

# MathWorks Math Modeling Challenge 2020

## Columbus North High School

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## M3 Challenge FINALIST—\$5,000 Team Prize

### JUDGE COMMENTS

This paper had a strong summary that gave details about the mathematical models that were developed as well as the results. In sharing their solution to Question 1 (Q1), this team made a big effort to incorporate previous trends via the use of various regression models in order to extrapolate forward in order to obtain values for operations and upfront costs associated with diesel and electric trucks. Then using their collection of cost parameters, the team identified the difference in cost between a new electric truck versus diesel which served as the basis for a logistic model that predicts the proportion of newly purchased trucks that are electric. This function was subsequently incorporated into a differential equation that models the change of the proportion of electric trucks over time; derivations were provided and the final model was solved numerically with a Runge-Kutta solver. Overall, this solution to Q1 was easy to follow with insightful graphs and an attempt at a sensitivity analysis. There was a very insightful discussion focused on the real world meaning of parameters in the logistic function they developed. Because the solution approach included the introduction of a number of functions, variables, and parameters, it could be helpful - and would further improve readability - to provide a summary of terms used in a graphic such as a table. More discussion of the results and a more thorough sensitivity analysis would provide the reader with additional insight regarding the solution.

This team's solution to Q2 really stood out! In particular, by considering the energy (and therefore the time) required to charge an electric truck, they identified the number of chargers needed first and then used this information to allocate chargers to individual stations. This was not a typical approach and led to more reasonable results than judges saw in other solutions where the number of stations was determined first, oftentimes resulting in a very large number of chargers per station. This team acknowledged that a large number of stations would be needed and made use of existing truck stops.

In solving Q3, this team found data online and developed a "motivation ranking" based on the level of environmental concern for each corridor. The team used a weighting system to combine motivation and regional financial interests to find a ranking of the transportation corridors. In this section, the team shared a great number of calculations. Readability could be improved by sharing a single (or "example") calculation of each value and then listing others in a table or another appropriate graphic.

#### Questions:

Q1: How much faith do you have in the accuracy of your model and the assumptions you made? What are some weaknesses of your model?

Q2: Can you provide more insight into where the values in Table 3.5.1 came from and how you derived  $T_{\text{daily}}$  values? How did you determine the number of truck stops on each corridor? Which parameter(s) would you vary if performing a sensitivity analysis?

Q3: Why did you choose 0.1 and 0.9 as the weights? What other attributes/factors would you have included if you had more resources available?

# Shocking Transportation

## Executive Summary

A staple of American highways, diesel semi-trucks tower over other vehicles cruising down the highway. While these giants of the road have established their purpose of transporting goods over the years, their method of achieving these goals is on the verge of a grand transformation—namely, their source of energy. As global climate change concerns rise, the push for electric semi-trucks has increased to replace the inefficient diesel engines currently in wide use. Corporations like UPS, PepsiCo, and Walmart are gravitating towards a fleet of zero-emissions semis.

To predict the future proportion of electric trucks on the highway, we created a multi-step model based on the economic incentives of electrification. We created and solved a differential equation to model the market's shift toward electric trucks because of market forces including cost and environmental concern. To model these market forces, we projected future trends for the upfront cost to purchase a diesel or electric vehicle and the cost of operating with each energy source. We then incorporated the difference between the electric and diesel semi-truck costs into a logistic function. Finally, we incorporated this logistic model into our differential equation and used the fourth order Runge-Kutta method to simulate future electrification. From this model, we found that after 5 years, 32.7% of semi-trucks would be electric; after 10 years, 59.2% would be electric; and after 20 years, 80.7% would be electric.

Considering the sustainability of such environmental efforts, we also formulated an algorithm that estimates the number of chargers and charging stations that need to be installed along a specific route to sustain long haul semi-truck traffic using inputs of daily truck traffic, workday length, route length, average truck speed, energy consumption rate, energy used to charge at night, and battery charge rate. We then tested our model with the following corridors: San Antonio, TX to/from New Orleans, LA; Minneapolis, MN to/from Chicago, IL; Boston, MA to/from Harrisburg, PA; Jacksonville, FL to/from Washington, DC; Los Angeles, CA to/from San Francisco, CA. Our results showed that the Jacksonville, FL to/from Washington, DC route required 87.2 charging stations and 69.6 chargers per station, the maximum level of installation required. In contrast, the Boston, MA to/from Harrisburg, PA route required the minimum amount of installation with only 48.2 charging stations and 48.6 chargers per station.

Finally, we created a model that ranks corridors in the order that they should be developed for electric charging stations. This model ranks the corridors in descending order of the level of environmental consciousness in communities surrounding the corridor and in ascending order of financial returns on initial investment to implement charging stations. For the final rankings, we found a weighted average of the rankings, weighting the community motivation to pursue eco-friendly behaviors at 0.1 and financial rankings at 0.9 of the final corridor rankings. Testing our model for short-term (5 years) and long-term (20 years) projections, we also weighted the financial rankings for the two types of projections to account for half of the corridors' average financial rankings. For the given corridors, we concluded that the rankings are as follows: (1) Boston, MA to/from Harrisburg, PA; (2) Los Angeles, CA to/from San Francisco, CA; (3) San Antonio, TX to/from New Orleans, LA; (4) Minneapolis, MN to/from Chicago, IL; (5) Jacksonville, FL to/from Washington, DC.

## **Table of Contents**

### **1 - Restatement of Problem - pg. 2**

### **2 - Shape Up or Ship Out - pg. 3**

#### **2.1 Introduction: Using Differential Equations to Model Market Shift - pg. 3**

#### **2.2 Diesel Semi-trucks - pg. 3**

#### **2.3 Electric Semi-trucks - pg. 5**

#### **2.4 Relationship between replacement proportion and cost - pg. 7**

#### **2.5 Derivation of Differential Equation for Electric Truck Proportion - pg. 8**

#### **2.6 Fourth Order Runge-Kutta Computation - pg. 9**

#### **2.7 Analysis and Conclusions - pg. 9**

### **3 - In It for the Long Haul - pg. 11**

#### **3.1 Introduction - pg. 11**

#### **3.2 Variables - pg. 11**

#### **3.3 Balancing Energy Consumption - pg. 11**

#### **3.4 Allocations of Chargers to Stations - pg. 12**

#### **3.5 Results - pg. 13**

### **4 - I Like to Move It, Move It - pg. 13**

#### **4.1 Basic Model of Corridor Rating - pg. 13**

#### **4.2 Motivation Rank - pg. 14**

#### **4.3 Finance Rank - pg. 15**

#### **4.4 Analysis and Conclusions - pg. 16**

### **5 - Conclusions - pg. 20**

### **6 - References**

## **1 - Restatement of the Problem**

The problem we addressed is as follows:

1. Assuming that all necessary electric charging stations have been installed in the proper locations, develop a mathematical model that predicts the percentage of semi-trucks that will be electric in 5, 10, and 20 years from 2020. Consider the effects of supply and demand on the transition from diesel to electric semi-trucks based on the changing prices of electric semi-trucks.
2. Create a mathematical model that estimates the number of chargers and charging stations that need to be installed along a specific route to sustain long haul semi-truck traffic, excluding regional and short haul traffic. Test the model using the following corridors:
  - a. San Antonio, TX to/from New Orleans, LA
  - b. Minneapolis, MN to/from Chicago, IL
  - c. Boston, MA to/from Harrisburg, PA
  - d. Jacksonville, FL to/from Washington, DC
  - e. Los Angeles, CA to/from San Francisco, CA

3. Based on the financial benefits and community support of establishing an electric trucking infrastructure in an area, develop a ranking system that can organize corridors in order of priority. The top-ranked corridor should be developed first.

