PREVIEW PAPER: EXCELLENT

This team produced a well balanced paper and has a strong response to all three questions. The summary is excellent, and for each question the team provided a good analysis of their models including verification of the model as well as a description of the strengths and weaknesses. The writing is quite good, and the paper is well organized. For each question the team did a good job of describing and motivating each model. One criticism of the model in question one, though, is that they add the losses over different parts of the production and consumption process when those losses should be compounded. Despite that, the team did a great job of using and describing a wide variety of different stages where losses could be identified.

Better ATE Than Never:
Reducing Waste Food

Team Number: 11071

March 2018

1 Executive Summary

Every year, about 40% [9] of the food produced in the United States is wasted, while more than 42 million people in the country suffer from food insecurity. Not only is a great amount of food wasted, but the production of this wasted food also results in the indiscriminate exploitation of resources such as water and fertilizers. The primary reason behind said wastage of food is people's unwillingness to eat seemingly "unattractive" food. While this kind of food is deemed unwanted, in reality, it is perfectly edible.

It is a goal of conservationists to implement the waste to feed the food-insecure population in the United States. To begin with, it was assumed that not all wasted food is repurposed, hence, only the wasted food at the processing, distribution, and consumption levels would have any effect on the model. By applying this model to the population of Texas, it was concluded that Texas can indeed reasonably feed its food-insecure population using wasted food.

The following analysis was to determine how much food is wasted by certain households with different specifications through mathematical modeling. Since food intake is affected by both age and income, the model focuses on finding a relationship between food consumption, household income, and age of the individuals. To carry out this task, two separate functions were formulated, each aimed to relate the age and income respectively to the food expenditure, overall evaluating the quantity of food wasted by each household. The results indicated that both functions of age and income on the total amount of food expenditure could be modeled as quadratic relations. Using multivariable analysis, it was found that age has the highest incremental impact on total food expenditure, but overall the number of household members was the most influential factor in the total amount of food wasted.

The last proposal created strategies to repurpose the maximum amount of waste in the most cost-efficient way, by applying it to our own community: Fairfax County, Virginia. We chose to address the problems in our community and developed two models to find the cost of maintaining different food repurposing strategies. These two models included a food drop-off and a food-pick up system, one of which would be implemented at numerous facilities located in several locations across the county while the latter of the two involves a singular facility with pickup trucks rather than locations. It was determined that the pick-up system was the most effective for the Fairfax County community.

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2 Part I: Just Eat It!

2.1 Restating the Problem

The question asks one to:

- Design a mathematical model that determines the ability, or lack thereof, of a state to support its food-insecure population
- Define the relation between
  - The amount of food that is wasted
  - The food-insecure population of a state
- Validate the model by applying it to the state of Texas.

2.2 Assumptions and Justification

- Assumption A: The demographics of the food-insecure population are negligible.
  - Justification: The average food insecure population, though having a varied food intake based on demographic, can be represented by a single number when they are averaged, as higher and lower extremes will cancel out.
- Assumption B: Wasted food lost to agricultural production or post-harvest handling and storage cannot be repurposed for the food-insecure population.
  - Justification: Food waste at the production and handling levels is damaged to the point it can no longer be salvaged for consumption by the public, thus only the waste at the processing, distribution, and consumption levels will be considered.
- Assumption C: The percentage of food produced is evenly distributed throughout the year, not accounting for harvest seasons.
  - Justification: This assumption is plausible because due to surplus food supply, food produced and consumed remains more or less even across twelve months, regardless of harvest season.
- Assumption D: Each state's specialized food output is negligible, and each food group is evenly distributed across the United States.
  - Justification: Specialized food output refers to the dominant food produced by a certain state. Though states specializing in a certain food or food group will have a more abundant supply of that item as opposed to the other states, it can be assumed that the supply will be allocated to each state such that all the states will have an equal quantity of said food groups.
- Assumption E: The average person eats three meals a day.
  - Justification: Logistically people will eat an average of three meals a day, those being breakfast, lunch, and dinner with the addition of beverages.

