

# Moody's Mega Math Challenge* 

 A contest for high school studentsSIAM
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## PREVIEW PAPER: EXCELLENT

This team provided a very good executive summary. The formatting is difficult to read, and it includes a missing citation. They do provide a good overview of the problem, discuss how they approached the problem, and also provide specific results. Overall their summary was one of the better ones seen during the pre-triage session.

With respect to the first question, the team describes the data. Their graphs are appropriate and correctly annotated. They do not provide a good description of the graphs in the narrative, though. The team clearly denoted their cut off points for different time and distance levels, but they did not give a strong motivation as to how they picked those cut offs. The team clearly stated their final result in the form of an easily readable table.

The team did a nice job of describing their assumptions and their approach. They clearly stated the model and described how it was developed. Their final result was displayed in a table, but they provided a large number of significant digits. It was not immediately clear how to read and use the table.

The team gave a brief outline and a short discussion on how they approached the third question. The discussion is short, and again it is not clear how to interpret their final table. Unfortunately, the team was not able to provide a strong conclusion nor an analysis of their model within the allotted time.

# Moody's Mega Math Challenge 2016 

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## 1 Executive Summary

Car sharing is one of the fastest growing areas of the transportation and automotive industry; it involves billions of dollars of invested capital, millions of participating users, and hundreds of thousands of cars [?]. As these numbers are sure to increase in the future, it is vital to identify new markets and new strategies for providing car-sharing systems. In order to do so, the driving habits of Americans must be studied, and the merits of the various car-sharing systems must be compared. We were asked to find the percentages of current American drivers in nine separate categories: all possible combinations of low, medium, and high daily driving distance; and low, medium, and high daily driving time. To find driving distance, we used exponential regression to find the function of daily driving distance per capita as a function of population density, and then used that function to translate census population density data by county into daily driving distance per capita by county. To find driving time, we used census data of daily travel time by county. Overall, the highest percentages were $32.11 \%$ in high miles per day and medium time per day, and $28.94 \%$ in medium miles per day and medium time per day. Next, we were asked to assess which of four car-sharing methods (Round Trip, One-Way Station Model, One Way Floating Model, and Fractional Ownership) would garner the most participation from the public in four cities: Poughkeepsie, NY; Richmond, VA; Riverside, CA; and Knoxville, TN. We assessed the degree of participation using two factors: the cost of the car-sharing program, and the convenience of the program (based on the distance of a shareable car from the average resident). We defined a cost coefficient for each method in each city by taking into account total cost per person per year in relation to the median household income for that city [10]; the lower the cost coefficient, the more desirable the program was, in the model, to the residents. We also defined a distance coefficient in terms of the average distance that a resident would have to travel to use one of the methods; once again, the lower the distance coefficient, the more desirable the program. To find an overall desirability coefficient, we weighted the cost coefficient twice as much as the distance coefficient. For the three larger cities, Knoxville, Riverside, and Richmond, Fractional Ownership was the most desirable program. In the smallest city, Poughkeepsie, the most desirable program was the One-Way Station Model. However, to rank the cities in terms of economic viability for a prospective car-sharing company, the Fractional Ownership program is not a business model; so the ranking overall was, in order from best to least, Poughkeepsie (One-Way Station), Riverside, Richmond, and Knoxville (all three One-Way Floating). Finally, we were asked to update our model from the second part to account for advances in technology on the future of car-sharing. We accounted for self-driving cars and renewable energy in the form of electric-powered cars in our updated model, which used the same coefficients as the original. The updated ranking of cities was Riverside (One-Way Floating), Poughkeepsie, Richmond, and Knoxville (all three One-Way Station). Therefore, going forward, our recommendation is that a prospective car-sharing company proceed in implementing a One-Way Floating Model in Riverside, California.

