MathWorks Math Modeling Challenge

A CONTEST FOR HIGH SCHOOL STUDENTS
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PREVIEW PAPER: AVERAGE

This summary for this paper did not have a good overview of the problem, nor did it adequately describe the approaches used by the team. On question 1 the comparison between cigarettes and vaping was not clear. Moreover the team uses linear models that initially have negative values describing the extent of vaping. For questions 2 and 3 little use of mathematical models were stated, and the team provided graphs and results without citations or discussion. For question 3 they provided numbers to obtain ratios and simply stopped.

The team briefly discusses taxes but do not convey an understanding of what taxes are. They also include subjective political claims. Near the end the team includes a citation that has little to do with the question they are examining.

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The Vapes of Wrath - Summary

Our first model predicts that by the year 2029, 45.79 million people will be vaping in America. It follows a similar pattern to the beginning stages of cigarette usage. Through an analysis of data and societal implications, it is evident that vaping is following a similar path to smoking and should be treated as such.

As far as question two, the likelihood that a given individual will use a given substance is based on many factors such as genetic predisposition, economic status, social setting, health issues, and the availability of given drugs in urban and rural areas and the subsequent difference in prices. Using the models, out of a pool of 300 high school seniors with varying characteristics, it can be estimated, by averaging varying characteristics, that 83 individuals will use nicotine, that 135 individuals will use marijuana, that 90 individuals will drink alcohol, and 36 individuals will take opioids.

Within question 3, the fact that nicotine, marijuana, alcohol, and opioids all have differing effects on human health were considered. However, these effects are not always directly comparable and so a robust metric has to be used to analyse their relative severity. Using the models, we found that opioids have the most detrimental impact to human health, followed by nicotine, marijuana, and alcohol.

From Grape Juice to Vape Juice

1. Introduction

For the first question, “Build a mathematical model that predicts the spread of nicotine use due to vaping over the next 10 years”, we believe that the problem wanted us to find the number of people using nicotine each year for the next 10 years and then use this to find the change in nicotine use. Since the question specifies the spread of nicotine use due to vaping we looked only at sources of vaping. To start we created a mind map focusing on nicotine use between generations alongside overall increases in vape usage. We also focused on the availability of nicotine in terms of supply and demand, and possible legislation that could affect the spread of nicotine.

For the second question, “Create a model that shows the likelihood that a given individual will use a given substance”, we believe that the problem wanted us to create a model that could work with multiple groups since we needed to use the solution to determine how many seniors would use a certain drug so we did the research by age. This mass amounts of research showed us how drugs are spreading to so many high school students and to so many people in general.

For the third question, “Develop a robust metric for the impact of substance use… use your metric to rank the substances mentioned in question #2”, we believe the problem wanted us to use our own opinions along with some data to rank each drug based on how much of a negative impact they had on the user. This allowed us to see how much an impact using a drug can have on a person’s career, mental health, relationships, jail-time, and how it could influence their dependency on the drug. This small amount of categories showed us how much of a huge impact drug use can have on an individual’s overall life.

2. For each of Questions 1, 2, and 3

Question 1: Darth Vapor

The Model

Assumptions:

1. The model is taking place within the United States.
   Justification: due to the nature of the question, it was assumed that the model would be taking place in the United States.
2. The rate vape usage in the United States is consistent with that of the world
   Justification: There is no data to suggest that the increase of vaping in the United states is inconsistent with that of the world. Vaping is so recent that it appears to be increasing at the same rate of the world.

3. Vape tax will be similar to cigarette tax (44.3%).
   Justification: From articles read, vapes are trying to be added to tobacco laws, which would eventually lead to them being added to a cigarette tax.
4. Supply and Demand for Vape is at market equilibrium
   Justification: Based on the theory of Price Mechanism in microeconomics, consumers and producers use a series of signals and incentives so the market always rests at equilibrium. Based on this we can conclude that the market for vape is approximately at or near market equilibrium.

