Moody's Mega Math Challenge®

A contest for high school students

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PREVIEW PAPER:
ABOVE AVERAGE

This team's executive summary was lacking. The team did not provide insights into how they approached the problem in their summary. They did provide some conclusions, but they did not give a good overview of the questions addressed in the report. With respect to the first question the team clearly described how they arrived at their cut off points for the different time and distance levels, and they clearly described how they calculated their final distribution. The team did not explicitly note that they are assuming that commuting time and distance are independent quantities.

The team provided a thorough treatment for the second question. They note the costs associated with providing various types of car sharing programs, and they provided a model of the value that is provided to customers. The team's graphs are annotated, and the axes are clearly labelled. The team also provided a discussion of the sensitivity of their model with respect to a change in the cost associated with operating the vehicles.

The team was not able to provide a good discussion of their response to the third question within the allotted time.

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Share and (Car) Share Alike
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Contents

Summary

Car-sharing provides people with access to a fleet of vehicles they can rent on an hourly or daily basis. Typically, renters have memberships to the car-sharing corporations and rent the cars via their phones or the internet. To rent, members locate one of the strategically located rental stations, open the doors some sort of card or key, and drive off. Some benefits of participating in car-sharing include serving as a substitute for outright car ownership and providing workers with access to a vehicle for use during business hours. In spite of its many benefits, car-sharing retains a small sliver of the sharing economy compared to more popular segments like ride-hailing [11]. Nonetheless, companies such as General Motors are seeking to expand into the car-sharing industry. Increases in levels of car-sharing can decrease oil dependence, pollution, and traffic congestion.

This study seeks to portray the distribution of the proportion of drivers in the U.S. based on daily time spent driving and miles driven. These distributions are then applied to study the possible revenue a business could generate in the cities of Richmond, Riverside, Knoxville, and Poughkeepsie. The salient factor that was implemented into the models was the income bracket of the city; this determined the likelihood that the consumers would opt for the highest convenience option over the cheapest one. The results showed that Riverside had the highest potential for revenue, and the One-Way Sharing Floating service provided the best outcome.

Considering electric and autonomous vehicles, autonomous electric vehicles provide the best jockeying service due to reduced maintenance cost as well as the lack of the jockey’s fee. This is the best solution for both the business as well as the consumer.

Introduction

Background

Right before the Great Recession, the number of cars owned by an average American household was 2.28 cars. However, with the innovation of ride-sharing services, new companies, such as Uber, Lyft, and Sidecar, are “disrupt[ing] the paradigm” of vehicle ownership as a usual indicator of consumption [1]. Americans, especially younger generations like the Millennials, are moving away from ownership of their vehicles towards sharing services [1]. These car-sharing companies manifest the opportunity to utilize private cars without the responsibility and burden of owning one. Companies like Uber collaborate with regular vehicle owners who voluntarily perform on-demand car service. These vehicle owners act as private drivers for people who request them. Other companies, such as Zipcar, allow its customers to reserve Zipcars for as little as one hour or as much as seven days at a time for personal usage [2]. As shown in the successes of these rising companies, “renting can trump ownership” [1].

According to General Motor’s president, Dan Ammann, the car industry is predicted to change more in the next five years than in the past fifty years [3]. Americans are becoming more and more cautious of large expenditures as they juggle to “monetize their time and their assets” as well as save money in the weak economy [1]. Arun Sundararajan, a professor at NYU’s Stern School of Business and an expert on the sharing economy, also notes that Americans are becoming increasing interested in the concept of car-sharing, because they are compelled by the “variety and expansion of choices” offered [1]. Rather than being restricted to one car, people can experience driving a multitude of cars of their liking—a very enticing option.

Due to the social shift, many car manufacturing companies are attempting to stay in line with evolving demands by developing or investing in their own unconventional transportation methods. Similar to Zipcar, General Motors’ new service, Maven, will offer its customers the ability to reserve and access vehicles through the ease and convenience of a smartphone app [4]. This year, Ford Motor Co. will also launch a pilot program in Austin, Texas in efforts to keep up with the wants of its consumers. This car-sharing service will lease vehicles to self-organized groups of three to six people [5]. With the rising nature of these companie, they are bound to compete with the existing players (i.e. Uber and Lyft) of this car-sharing field.

Restatement of the Problem

To analyze the potentials of car-sharing and evaluate their function in different cities around the United States, the following problems were addressed:

1. Develop a model to determine the percentage of current U.S. drivers that exist within one of the nine categories formed by the combinations of the amount of time a car is used (low, medium, high) and the mileage driven per day (low, medium, high).

2. Evaluate which car-sharing option—round trip car sharing, one-way car sharing floating model, one-way car sharing station model, fractional ownership—will garner the most participation in a particular city.
3. Develop a method to rank cities based on their predicted car-sharing option so that a car-sharing company can decide whether or not to move forward with investigating legalities and implementation issues.
4. Create an adjusted model to account for the potential emergence of automobile technologies in the car-sharing industry (i.e. self-driving or environmentally friendly vehicles) and use the new model to re-rank cities.

Model 1: Categorizing Current Drivers in the United States

Analysis of the Problem

Automobiles entered the transportation market in the 1900's as a luxury for the wealthy. However, with increasing popularity and advancements in manufacturing, they have become cheaper and more accessible to Americans of various classes, not simply the rich. Due to cars, the gap between rural life and urban life have narrowed as the ability to populate and urbanize various locations have increased, as long as roads are available. Outward expansion into suburban areas have additionally been emphasized by the emergence of highways, which further encourage industries to relocate to where land is less expensive in comparison to that of cities [6]. The construction of better roads and infrastructure manifests the ability to commute to and from work, drive around the city, as well as travel throughout the nation.

From the late 1940's to early 1980's, there was a drastic increase in drivers within the United States. Approximately 88.6% of the United States population in 1983, ages sixteen and above, were licensed drivers, a 31.6% increase in comparison to those of 1949. Since 1949 to 2012, average vehicle miles traveled (VMT) rose by 6,861 miles, as well. Average American driver within this century, thus, annually travel almost twice as far as they did in the mid-20th century [7].