## 2 – Part I: Shape Up or Ship Out

### 2.1 Introduction: Using Differential Equations to Model Market Shift

In order to model the rate of adoption of electric trucks, we created a multi-step model based on the economic incentives of electrification. First, we projected the upfront and operational costs of diesel and electric trucks into the future. Next, we modeled the proportion of replacements that are electric as a function of the cost difference between electric and diesel trucks.

Once these had been established, we modeled the proportion of electric trucks on the highway over time using a differential equation based on the replacement rate of trucks and the relative demand for electric trucks as derived from the previous steps. We solved the differential equation, yielding an expression for the share of electric trucks as a function of time.

Finally, we simulated the future adoption of electric trucks on a spreadsheet using the fourth order Runge-Kutta method on the previous differential equation. We used the results of this analysis to project future proportion 5, 10, and 20 years in the future, and to analyze the sensitivity of our model.

### 2.2 Diesel Semi-trucks

To calculate the cost of buying and running a diesel semi-truck, we first modeled the upfront cost to actually purchase a diesel semi-truck. Then, we needed to add the operational costs to the upfront costs. The total operational costs for diesel semi-trucks consist of 39% for fuel; 26% for driver salary; 17% for the truck cab and trailer; 10% for repairs and maintenance; 4% for insurance; 3% for tires; and 2% for permits, licenses, and tolls [1].<sup>1</sup> We decided to only model the cost of diesel fuel because fuel is the most influential factor over a diesel semi-truck's cost of operation. Additionally, we assumed all of the other factors besides energy source are the same between diesel semi-trucks and electric semi-trucks since both diesel and electric semi-trucks perform the same tasks for similar amounts of time. Modeling the source of energy is sufficient for finding the difference in cost between diesel and electric semi-trucks.

#### 2.2.1 Upfront Cost of Diesel Semi-trucks

In order to calculate the upfront cost of purchasing a diesel semi-truck, we graphed the average price of new Class 8 trucks over the years 2010 to 2018 as years since 2000 [17]. We assumed that trends in Class 8 trucks are representative of those in semi-trucks because semi-trucks are typically Class 8 trucks [8]. We modeled the data with an exponential function since inflation and technological advancements tend to act exponentially. In this case, inflation is increasing at a greater rate than technological advancements, leading to a general increase in the upfront cost of diesel semi-trucks:

$$dieselUpfrontCost = 100460e^{0.011(year-2000)} \quad (\text{Eq. 2.1})$$

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<sup>1</sup> Numbers may not add up to 100% due to rounding.

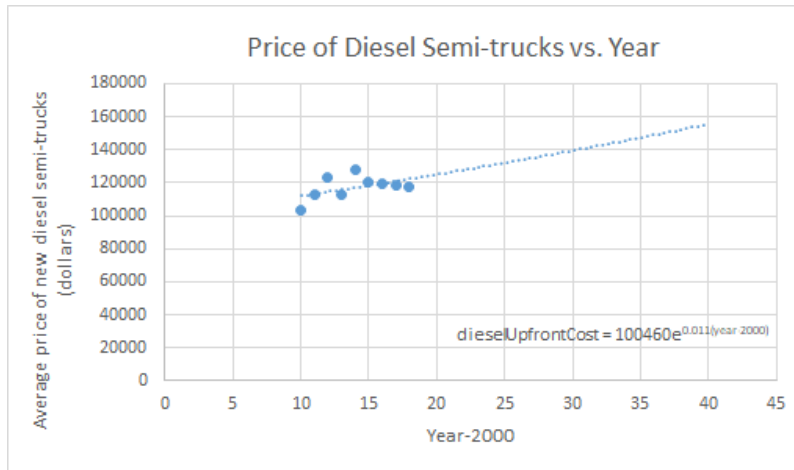


Figure 2.2.1.1 is an extrapolation of the price of diesel semi-trucks over the years since 2000.

### 2.2.2 Diesel Fuel Cost

In order to calculate the price of diesel fuel in future years, we first graphed the U.S. diesel retail price per gallon over the years 1995 to 2018 as a function of the year since 2000 [11]. In order to get the price per mile rather than price per gallon, we took the original data in price per gallons and multiplied by the inverse of the average fuel efficiency (1/5.98 gallons per mile) to end up with data in dollars per mile [8]. Then, we found the exponential function because inflation tends to be exponential. Furthermore, the price will increase exponentially because diesel fuel supplies will start to run out subsequently increasing its cost. Multiplying this equation by the miles traveled per year by the diesel semi-truck will give the total cost for the year spent on diesel gas:

$$dieselCost = 0.2516e^{0.0528(year-2000)} * milesPerYear \quad (Eq. 2.2)$$

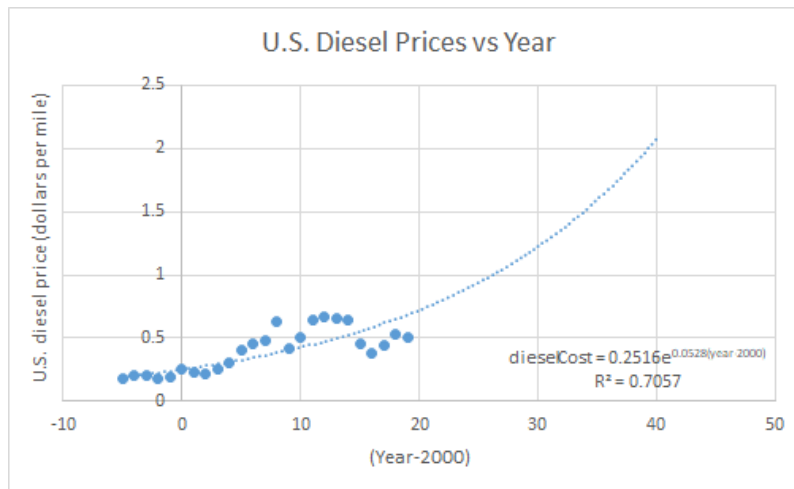


Figure 2.2.2.1 is an extrapolation of the price of diesel fuel in dollars per mile over the years from 2000.