We started to create our model by just taking into account trends in past populations. We first attempted to create a function to describe the amount of American adults that will vape in a certain amount of years. In order to do so, we attempted to find a line of best fit for three data points about amount of vapers in 2016, 2017, and 2018. This line of best fit ended up being linear. The data we used was global, so we had to take the number of American vapers and divide that by the number of worldly vapers in order to decide what to multiply the whole world function by to get the American vapers per year. The initial function we created was the following.

$Vape(t) = (3.5t + 30.333)(.569)$

where t is the number of years after 2015.

We then used this function to find out the amount of vapers there would be in the 10 years we would be modeling.

Then we created a second function to model the amount of high school students that will be vaping in the next ten years. We did this in the same way as the first function, by using data in order to create a predictive line of best fit. We used the data provided about the percentage of high school students in order to figure out how many high school students vape based on year in order to determine the line of best fit of the rate of vape of the amount of high school students increasing. This turned out to be the function:

$Vape_o(t) = .809t - 1.299$

where t is the number of years after 2011.

Then we found the combination of these two models by adding the points for the same years together and that turned out to be:

$Vape_x(t) = 2.8t + 30.39$

where t is the number of years after 2019.

Then we attempted to take into account the availability of vapes over the years. In order to do so we determined the demand of the market for vapes if a tax was imposed. Based on the assumption that vape will be taxed similar to that of cigarettes, we calculated the drop in demand that will result from

the tax. The current retail price of cigarettes is $5.51 and 44.3% is made up of excise taxes. This means the tax is $2.44 and without the tax, cigarettes are $3.07. The tax is equal to 80% of the cigarette price. By carrying this percentage to the current price of e-cigarettes we found the approximate amount of the tax which is $6.39. Using a supply-demand graph we found the decrease in demand from a $6.39 tax. The current equilibrium is at $7.99 for an e-cigarette and the quantity is around 33.1973 million people using e-cigarettes. The equation for demand can be modeled using Quantity demanded = a-b*Price. The equation for supply can be modeled using Quantity supplied = -c+d*Price. In 2016 the demand for e-cigarettes was approximately 19.915 million people and the price for e-cigarettes was $14.36.

Using these two points we found the slope to be (change in quantity/change in price) = (33.1973-19.915)/(7.99-14.36) = -2.08513

Then we solved for the rest of the equation by plugging in the slope and a set of points.

33.1973=a-2.08513(7.99). a = 36.5752

So the equation for quantity demanded is Qd = 36.5752-2.08513*P

To find the equation for quantity supplied we found that the cost to produce and e-cigarette was approximately $1. This means at P=1 quantity supplied must be 0 because companies are unable to earn a profit and will go out of business. Using these two points we found the slope to be = (33.1973-0)/(7.99-1) = 4.74926

Then we solved for the rest of the equation.

0=c+4.74926(1). c = -4.74926

So the equation for quantity supplied is Qs = -4.74926+4.74926*P

To see the effect of the tax we put modified the equation for quantity supplied so that Qs = -4.74926+4.74926*(P-6.39). Our new equation becomes Qs =-30.3573+4.74926

Then we can find the new equilibrium by equating Qs and Qd,
-30.3573+4.74926*P=36.5752-2.08513*P
P = $9.79
and then plugging price into one of our equations.
Q = 36.5752-2.08513*9.79 = 16.1543
Market Equilibrium = (16.1543, $9.79)

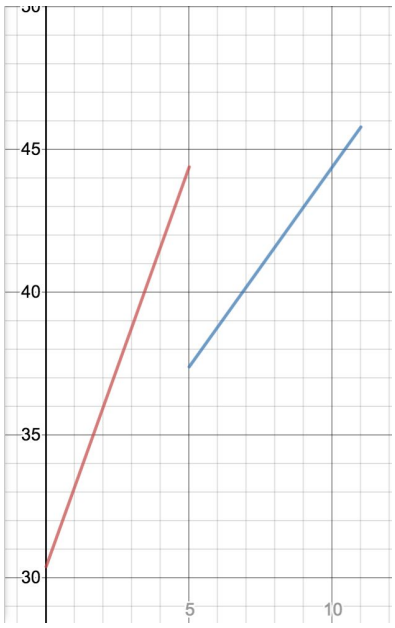
So, with the tax, the quantity demanded for e-cigarettes decreases from 33.1973 million people to 16.1543 million people.