2.3 Constructing the Model

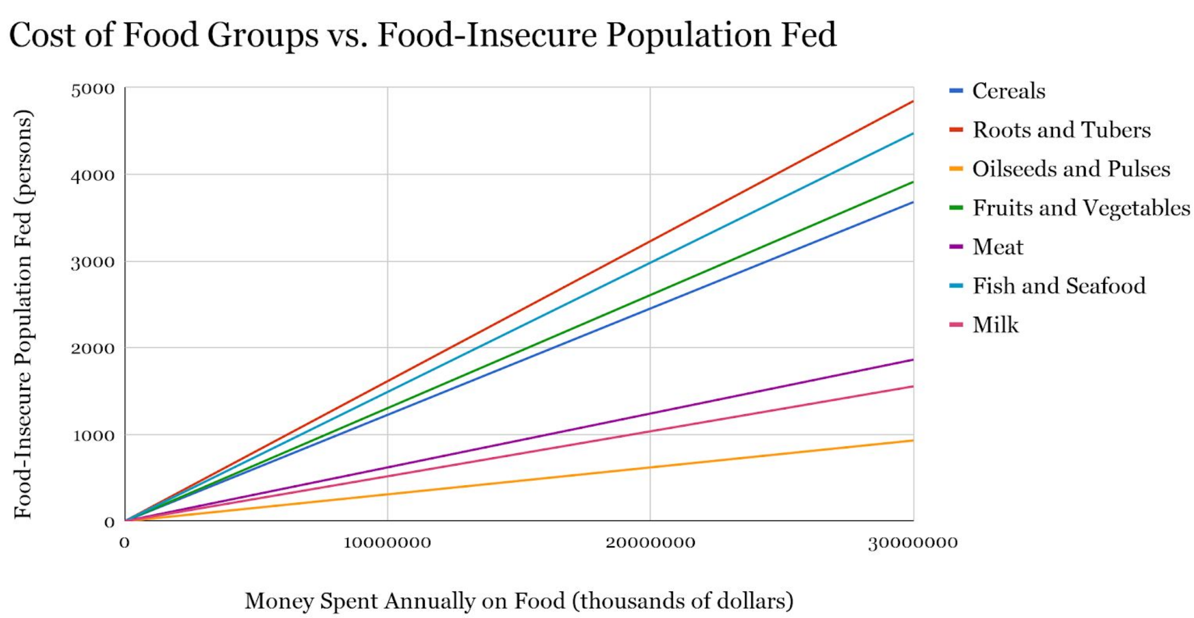
In order for a state to be able to feed its food-insecure population, its wasted food has to be greater than or equal to the amount of food the food-insecure population would eat. However, not all wasted food can be repurposed; food lost to agricultural production or post-harvest handling may have contaminations. This means that the majority of food is wasted by processing, distribution, and consumption. Each of these constitutes its own percentage of all food produced in the year: *p, d,* and *c,* respectively. With *F* being the total cost of food produced in dollars for that particular state, this can be modeled by the following inequality:

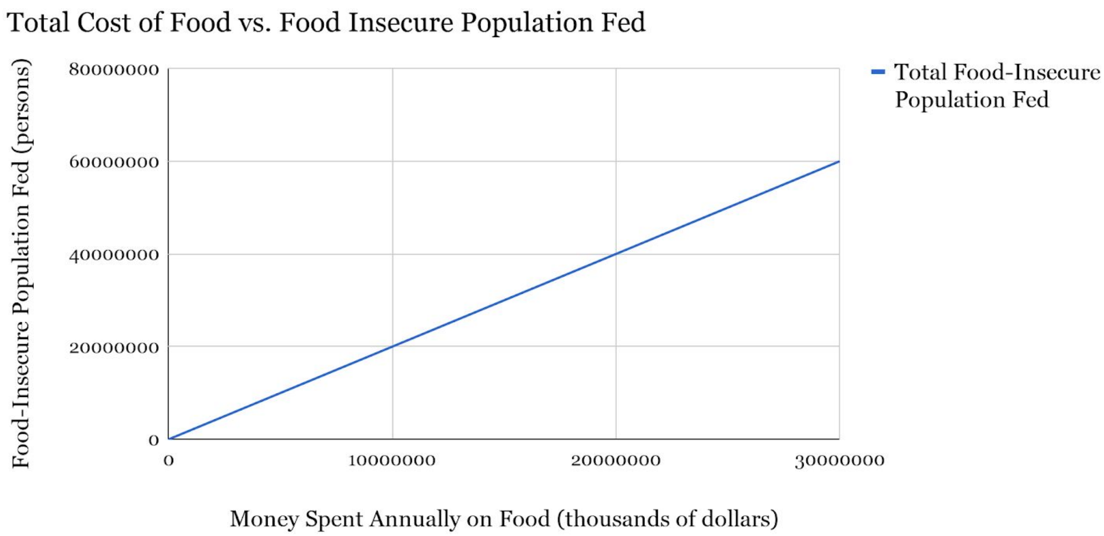
$$(p + d + c) \cdot F \geq P_I \cdot A$$

where P_I is the population of food-insecure people in the region, and *A* is the average cost of food a person in the United States eats in a year: \$3,219.30 [1]. This can be rearranged to display the food-insecure population's relation to the minimum amount of total food that must be produced.