Given the vastness of drivers within the United States and their diversity, how can this portion of the national population be distinguished into simple subcategories?

Assumptions for Model 1

1. Time spent driving on the road can be represented by the time people spent driving to work.

It is sensible that each week, drivers' commute time is largely made up of driving to work. Furthermore, the value found for mean time spent driving to work each day was very close to the average commute travel time published by the U.S. Department of Transportation; however, standard deviations were not provided by the U.S. Department of Transportation, so alternate sources had to be found.

Design of the Model

The intent of Model 1 was to determine the percentage of current U.S. drivers who fall in the category of low, medium, or high within the parent categories of time using the car in minutes and miles driven per day. The daily driving times of low, medium, and high could be described by matrix A_{3x1}.

$$A_{3x1} = \begin{bmatrix} Low \\ Medium \\ High \end{bmatrix}$$

And the daily miles driven could be described by matrix B_{1x3}.

$$B_{1x3}[Low \quad Medium \quad High]$$

Both matrices would contain percentages of the distribution of those categorized as "low, medium or high." The ultimate goal would be to obtain matrix C_{3x3}, which would be the product of matrices A and B. Matrices A and B could be multiplied because the data that filled each matrix were found independent of one another, and the column number of matrix A equaled the row number of matrix B.

Within each item in the matrix, let the first letter represent "low," "medium," or "high" for the daily driving times and the second letter represent "low, medium, or high" for the daily miles driven per individual. Let *L* be low, *M* be medium, and *H* be high. For instance, *LL* would be the equivalent of the proportion of drivers who use their vehicle for a "low" amount of time and are "low" mileage drivers.

$$AB = C_{3x3} = \begin{bmatrix} LL & LM & LH \\ ML & MM & MH \\ HL & HM & HH \end{bmatrix}$$

The low, medium, and high values for the categories of "time" and "miles driven" were found by the following procedures.

Finding Matrix A for Daily Driving Times per Person in U.S.

Data was found by the U.S. Census Bureau on daily travel time to work and the proportion of people who fell within each frequency interval [8]. The justification for why travel time to work could represent daily driving time can be found in Assumption 1 of Model 1. The data itself can be found in Table 1 in the Appendix.

The goal within this procedure was to use the proportion distribution per each travel time interval to determine thresholds for high, medium, and low travel times. This was accomplished by determining the descriptive statistics of the data set; given the mean and margin of error, these statistics could be found. The margin of error (MOE) at 90% confidence was given to be 0.1.

$$MOE = critical\ value * \frac{s}{\sqrt{n}}$$

where s is the standard deviation of the sample, n is the sample size (132,674), and the critical value for a 90% confidence interval (CI) is 1.65. Plugging in values for MOE, critical value, and sample size, the standard deviation of this sample was found to be 22.07 minutes. A reasonable threshold value for "low" travel times was determined to be the upper bound of the interval in which one standard deviation below the mean fell in. Thus, the proportion of "low" travel times equaled the proportion of people whose travel times were less than 10 minutes. Similarly, the "high" threshold value was determined by using the lower bound of the interval in which one standard deviation above the mean fell in. This threshold was found to be 45 minutes and above, and the proportion of "high" travel time drivers was calculated to be 0.146. This left the "medium" travel time drivers to be designated as greater than 10 minutes and less than 45 minutes, and accounted for 0.72 of the sample.

Thus, the values for matrix A could be inputted.

$$A_{3x1} = \begin{bmatrix} Low \\ Medium \\ High \end{bmatrix} = \begin{bmatrix} 0.134 \\ 0.72 \\ 0.146 \end{bmatrix}$$

Finding Matrix B for Total Daily Driving Distance per Person in U.S.

Data for the average number of miles per trip in one day was downloaded from the U.S. Department of Transportation [9]. From the data set titled "ASCII.csv," the desired data – average number of miles per trip in one day for one person – was isolated, and the mean (9.511 miles/trip/day) and sample size (1,048,575 individuals) were found and recorded. To convert 9.511 miles/trip/day to the desired units of miles/day, the value was multiplied by 3.79, the average number of trips per person in one day provided by a U.S. Department of Transportation national travel survey. This yielded an average of 36.047 miles/day for a driver; the calculated value was verified by the value provided in the U.S. Department of survey study of 36.13 miles/day. The method used in Procedure 1 could not be used to determine threshold values as the sample data was extremely right skewed (see Graph 1 in Appendix). This graph was provided by John Krumm's report and further study of the 2009 U.S. National Household Travel Study [10]. Thus, to determine the thresholds for "high, medium, and low" daily driving distances, it was concluded that the best method was to divide the difference between maximum trips per day of 50 and minimum trips per day of 2 by three ((50-2)/3) to give even intervals for each category. "Low" was therefore 18 trips/day or less, "medium" was between 18 and 34 trips/day, and high was between 34 and 50 trips per/day. Each threshold value was multiplied by 3.79 trips/mile to yield the following values: Low: Less than or equal to 68.22 miles/day; Medium: Greater than 68.22 and less than or equal to 128.86 miles/day; High: Greater than 128.86 and less than 189.5 miles/day.

