, the question that remains is what is the most effective form of internet? Whether it be cable, fiber-optic lines, satelites, or mobile broadband, finding the most productive form of internet will destroy the digital divide. Regardless of this question, the currency spent on this form of service has definitely decreased in the United States and United Kingdom.

Based on the price decrease, we predicted the cost of mbps over the next ten years. The predicted costs were calculated by the use of a formula we made based on the decreased cost over the last twenty years. Our model involves natural exponential decay, causing the results to be as accurate as possible. We also searched for other factors to ensure that the price of bandwidth would never be free. The difference of the costs were used to calculate an authentic natural exponential decay formula. On top of that fact, our model includes the United States and the United Kingdom. Both of these countries were included to show a similar narrative of the decreasing price for mbps.

When looking at the drastic differences inferred from the first problem, a model for mbps used to predict a given household's need for the internet over the course of a year. This model helps further correlate an understanding of the decreased price of mbps and how much is used in any household's situation. The model can fit any percentage of internet needed. Whether it be 99% or 90%, the model perfectly shows that depending on the situation, any family can find affordable bandwidth so long as it covers only their needs.

On the other hand, we also created a model for mobile broadband usage that optimally distributes cellular internet nodes with examples in three different regions. The model takes into account population and demographic data of the region causing us to be able to

calculate the bandwidth needs of said regions. Our model shows how to evenly distribute the towers so they can be optimally distributed.

Contents

I The Cost of Connectivity

1 The Problem

Build a model to predict the cost per unit of bandwidth in USD or GBP per Mbps over the next 10 years for consumers in the United States and the United Kingdom.

2 Assumptions

2-1 Internet prices only encompass bandwidth speed and do not include maintenance or servicing fees.

- Justification: Maintenance and serving fees vary by ISP (Internet Service Provider) and therefore would be difficult to account for in our model.

2-2 Technology will continue its current rate of progression.

- Justification: There is no way to tell how technology will develop in future years, so it is assumed that technology will continue to progress at approximately the same rate.

3 Variables Used

Symbol	Definition	Units	Value
P_s	Initial US Price of Bandwidth (2000)	USD/Mbps	28.13
P_k	Initial UK Price of Bandwidth (2000)	GBP/Mbps	5.60
r_s	Rate of Change for US	-	-0.1892
r_k	Rate of Change for UK	-	-0.2802
t	Time	Years	-
$C_s(t)$	Total US Cost	USD	-

$C_k(t)$	Total UK Cost	GBP	-

4 Developing the Model

Using the data, we developed a natural exponential decay model. We used a continuously compounding formula represented as:

$$A(t) = P e^{rt}$$

Using the variables P_s, r_s, t, and $C_s(t)$ we found the formula needed to make this natural logarithmic function. Our initial value represented as P_s, is found by the data found in 2000 which is $28.13 per Mbps [1]. Finding the difference between 2000 and 2020 we receive our "t" value which is 20. We also know that $C_s(20)$ would equal $0.64. Solving for r_s yields -0.1892. This value can be inserted into the exponential decay model. Therefore, we can conclude that the price in dollars per Mbps in the United states per year can be represented as:

$$Cs(t) = 28.13e^{(-0.1892)t}$$

Next we used the variables P_k, $r_{k,}$, t, and $C_k(t)$ to find the formula needed for the natural logarithmic of the United Kingdom's megabyte price. Same as before we found P_k, our initial value, to be £5.60 in 2007 [3]. The value C_k will be £0.36 based on a megabyte's price in 2017 [2]. The value for t would be temporarily set to 10 to solve for r_k making it -0.2802 which is our rate of decay. The "t" value will be subtracted by 7 to set the year to 2000 for an easier approach. The formula can then be represented as:

$$Ck(t) = 5.60e^{(-0.2802)(t-7)}$$

5 Executing the Model

After finding the formula for both the United States and United Kingdom's natural logarithmic functions we can then create a table of values and models. We will start with creating a table for the United States. The x-value will be the "t" variable indicating time while the y-value will be $C_s(t)$ indicating the total cost. The time will also start on 21 and go to 31 because it is the years after 2000. The function is represented by Table 1:

t	$C_s(t)$
21	0.53
22	0.44
23	0.36
24	0.30
25	0.25
26	0.21
27	0.17
28	0.14
29	0.12
30	0.10
31	0.08