## 2 Introduction

### 2.1 Background

Ever since Henry Ford released his Model T to the public in 1910 , people have desired to make transportation more convenient and efficient both economically and environmentally. Yet since the Model T, the automobiles have become a constant in society. Recently, due to an increase in concern over the environmental impact of cars and the growing demand for cheaper transportation, a new industry, car sharing, has emerged. Companies such as Zipcar have allowed people to pay for a car by the hour at an affordable rate and pick them up at convenient locations. Car sharing makes it easier for people to travel without actually owning a car, helping customers to save money. In addition, it reduces the number of cars on the road, therefore reducing traffic, emissions, and dependence on oil.
There are four major types of car sharing. In Round Trip Car Sharing, the customers reserve cars in advance for a specific time window. When their reservation begins, they go and pick the car up at a designated location and can take it wherever they want. At the end of the reservation, the car must be returned back to the same location from which it was originally taken. The customers have to pay fixed rates for the time they spent with the car, generally per hour. Gas prices and insurance are included in the rate paid, and if the car is returned late there will be a late fee charged. The second method of car sharing, the One Way Car Sharing Floating Model, provides far more flexibility. Generally, the customer is only required to reserve a car 30 minutes in advance and can do so using a mobile app. With this type of car sharing, buyers do not have to return the car to the same location they picked it up from, and the end time to the reservation can be extended as many times as the users wish. For this reason, it is a one way trip with guaranteed parking at the end, where the trip can finish in one of many designated locations providing for much more mobility. Gas, insurance, and parking are all included into the rate, usually calculated either by the hour or by the half-hour. The third option, the One Way Car Sharing Station Model, is similar to the floating model except slightly more restricting. Instead of being able to end the trip in one of many locations like in the floating model, in the station model one must return the car to one of the car sharing agency's stations, usually a garage or parking lot where many cars are held. In addition, there is a much more strict end to the time of the reservation than in the floating model. The rate includes gas, insurance, and parking. The final car sharing method, Fractional Ownership, is the more traditional method of sharing a car between multiple people. In fractional ownership, multiple people purchase and physically share a car amongst them. In this type of car sharing, the multiple owners must divide up and pay for insurance, gas expenses, maintenance, and parking as any normal car owner would. This option also has the built in drawback that only one person can be using the car at once, restricting its potential use. In our analysis, we assess which of these methods of car sharing is most cost effective and would generate the most participation.

### 2.2 Restatement of the Problem

In this analysis, we were first requested to build a mathematical model that determined the percentage of current U.S. drivers in the categories of low, medium, and high distances traveled daily; and low, medium, and high times spent driving per day. In all, there would be nine separate categories, dividing the number of drivers in the United States into categories valuable to car-sharing companies. Then, we were requested to develop a model to analyze four different ride-sharing systems and determine which one would garner the most participation in the following cities: Poughkeepsie, NY; Richmond, VA; Riverside, CA; and Knoxville, TN. Next, we had to rank the cities based on the viability of the ride-sharing programs for a company looking to develop in a new market. Finally, we had to update our model to reflect advances in technology, including self-driving cars and renewable energy, and re-rank the four cities.

### 2.3 Global Assumptions and Justifications

1. Assumption: Age, race, income, and gender do not have to be considered when working with the average miles traveled in relation to population density.
Justification: The neglect of these demographics is inconsequential because we are working with averages in a national scope. As a result, individual demographics are worked into the average and thus considered proportionally in the national scope. For example, some areas may have a higher population density of white people and other areas may have a higher population density of Hispanics and as a result, each area would balance the other and remain consistent on a national average.
2. Assumption: The time traveled by a car sharing vehicle is twice the length of the mean commuting time.
Justification: The mean commuting time is from US Census Data from 2010[15] and is specific for all four cities (Poughkeepsie, NY; Richmond, VA; Riverside, CA; Knoxville, TN). Data specificity accounts for traffic patterns in the cities. Although some driving patterns and commutes may take longer than the mean, other habits will take less time.