These thresholds were reasonable because they would yield a realistic expected frequency of proportions within each interval: a large proportion in the "low" category, a lesser proportion in "medium," and a very small proportion in "high." Due to the large amount of data, a Visual Basic program was written to determine the number of trials in the sample that fell within each category range. Because of the large sample size, the data is not included in the appendix, but can be downloaded from the U.S. Department of Transportation [9]. Within each category of low, medium, or high, the proportion of the sample that fell within each category was found, and the distribution was entered into matrix B.

$$B_{1x3}[Low \quad Medium \quad High] = [0.896 \quad 0.062 \quad 0.042]$$

Lastly, matrix C of proportions could be found by multiplying matrix A and matrix B.

$$AB = C_{3x3} = \begin{bmatrix} LL & LM & LH \\ ML & MM & MH \\ HL & HM & HH \end{bmatrix} = \begin{bmatrix} 0.120064 & 0.008308 & 0.005628 \\ 0.64512 & 0.04464 & 0.03024 \\ 0.130816 & 0.009052 & 0.006132 \end{bmatrix}$$

Justification and Testing of the Model

As expected, the proportions in matrix C sum to one. This verifies that calculations were performed correctly. Due to the high proportion of low travel distances, combinations that include low travel distances make up the majority of the distribution. This makes sense, as many drivers commute to nearby places for work, school, errands, and shopping. Furthermore, it makes sense that very few drivers out of the population of drivers in the U.S. commute long distances and spend a long time on the road on a daily basis. Due to these considerations, matrix C was deemed valid.

Model 2: Finding Car-Sharing Options that Result in Most City Participation

Analysis of the Problem

Recently, car-sharing has emerged as an alternative means for travel and is becoming increasingly popular with more companies, such as Zipcar and Modo, saturating the market. Car-sharing is a service that provides people with access to a fleet of vehicles they can rent on an hourly or daily basis. Typically, renters have memberships to the car-sharing corporations and rent the cars via their phones or the internet. To rent a car, members locate one of the strategically located rental stations, open the doors with an electric key card, and drive off. Some benefits of participating in car-sharing include serving as a substitute for outright car ownership and providing workers with access to a vehicle for use during business hours. In spite of its many benefits, car-sharing retains a small sliver of the sharing economy compared to more popular segments like ride-hailing [11].

New research from Susan Shaheen, the director of Innovative Mobility Research at the University of California, Berkeley's Transportation Sustainability Research Center, found approximately a third of car-sharing automobiles in America were available as part of one-way car-sharing services as of last January. However, various other business models are used, such as round trip car-sharing, one way sharing (floating model and station model), and fractional ownership [12].

In round trip car-sharing, vehicles are rented by the hour, mile, or day and are returned to the starting point. As for one way car-sharing, there are two common methods: the floating model and the station model. For the floating model, cars are rented on-demand and driven off to the intended destination. Once returned, the car is manually repositioned to the nearest station by a "jockey." However, rather than have an employee return the car back to its original location, in the station model, the customer is tasked with the responsibility of returning the car to the appropriate station. Fractional ownership is a more literal interpretation of car-sharing; one vehicle is jointly owned by multiple owners in order to split the cost of the private car [12].

According to a study conducted by the Transit Cooperative Research Program, the target demographic for car-sharing participants include people who are highly educated, live in dense urban areas, have a middle or high income, young, and highly environmentally and socially-conscious. People who share these attributes are typically early adopters to new technologies [13].

With the evolution of consumer demands in recent years, there has been a surge in car-sharing services. Model 2 seeks to investigate the appropriate implementation of various car-sharing options in four cities in the United States: Poughkeepsie, NY, Richmond VA, Riverside, CA, and Knoxville, TN.

Assumptions for Model 2

1. The ride-sharing business has enough initial capital to establish the business and run without failing

Design of the Model

The purpose of the second model was to illustrate how effective the various car-sharing business models were in each of the four markets by viewing the potential maximum revenue for each method and scaling them to a national reference (United States income distributions). The total expenditures for a company to run their service in each city was explicated by the following equation:

$$C_{total} = C_{car} + C_{maintenance} + C_{fuel}$$

in which

$$C_{total} = total\ cost\ (\$)$$

$$C_{car} = purchasing\ cost\ of\ vehicle\ (\$) = \$18{,}619.22$$
$$C_{maintenance} = \ maintenance/service\ costs\ of\ vehicle\ (\$)\text{, and}$$
$$C_{fuel} = operating\ fuel\ costs\ of\ vehicle\ (\$).$$

The relationship of the total revenue of the corporation to the cost was described by the formula

$$R_{total} = \ u * C_{total}$$

in which

$$R_{total} = total\ revenue\ (\$)$$
$$\text{and } u = markup\ constant = 1.5.$$

Finding the Total Cost of Running the Cars per Unit

Using information gathered from the AAA and other vehicle-related sources on the annual person miles travelled (PMT) per capita, cost of a compact vehicle, cost of servicing the vehicle, and oil prices, the cost per hour and mile for a company to conduct its business was calculated using:

$$H = \frac{C_{total}}{t} = \frac{C_{car} + C_{maintenance} + C_{fuel}}{t}$$
$$\text{and } M = \frac{C_{total}}{d} = \frac{C_{car} + C_{maintenance} + C_{fuel}}{d}\text{, respectively,}$$

in which

$$t = time\ (hr)$$
$$d = distance\ (mi)$$

In particular to the equations, C_{car} was only valid for only the first 24 months because the vehicle, a 2015 Honda Civic LX, was purchased for $18,240 with a 10% down payment and financed with an APR of 0.9% for that time period. Additionally, $C_{maintenance}$ included the expenditures needed to service a vehicle up to 75,000 service miles, which cost approximately $1,850. C_{fuel} consisted of the amount of money needed to sustain the fuel for the car. The chosen vehicle had a combined highway/city fuel efficiency of 33 miles per gallon, and as of February 27, 2016, the United States national average gas price per gallon was $1.74.