$$P_I \leq \frac{(p + d + c) \cdot F}{\$3,219.30}$$

Graphing P_I for each of the values of *p, d,* and *c* across various food groups (cereals, roots, oilseeds, fruits, vegetables, meats, milk; and their cumulative sum) will estimate how much food would need to produced to support the food-insecure population in a given region.





2.4 Applying the Model

Applying this model to the food-insecurity production in Texas displays whether or not Texas has enough average wasted food to support their food-insecure population. Based on the Texas Food Data [7], one would substitute P_I for 4,320,050 to represent the estimated number of food-insecure individuals. Additionally, one should substitute F to be amount of money spent on all commodities in the state of Texas: $20,897,606,000 [3]. The denominator, A, should be redetermined to be $3,230.25 [7], which is calculated by multiplying the average cost per meal in the state of Texas by 1,095 (3 meals multiplied by 365 days). Once the value for the wasted food of one food group is calculated, it should then be summed to figure out total wastage in comparison to total population of food-insecure people.

$$P_I \leq \frac{(p + d + c) \cdot F}{\$3{,}230.25}$$

Food Group	p	d	c	A (dollars)	F (dollars)	$\frac{(p+d+c) \cdot F}{A}$
Cereals	0.105	0.02	0.27	3,230.25	20,897,606,000	2,555,391.802
Roots and Tubers	0.15	0.07	0.3	3,230.25	20,897,606,000	3,364,060.094
Oilseeds and Pulses	0.05	0.01	0.04	3,230.25	20,897,606,000	646,934.6335
Fruits and Vegetables	0.02	0.12	0.28	3,230.25	20,897,606,000	2,717,125.461
Meat	0.05	0.04	0.11	3,230.25	20,897,606,000	1,293,869.267
Fish and Seafood	0.06	0.09	0.33	3,230.25	20,897,606,000	3,105,286.241
Milk	0.012	0.005	0.15	3,230.25	20,897,606,000	1,080,380.838

Based on the table, the population of food-insecure people needs to be less than the sum of the values in the rightmost column: 14,763,048.34. The 2015 data for the food-insecure population says that food-insecure population was only 4,320,050, which verifies that P_I is less than the amount of people that Texas could feed with its annual waste.

2.5 Discussing the Model

The model returns a boolean value: enough, or not enough; yes, or no. Thus, evaluating the function at extreme values of P_I does not return any beneficial information. However, it does give the exact value of the remainder, R, of the people that will not be fed if P_I is greater than the amount of people that can be fed.

$$R(P_I) = P_I - \frac{(p + d + c) \cdot F}{A}$$

As this function is evaluated around smaller and smaller areas with significant populations of food-insecure people, the remainder will help local organizations, such as food banks and various charities, to have a proper estimate of how much food they may need to contain to feed their population.

3 Part II: Food Foolish?

3.1 Restating the Problem

The task at hand is to create a mathematical model that determines the amount of food waste a household generates in a year, based on the following demographic factors:

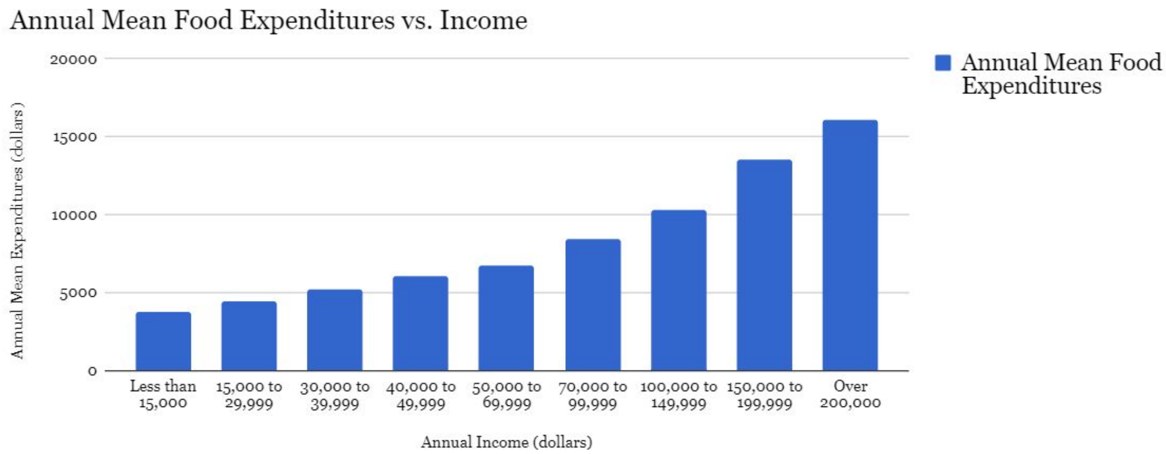
- Ages of everyone in the household
- Annual household income