Once we figured out how much the market would decrease after the tax was imposed, we needed to figure out when the tax was imposed. To do this we took into account legislation and political parties.

Congress is predicted to go to the democrats in 2020. Democrats are more likely to vote for regulation and taxation because of their platform and beliefs. This leads us to believe that the tax will go into effect around 2022. This changes the function into a piecewise function where:

$Vape_x(t)$= { 0 < t < 5 ⟶ 2.8t + 30.39 ; 5 < t < 11 ⟶ 1.4t +30.39 for t ∈ natural numbers }

This leads the graph of the model to look like:



Solution(s)

Comparing our model to the smoking model provided, it seems to mirror the beginning stages of smoking prevalence with the slope because of the steep increase, followed by a sharp decrease, and then a gradual decrease of the increase due to an event. In our case, this event is a vape tax, but in theirs it was the confluence of evidence linking smoking to cancer. It is hard to compare the graph given because vaping is relatively new and we only have a few years with which to work. Smoking was established when the graph provided was created with an almost 100 year coverage, whereas ours only shows 10 years.

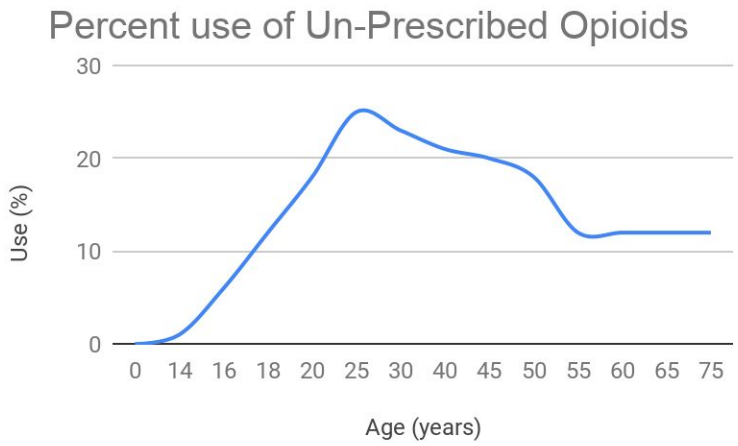
Discussion

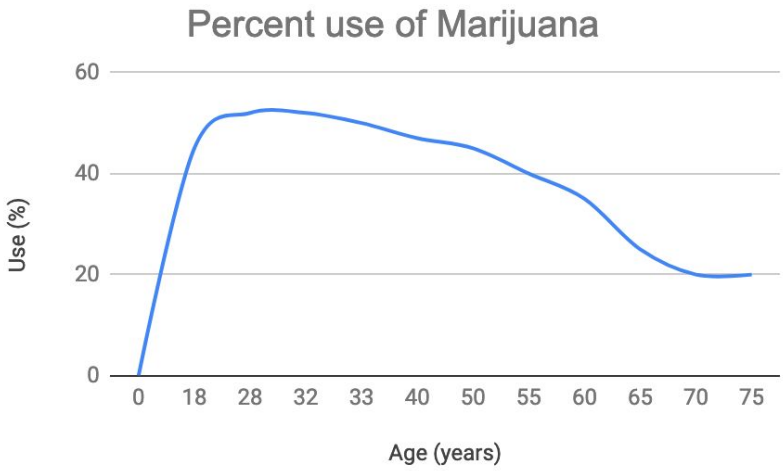
The model reveals how prevalent vaping will become in our society, akin to smoking almost. It reveals that it is not just a good way to get people out of smoking cigarettes, but it is actively something that is being used as a gateway. The model does take into account differing rates of change of generations. It also takes into account taxes and the political environment. The weaknesses with this model, though, is that it is linear, as well that it doesn't take into account how populations of people change because as the younger generation grows and the older, who generally does not vape as much, dies out, the prevalence will become heavier. If we could work on it for the next few months we would try to come