## 3 Who's Driving?

### 3.1 Assumptions

1. Assumption: We used data from 2000 from the Federal Highway Administration [12] to find the number of daily vehicle miles per capita in the United States, assuming that the values were representative of the current population of drivers in the United States [3].
2. Assumption: We assumed that the ratio of drivers to the total population of the United States is uniform for all states, allowing all percentages of total population to be extrapolated to also be percentages of current drivers.
3. Assumption: We used data from the 2000 US Census and its 5 -year estimates from the American Community Survey [15] to find the average travel time by county, assuming that the values were representative of current travel times.
4. Assumption: We assumed that the relation between daily driving miles per capita versus population density can be extrapolated from data from urban areas with total populations over 750,000 to the United States as a whole [12].
5. Assumption: We assumed that the data we used accounted for the correspondence between driving time and population density, as it all came from the same set of census and American Community Survey data by county; therefore, we assumed in our calculations that driving time and driving distance were independent of each other, as the connection had already been accounted for.

### 3.2 Approach

First, we found the equation of the function of daily driving miles per capita with respect to population density by performing exponential regression on data of daily driving miles per capita and population density from 47 urbanized areas with total populations over 750,000 [12].


Figure 1: Daily Vehicle Miles per Capita
[12]

Daily Driving Miles per Capita $=31.185339 * e^{0.000100 * P o p u l a t i o n ~ D e n s i t y ~} ; R^{2}=.330982$

Based on this model, we were able to calculate the daily driving miles per capita of all counties in the United States by inputting their population density, from the 2000 U.S. Census [15], into the equation from the model. A sample calculation, for Bergen County, New Jersey, is shown:

Daily Driving Miles per Capita $=31.185339 * e^{.000100 *(3884.5 p e r s o n s / s q u a r e m i l e)}=21.147 \mathrm{miles}$
The weighted average for daily driving miles per capita, calculated by multiplying the daily driving miles per capita of each county by the fraction of that countys population over the total population of the country and then finding the sum of those values, was 28.03 miles, which is only a $4.01 \%$ error from the average daily driving miles per capita of 29.2 miles reported by the American Driving Survey [13].

Taking the values for daily driving miles per capita under 20 miles as outliers (as they accounted for less than $5 \%$ of the total population) and grouping them into the low category, we divided the remaining data into three categories, which we assigned as low, medium, and high: 20-25 miles per day; 25-30 miles per day; and more than 30 miles per day.

| Miles per day | Population (people) | Percentage |
| :--- | ---: | ---: |
| $20-25$ miles | $36,560,942$ | 12.7648534 |
| $25-30$ miles | $118,430,560$ | 41.3487358 |
| $30+$ miles | $131,427,315$ | 45.8864108 |

Figure 2: Breakdown of Population into Low, Medium, and High Categories of Driving Distance per Day

Although the percentages were calculated based on total population, they are also representative of the total number of drivers in the United States given our assumption that the calculated ratio of drivers to total population for the United States, 0.67742 [12], remains constant for all areas and all variations in population density across the country.

Then, we took data from the American Community Survey on daily driving time by country [15] and once again found a weighted average using census data of population by county. A sample calculation of a value, which were then all summed to find the weighted average of 25.322 minutes per day. Disregarding the values under 10 minutes per day as insignificant outliers, and grouping them with the low category, we divided the remaining range of times into three categories, for low, medium, and high: 10-20 minutes per day; 20-30 minutes per day; and more than 30 minutes per day.

Once again, while the percentages were calculated based on total population, they are also representative of the total number of drivers in the United States, as the population was only used to weight the values of each county, so the ratios of drivers would have been

| Time per day | Population (people) | Percentage |
| :--- | ---: | ---: |
| $10-20$ minutes | 38779302 | 13.53910537 |
| $20-30$ minutes | 200453348 | 69.98473053 |
| $30+$ minutes | 47191755 | 16.4761641 |

Figure 3: Breakdown of Population into Low, Medium, and High Categories of Driving Time per Day
[12] [15]
equivalent to the percentages of the total population.