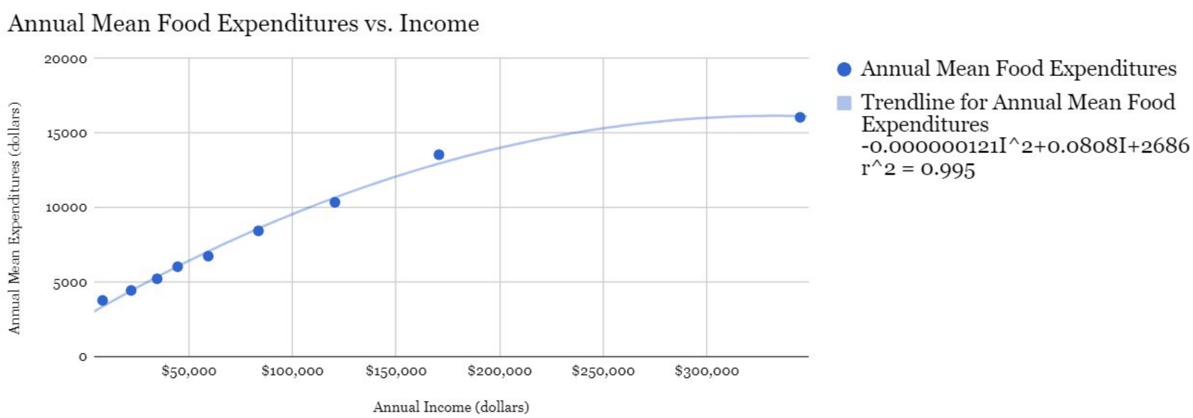
3.2 Assumptions and Justifications

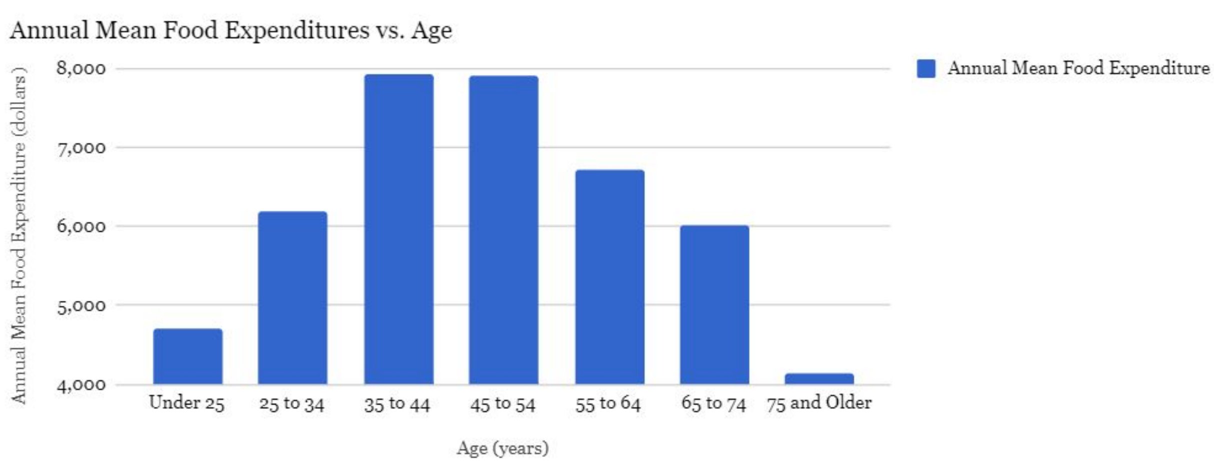
- Assumption A: The average percentage of food wasted remains constant for all individuals.
    - Justification: If the average value of food wasted per person in the United States is used, it can be assumed that this value applies to every individual.
- Assumption B: Each household will have an equal amount of each food group.
    - Justification: In order to meet the needs of a nutritional diet from their previous food-insecure diet, each household must consume all food groups; thus each will be evenly distributed amongst households to ensure the maximum amount of food based off each food group.
- Assumption C: Males and females of the same age eat approximately the same amount.
    - Justification: While it is known that males will typically consume more than females, the total amount consumed by both genders can be represented by a singular number as higher and lower extremes will center around the same average number for each age range [6].
- Assumption D: People who have higher incomes are more compliant with wasting food than those with lower incomes.
    - Justification: The people that have a higher likelihood of wasting food would be that of those who earn more annually, as a greater access to expendable wealth causes a person to be more ignorant and careless when it comes to throwing away leftover food than that of a person who saviors every penny.