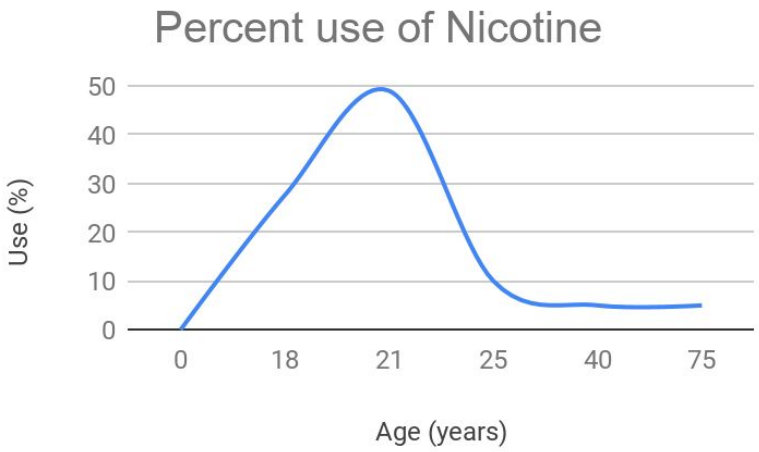
up with a way to take in more variables such as revealing research about vaping being exposed, as it did with cigarettes, or with cities versus rural areas. I do think our model provides a good rough estimate of the direction that vaping is going, which gives valuable insight into society.

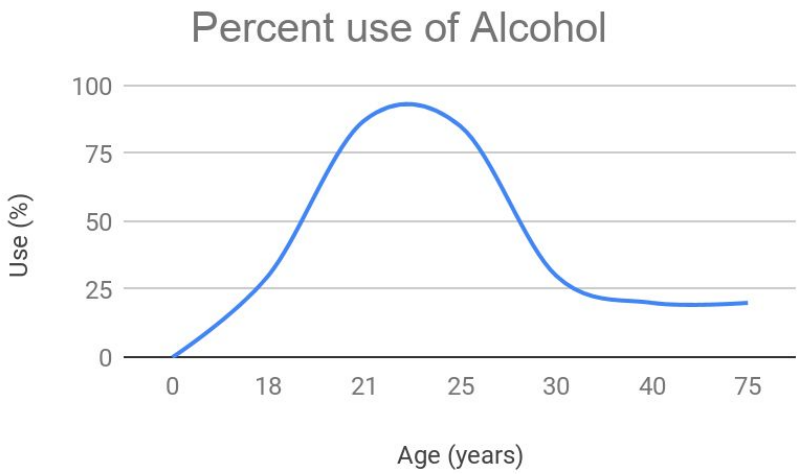
Question 2: Above or Under the Influence?

The Model









In order to create a model that most closely simulates the likelihood that a given individual will use a given substance, assumptions were undertaken in order to provide the closest estimate to the solution. The assumptions taken include referring to all high school seniors as being eighteen years of age, using 2018 as the current year, drug prices remaining stable and the same across the United States, the relationship between the dissolution of high school cliques and the subsequent increase in the mental health of students cancelling each other out, and that laws are upheld at all times in all aspects of society. The variables used in the model include the age of the individual compared to the likelihood that they will use said drug beyond that point in their life. All of our graphs begin at the age of zero and progress to the age of seventy-five due to the fact that we found that the percent chance of a given individual using a given drug after a certain age appeared to level off.