Therefore, the values were inputted as such and resulted in:

Example Calculation of Car Rental ($/hour) and Revenue ($)

$$H = \frac{\$18{,}619.22}{24\ mo} + \left[\left(\frac{75{,}000\ mi}{10{,}658\ \frac{mi}{year}}\right) * \$1{,}850\right] + \left[\left(\frac{10{,}658\frac{mi}{year}}{33\ mpg}\right) * \$1.74\right] = \frac{\$1.156}{hr}$$

$$\frac{R_{total}}{hr} = \ u * \frac{C_{cost}}{hr} = \frac{\$1.734}{hr}$$

Example Calculation of Car Rental ($/mile) and Revenue ($)

$$M = \frac{\$18{,}619.22}{2 * \left(10,658\,\frac{mi}{year}\right)} + \frac{\$1{,}850}{75{,}000\;mi} + \frac{\$1.74}{33\;mpg} = \frac{\$0.951}{hr}$$

$$\frac{R_{total}}{mi} = u * \frac{C_{cost}}{mi} = \frac{\$1.426}{mi}$$

Finding the Value of each Service to the Consumer

To determine the value *V* for a customer, the cost *C* of the individual car-sharing model was combined with a reduction factor of convenience Q that was multiplied by an affluence constant *k* in order to incorporate the value of convenience to each car-sharing model. The cost *C* within the equation for value *V* was calculated using a factor for cost per hour *H* for the consumer that used calculated car maintenance per hour with a 50% markup (u) to generate revenue multiplied the time *T* that the car is used by a consumer. The time *T* that a consumer would drive the car was calculated using the probability values for low, medium, and high driving times from model one. Using the probability values, an expected value was calculated by multiplying the average driving time for each category (low, medium, and high) by the probability value and taking the sum of the three resulting products. This expected value was multiplied by cost per hour *H* in order to determine the cost for the consumer and thus, the revenue for the business, for a given model.

The convenience factor *Q* was calculated by assigning values to each car-sharing model, with the lower values representing lesser convenience and greater values representing more convenience for the consumer. The convenience values for *Q* were assigned as follows: one way sharing: floating model = 40, round trip car-sharing = 30, one way sharing station = 20, and although not used in this model, the fractional ownership model = 10 due to its inconvenience for a single consumer. The convenience factor was also multiplied by *k*, the affluence constant, which represented the willingness of a consumer to pay more money for a more convenient experience in a car-sharing model. The value *k* was calculated as a proportion that represented the position of a given city's median annual household income within the inter-quartile range IQR of the annual household income of the United States from a value 0-1 in which 0 represented the city's median annual household income being at or below the 25th percentile of United States annual household income and 1 represented the city's median annual household income being at or above the 75th percentile of the United States annual household income. As a result, a *k* value of 0.5 represented a city in which the median annual household income was equal to the median annual household income of the United States. This relationship among value, total cost, *k*, and the convenience factor was described with:

$$C_{total} = H * t,$$
$$C_{total} = M * d,$$
and $V = C - (k * Q)$

In the one way sharing: floating model, an additional cost was added to represent the cost of returning the car to the station. This value was obtained by multiplying the federal minimum wage, \$7.25, by 1.5 to account for one man-hour and the 50% profit margin *u*. Also, in round trip car-sharing, the cost per hour *H*, which represents the time required for the one way leg of the trip, was doubled to account for the returning leg of the trip, assuming traffic flow would remain unchanged for both legs of the trip. Also, for the round trip, the time spent at the destination was included at the levels: low, medium, and high. These values were 1.6 hours, 2.5 hours, and 8.9 hours, respectively. By multiplying the values before calculated the expected value, the time spent at the destination in between transit during a round trip was incorporated into the cost of the round trip car-sharing.

Refer to Tables 2.1 and 2.2 as well as Graphs 2.1 and 2.2 in the Appendix for a visualization of the data and calculation of the model.

Model 3: Ranking Cities

Design of the Model

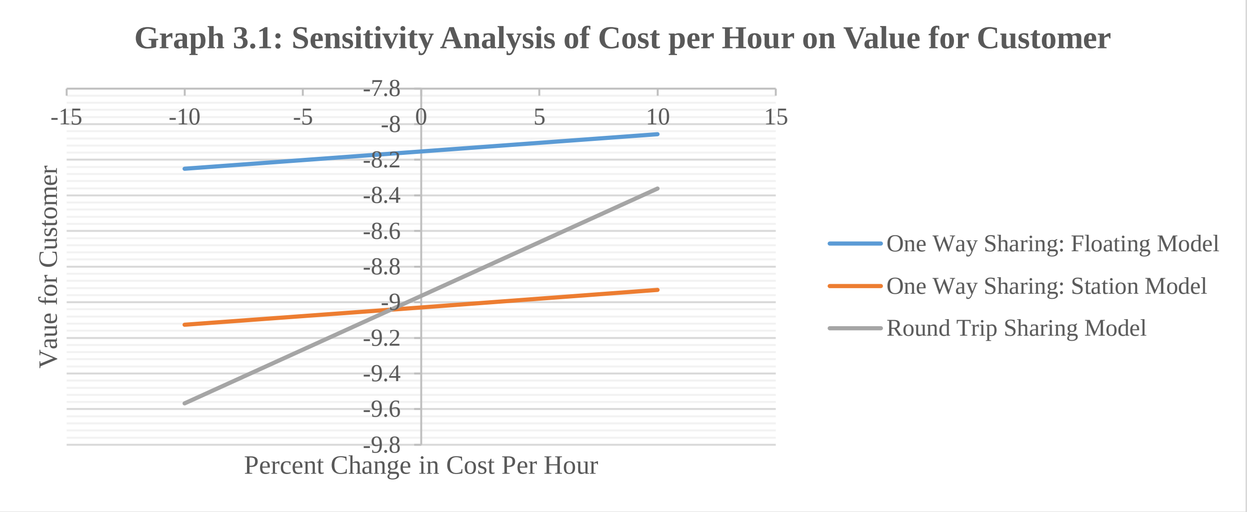
The purpose of the model was to determine which of the four given cities would yield the highest revenue for a car-sharing business. Out of the four suggested car-sharing models, only three were used because the fourth, divided private ownership of a car, would not create revenue for a car-sharing business. In order to determine the distribution of the population that fall into the low, medium, and high travel time categories, the population of the city was multiplied by the travel time matrix. To calculate the revenue of each car-sharing model, the cost equation from Model 2 was used, resulting in values that represented revenue in dollars that the business could earn for a given car-sharing model in one hour at max city usage. The revenue from each car-sharing model was determined by calculating *k* values for the first and third quartiles of the city. These provided bounds in which the optimal model was evaluated with the following equation:

$$Revenue = (\Delta k * \frac{1}{2} population * cost)$$

The model of greatest value for consumers was determined between the two *k* values, and out of the multiple car-sharing models that were present in the interval, the model that provided the greatest revenue was chosen a the best car-sharing model for the city.