### 2.2.3 Total Cost of Diesel Semi-trucks

The total cost of diesel semi-trucks is the upfront cost to purchase the diesel semi-truck plus the operational cost per year which we simplified to the cost of diesel fuel.

$$totalDieselCost = dieselUpfrontCost + dieselCost \quad (\text{Eq. 2.3})$$

Substituting the previously found equations, our final equation is

$$totalDieselCost = 100460e^{0.011(year-2000)} + (0.2516e^{0.0528(year-2000)} * milesPerYear) \quad (\text{Eq. 2.4})$$

## 2.3 Electric Semi-trucks

To calculate the cost of buying and running an electric semi-truck, we first modeled the upfront cost to purchase an electric semi-truck. Then, we needed to add the operational costs to the upfront costs. We assumed all of the factors of operational cost besides energy source are the same between diesel and electric semi-trucks because both diesel and electric semi-trucks are performing the same tasks for similar amounts of time. Therefore, modeling the source of energy is sufficient in finding the difference in cost between diesel and electric semi-trucks.

### 2.3.1 Upfront Cost of Electric Semi-trucks

In order to calculate the upfront cost of an electric semi-truck, we decided to focus on the change in cost of the car battery as materials and other factors will not have as significant of an effect on the price. Since electric semi-trucks can travel up to 500 miles on one charge, we assumed most semi-trucks will use a 500 kWh battery [5]. Based on this fact and data we found on the price of car batteries in dollars per kWh, we graphed the price of a 500 kWh car battery over the years 2016 to 2020 as years since 2000 [5]. We modeled the data with an exponential function because inflation and technological advancements tend to act exponentially. In this case, technological advancements are increasing at a greater rate than inflation, leading to a decrease in the cost of an electric semi-truck battery. Furthermore, we inferred by the shape of the curve that the cost of the battery will level out at a price somewhere above zero. Excel was unable to fit an exponential curve with a horizontal asymptote above zero, so we subtracted a constant from all data points until the  $R^2$  value of this new variable cost was at its greatest. We used that equation, adding the constant to shift the data points back to their original values. To account for the cost of materials and other factors that factor into the total up front cost of the electric semi-trucks, we added another constant to include the costs for the ensure the total cost of the electric semi-truck in 2020 would equal the current average cost of an electric semi-truck, \$165,000 [5]:

$$electricUpFrontCost = baselineCost + variableCost = 156,500 + 8 * 10^8 e^{-0.579(year-2000)} \quad (\text{Eq. 2.5})$$

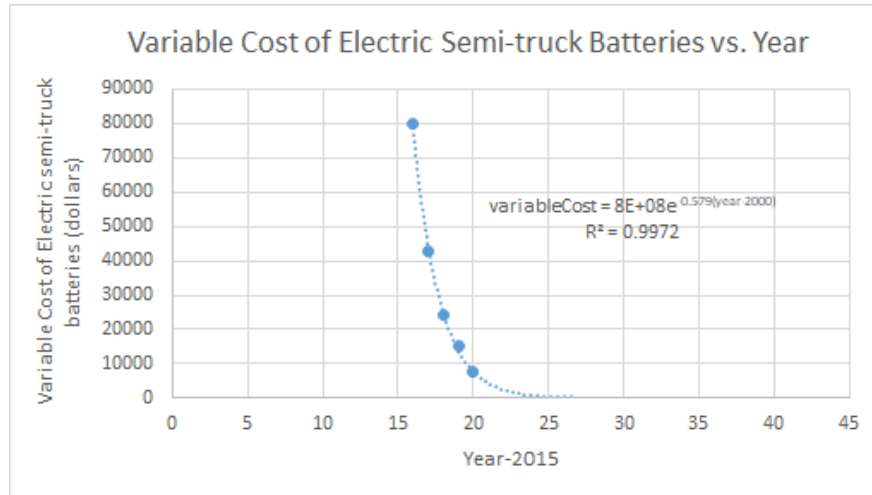


Figure 2.3.1.1 is an extrapolation of the variable cost of electric car batteries in dollars over years since 2000

### 2.3.2 Energy Cost

In order to calculate the cost of charging electric trucks in the future, we first graphed the U.S. average electricity price from 1990 to 2018. In order to get the price per mile instead of price per kWh, we multiplied by the energy used each mile, which we found to be 2 kWh/mi [5]. Then, we found the exponential function because inflation tends to be exponential. Multiplying this equation by the miles traveled per year by the semi-truck will give the total amount of money spent on energy throughout the year:

$$energyCost = milesPerYear * 0.1547e^{0.0199(year-2000)} \tag{Eq. 2.6}$$

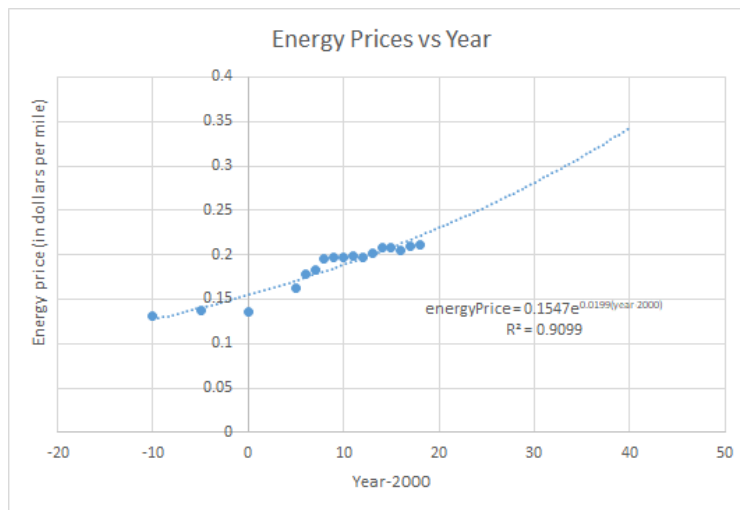


Figure 2.3.2.1 is an extrapolation of the price of energy as

### 2.3.3 Total Cost of Electric Semi-trucks

The total cost of electric semi-trucks is the upfront cost to purchase the electric semi-truck plus the operational cost per year, which we simplified to the cost of energy.

$$totalElectricCost = electricUpfrontCost + energyCost * lifetime \quad (Eq. 2.7)$$

Substituting the previously found equations, we arrived at our final equation:

$$totalElectricCost = 156,500 + 8 * 10^8 e^{-0.579(year-2000)} + (0.1547e^{0.0199(year-2000)} * milesPerYear * lifetime) \quad (Eq. 2.8)$$

## 2.4 Relationship Between Replacement Proportion and Cost

Every time a truck reaches the end of its life cycle, it can be replaced with either a diesel semi-truck or an electric semi-truck. The ratio between diesel trucks bought and electric trucks bought depends primarily on the cost difference and on environmental factors. We defined  $p_e$  as the proportion of newly purchased trucks that are electric (this is not the same as the proportion of *existing* trucks which are electric; that will be derived in the next section). For the purpose of this analysis, we will define the cost difference as follows:

$$\Delta c = electricCost - dieselCost \quad (Eq. 2.9)$$

To determine the form of the relationship between  $p_e$  and  $\Delta c$ , we looked at the expected behavior for extreme values of  $\Delta c$ . In particular, the relationship must have the following properties:

1.  $p_e$  must go to 0 as  $\Delta c$  goes to positive infinity.
  - a. Justification: No business will buy electric trucks if they are trillions (or any other extremely large finite number) of dollars more expensive than diesel trucks.
2.  $p_e$  must go to 1 as  $\Delta c$  goes to negative infinity.
  - a. Justification: No business will buy diesel trucks if they are trillions (or any other extremely large finite number) of dollars more expensive than electric trucks.
3. There is some value  $\Delta c = b$  for which  $p_e = 0.5$ . This value represents a “bias” towards either electric or diesel trucks.
  - a. Justification:  $p_e$  will be modeled as a continuous function ranging from 0 to 1, so it must pass through 0.5 for some value of  $\Delta c$ .