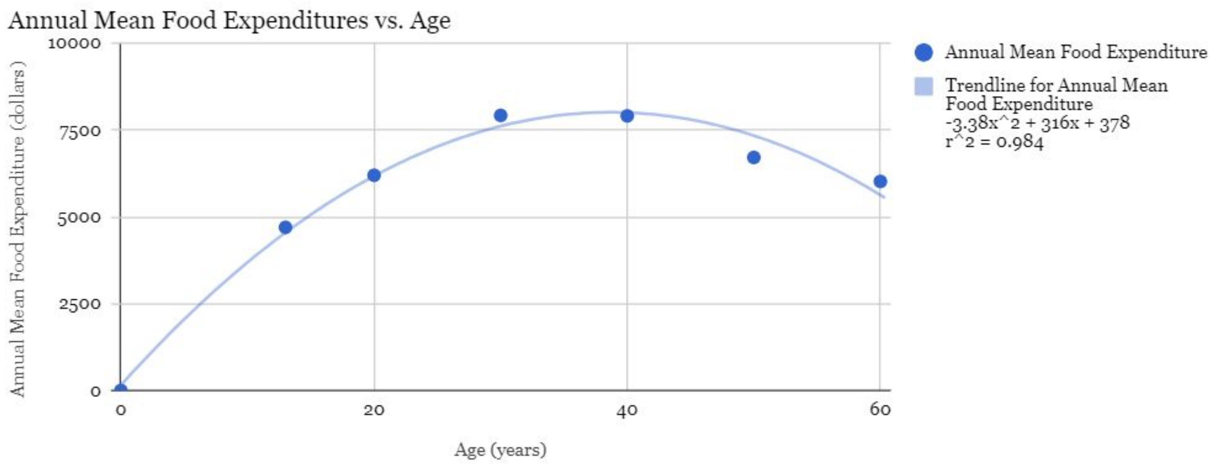
3.3 Constructing the Model

The predominant factors in the food intake of a civilian are their annual income and their age. The conjunction of these two factors results in a multivariable function F that gives an estimate of how much any given person wastes in food on average in units of meals. F is based on two other functions based on age and income, $f(a)$ and $f(I)$, where $f(a)$ returns the amount of money people spend on food based on their age (a), and $f(I)$ returns the amount of money people spend based on their income (*I*). For values of *a* and *I* greater than zero, both functions can be modeled as quadratic with squared coefficients of determination greater than 0.95.









$$f(I) = -0.000000121I^2 + 0.0808I + 2686$$
$$f(a) = -3.38a^2 + 316a + 378$$

To place both of the functions in units of meals, each function will be divided by the average cost of a meal in the United States (c). In order to get an accurate representation of how these to functions coincide, they can be averaged and multiplied by the percent of food that the average human wastes, resulting in F.

$$F(a, I) = (0.1347)\left[\frac{f(a) + f(I)}{2c}\right]$$
$$F(a, I) = (0.1347)\left[\frac{-3.38a^2 + 316a - 0.000000121\,I^2 + 0.0808I + 3{,}064}{2(2.94)}\right]$$

While the average is not the most intuitive method of amalgamating the two functions accurately, it does serendipitously account for the general willingness of lower-income people to conserve more of the food. For example; given lower values of I, F should evaluate to a much lower level, but the average of $f(a)$ and $f(I)$ will increase the value of F. This increase is relatively consistent with the propensity of lower-income people to conserve their food

However, this only accounts for one person in the household, when it is necessary to look at all people in the household. Assuming that the household income is constant for each family, the total wasted food can be represented as a summation of the wasted food by each constituent.

$$\sum_{i=1}^{n} F(a_i, I)$$

where n is the number of people in the household and a_i represents the ages of each of the household members.