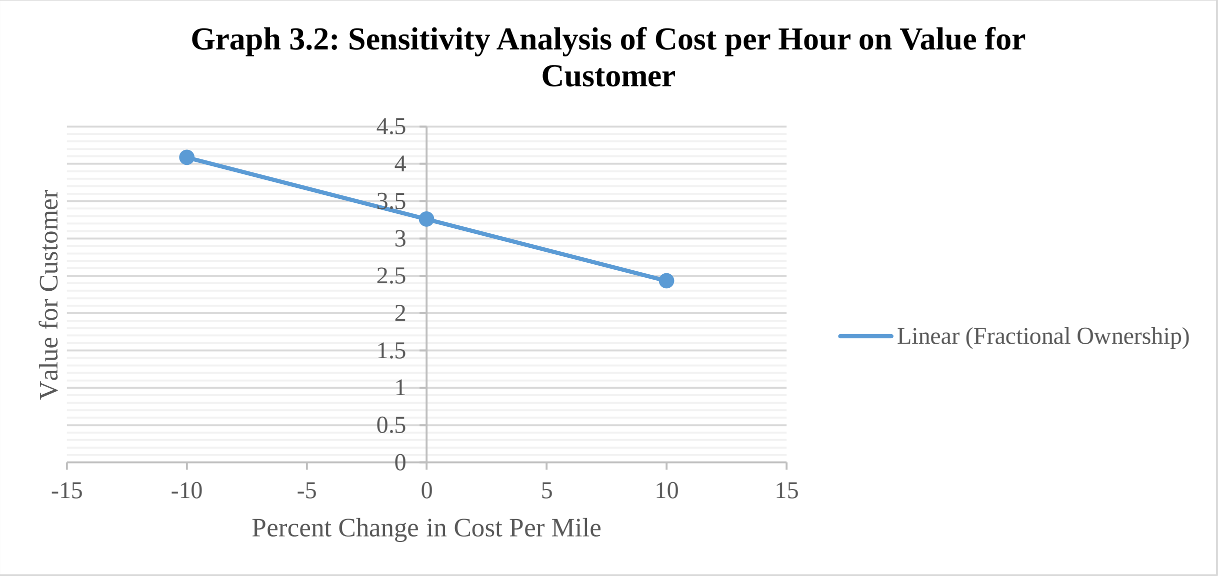
Implications and Conclusions

This model incorporates elements from the first and second models, using proportions of drivers and the value of each car-sharing model in each city in order to create a model that applies these two models to a real world application. By calculating the optimal car-sharing model for each city and calculating the potential revenue for each car-sharing model and city, this model presents an efficient and optimal method by which a company looking to create a car-sharing business can introduce itself into the optimal market with an effective car-sharing model. As concluded from the model, the revenues for each car-sharing model for each city can be found in table 3.1. It can be concluded that the most car-sharing model is the one way sharing: floating model. The market that presents the greatest potential for revenue is Riverside, CA, and a company looking to create a business would most likely experience the greatest levels of success by introducing the one way sharing: floating model to Riverside, CA.



For the one way sharing: floating model, $V = 0.9174H – 8.1536$, for the one way sharing: station model, $V = 0.9714H – 9.0286$, and for round trip car-sharing, $V = 6.0356H – 8.9644$, where H is hours and V is value.

A sensitivity analysis was conducted to determine the robustness of the model. The results showed for the one way sharing models that a 10% change in cost per hour resulted in an approximate change in value for the customer of 0.1. Since the change in value for the customers were relatively low, the model is fairly robust and gives a fairly accurate representation of the expected value for the customer despite slight errors in the cost per hour data. For round trip car-sharing, the approximate change in value for customer per 10% of change in cost per hour was 0.6. The change for round trip car-sharing was greater compared to that was the one way sharing models. However, this change was still comparatively low and indicates that it also represents the behavior well despite slight errors in the cost per hour data.



$V = -0.0826D + 3.2593$, where D is distance in miles.

The results for this sensitivity analysis of the fractional ownership model also showed the effect of a 10% change in cost per hour on the value for the customer. Since the resulting change in value is -0.008, the model would be robust to errors in cost per mile.

Model 4: Adjusting for Self-Driving and Environmentally Friendly Vehicles

Analysis of the Problem

Due to the release of greenhouse gases, climate change is one of the most significant global challenges faced today. As many know, the emissions produced by the burning of fossil fuels are the fastest rising cause of greenhouse gases, and they only expected to rise in the future [Ward-Jones, Richard. (2008). Environmentally Friendly Cars: Promoting and increasing their use in the UK. Earth & Environment, 3, 282-317. Retrieved from http://www.see.leeds.ac.uk/misc/ejournal/3,282-317.pdf]. Studies have shown that car-sharing can increase sustainability in transportation via promoting the usage of public transit and decreasing vehicle miles traveled (VMT), reducing emissions [13]. Through trading car ownership for car access, customers of car-sharing services have the potential to save on the overall cost of transportation as well as gain the opportunity to utilize a range of cars.

With the emergence of automobile technologies, however, car-sharing contains the potential to be taken to a new levels. Efficient integration of electric cars into car-sharing can eliminate emissions altogether. Recently, BlueIndy is investing approximately $41 million in what it plans to be the country's largest electric car-sharing service in Indianapolis [Tuttle, Brad. (2015, September 2). Groundbreaking Electric Car Sharing Program Launches in an Unlikely City. Time, Inc. Retrieved from http://time.com/money/4019200/electric-car-sharing-indianapolis/].