Given these requirements, the logistic function was the most natural and logical choice to model the relationship. We start with the following general logistic function with parameters  $a$  and  $b$ :

$$p_e = \frac{1}{1 + e^{(\Delta c - b)/a}} \quad (Eq. 2.10)$$

Let us now consider what these parameters mean. The parameter  $a$  controls how sensitive  $p_e$  is to changes in  $\Delta c$ ; smaller values of  $a$  make the proportion more sensitive to small differences in cost. The parameter  $b$  functions as a “bias,” shifting the curve left or right. In this case, a positive value of  $b$  corresponds to a bias in favor of electric trucks. For example, a value of  $b = \$10,000$  would mean that companies buy equal amounts of electric and diesel trucks even if electric trucks are \$10,000 more expensive and that they favor electric trucks when prices are equal. Thus,  $b$  can be interpreted as the equivalent monetary value which businesses place on the environmental and other advantages of electric trucks.

Since electric trucks are a fairly recent innovation, sufficient data was not found to empirically determine the values of  $a$  and  $b$ . However, using the interpretations above, we formed reasonable ballpark estimates for  $a$  and  $b$  and tested multiple value combinations around these ballpark



values. This gives a range of predictions based on variations in the future buying habits of companies. Our results also reveal that variations in  $a$  and  $b$  only very slightly affect the final results of the model. Because of this lack of sensitivity, uncertainty in the exact parameter values is acceptable.

Ballpark estimate for  $a$ : 5% of the cost of a diesel truck ( $a = \$5871.5$ )

Interpretation: A 10% difference in cost is enough to choose the cheaper option roughly 88% of the time. This is fairly reasonable given that a 10% cost difference is fairly significant, but not prohibitively expensive if a company has a strong preference. The model is very insensitive to different choices for this value.

We chose a baseline value of 0 for  $b$ , meaning that there is no preference apart from cost.

However, we also tested the model for a positive value of  $b$ , representing a scenario where businesses favor electric trucks for environmental reasons.

## 2.5 Derivation of Differential Equation for Electric Truck Proportion

In this section, we derived and solved a differential equation for the time evolution of  $P(t)$ , the proportion of trucks, which are electric as a function of time.  $r$  is the rate at which trucks need to be replaced (specifically,  $r$  is the proportion of all trucks which go out of service each year). At equilibrium, we must have  $r = 1 / \text{lifespan}$ , where *lifespan* is the average longevity of a truck. The lifespan of a truck is about 12 years, so  $r = 1/12$  for this problem [8].

The number of electric trucks entering the fleet each year, as a proportion of the total fleet size, is equal to  $rp_e$  (the replacement rate times the proportion of new trucks which are electric).

Similarly, the number of electric trucks exiting the fleet each year is  $rP$  (the rate at which existing trucks retire times the proportion of existing trucks which are electric). Therefore, the total rate of change  $dP/dt$  is given by the following differential equation:

$$\frac{dP}{dt} = rp_e - rP \quad (\text{Eq. 2.9})$$

Note that this equation is *not* separable because both  $P$  and  $p_e$  are functions of time, not constants ( $p_e$  as a function of time was obtained by plugging in the cost projections to the equation in the previous section).

We then rewrote the differential equation and solved it using the general method for linear differential equations, with integrating factor  $\mu(t)$ :

$$\begin{aligned} P' + rP &= rp_e \\ \mu &= e^{\int r dt} = ke^{rt} \\ \mu P' + \mu rP &= \mu rp_e \\ (\mu P)' &= \mu rp_e \\ \mu P &= \int \mu rp_e dt \\ e^{rt} p &= \int e^{rt} rp_e(t) dt \\ P &= \frac{\int e^{rt} rp_e(t) dt}{e^{rt}} \end{aligned} \quad (\text{Eq. 2.10})$$

This gives an explicit solution for  $P(t)$ , the proportion of trucks which are electric as a function of time. Given the formula for  $p_e(t)$  from the previous sections, the integral in the numerator can

be computed to arbitrarily high precision, making this expression useful for computing the proportion for a particular year. Our numerical predictions are given in section 2.7.

## 2.6 Fourth Order Runge-Kutta Computation

Using the differential equation from the previous section, we simulated the future electric truck proportion on a spreadsheet using the fourth order Runge-Kutta method [14]. We used a step size of 0.1 years to improve the accuracy of the projection. The Runge-Kutta method has two main advantages: It provides much better accuracy than Euler's method, and it makes it easier to compute many consecutive values without repeatedly evaluating the integral in Eq. 2.10. The results of our Runge-Kutta computation are presented and analyzed in the next section.

## 2.7 Results, Analysis, and Conclusions

Model outputs for 5, 10, and 20 years:

5 years: 32.7% electric

10 years: 55.6% electric

20 years: 80.7% electric

Our model predicts over 50% adoption of electric trucks by 2029 and over 75% adoption by 2037.

The following graphs show the proportion of electric trucks over the next 25 years for different parameter values. All three graphs show rapid growth at first, followed by a decreasing growth rate.

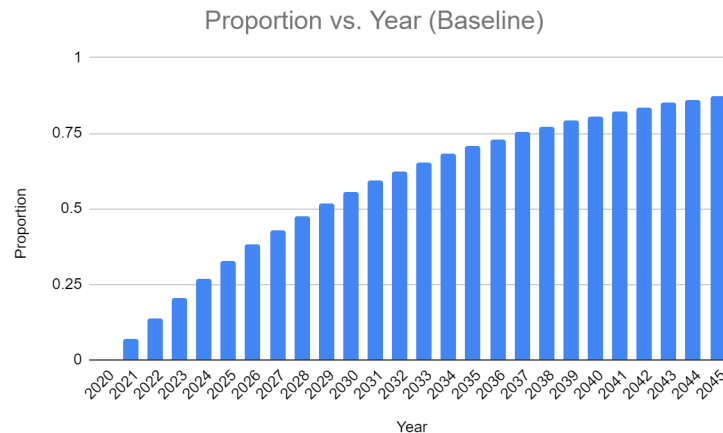


Figure 2.5.1 displays the proportion over 25 years using the baseline values.