Table 1: Cost decay from 2021 to 2031 in the US

The table's values can then be used to create a graph of what the decay would look like. This can be represented by Figure 1:

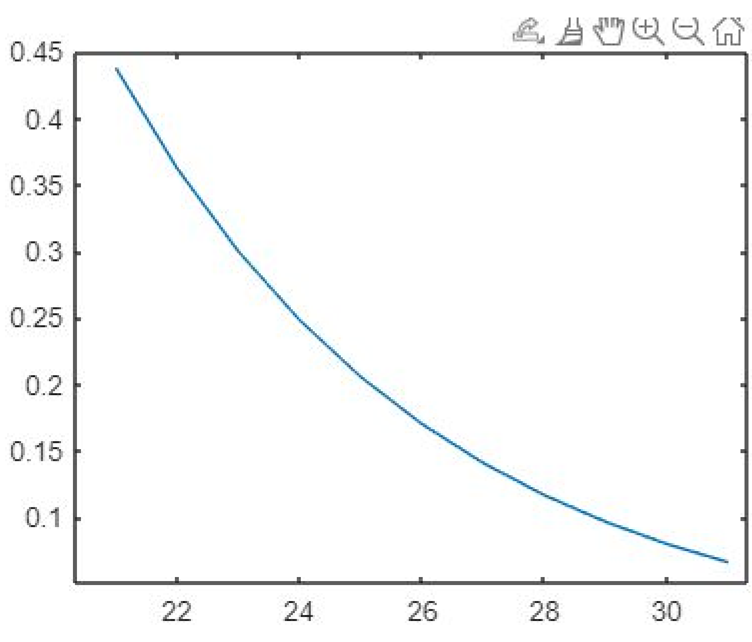

Figure 1: Graph of Mbps cost decay in the US

A table can also be created for the decay of cost in the United Kingdom. Doing the same method as before where t is the x-value and $C_k(t)$ will be the y-value. The time will also have the same principle where the number corresponds with the last two digits of the year (example: 22 = 2022). This can then be represented as Table 2:

t	$C_k(t)$
21	0.11
22	0.08
23	0.06
24	0.05
25	0.04
26	0.03
27	0.02
28	0.02
29	0.01
30	0.01
31	0.01

Table 2: Cost decay from 2021 to 2031 in the UK

The graph can be found by taking the values from Table 2 and plotting them on a coordinate plane The graph is represented by Figure 2:

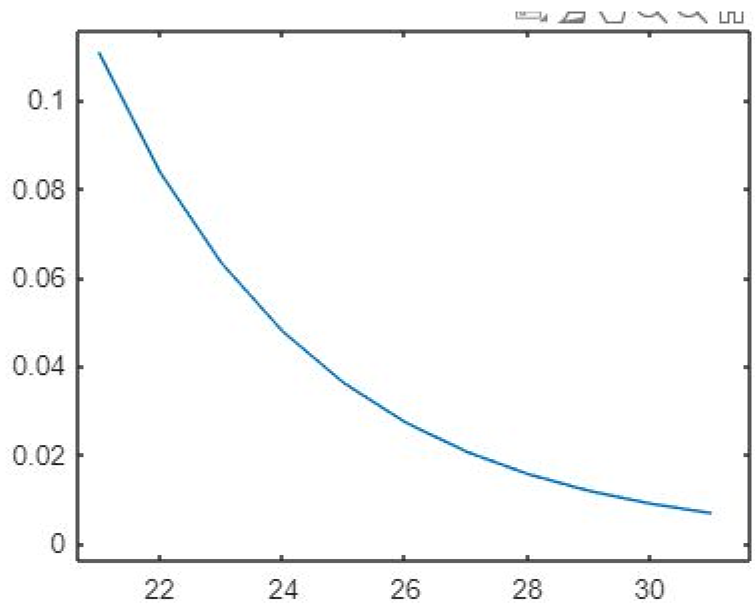

Figure 1: Graph of Mbps cost decay in the UK

II Bit by Bit

1 The Problem

Create a flexible mathematical model to predict a given household's need for the internet over the course of a year. Apply your model to the example households listed below and determine the minimum amount of required bandwidth that would cover their total internet needs 90% of the time. What about 99% of the time?