Solution(s)

The models were used so that we could answer the question by regarding a compilation of different sets of data. Each of these models gave the age and percent chance that they would use a given drug after a

certain age. Our solution is dependent on the interactions between different age ranges and the inability of a human mind to perform risky behaviours and non-habited behaviours after a certain point in time, hence, why the graphs depict curves that reach a vertex point and then decline at differing rates due to differing drug and alcohol statistics. By using our models, we were able to determine that out of a group of 300 high school seniors, assumed to be 52% lower income and 48% higher income due to US Census Statistics on the percentage of high school seniors on the free or reduced meal plan, 83 are likely to use nicotine, 135 are likely to use marijuana, 90 are likely to drink alcohol, and 36 are likely to take opioids. These values were made with the assumption that the 300 high school students were reminiscent of the demographics of the United States as a whole and that the dissolution of high school cliques and the improvement of the mental health of students due to that cancel each other out as well as the assumption that drug prices remain constant around the United States.

Discussion

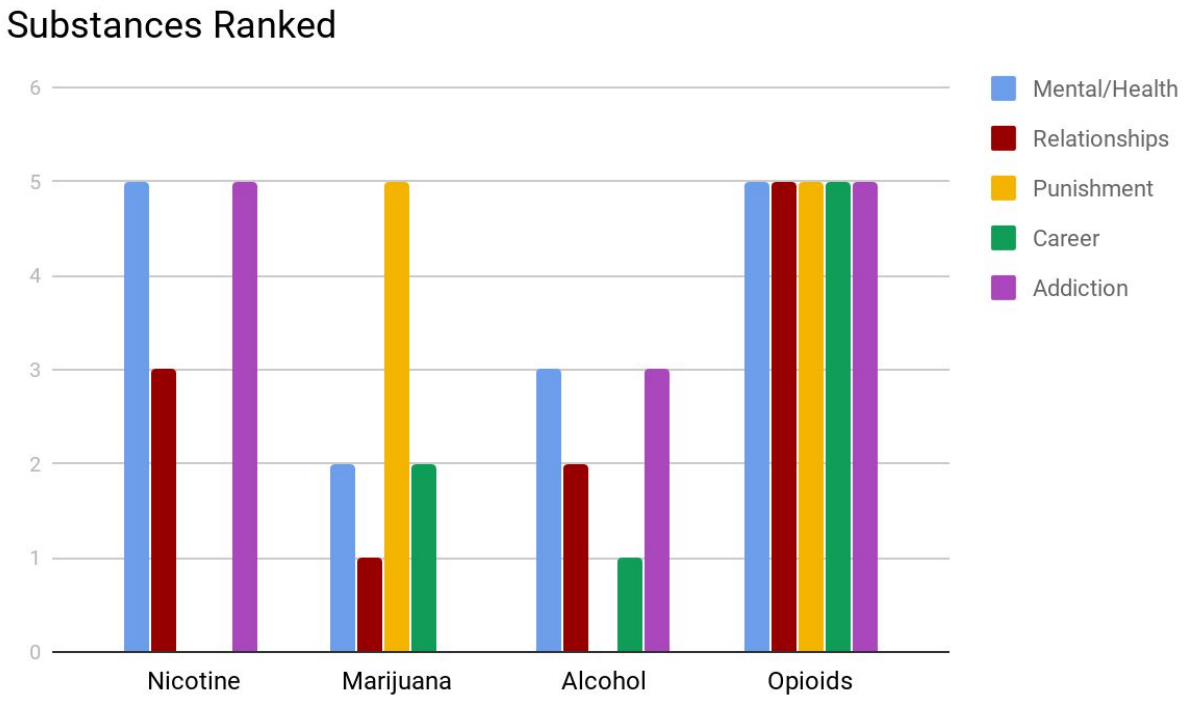
The different models we employed to determine the relationship between the age of a given individual and the likelihood that they were going to use a given drug were able to show common themes concerning the age of a given individual when they lose their drug innocence. By comparing the models, it is evident that a given person is most likely to use a given drug between the ages of 25 and 35. However, these are not definite solutions to the problem because the data was used in such a way as to estimate the overall percent chance of a given individual using a given drug at a certain age for a decade and only estimating the percentages based on the connection of points on the graph rather than finding values for each. This includes the fact that we had to manually deduct values in order to determine the expected shorter life expectancy of smokers from the decades of the middle of the 20th century. For example, when researching data about the susceptibility of lower income status and higher income status individuals and the types of drugs that they do, we found that individuals with a lower income status are more likely to smoke marijuana and nicotine devices while individuals with a higher income status are more likely to take opioids with alcohol being fairly equal across all economic status'. We also had to take into account individuals who became addicted to opioids through their prescriptions and then took them as well as the specific social circles individuals are in and whether society's progression away from high school cliques result in addiction spreading further due to interactions between more people or less because of decreased loneliness and bullying. Finally, we had to analyse whether genetics had a profound impact on addiction rates among the population and we discovered that genetics accounts for great inclination to begin smoking.