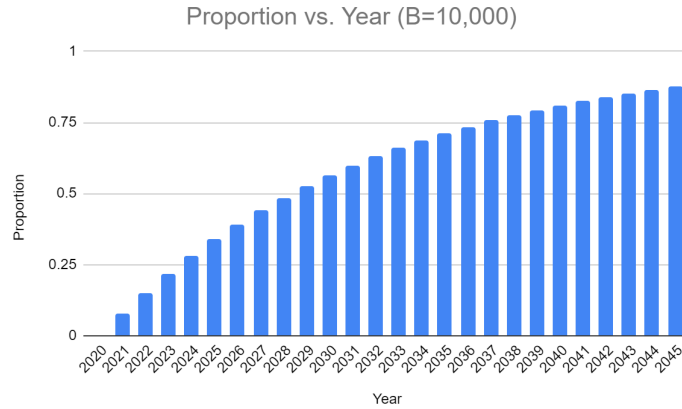


Figure 2.5.2 shows the results over 25 years using the baseline value of parameter  $a$  and changing the bias towards electric vehicles to \$10,000.

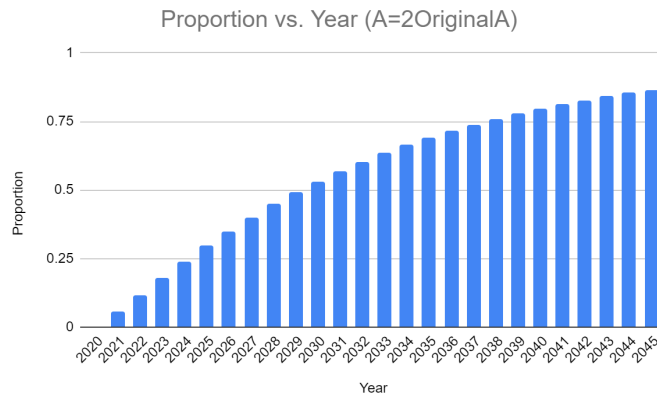


Figure 2.5.3 shows the results when parameter  $a$  (inverse sensitivity to cost) is doubled.

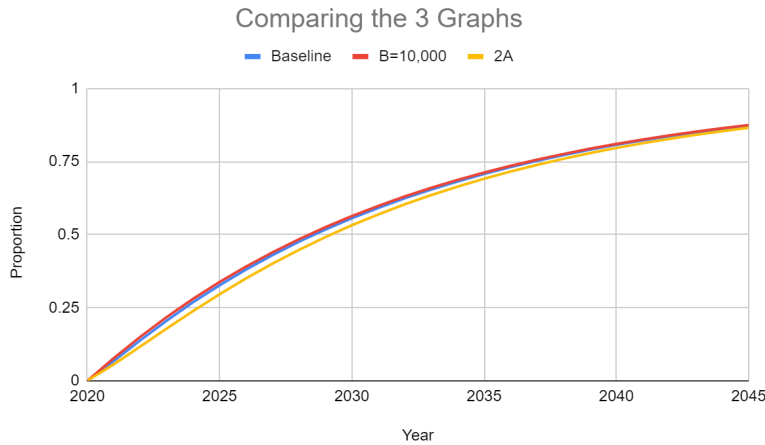


Figure 2.5.4 is a comparison of results for different parameter values.

The final graph shows a comparison of the model results for variations of the parameter values. All three configurations behave similarly, showing low sensitivity to variation in parameters. Adoption lags behind slightly when sensitivity to cost is decreased, and pulls ahead slightly when businesses value the environmental benefits of electric trucks. Another key result is that

the cost of electric trucks quickly becomes far lower than the cost of diesel trucks, easily outcompeting them even without any environmental bias.

## 3 – Part II: In It for the Long Haul

### 3.1 Introduction

In this section, we derived a model for the number of charging stations and chargers required to support 100% electric long haul trucking on a given corridor.

### 3.2 Variables

Note: Single letter variables will be used in this section to make large equations more readable.

$v$  - Average truck speed (miles per minute)

$T_{daily}$  - Number of trucks passing through a route per day

$L$  - Route length (miles)

$N$  - Number of charging stations on a route

$c$  - Average number of chargers per charging station

$E(v)$  - Rate at which a truck consumes energy while driving, as a function of truck speed (kWh per minute)

$r$  - Rate at which a truck's battery charges (kWh per minute)

$t_{drive}$  - Total amount of time each truck spends driving on a given route (minutes)

$t_{charge}$  - Total amount of time each truck spends charging on a given route (minutes)

$E_{battery}$  - Total energy storage of a truck battery (kWh)

$E_{night}$  - Energy used each night to charge a truck's battery to 100% (kWh)

$E_{in}$  - Total daily energy intake of an electric truck (kWh)

$E_{out}$  - Total energy spent daily by an electric truck; equal to  $E_{in}$  (kWh)

$W$  - Length of workday for trucker

### 3.3 Balancing Energy Consumption

The first constraint on electric trucking is that a truck's daily energy usage must be equal to its daily energy consumption. For the purposes of this model, we assumed that trucks charge up from 20% to 100% overnight and charge as needed during the day, typically staying between 20% to 80% during the daytime. This is reasonable because 20% to 80% is the recommended operating range for electric truck batteries [8]. During the night (when the driver is resting), there is plenty of time to charge, so the batteries can be charged to 100% to minimize the amount of time spent charging during the day. Daytime charges should stop at 80% to ensure optimal charging speed, as operating time is valuable during the day.

Given this charging pattern, the energy intake of a single truck is given by the following equation:

$$E_{in} = E_{night} + t_{charge}r = 0.8E_{battery} + t_{charge}r \quad (\text{Eq. 3.1})$$

In Eq. 3.1, the total intake is equal to the nighttime energy intake (80% of the battery capacity as established in the previous section) plus the energy intake from charging on the route, which is given by the charging rate multiplied by the time spent charging.