2 Assumptions

2-1 No member of the household is consuming more than one form of media at a time.

- Justification: For most households, only one person consumes a certain type of media at a given time.

2-2 If an Internet Service Provider offers a plan with a download speed sufficient for a particular household, the plan will also include a correspondingly sufficient upload speed.

- Justification: Upload and download speeds are offered by ISPs in plans that vary according to the proportional needs of end-users.

2-3 The model does not take into account different ISPs or technologies.

- Justification: Different ISPs may differ in their cost per unit of bandwidth.

However, their prices will still decrease over the span of many years at a consistent rate. Therefore, our model will remain relatively accurate for different ISPs.

2-4 Each household member is content with SD (Standard Definition) video

- Justification: Our model is only concerned with the minimum required bandwidth for a household, therefore HD (High Definition) video streaming can be safely ruled out as unnecessary.

2-5 For an activity with a bandwidth usage value that is represented by a range, we assume that the range is normally distributed.

- Justification: While there may be peaks and lows for the bandwidth consumption of an activity, on average, it stands to reason that the activity will be consuming an average amount of bandwidth.

3 Variables Used

Symbol	Definition	Units	Value
n	Number of Household Members	People	-
m	Average Bandwidth Consumption of Maximum Bandwidth Consuming Activity (Video Streaming)	Mbps	3.5
$S(n)$	Required Actual Speed	Mbps	-
A_D	Actual versus Advertised DSL Speed	Percent	82.368
A_C	Actual versus Advertised Cable Speed	Percent	94.959

4 Developing the Model

The required bandwidth for a household can be represented as the sum of the bandwidths needed for each member. Examining the data, we determined that for all age groups, video consumption was the activity that required the most bandwidth. Therefore, a model for the required bandwidth usage for a household of can simply be represented as:

$$S(n) = n * m$$

If we want to consider the bandwidth of a service plan that is sufficient 90% of the time, we simply multiply S(n) by .9, or for a plan to be sufficient 99% of the time, we multiply S(n) by .99. Lastly, we also found it to be important to adapt the result of our model to fit an advertised speed. Based on [4], there is an average discrepancy of 17% among DSL service plans and 5% among cable service plans between the advertised speed and the actual speed. Therefore, for a given household, we can represent the required advertised speed as $\frac{S(n)}{A_D}$ if they subscribe to a DSL service plan, and $\frac{S(n)}{A_C}$ if they subscribe to a cable service plan.

5 Executing the Model

We will now use this model to calculate the sufficient advertised internet speeds for a few sample households:

1. A couple in their early 30's (one is looking for work and the other is a teacher) with a 3-year-old child.

   n = 3

   S(n) = 3 * 3.5 = 10.5 Mbps

   .9 * S(n) = 9.45 Mbps

   $$\frac{S(n)}{A_D} = \frac{9.45}{0.82368} = 11.4729$$

   $$\frac{S(n)}{A_C} = \frac{9.45}{0.94959} = 9.9517$$

   .99 * S(n) = 10.395 Mbps

   $$\frac{S(n)}{A_D} = \frac{10.395}{0.82368} = 12.6202$$

   $$\frac{S(n)}{A_C} = \frac{10.395}{0.94959} = 10.9468$$

2. A retired woman in her 70's who cares for two school-aged grandchildren twice a week.

   n = 3

S(n) = 3 * 3.5 = 10.5 Mbps

.9 * S(n) = 9.45 Mbps

$$\frac{S(n)}{A_D} = \frac{9.45}{0.82368} = 11.4729$$

$$\frac{S(n)}{A_C} = \frac{9.45}{0.94959} = 9.9517$$

.99 * S(n) = 10.395 Mbps

$$\frac{S(n)}{A_D} = \frac{10.395}{0.82368} = 12.6202$$

$$\frac{S(n)}{A_C} = \frac{10.395}{0.94959} = 10.9468$$

3. Three former M3 Challenge participants sharing an off-campus apartment while they complete their undergraduate degrees full-time and work part-time.