3.4 Applying the Model

The model can be applied to four unique situations in which a household's waste is evaluated based on various characteristics of that family.

Assuming the following ages:

- Toddler: 3 years of age
- Parent: 30 years of age

a	*c* (dollars per meal)
Toddler (3 years)	2.94
Parent (30 years)	2.94

Total household income (dollars)	$20,500

$$\sum_{i=1}^{n} F(a_i,\ I) = F(3, 20500) + F(30, 20500) = 127.99 + 254.45 = 382.44\ \frac{meals}{year}$$

Assuming the following ages:

- Parents: 50 years of age
- Teenagers: 15 years of age

a	*c* (dollars per meal)
Parent 1 (50 years)	2.94
Parent 2 (50 years)	2.94
Teenager 1 (15 years)	2.94
Teenager 2 (15 years)	2.94

Total household income (dollars)	$135,000

$$\sum_{i=1}^{n} F(a_i,\ I) = 2F(50, 135000) + 2F(15, 135000) = 2(360.72) + 2(437.93) = 1{,}597.30\ \frac{meals}{year}$$

Assuming the following ages:

- Elderly: 70 years of age

a	c (dollars per meal)
Elderly 1 (70 years)	2.94
Elderly 2 (70 years)	2.94

Total household income (dollars)	$55,000

$$\sum_{i=1}^{n} F(a_i,\ I) = 2F(70, 55000) = 2(290.93) = 581.87\ \frac{meals}{year}$$

Given the following age:

- Adult: 23 years of age

a	c (dollars per meal)
Adult (23 years)	2.94

Total household income (dollars)	$45,000

$$\sum_{i=1}^{n} F(a_i,\ I) = F(23, 45000) = 273.41\ \frac{meals}{year}$$

In order to determine which factor has the highest varying effect on F, the partial derivative gives the rate at which each of the variables changes with respect to the function. The partial derivative with the largest absolute value of the leading coefficient will have the largest impact for incremental changes in each variable.

$$\left|\frac{\partial F}{\partial I}\right| = \left|(-5.54\ x\ 10^{-9})I\ +\ 0.0019\right|$$
$$\left|\frac{\partial F}{\partial a}\right| = \left|-0.1549a\ +\ 7.239\right|$$

Evidently, because the leading coefficient with a larger absolute value is $\frac{\partial F}{\partial a}$, the age has the highest incremental impact on the amount of total food expenditures per household. However, based on the above data, the most influential impact on the entire function F is the number of household members.

3.5 Verifying the Model

If extreme values of age and income are plugged in, it can be seen that the income vs. annual food expenditure function, $f(I)$, reaches a maximum value at I = \$333,884.30 based on the location of the vertex. For extreme values of income greater than this value, this will not accurately represent the true limit of food purchasing. While quadratic regression was more accurate for the intervals of of income given, a logarithmic function would likely be more accurate as income values supercede seven figures.

Similarly, $f(a)$ reaches an x-intercept at $a = 94.67$, implying that people who are above 95 years of age buy negative values of food, something that is impossible save through indebtedness. While the majority of the population does not fit into this small age group, the model should not be applied to those above eighty years old.

4 Part III: Low-Cost Repurposing of Wasted Food

4.1 Restating the Problem

Repurposing food has different capabilities in different locations. This problem asks to do the following:

- Present various solutions for food repurposing
- Analyze the cost efficiency of these solutions in a community