The energy spent is given by the following equation:

$$E_{out} = t_{dr} * E(v) \quad (\text{Eq. 3.2})$$

Eq. 3.2 is the driving time multiplied by the rate of energy use while driving. For the charging stations to support electric vehicle traffic, the energy intake must equal the energy spent because if the energy spent was greater than the energy intake on any given day, then the trucks would run out of energy (drop below the acceptable charge range) before reaching the end of the route. Therefore, we must have the following equation:

$$E_{in} = E_{out} \quad (\text{Eq. 3.3})$$

Substituting Eqs. 3.1 and 3.2 into Eq. 3.3 yields the following equation:

$$E_{night} + t_{charge}r = t_{dr} * E(v) \quad (\text{Eq. 3.4})$$

Next, we will relate the number of chargers on a route to the required charging time. The total amount of truck-minutes spent charging on a route is given by  $T_{daily} * t_{charge}$  (the number of trucks multiplied by the time spent by each truck). If the chargers are fully utilized, then the amount of truck-minutes spent charging is also given by  $Nc * W$ , since  $Nc$  (number of stations multiplied by chargers per station) gives the total number of chargers and  $W$  is the length of a workday, the period when truckers are active. Equating the two gives Eq. 3.5:

$$T_{daily} t_{charge} = NcW \quad (\text{Eq. 3.5})$$

The time spent driving is equal to the length of the route divided by average driving speed:

$$t_{driving} = \frac{L}{v} \quad (\text{Eq. 3.6})$$

Combining Eqs. 3.4 and 3.6, we have the following equation:

$$t_{charge}r + E_{night} = \frac{L}{v}E(v) \quad (\text{Eq. 3.7})$$

Rearranging Eq. 3.7 gives the following expression for charging time:

$$t_{ch} = \frac{(L/v)E(v) - E_{night}}{r} \quad (\text{Eq. 3.8})$$

Finally, we can combine Eqs. 3.5 and 3.8 and rearrange to derive the following result for the total number of chargers needed along the route:

$$Nc = \frac{T_{daily}}{W} \left( \frac{(L/v)E(v) - E_{night}}{r} \right) \quad (\text{Eq. 3.9})$$

### 3.4 Allocation of Chargers to Stations

The analysis in the previous section shows that energy considerations yield a required value for the total number of chargers on a route but do not constrain how many stations these chargers are allocated to. Thus, other factors must be considered in deciding how to group the chargers into stations. We based our decision on the practical consideration that charging stations should ideally be constructed in existing truck stops to utilize existing exits and parking. This provides a lower limit on spacing.

Upon applying our model to the five traffic corridors provided (shown in the next section), it became clear that the number of chargers was very large (in the thousands) for each route. Thus, it was necessary to place a charging station at every existing truck stop to keep the number of chargers per station to a reasonable number. The results for the number of stations and chargers are given in the next section.

### 3.5 Results

The following table shows the results for each corridor. The bolded values are ones we're most concerned about, with  $N$  being the number of locations along the corridor that will install charging stations and  $c$  the number of charging stations that are at each location. The total cost is the cost of installing all the required chargers.

Corridor	r (kW h / min)	E(v) (kWh / min)	v (miles per min)	L (mi)	E_tot (kWh)	W (min)	Long haul trucks per day	N*c	d (mi)	Number of stations	Chargers per station	Cost per charger	Total cost (\$)
<b>SanAn to NOLA</b>	1.83	1.83	0.92	544	440	840	9,909	4169.5	8.097	<b>67.2</b>	<b>62.1</b>	52500	<b>218,898,818</b>
<b>Minn to Chi</b>	1.83	1.83	0.92	408	440	840	17,550	4284.9	8.097	<b>50.4</b>	<b>85.0</b>	52500	<b>224,959,091</b>
<b>Bos to Hburg</b>	1.83	1.83	0.92	390	440	840	10607	2341.8	8.097	<b>48.2</b>	<b>48.6</b>	52500	<b>122,944,773</b>
<b>Jax to DC</b>	1.83	1.83	0.92	706	440	840	9,611	6065.8	8.097	<b>87.2</b>	<b>69.6</b>	52500	<b>318,457,023</b>
<b>LA to SF</b>	1.83	1.83	0.92	382	440	840	11240	2364.8	8.097	<b>47.2</b>	<b>50.1</b>	52500	<b>124,150,909</b>

Table 3.5.1 displays the result for the number of stations and number of locations along each corridor.

## 4 – Part III: I Like to Move It, Move It

### 4.1 Basic Model of Corridor Rating

To rank the trucking corridors in order of priority for charging station installation, we created a model to calculate a rating value for each corridor based on the level of encompassing states' motivation to install charging stations and the projected profit of the corridor over a certain period of time in years. The corridor rating is a weighted average of the rankings of the corridors in motivation and profit compared to the other corridors in question. The weightings are user inputs depending on which ranking the user values most:



<b>New Jersey</b>	6	<b>North Carolina</b>	19	<b>Ohio</b>	32	<b>Arkansas</b>	44
<b>Nevada</b>	7	<b>Colorado</b>	19	<b>Iowa</b>	33	<b>Wyoming</b>	45
<b>Oregon</b>	8	<b>Michigan</b>	21	<b>Utah</b>	34	<b>Mississippi</b>	46
<b>Minnesota</b>	9	<b>Pennsylvania</b>	22	<b>Texas</b>	35	<b>Alabama</b>	46
<b>Maine</b>	9	<b>Idaho</b>	23	<b>Georgia</b>	36	<b>Kentucky</b>	46
<b>Washington</b>	11	<b>South Carolina</b>	23	<b>Nebraska</b>	37	<b>West Virginia</b>	49
<b>Hawaii</b>	11	<b>Montana</b>	23	<b>Kansas</b>	38	<b>Louisiana</b>	50
<b>Vermont</b>	13	<b>Virginia</b>	26				

Table 4.2.2.1 ranks the states in order based on how much their community cares about the environment using data from 2019. The states are in descending order of environmental consciousness.

The *stateMotivation* is equal to the ranking given by the table above.

#### 4.2.3 Motivation Rating and Rank

A weighted average can be calculated for the corridor, using the *proportionInState* and the *stateMotivation*, to find a rating which can be used to rank the corridors based solely on how much the community cares about the environment:

$$motivationRating = \sum^{eachState} proportionInState * stateMotivation \quad (\text{Eq. 4.4})$$

The smallest *motivationRating* will result in a *motivationRank* of 1 and continue sequentially until all of the corridors being compared have been ranked.

### 4.3 Finance Rank

The total returns on investment in installations, represented by *financeReturn*, in U.S. dollars, can be given by the following equation:

$$financeReturn = operationProfit - installationCost \quad (\text{Eq. 4.5})$$

A negative *financeReturn* value means more money has been spent than earned back. The *financeRank* is then the ranking of one corridor compared to other corridors in returns on investment in developing a system of charging stations.