In addition to electric cars, companies, such as Lyft, have declared that they intend to operate with self-driving cars in the near future. Lyft, which has significant backing from General Motors, will possibly even outsource Uber, a company that already designates a large portion of resources to autonomous driving research. The leader in autonomous driving technology, Google, has additionally communicated its interest in partnerships with car manufacturing companies in order to push for self-driving cars [3].

Through the assimilation of automobile advancements, such as self-driving cars, electric cars, and cars which run on alternative or renewable fuel, into car-sharing services, various advantages are achieved i.e. increased convenience and decreased maintenance costs, which Model 4 investigates.

Design of the Model

Electric vehicles have half the maintenance cost of a traditional gasoline vehicle [U.S. Department of Energy. (2011, June 10). The eGallon: How Much Cheaper Is It to Drive on Electricity?. U.S. Department of Energy. Retrieved from http://energy.gov/articles/egallon-how-much-cheaper-it-drive-electricity#]. To account for the reduction in maintenance cost in this model, the cost per hour *CPH* of each car-sharing model was reduced to 50% of the original value. Additionally, because of autonomous driving technology being installed in many electric vehicles, the fee for a jockey in the one way sharing: floating model was removed due to the car's ability to drive itself back to the station. After re-evaluating the cost for electric vehicles, the cost for consumers and the revenue for the business was recalculated. These new values were used to re-rank the cities and car-sharing models based on the potential for revenue using the new, electric car paradigm (see Table 4.1 in the Appendix).

Conclusions

As with any mathematical model, the situations were idealized with the assumptions, which constricted the scope of the realistic quality of the model. For example, in Model 2, factors such as demographics, other forms of public transport, relative cost of living, and demand throughout the day would have impacted the amount of revenue for each method. This is because for demographics, it is known that younger, more educated people are more likely to be attracted to newer technologies, such as car-sharing. Additionally, if there are other forms of viable public transport that are commonly-used in the communities, then there is greater competition for the newly established business, which would have made it more difficult for market penetration.
For Models 3 and 4, the one way: floating model "jockeying" service was the most viable in Riverside, CA since they had higher incomes, who had more disposable income compared to the other cities. Electric vehicles (EVs) and autonomous vehicles, especially electric, autonomous vehicles were the most cost-effective method because they had the low maintenance cost and there was no need to have a physical human working to drive the car back to the original, thus the salary of the "jockey" did not need to be accounted for; this option was advantageous to both the business and the consumer because if the business costs went down, so did the consumer costs.

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Appendix

Table 1.1: Daily Travel Time per Person

Travel Time to Work (min)	**Number of People**	**Proportion Distribution**	**Margin of Error (represents 90% CI)**
< 10	18,565	0.134	0.1
10 – 14	19,328	0.139	0.1
15 – 19	20,775	0.150	0.1
20 – 24	19,559	0.141	0.1
25 – 29	8,040	0.058	0.1
30 – 34	17,874	0.129	0.1
35 – 44	8,321	0.060	0.1
45 – 59	9,834	0.071	0.1
60 – 89	7,160	0.052	0.1
> 90	3,218	0.023	0.1
Total	**132,674**	**0.957***	**0.1**
Mean Travel Time	**25.1 min**		**0.1 min**

*This value is not equal to one due to the Census' rounding of their data [8]. To simplify calculations and models, it was assumed the proportions totaled to one.

Graph 1.1: Trip Distances

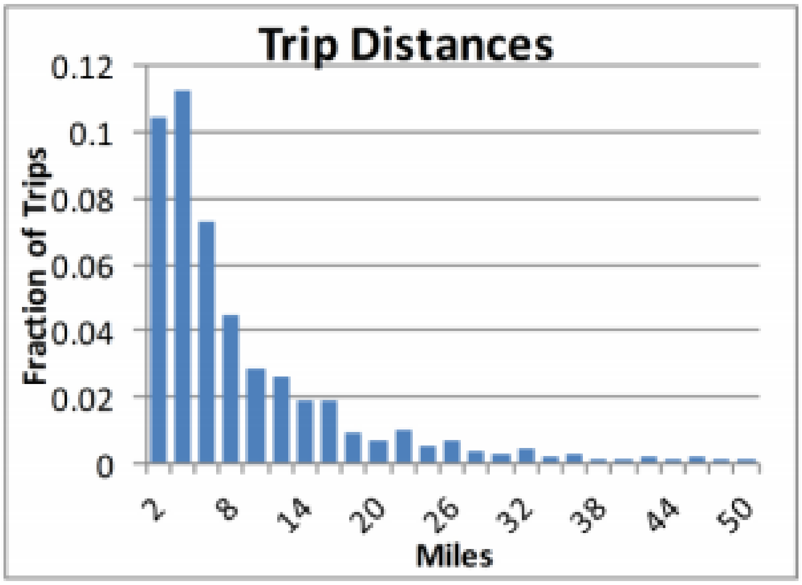


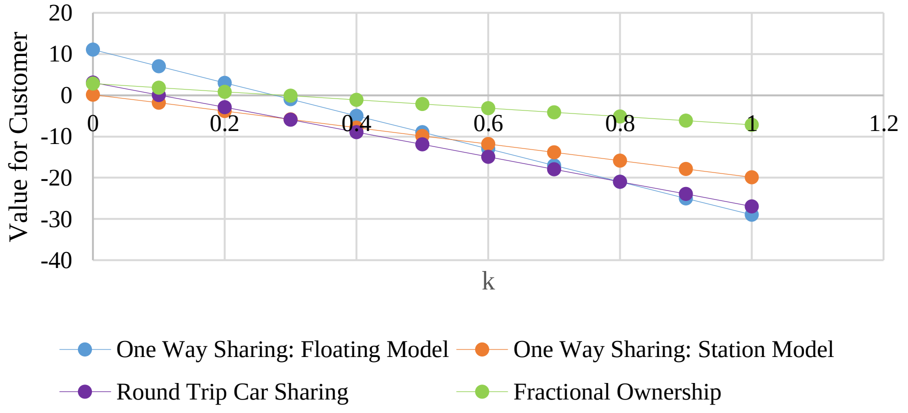
Table 2.1: Annual Household Income at 25th, 50th, and 75th percentile per City