### 3.3 Final Data Analysis for "Who's Driving"

Using these two sets of percentages, we compiled a chart of the nine categories of all combinations of daily driving time and driving distance.

| Time |  |  | Miles |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | Low | Medium | High |
|  |  |  | 12.77\% | 41.35\% | 45.89\% |
|  | Low | 13.54\% | 1.73\% | 5.60\% | 6.21\% |
|  | Medium | 69.99\% | 8.93\% | 28.94\% | 32.11\% |
|  | High | 16.48\% | 2.10\% | 6.81\% | 7.56\% |

Figure 4: Breakdown of Population into Low, Medium, and High Categories of Daily Driving Distance and Driving Time

$$
[12][15]
$$

## 4 Zippity do or don't?

### 4.1 Assumptions

1. Assumption: All car sharing vehicles are Honda Civics.

Justification: The Honda Civic is widely used in current car sharing services like Zipcar[16]. It also has a miles per gallon (MPG) rating of $31,21.6 \%$ greater than the national average[8]. This progressive MPG makes sense for a company looking to make investment into more cost effective cars.
2. Assumption: $\$ 4000$ per spot covers the cost of constructing a new parking lot Justification: The average parking lot cost is $\$ 4000$ per space[4]. Thus, the cost of a
parking lot can be found by multiplying $\$ 4000$ by the number of parking spaces.
3. Assumption: The amount of parking spaces required to facilitate one way floating car sharing is 1.5 x the amount of car sharing vehicles on the road.
Justification: If each car were to be parked on the road at the same time, the number of parking spaces required would be 1 x the amount of car sharing vehicles on the road. However, because there will be a number of vehicles on the road at any given moment, open spaces are required in order for the cars on the road to be able to park.
4. Assumption: The increase in the price of insurance for each car added by the Progressive Insurance Quote calculator given by [11] is proportional to the increase in the state average for insurance price given by [6].
Justification: The
5. Assumption: The cost of the car-sharing methods is weighted twice as much as the distance from the station location.
6. Assumption: The number of Zipcars present in major cities follows a direct proportion to the population of that city and is translatable to other cities.

### 4.2 Ranking Participation in Different Car Sharing Methods

We determined that the probability of a person to participate in any car sharing service or method would be directly related to the financial cost and the convenience. As the cost decreases, the likelihood of a person to participate should increase and as the convenience increases, the likelihood of a person to participate should increase. (Convenience defined as the greatest distance necessary to travel in order to take advantage of a car sharing service). We broke the problem into two separate models, cost to consumer and convenience, that would later come together into a final model.

### 4.2.1 Cost to consumer

Cost to the consumer is driven by the cost to the car sharing company in question. The company has initial costs to setup the car sharing infrastructure in a city, and it has recurring costs every year.

$$
\begin{equation*}
\text { initial cost }=\text { cars } * \text { price per car }+ \text { infrastructure cost } \tag{3}
\end{equation*}
$$

We assumed that the population of a city is directly proportional to the number of inhabitants. We plotted the populations of cities against the number of zipcars from Atlanta, Baltimore, Chicago, Denver, Detroit, Miami, New York City, Philidelphia, Pittsburg, Portland, Providence, Sacramento, Bay Area, Seattle, and Washington DC.