n = 3

S(n) = 3 * 3.5 = 10.5 Mbps

.9 * S(n) = 9.45 Mbps

$$\frac{S(n)}{A_D} = \frac{9.45}{0.82368} = 11.4729$$

$$\frac{S(n)}{A_C} = \frac{9.45}{0.94959} = 9.9517$$

.99 * S(n) = 10.395 Mbps

$$\frac{S(n)}{A_D} = \frac{10.395}{0.82368} = 12.6202$$

$$\frac{S(n)}{A_C} = \frac{10.395}{0.94959} = 10.9468$$

III Mobilizing Mobile

1 The Problem

Mobile broadband (e.g. 4G and 5G internet) is transmitted from towers or nodes. Develop a model that produces an optimal plan for distributing/placing cellular nodes in a region. The model should incorporate information regarding population and demographic data for the region and should take into account the bandwidth needs of the region. Demonstrate the flexibility of your model in the three hypothetical regions provided (see the data provided) or substitute with regions of your choosing.

2 Assumptions

2-1 All towers and nodes produce equal mobile broadband.

- Justification: All towers and nodes must produce equal mobile broadband to ensure an optimally distributed network.

2-2 There are a certain amount of towers and nodes for equal distribution.

- Justification: The problem is about equally and optimally distributing the towers and nodes. If there were infinitely many, there would be no need to equally distribute them across the regions.

2-3 Physical blocking of signal distribution will not be a factor.

- Justification: To truly factor in the physical blocking aspect we would need to look at a map and look at the geography and man made elements to make sure nothing gets in the way

3 Variables Used

Symbol	Definition	Units	Value
P_a	The Population of Region A	People	6327
P_b	The Population of Region B	People	15054
P_c	The Population of Region C	People	9505

S_a	Number of Sectors in Region A	Sectors	6
S_b	Number of Sectors in Region B	Sectors	7
S_c	Number of Sectors in Region C	Sectors	7
F_h	High Frequency Node	Mbps	2000
N_a	Total Nodes Needed for Region A	Nodes	-
N_b	Total Nodes Needed for Region B	Nodes	-
N_c	Total Nodes Needed for Region C	Nodes	-

4 Developing the Model

Using the given data [D8] we can find all the basic values for the variables $P_{a\text{-}c}$ and $S_{a\text{-}c}$. We took the total amount of people in that region and divided it by the number of sectors in that region. If all nodes were set to high frequencies then the total number of people in a sector will have to be divided 150. This can be represented by the following:

$$Na, b, c = (Pa, b, c/Sa, b, c)/150$$

5 Executing the Model

After finding our formula we will create a table to express our solutions. This can be represented by Table 3:

N_a	7
N_b	14
N_c	9

Table 3: Total number of nodes needed in Regions A, B, and C

Reference

[1]https://www.ncta.com/industry-data/92-decrease-in-price-per-megabit

[2]https://www.ispreview.co.uk/index.php/2017/05/average-cost-per-mbps-uk-fixed-broadband-speed-0-34-vs-0-85-globally.html

[3]http://muniwireless.com/2007/07/09/broadband-prices-per-megabit-around-the-world/

[4]https://www.fcc.gov/general/charts-measuring-broadband-america#chart5

[5]http://www.nabla.hr/IA-InterestCalculation3.htm#:~:text=A%20quantity%20is%20said%20to,proportional%20to%20its%20current%20value.&text=A%20quantity%20is%20said%20to%20be%20subject%20to%20exponential%20decay,in%20exponential%20decay%20k%20%3C%200.

[6]https://www.newamerica.org/oti/reports/cost-connectivity-2020/global-findings/#the-total-cost-of-connectivity-depends-on-the-scenario