#### 4.3.1 Cost of Installation

Since we assumed the route would only use DCFC chargers, the estimated cost of installing charging stations for a corridor is the average price of DCFC chargers times the total number of chargers, represented by *chargerNumber*. To find the average cost of installing a DCFC charger, we found the mean of the minimum and maximum prices in DCFC charger installations: \$15,000 and \$90,000 [2]. Thus, we calculated the average price to be \$52,500, giving us the following equation for cost of installation in dollars:



$$installationCost = 52,500 * numberOfChargers \quad (Eq. 4.6)$$

### 4.3.2 Operation Profit

We assumed that the profit of electric charging stations excluding the startup costs is similar to the profit of fueling stations (in this case, a station that provides gasoline and diesel fuel) since both electric charging stations and fueling stations provide similar products in a similar infrastructure. We then found the average revenue of diesel fuel, \$0.24 per gallon, with \$0.11 per gallon cents of profit, assuming the \$0.13 per gallon in utilities, rent, and wages for gas stations is true for diesel and electricity will result in a profit of \$0.10 per kWh [10; 6]. If each station will aim to make the same profit per semi-truck refueling with an electric as a diesel, then the station will need to charge \$0.10 per kWh given the average electric truck's battery holds 325 kWh and the average diesel truck holds 300 gallons [7; 2]:

$$operationProfit = 0.044 * timesCharged \quad (Eq. 4.7)$$

The *timesCharged* is a measurement of how many electric trucks will charge at a single station over the course of a year:

$$timesCharged = 365 * numberSemi * \left( \frac{distanceOfRoute}{200} \right) * \sum_n^{year} percentElectric \quad (Eq. 4.8)$$

Since fueling stations are open year round, then every day of the year they could be profitable. The *numberSemi* is the percentage of semi-trucks on the corridor which can be calculated by dividing the AADTT by the AADT. The *percentElectric* is the number of semi-trucks on the route that are electric which is calculated using Part I. The *year* is a measurement of how many years in the future the organization is wishing to examine. The *distanceOfRoute/200* is used to calculate the number of times each electric semi-truck needs to refuel on a route. The value 200 was calculated assuming that each semi-truck uses 80% of its charge between each charge [2].

### 4.3.3 Finance Return and Rank

As shown earlier, the financial return on investing from charging stations along a corridor is the difference between profit from operating the chargers for a certain number of years and the initial installation cost:

$$financeReturn = operationProfit - installationCost \quad (Eq. 4.9)$$

The greatest *financeReturn* value will result in a *financeRank* of 1 and continue sequentially until all of the corridors being compared have been ranked. The *operationProfit* will increase each year and begin to balance out the *installationCost*. While after one year the financial return for a smaller corridor may be greater, after 20 years the financial return for longer corridors will be greater.

## 4.4 Analysis and Conclusions

We calculated the motivation ratings and ranked them in ascending order for short-term projections looking 5 years into the future and long-term projections looking 20 years into the future. Our inputs were based on current data for the five corridors provided by MathWorks [3].

#### 4.4.1 Motivation Rank Analysis

##### San Antonio, TX to/from New Orleans, LA

The corridor travels through Texas and Louisiana, with approximately 26.8% of the corridor distance located in Texas and 73.2% of the distance located in Louisiana. Texas has a *stateMotivation* value of 35, while Louisiana has a *stateMotivation* value of 50:

$$motivationRating = 0.268 * 35 + 0.732 * 50 = 45.98$$

##### Minneapolis, MN to/from Chicago, IL

The corridor travels through Minnesota, Wisconsin, and Illinois with approximately 5.5% of the corridor distance located in Minnesota, 82.6% of the distance located in Wisconsin, and 11.9% of the distance located in Illinois. Minnesota has a *stateMotivation* value of 9, Wisconsin has a *stateMotivation* value of 13, and Illinois has a *stateMotivation* value of 28:

$$motivationRating = 0.055 * 9 + 0.826 * 13 + 0.119 * 28 = 14.57$$

##### Boston, MA to/from Harrisburg, PA

The corridor travels through Massachusetts, Connecticut, New York, and Pennsylvania, with approximately 13.4% of the corridor distance located in Massachusetts, 29.5% of the distance located in Connecticut, 18.7% of the distance in New York, and 38.4% of the distance located in Pennsylvania. Massachusetts has a *stateMotivation* value of 5, Connecticut has a *stateMotivation* value of 2, New York has a *stateMotivation* value of 3, and Pennsylvania has a *stateMotivation* value of 22:

$$motivationRating = 0.134 * 5 + 0.295 * 2 + 0.187 * 3 + 0.384 * 22 = 10.269$$

##### Jacksonville, FL to/from Washington, DC

The corridor travels through Florida, Georgia, South Carolina, North Carolina, and Virginia with approximately 1.4% of the corridor distance located in Florida, 17.9% in Georgia, 28.3% in South Carolina, 26.3% in North Carolina, and 26.1% in Virginia. Florida has a *stateMotivation* value of 39, Georgia has a *stateMotivation* value of 36, South Carolina has a *stateMotivation* value of 23, North Carolina has a *stateMotivation* value of 19, and Virginia has a *stateMotivation* value of 26:

$$\begin{aligned} motivationRating &= 0.014 * 39 + 0.179 * 36 + 0.283 * 23 + 0.263 * 19 + 0.261 * 26 \\ &= 25.282 \end{aligned}$$

##### Los Angeles, CA to/from San Francisco, CA

The corridor only travels through California, which has a *stateMotivation* value of 1:

$$motivationRating = 1 * 1 = 1$$

#### 4.4.2 Motivation Rank Conclusion

Based on the *motivationRating* values, the motivation rankings in ascending order for the corridors are as follows:

- |   |   |
|---|---|
| 1. Los Angeles, CA, to/from San Francisco, CA | 4. Jacksonville, FL, to/from Washington, DC |
| 2. Boston, MA to/from Harrisburg, PA          | 5. San Antonio, TX to/from New Orleans, LA. |
| 3. Minneapolis, MN, to/from Chicago, IL       |   |

#### 4.4.3 Finance Rank Analysis

We calculated the finance ratings and ranked them in descending order for short-term projections looking 5 years into the future and long-term projections looking 20 years into the future. Our inputs were based on current data for the five corridors provided by MathWorks [3]. The

*operationProfit* increases over time as the *percentElectric* changes and varies from corridor to corridor based on the *numberSemi*. We calculated the average *numberSemi* over the corridor to represent the route.

### **San Antonio, TX to/from New Orleans, LA**

Using the result from Part II, the cost for the installation of charging stations is as follows:

$$installationCost = 68 * 63 * 52,500 = 224,910,000 \text{ or } \$224.9 \text{ million}$$

Using the results from Part I and the provided data, the profit of operation is as follows:

$$operationProfit = 0.044 * 365 * 0.48 * 14,288 * (544/200) * \sum^5 percentElectric = 301,749 \text{ or } \$301,749 \text{ after 5 years}$$

$$operationProfit = 0.044 * 365 * 0.48 * 14,288 * (544/200) * \sum^{20} percentElectric = 3,140,146 \text{ or } \$3.14 \text{ million after 20 years}$$

$$financeReturn = -\$224,608,251 \text{ for 5 years and } financeReturn = -\$221,769,854 \text{ for 20 years}$$

### **Minneapolis, MN to/from Chicago, IL**

Using the result from Part II, the cost for the installation of charging stations is as follows:

$$installationCost = 51 * 86 * 52,500 = 230,265,000 \text{ or } \$230.3 \text{ million}$$

Using the results from Part I and the provided data, the profit of operation is as follows:

$$operationProfit = 0.044 * 365 * 0.48 * 16,022 * (408/200) * \sum^5 percentElectric = 253,777 \text{ or } \$253,777 \text{ after 5 years}$$

$$operationProfit = 0.044 * 365 * 0.48 * 16,022 * (408/200) * \sum^{20} percentElectric = 2,640,927 \text{ or } \$2.64 \text{ million after 20 years}$$

$$financeReturn = -\$230,011,223 \text{ for 5 years and } financeReturn = -\$227,624,073 \text{ for 20 years}$$