Figure 5: Population vs. Zipcars in Major Cities

We then used excel to regress a best fit line for the data.

$$
\begin{equation*}
\text { cars needed }=0.000287 * \text { population } \tag{4}
\end{equation*}
$$

The infrastructure for the Round Trip and One Way (Non-Floating) car sharing methods is the cost to build the parking spaces in a parking lot. The cost of a parking lot is $\$ 4,000$ per space[?].

$$
\text { infrastructure cost }=4000 * \text { cars }
$$

The One Way (Floating) car sharing method does not require parking lots to be built as it works off of preexisting parking spaces. This cost is discussed later in this section.
This model applies to the 3 car sharing methods that can be implemented by a company. They are Round Trip, One Way (Non-Floating), and One Way (Floating). The initial cost for Fractional Sharing follows a different initial.

$$
\begin{equation*}
\text { initial cost }=\frac{\text { price of car }}{\text { number of people sharing }} \tag{5}
\end{equation*}
$$

There are also recurring cost that must be paid every year. For Round Trip, One Way (Non-Floating), and Fractional Sharing car sharing methods, companies or individuals need to pay for gasoline and car insurance.

$$
\begin{equation*}
\text { recurring cost }=\text { cars } * \text { [insurance cost per car }+ \text { gasoline per car per year }] \tag{6}
\end{equation*}
$$

The number of cars comes from equation (2) and the cost for insurance was acquired from a progressive car insurance calculator[11]. The gasoline cost calculated using the following

|  | Round Trip | One Way (Non-Floating) | One Way (Floating) |
| :---: | :---: | :---: | :---: |
| Poughkeepsie | $\$ 163,908.84$ | $\$ 163,908.84$ | $\$ 163,908.84$ |
| Richmond | $\$ 1,145,441.38$ | $\$ 1,145,441.38$ | $\$ 1,145,441.38$ |
| Riverside | $\$ 1,693,810.332$ | $\$ 1,693,810.332$ | $\$ 1,693,810.332$ |
| Knoxville | $\$ 98,0435.85$ | $\$ 98,0435.85$ | $\$ 98,0435.85$ |

Table 1: Calculated Initial Costs for Companies

|  | 1 | 2 | 3 | 4 | 5 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Poughkeepsie | $\$ 18,640.00$ | $\$ 9320$ | $\$ 6213.33$ | $\$ 4660.00$ | $\$ 3728$ |
| Richmond | $\$ 18,640.00$ | $\$ 9320$ | $\$ 6213.33$ | $\$ 4660.00$ | $\$ 3728$ |
| Riverside | $\$ 18,640.00$ | $\$ 9320$ | $\$ 6213.33$ | $\$ 4660.00$ | $\$ 3728$ |
| Knoxville | $\$ 18,640.00$ | $\$ 9320$ | $\$ 6213.33$ | $\$ 4660.00$ | $\$ 3728$ |

Table 2: Calculated Initial Cost for Fractional Owners per Person
equation.

$$
\begin{equation*}
\frac{\text { Cost of Gasoline }}{\text { year }}=\frac{\text { Miles Driven }}{1 \text { day }} \cdot \frac{365 \text { days }}{1 \text { year }} \cdot \frac{\text { Honda Civic Gallons }}{1 \text { mile }} \cdot \frac{\text { Price in } \$}{1 \text { Gallon }} \tag{7}
\end{equation*}
$$

Average miles driven per day was taken from census data[15], and gas prices were found specifically for the different cities[?]. Reacquiring costs differed slightly for the One Way (Floating) car sharing solution, as instead of the one time payment for building parking lots, there are reoccurring payments to the municipalities for parking spots. So the model changes.

$$
\begin{aligned}
\text { recurring cost }=\text { cars } *[\text { insurance cost per car } & + \text { gasoline per car per year }] \\
& + \text { cost per parking spot } * \text { parking spots }
\end{aligned}
$$

The cost per parking spot was taken from the current price that zipcar pays to the city of Boston for their parking spots. We estimated the number of spots needed to be one and a half times the number of cars, so that a consumer can rent a car, and a have at a number of guaranteed open spaces where they can park it.