4.2 Assumptions and Justifications

- Assumption A: Grocery stores and commercial businesses throw away all food that is deemed unappealing.
    - Justification: This is a plausible assumption because consumers tend not to buy food that doesn't look appealing, thus compelling the stores and businesses to discard the food [10].
- Assumption B: Locations can be categorized into three types: urban, suburban, and rural.
    - Justification: Based primarily on the varying population density of a given location and proximity of certain types of infrastructure from residential areas, locations are classified into these three types.
- Assumption C: Population is evenly distributed throughout a city.
    - Justification: Although such a statement would be deemed as incorrect, to consider other distributions of the population would be inefficient and would neglect those who desire to donate at other locations. Additionally, even though houses will not be uniformly distributed, the likelihood that neighborhoods are approximately uniformly distributed is extremely high.
- Assumption D: People are only willing to drive two to four miles to their nearest grocery store.
    - Justification: Since grocery stores are seen to people as a commonplace, people will not tend to go out of their way to go to such a place. Therefore, the majority of people will not go past approximately five miles to buy groceries, let alone donate food that would have otherwise been wasted.
- Assumption E: People will be willing to volunteer at a food repurposing facility.
    - Justification: Based on the fact that hundreds of thousands of people are willing to volunteer at other non-profit organizations, it is safe to assume there would be at least a handful of people who would be willing to volunteer for such a facility.

4.3 Cultivating Viable Strategies for Repurposing Wasted Food

For the purposes of this inquiry, two simple strategies were developed to attempt to mitigate the amount of wasted food:

I. Food Drop-Off

II. Food Pick-Up

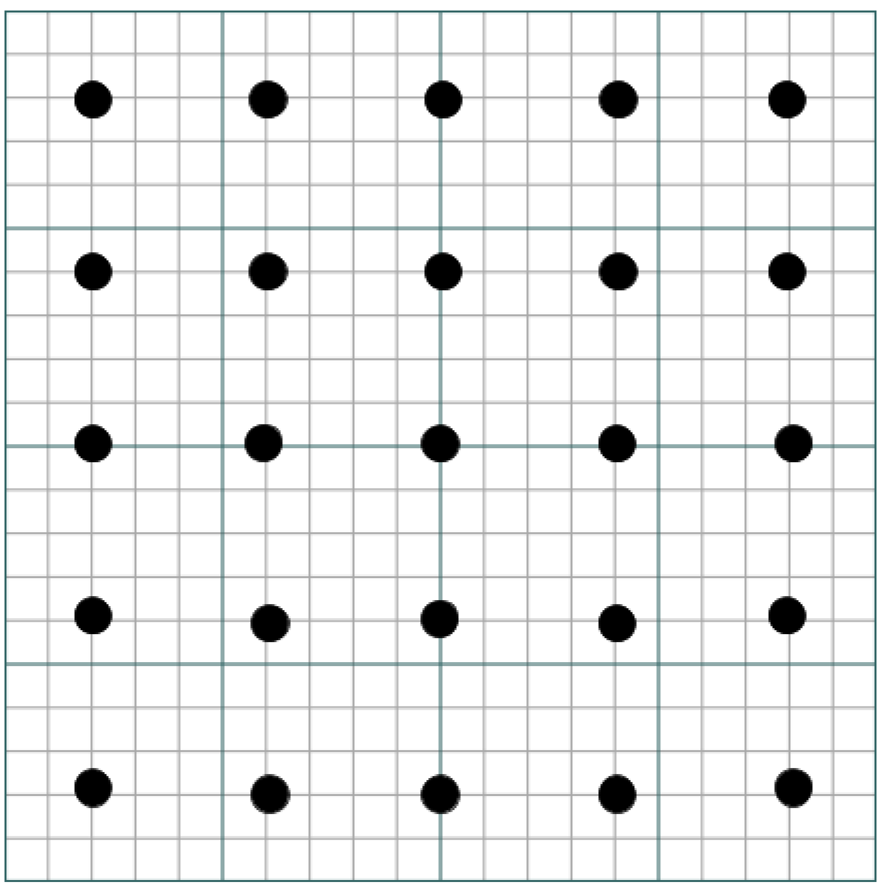
These two systems were analyzed in Fairfax County, Virginia, a relatively suburban district outside of Washington, D.C.

4.4 Constructing the Model

I. Food Drop-Off

Food drop-off is a system in which people will donate their unwanted, untainted food items by driving to their nearest food drop-off center located at a grocery story. The main factor that influences its efficacy is the resident's willingness to drive to that center. In order to calculate this, the area of the location must be divided into a rectangular grid in which the gridlines represent roads and the vertices represent potential housing locations.

The area of Fairfax County is 406 mi^2, which can be estimated as 400 mi^2 to create an even 20 mi x 20 mi square grid. Assuming that the average human is willing to drive only two to four miles to get to their nearest grocery store [11], a probable grid layout would be the following:



While this does not necessarily optimize the amount of food drop-offs that can be placed in the city, the layout displays that an estimate of fifteen food drop-off locations would be the ideal situation for Fairfax County given that the population is evenly distributed.