Question 3: Ripples

The Model

	Mental/Health	Relationships	Punishment	Career	Addiction
Nicotine	5	3	0	0	5

Marijuana	2	1	5	2	0
Alcohol	3	2	0	1	3
Opioid	5	5	5	5	5



Nicotine: 13/25 = 52%

Marijuana: 10/25 = 40%

Alcohol: 9/25 = 36%

Opioids: 25/25 = 100%

*none of which these substances are prescribed

Discussion

In order to assess the ranking of each of the substances' impacts on an individual, we redefined the word "impact" into five categories. We also only focused on the impact on the user exclusively i.e. not the economy or others surrounding the user. We first started by taking into account the impact of the substance on the user's overall motivation and work environment. The use of substances can impact

an individual's current job and any possible future job opportunities. It may also lead to dropping out of school as an adult. The use of these substances also impact mental health and the biological brain makeup. This also includes the cost of treatment for addiction or other side effects that may come along with the substance such as cancer, depression, withdrawls, etc. An individual's relationship with their significant others and family is also an impact to take into consideration. The average addiction rate was also ranked as it affects the individual financially as well. In addition to all of these variables and factors, we took into account the federal government's view on these substances such as jail sentences for punishment. Within these categories, we also determined which would fall under financial impacts and non-financial impacts with possible range of error granted.

With these assumptions being stated, we rated each category for each substance one through five. By asserting the impact of opioids having the worst possible impact on an individual, we ranked the other substances in comparison to opioids. For example, the closer the percentage of the substance impact is to one, the worse the impacts are. Nicotine has 52% of an impact on an individual on the scale of the impact of opioids. Marijuana is 40% and alcohol is 36%. The fact that all of these substances are affordable in most cases was also taken into consideration. These rankings were determined to be the average. So depending on the individual's personal experience affects the result. For nicotine, the robust value would be f(52%, x). Marijuana would be f(40%, x), alcohol f(36%, x), and opioids f(100%, x).

What's concrete about this model is that it is based on forms of evidence to back our claims. However, with a robust metric, the model could also be argued to be subjective. To take qualitative information and convert it into quantitative data, a range of error will be present. To rank the substances, we would need materials that are simply not available to us. To ensure a correct model, self-conducted experiments and surveys to find more data would be of aid. There also needed to be stated assumptions as the amount of variables and factors that could sway our results are endless. With more time for research to develop, we'd hope to definitely conclude that this model is a hundred percent accurate across the United States.