### **Boston, MA to/from Harrisburg, PA**

Using the result from Part II, the cost for the installation of charging stations is as follows:

$$installationCost = 49 * 49 * 52,500 = 126,052,500 \text{ or } \$126.1 \text{ million}$$

Using the results from Part I and the provided data, the profit of operation is as follows:

$$operationProfit = 0.044 * 365 * 0.48 * 9,293 * (390/200) * \sum^5 percentElectric = 140,700 \text{ or } \$140,700 \text{ after 5 years}$$

$$operationProfit = 0.044 * 365 * 0.48 * 9,293 * (390/200) * \sum^{20} percentElectric = 1,464,199 \text{ or } \$1.46 \text{ million after 20 years}$$

$$financeReturn = -\$125,911,800 \text{ for 5 years and } financeReturn = -\$123,588,301 \text{ for 20 years}$$

### **Jacksonville, FL to/from Washington, DC**

Using the result from Part II, the cost for the installation of charging stations is as follows:

$$installationCost = 88 * 70 * 52,500 = 323,400,000 \text{ or } \$323.4 \text{ million}$$

Using the results from Part I and the provided data, the profit of operation is as follows:

$$\text{operationProfit} = 0.044 * 365 * 0.48 * 9,515 * (706/200) * \sum^5 \text{percentElectric} = 260,789 \text{ or } \$260,789 \text{ after 5 years}$$

$$\text{operationProfit} = 0.044 * 365 * 0.48 * 9,515 * (706/200) * \sum^{20} \text{percentElectric} = 2,713,895 \text{ or } \$2.71 \text{ million after 20 years}$$

$$\text{financeReturn} = -\$323,139,211 \text{ for 5 years and } \text{financeReturn} = -\$320,686,105 \text{ for 20 years}$$

**Los Angeles, CA to/from San Francisco, CA**

Using the result from Part II, the cost for the installation of charging stations is as follows:

$$\text{installationCost} = 48 * 51 * 52,500 = 128,520,000 \text{ or } \$128.5 \text{ million}$$

Using the results from Part I and the provided data, the profit of operation is as follows:

$$\text{operationProfit} = 0.044 * 365 * 0.48 * 13,974 * (382/200) * \sum^5 \text{percentElectric} = 207,233 \text{ or } \$207,233 \text{ after 5 years}$$

$$\text{operationProfit} = 0.044 * 365 * 0.48 * 13,974 * (382/200) * \sum^{20} \text{percentElectric} = 2,156,570 \text{ or } \$2.15 \text{ million after 20 years}$$

$$\text{financeReturn} = -\$128,312,767 \text{ for 5 years and } \text{financeReturn} = -\$126,363,430 \text{ for 20 years}$$

**4.4.4 Finance Rank Conclusion**

Based on the *financeReturn* for 5 years we would rank the corridors in the following order:

- |  |  |
|--|--|
| 1. Boston, MA to/from Harrisburg, PA         | 4. Minneapolis, MN to/from Chicago, IL     |
| 2. Los Angeles, CA to/from San Francisco, CA | 5. Jacksonville, FL to/from Washington, DC |
| 3. San Antonio, TX to/from New Orleans, LA.  |  |

Based on the *financeReturn* for 20 years we would rank the corridors in the following order:

- |  |   |
|--|---|
| 1. Boston, MA to/from Harrisburg, PA         | 4. Minneapolis, MN to/from Chicago, IL  |
| 2. Los Angeles, CA to/from San Francisco, CA | 5. Jacksonville, FL to/from Washington, |
| 3. San Antonio, TX to/from New Orleans, LA   |   |

**4.4.5 Corridor Rank Analysis**

For the purposes of this analysis, we weighted the financial impact more heavily than public opinion with *motivationWeight* equal to 0.1 and *financeWeight* equal to 0.9 (the *financeRank* after 5 years and after 20 years are each weighted as 0.45 of the final ranking) because it is primarily financial incentives that would motivate the installation of chargers. However, users of our model can adjust the weighting factors to match their priorities.

**San Antonio, TX to/from New Orleans, LA**

Combing the rankings from the results above, we get the following:

$$\text{corridorRanking} = 0.1 * 5 + 0.45 * 3 + 0.45 * 3 = 3.2$$

**Minneapolis, MN to/from Chicago, IL**

Combing the rankings from the results above, we get the following:

$$\text{corridorRanking} = 0.1 * 3 + 0.45 * 4 + 0.45 * 4 = 3.9$$

**Boston, MA to/from Harrisburg, PA**

Combing the rankings from the results above, we get the following:

$$\text{corridorRanking} = 0.1 * 2 + 0.45 * 1 + 0.45 * 1 = 1.1$$

**Jacksonville, FL to/from Washington, DC**

Combing the rankings from the results above, we get the following:

$$\text{corridorRanking} = 0.1 * 4 + 0.45 * 5 + 0.45 * 5 = 4.9$$

**Los Angeles, CA to/from San Francisco, CA**

Combing the rankings from the results above, we get the following:

$$\text{corridorRanking} = 0.1 * 1 + 0.45 * 2 + 0.45 * 2 = 1.9$$

**4.4.6 Corridor Rank Conclusion**

Based on the *corridorRanking*, we would rank the corridors in the following order:

- |  |  |
|--|--|
| 1. Boston, MA to/from Harrisburg, PA         | 4. Minneapolis, MN to/from Chicago, IL     |
| 2. Los Angeles, CA to/from San Francisco, CA | 5. Jacksonville, FL to/from Washington, DC |
| 3. San Antonio, TX to/from New Orleans, LA   |  |

**5 - Conclusions**

In conclusion, assuming the infrastructure for electric semi-trucks already exists, we found that after 5 years, 32.7% of semi-trucks would be electric. After 10 years, 59.2% would be electric and after 20 years, 80.7% would be electric. Our results for the number of required stations and chargers showed that the Jacksonville, FL to/from Washington, DC route required 87.2 charging stations and 69.6 chargers per station; the Boston, MA to/from Harrisburg, PA route required 48.2 charging stations and 48.6 chargers per station; Los Angeles, CA to/from San Francisco, CA route required 47.2 charging stations and 50.1 chargers per station; San Antonio, TX to/from New Orleans, LA route required 67.2 charging stations and 62.1 chargers per station; and Minneapolis, MN to/from Chicago, IL route required 50.4 charging stations and 85.0 charges per station. For the given corridors, we concluded that the rankings for targeted development are as follows: (1) Boston, MA to/from Harrisburg, PA; (2) Los Angeles, CA to/from San Francisco, CA; (3) San Antonio, TX to/from New Orleans, LA; (4) Minneapolis, MN to/from Chicago, IL; (5) Jacksonville, FL to/from Washington, DC.

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