When computing the cost of each food repurposing facility, the cost of workers, management, and rent will have to be taken into consideration, as well as the number of locations in the county. Using these factors, this function can be deduced:

$$B = g(w + r + m)$$

Where B is the total cost of expenses of all of the food facilities in the county, g representing the number of food packaging facilities in the county, w being the workers' salary, r being the cost of rent for said facility, and m being the cost of management at every facility. Using the grid method above, it was found that 15 locations would be optimal. If it is assumed that the workers are volunteers, their salary will be reduced to $0. If the facility is 500 sq feet, the cost of rent will be approximately $5,500. If there is one manager who is paid minimum wage of $7.25, and works a typically forty hour week, and if there are four weeks in a month, a modified function for monthly costs can be made to be:

$$B = 15(0 + 5,500 + (7.25 \bullet 40 \bullet 4))$$

Using this functions, the total cost of the all of the facilities per month will be $99,900.

II. Food Pick-Up

The food pick-up system is one analogous to a neighborhood garbage pick-up system. It will pick up unwanted food from participating families once a week. Once the food is brought back to the repurposing facility, it will be inspected for any impurities. The food will be packaged and resold at a significantly lower price.

The food pick-up will ask its constituents to pay a monthly fee in order to function. The amount that people will be willing to spend can be equated to the cost of gas per week they would have spent if they had to drop off the food on their own. If the average cost of gas per mile is $0.59, and the nearest food drop-off station would be an average of three miles away, people would, on average, be willing to pay $1.77 per week to have their food picked up for them.

The number of households in Fairfax county is 405,837 as of 2017 [11]. If every truck driver reaches between 500 and 800 homes per week [11], then the pick-up system will need to employ between 507 and 812 workers. Changing the function above slightly, the model for the monthly cost of the food pick-up system can set as a function of the percent of county-wide participation (x):

$$B(x) = 1[(a \bullet c) + r + (c \bullet 40 \,\frac{hours}{week} \bullet 4 \,\frac{weeks}{month})] - x(P)(n)$$

with a being the average number of workers, c bring the salaries of the workers, r being the cost of rent for 1 facility, P being the total population of Fairfax County, and n being the percent of the population that chooses to participate.

$$B(x) = 1[\,\frac{507 + 812}{2}\,(\$7.25) + 5{,}500 + (\$7.25 \bullet 40 \bullet 4)] - [x(1{,}142{,}234)(\$7.08)]$$

In order to find the minimum amount of county-wide participation necessary to fund the pick-up system on its own, the function B was equated to zero and manipulated to find x. Solving for x, the value ended up being a miniscule number of 0.0141, intimating that 1.41% of the people in Fairfax County would need to participate in order to support the most elementary functions of the food pick-up.

4.5 Discussion of Strategies

Of the two models, both are effective for various city types. However, in the context of suburban Fairfax County, the pick-up method is more effective because of its total cost in relation to the number of people that would need to participate.

Although both strategies discussed and calculated for above are extremely viable options, overall, both could be seemingly improved on if expenses were deduced to more in-house options, specifically if costs were reduced by not using storage facilities but instead directly utilizing grocery stores. Grocery stores could directly assist with repurposing wasted food by providing those that are under the poverty threshold [12] a “food card” indicating the option to buy foods at a discounted price, more specifically the foods that the store has deemed inedible due to the distorted and misshapen forms some produce takes [10]. This way, grocery chains can obtain more revenue through additional customers via the food card in addition to lessening the amount of food that would have been wasted otherwise. Overall, such an idea would be highly useful to explore in the future.

5 Conclusion and Evaluation

5.1 Strengths in the Models

Given the higher concentration of populations in the middle range of each set of data, the majority of the models for this problem were accurate and provided insight into the relation between the overall food expenditure and the given variables. Furthermore, the r^2 of the determined functions all had a value greater than 0.95, thus implying a strong correlation between the data and the regression line making the estimated calculations extremely close to the real answer.

5.2 Weaknesses in the Models

One weakness of model number two is that it does not provide an accurate output for some extreme values. Another weakness was that we assumed that the cost per meal was $2.94 (according to a study conducted in 2015). Since this was how much it costed three years ago, the cost per meal today may slightly differ. Additionally, although strategies of food pick-up and food drop-off are feasible, the idea of implementing within grocery stores might have had a more effective outcome; but since only mentioned and not fully explored through a model, there could have been more done to elaborate on such an idea.

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