|  | Round Trip | One Way (Non-Floating) | One Way (Floating) |
| :---: | :---: | :---: | :---: |
| Poughkeepsie | $\$ 8,863.35$ | $\$ 8,863.35$ | $\$ 37,749.65$ |
| Richmond | $\$ 53,123.81$ | $\$ 53,123.81$ | $\$ 254,989.42$ |
| Riverside | $\$ 125,773.59$ | $\$ 125,773.59$ | $\$ 424,280.40$ |
| Knoxville | $\$ 50,294.16$ | $\$ 50,294.16$ | $\$ 223,080.20$ |

Table 3: Calculated Recurring Costs for Companies
The following equation relates time and cost per hour for when a company will break even on its investment.
initial cost + recurring cost $*$ year $=$ hours driven per year $*$ cost per hour $*$ years

|  | 1 | 2 | 3 | 4 | 5 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Poughkeepsie | $\$ 503.15$ | $\$ 251.58$ | $\$ 167.72$ | $\$ 125.79$ | $\$ 100.63$ |
| Richmond | $\$ 503.15$ | $\$ 251.58$ | $\$ 167.72$ | $\$ 125.79$ | $\$ 100.63$ |
| Riverside | $\$ 503.15$ | $\$ 251.58$ | $\$ 167.72$ | $\$ 125.79$ | $\$ 100.63$ |
| Knoxville | $\$ 503.15$ | $\$ 251.58$ | $\$ 167.72$ | $\$ 125.79$ | $\$ 100.63$ |

Table 4: Calculated Recurring Cost for Fractional Owners per Person

Hours driven per year was calculated by taking the average time spent driving per day, global assumption $\# 2$, and multiplying it by 365 days per year. The equation can then be rearranged to find the cost per hour necessary in order for a company to recieve a full return on their investment.

$$
\text { cost per hour }=\frac{\text { initial cost }+ \text { recurring cost } * \text { year }}{\text { hours driven per year } * \text { years }}
$$

We then translated the cost per hour into the "cost coefficient".

|  | Round Trip | One Way (Non-Floating) | One Way (Floating) |
| :---: | :---: | :---: | :---: |
| Poughkeepsie | $\$ 7.38$ | $\$ 7.38$ | $\$ 15.81$ |
| Richmond | $\$ 9.79$ | $\$ 9.79$ | $\$ 21.58$ |
| Riverside | $\$ 8.58$ | $\$ 8.58$ | $\$ 17.27$ |
| Knoxville | $\$ 10.78$ | $\$ 10.78$ | $\$ 23.34$ |

Table 5: Calculated Cost per Hour for Full Return on Investment in 10 Years

$$
" \text { cost coefficient" }=\frac{\text { costperhour }}{\text { medianincome }}
$$

This is essentially the inverse of the desirability to use the service. This is because as the hourly cost increases, the "cost coefficient" increases as well, meaning the service is less desirable.

### 4.2.2 Convenience

We found the maximum distance needed to travel to a car pickup location to be the following.

$$
\begin{equation*}
\text { Maximum Distance }=\frac{1}{2} \cdot \sqrt{\frac{2 * \text { square miles }}{\text { total pickup locations }}} \tag{8}
\end{equation*}
$$

The lower the maximum distance needed to travel, the higher the desirability is for use of the service. This is called the "Distance Coefficient".

### 4.2.3 Combining Cost and Distance Coefficients

The two properties that influence the desirability of using a service are the Cost Coefficients and Distance Coefficients. We then combined the two coefficients by multiplying them
together. However we raised the Cost Coefficient to the 2nd power in order to weight it more heavily, as it would influence the consumers decision greater.

$$
\begin{equation*}
\text { Desirability }=(\text { Cost Coefficient })^{2} *(\text { Distance Coefficient }) \tag{9}
\end{equation*}
$$

The final Desirability coefficients are below for each car sharing method and each city. The lower the value, the higher the Desirability.