City	**Percentile**	***Annual Household Income ($)**
Richmond, VA	25%	19,999.50
	50%	40,496
	75%	75000
Knoxville, TN	25%	18,333
	50%	33, 595
	75%	62,499.5
Riverside, CA	25%	29999.50
	50%	56089
	75%	87,499.50
Poughkeepsie, NY	25%	19,999.50
	50%	38,973
	75%	75,000

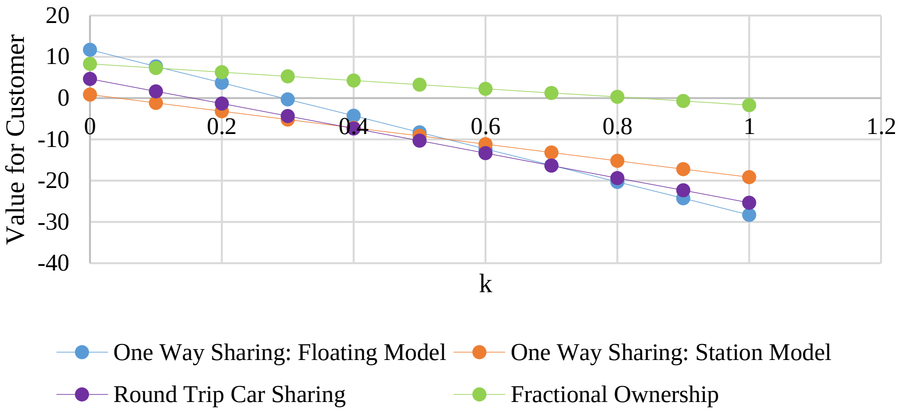
Table 2.2: Annual Household Income at 25th, 50th, and 75th percentile for the U.S.

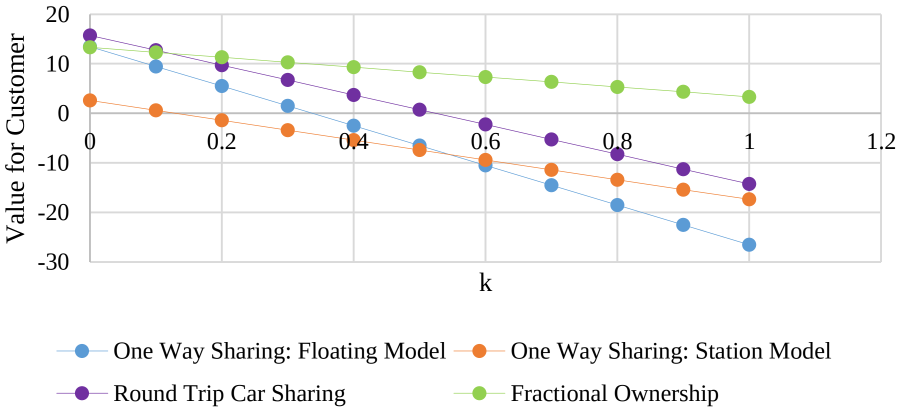
Country	**Percentile**	***Annual Household Income ($)**
U.S.	25%	26.999.80
	50%	53,657
	75%	99999

*Unfortunately, because the U.S. Census Bureau information displayed the income distributions within income brackets, actual values for the 25th and 75th quartiles were unable to be accessed for both the cities and the nation. These values were approximated to the nearest half-bracket.

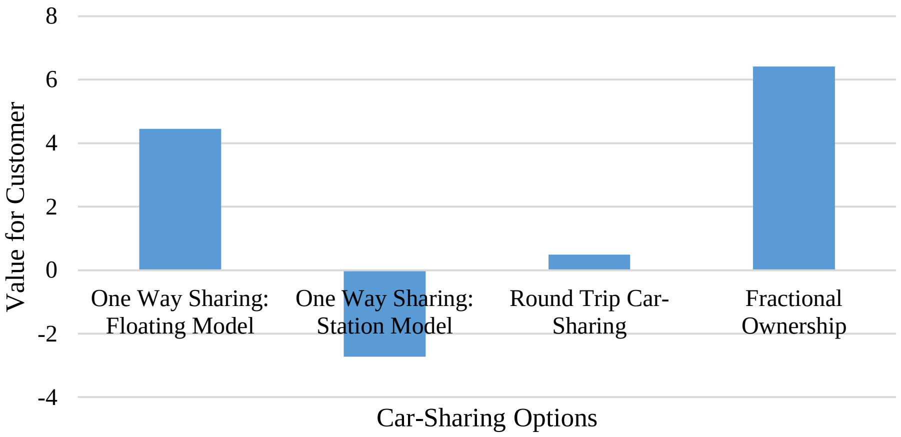


Graph 2.1: Effectiveness of Affluence Constant (k) Car-Sharing Options for Low Amount of Time (0 - 10 minutes)

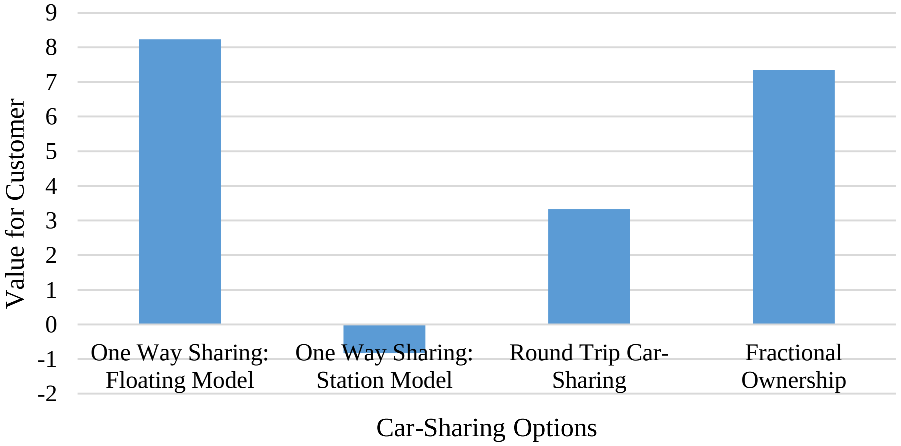

Graph 2.2: Effectiveness of Affluence Constant (k) Car-Sharing Options for Low Amount of Time (10 - 45 minutes)