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3. Conclusions

Throughout our investigation into these pressing issues we discovered many problems along the way that we don't notice in our everyday lives. In question one, while exploring the numerous ways that we could predict nicotine use with vapes over the next 10 years, we were able to dig under the surface and see the real issues at hand. Vapes are not only just a "safe" alternative for older smokers but a dangerous gateway drug. More youths were using vapes and were becoming addicted to nicotine, this showed us just how misguided and unforgiving society can truly be. Then in question two, we saw that drug use as a senior could lead to even heavier drug use in the future leading so many to start their lives in the wrong direction. Though, we were happy to see how nicotine use has gone down in recent decades yet this was washed away by the staggering amount of nicotine use with e-cigarettes. Finally, in question 3, we outlined how drugs influence an individual's overall life and this had the largest impact on our team. With the opium crisis occurring, we knew how bad the situation is but with further

research we experienced what a person with an opioid addiction has to go through, this is why we stated opioid use as the biggest issue. When someone is addicted to opium; their relationships deteriorate along with their mental health, and if they get caught, any chance of getting a job. This entire project has left us with a new perspective on these issues and have educated us on the effects of drugs that are greatly affecting so many people around us.

4. References

"Prices Of Cigarettes By State." *Fair Reporters*, 3 Apr. 2017, fairreporters.net/health/prices-of-cigarettes-by-state/.

Galvin, Gaby. "Americans Are Buying E-Cigarettes At An Increasing Rate." *U.S. News & World Report*, U.S. News & World Report, 2 Aug. 2018, 12:00 pm, www.usnews.com/news/healthiest-communities/articles/2018-08-02/e-cigarette-sales-have-surged-immensely-in-the-us-cdc-study-shows.

Cook, Colin. "How High Are Cigarette Tax Rates in Your State?" *Tax Foundation*, Tax Foundation, 26 July 2018, taxfoundation.org/state-cigarette-tax-rates-2018/.

"Wolfram Alpha."https://www.wolframalpha.com/.

Gordon, Lydia, and Shane MacGuill. "Growth in Vapour Products." *Euromonitor International Blog*, Euromonitor International, 1 Nov. 2017, blog.euromonitor.com/growth-vapour-products/.

McPhail, C., et al. "Robustness Metrics: How Are They Calculated, When Should They Be Used and Why Do They Give Different Results?" *Earth's Future*, Wiley-Blackwell, 8 Jan. 2018, agupubs.onlinelibrary.wiley.com/doi/full/10.1002/2017EF000649.

Blair, Paul. "New CDC Data: More Than 9 Million Adults Vape Regularly in the United States." *Americans for Tax Reform*, 9 Nov. 2015, 7:53 AM, www.atr.org/new-cdc-data-more-9-million-adults-vape-regularly-united-states.

"Electronic Cigarettes." *Centers for Disease Control and Prevention*, Centers for Disease Control and Prevention, 3 Dec. 2018, www.cdc.gov/tobacco/basic_information/e-cigarettes/index.htm.

Jones, Lora. "Vaping - the Rise in Five Charts." *BBC News*, BBC, 31 May 2018, www.bbc.com/news/business-44295336.

Mincer, Jilian. "E-Cigarette Usage Surges in Past Year: Reuters/Ipsos Poll." *Reuters*, Thomson Reuters, 10 June 2015, www.reuters.com/article/us-usa-ecigarette-poll-analysis/e-cigarette-usage-surges-in-past-year-reuters-ipsos-poll-idUSKBN0OQ0CA20150610.

"Total Population by Child and Adult Populations | KIDS COUNT Data Center." *KIDS COUNT Data Center: A Project of the Annie E. Casey Foundation*, Aug. 2018, datacenter.kidscount.org/data/tables/99-total-population-by-child-and-adult#detailed/1/any/false/871,870,573,869,36,868,867,133,38,35/39,40,41/416,417.

"The NCES Fast Facts Tool Provides Quick Answers to Many Education Questions (National Center for Education Statistics)." *National Center for Education Statistics (NCES) Home Page, a Part of the U.S.*

Department of Education, National Center for Education Statistics, 2018, nces.ed.gov/fastfacts/display.asp?id=372.

blu. “Disposable E-Cigarettes & Vape Pens | Blu®.” *Blu US*, Blu, www.blu.com/en/US/e-cigs/blu-disposable?countryselect=tru.