|  | State | Round Trip | One Way (Non-Floating) | One Way (Floating) | Fractional Ownership |
| :--- | :--- | ---: | ---: | ---: | ---: |
| City |  |  |  |  |  |
| Poughkeepsie | New York | 0.008462770048 | 0.005984082088 | 0.01070940034 | 0.0009191885434 |
| Richmond | Virginia | 0.02622916755 | 0.01854682224 | 0.01327037069 | 0.002245346347 |
| Riverside | California | 0.02072371343 | 0.0146538783 | 0.007181100302 | 0.00005552150282 |
| Knoxville | Tennessee | 0.0493689054 | 0.03490908779 | 0.02604730591 | 0.001182065173 |

Figure 6: Final Desirability Coefficients

### 4.2.4 Analysis

The Fractional Ownership method was the most desirable in all cases. Despite the large cost associated with buying a vehicle, the close proximity to the car and distance coefficient outweighed the very large cost coefficient. However a company would be unable to profit from a Fractional Ownership car sharing method, so we also ranked the commercial options.

1. The top rank is One Way Non-Floating in Poughkeepsie

## 5 Road Map to the Future

### 5.1 Assumptions

1. Assumption: The data which predicts the future price of the Chevy Volt upon release [7].
Justification: The Chevy Volt will be one of the more affordable models of self-driving cars. Thus, it is most similar in price to the currently available models in Zipcar and will be a more plausible choice when self-driving cars reach the car-sharing industry.
2. Assumption: The translation from gallons to eGallons and miles per gallon and miles per eGallon [2] were accurate predictions of the future.
Justification: The price of an eGallon is a measure of how much it costs to drive the same distance in an electric vehicle as in a regular car. The price of the eGallon was calculated by figuring out how much it would cost to drive an electric vehicle the same
distance as a regular car. Therefore, because distance is constant, this conversion is accurate.
3. Assumption: The predictions for each individual state [2] could be accurately applied to the specific cities in focus.
Justification: The predictions for each state are averages of the gas prices all over each state. Thus, these predictions can be used for the specific cities in the corresponding state because the average cost will take into account the flexibility of gas prices at the different local gas stations in the vicinity of each city.

### 5.2 Factors in Calculating Desirability

The factors in determining the desirability of each different car sharing method was based upon the coefficient of cost to the consumer, $\frac{\text { cost }}{\text { median income }}$ and coefficient of distance to the location as calculated in the section Ranking Participation in Different Car Sharing Methods. The equation we designed weighted the coefficient of the financial cost to the consumer as twice that of the coefficient of the distance from the station as explained in the section Ranking Participation in Different Car Sharing Methods.

$$
\begin{equation*}
\text { desirability }=(\text { cost coefficient })^{2} *(\text { distance coefficient }) \tag{10}
\end{equation*}
$$

The lower the coefficient of desirability, the more desirable the method is in that location.

### 5.3 Final Desirability Coefficients

The table below displays the final factors of the coefficients for each city.

| City | State | Round Trip | One Way (Non-Floating) | One Way (Floating) |
| :--- | :--- | ---: | ---: | ---: |
| Poughkeepsie | New York | 0.02658861143 | 0.01880098744 | 0.01981825309 |
| Richmond | Virginia | 0.08326943499 | 0.05888038215 | 0.0243542826 |
| Riverside | California | 0.05533479801 | 0.03912761091 | 0.01242217581 |
| Knoxville | Tennessee | 0.1526848377 | 0.1079644841 | 0.04750148131 |

Figure 7: Final Factor Coefficients per City

In Figure 2 above, the cells highlighted in yellow are the most desirable location for companies to establish the highlighted method. According to the figure, a prospective carsharing company should move forward in investigating legalities and implementations in the cities and their corresponding methods ranked as follows:

1. Riverside, Virginia using the One Way (Floating) method.
2. Poughkeepsie, New York using the One Way (Non-Floating) method.
3. Richmond, Virginia using the One Way (Non-Floating) method.
4. Knoxville, Tennessee using the One Way (Non-Floating) method.

### 5.4 Analysis

## 6 Conclusion

## References

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