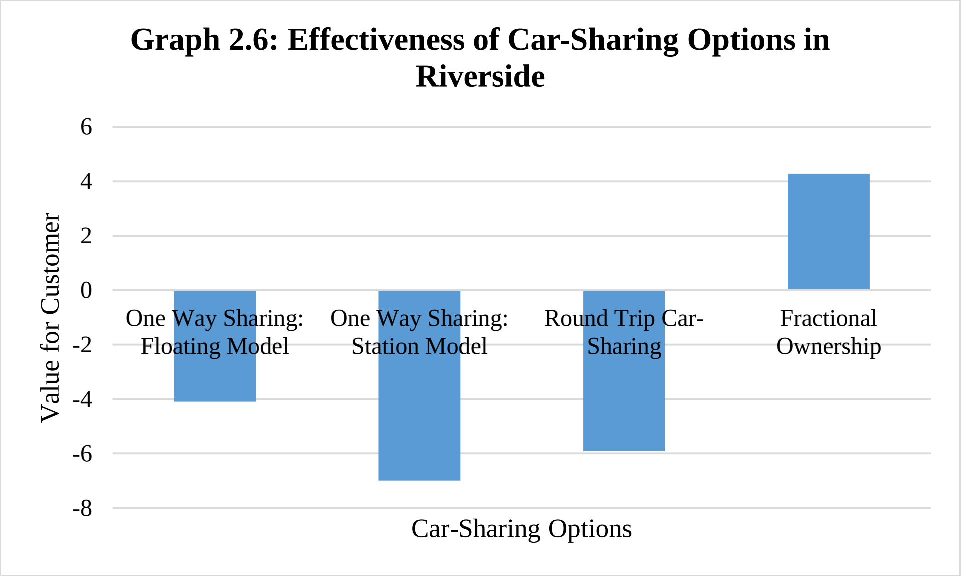

Graph 2.3: Effectiveness of Affluence Constant (k) Car-Sharing Options for Low Amount of Time (45+ minutes)

Graph 2.4: Effectiveness of Car-Sharing Options in Richmond



Graph 2.5: Effectiveness of Car-Sharing Options in Knoxville





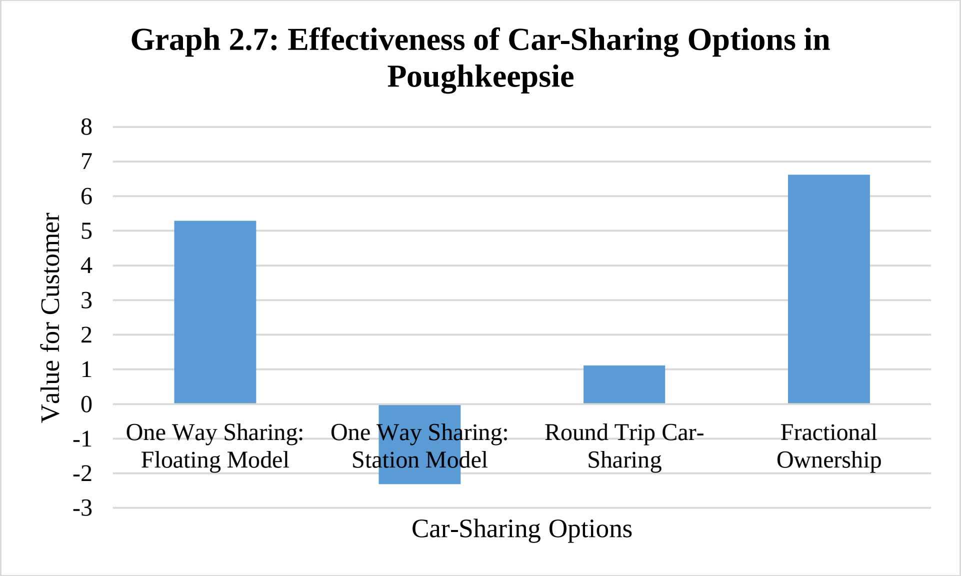


Table 3.1: Max Potential Revenue per City

City	**Car-Sharing Option**	**Max Potential Revenue ($/hr)**
Richmond, VA	Round Trip Car-Sharing	49057.7377
	One Way Sharing: Floating Model	98719.5086
	One Way Sharing: Station Model	63730.23499
Knoxville, TN	Round Trip Car-Sharing	0
	One Way Sharing: Floating Model	0
	One Way Sharing: Station Model	54151.49236
Riverside, CA	Round Trip Car-Sharing	71948.25606
	One Way Sharing: Floating Model	324045.7881
	One Way Sharing: Station Model	72208.77225

Poughkeepsie, NY	Round Trip Car-Sharing	6871.141323
	One Way Sharing: Floating Model	13826.88495
	One Way Sharing: Station Model	8926.205562

Table 4.1: Adjusted Max Potential Revenue per City for Electric, Self-Driving Cars

City	Car-Sharing Option	Max Potential Revenue ($/hr)
Richmond, VA	One Way Sharing: Floating Model	34786.68618
	One Way Sharing: Station Model	5073.49045
Knoxville, TN	One Way Sharing: Floating Model	21762.67138
	One Way Sharing: Station Model	5312.869438
Riverside, CA	One Way Sharing: Floating Model	61115.42177
	One Way Sharing: Station Model	0
Poughkeepsie, NY	One Way Sharing: Floating Model	4872.304514
	One Way Sharing: Station Model	710.6049221