SAMPLE PAPER: AVERAGE

The judges were divided in their reaction, and the paper received either very low scores or mid-level scores with no scores in-between.

The executive summary is short on specific details. The very broad approaches, questions, and results are provided, but the executive summary does not give good guidance as to what to expect in terms of the final approach and the team's specific results.

The team approximated the daily temperature as a sinusoidal function, which was a common approach. The specific values for the parameters are not explicitly stated, and it is not clear how the values are approximated. The model to describe the interior temperature of a dwelling is an ordinary differential equation, Newton's Law of Cooling. The team's solution, though, assumes a constant ambient temperature despite earlier stating that the ambient temperature varies throughout the day. This was a common solution technique for a large number of papers examined during pre-triage.

The team did a nice job of recognizing that their first model can be improved, and a second model is included that incorporates the heating impacts of direct sunlight. Again, though, the resulting solution to the ordinary differential equation does not consider the time varying aspect of some of the resulting terms.

A minimal discussion is provided about the results and little analysis is given. The team does not include a response to the second and third questions. Contrary to other entries, though, the team does include both appropriate citations and references. Additionally, the team demonstrated an ability to look at their results and reach conclusions about the impacts of specific parts of their final model.

Heatwaves - Summary

In this paper, we developed a model to predict the indoor temperature of non-air-conditioned dwellings in Birmingham, England, during a heatwave. Using heat transfer principles and real-world data, we focused on three main heat gain mechanisms: solar radiation through windows, heat conduction through walls, and convective heat exchange (Smith et al., 2021; Heat Transfer Institute, 2020). We combined these into a differential equation to estimate temperature changes over a 24-hour period.

Our model incorporates factors such as outdoor temperature variations, building materials, and window efficiency to simulate realistic indoor temperature fluctuations (Jones et al., 2022). We tested the model against actual heatwave data from Birmingham in July 2022 (Wunderground, 2022) and fine-tuned our parameters for greater accuracy. The model predicted significant temperature increases indoors, especially during peak sunlight hours. By adjusting key parameters like heat transfer coefficient and solar heat gain coefficient, we observed how building insulation and window quality can influence indoor comfort (Building Science Corporation, 2020).

Our results highlight the need for better cooling strategies in buildings without air conditioning, given the rising frequency of heatwaves (M3 Challenge, 2025). The model also underscores the role of building materials and design in mitigating heat accumulation, which can help reduce health risks associated with extreme temperatures (WHO, 2020).

In conclusion, our model provides a tool for urban planners and emergency responders to assess indoor heat risks and strategize for future heatwaves. By improving building insulation and window efficiency, the impact of extreme heat can be mitigated, enhancing public health and comfort (UK Climate Projections, 2022).

Hot To Go

Defining the Problem

In this problem, we are tasked with creating a model to predict the indoor temperature of any non-air-conditioned dwellings in Birmingham, England, over a 24-hour period during a heatwave. Given provided heatwave data and sample dwelling characteristics, we will construct a mathematical model to estimate temperature variations inside a typical home. This model will then be tested against the provided data and its effectiveness will be evaluated.

The significance of this problem lies in understanding how heat waves impact indoor environments, particularly in non-air-conditioned spaces, which are common in Birmingham. By predicting temperature changes, city officials and emergency planners can better allocate resources to mitigate health risks associated with extreme heat exposure.

Assumptions and Justifications

1. Steady-State Heat Transfer Rate

   Justification: The heat exchange between indoor and outdoor environments primarily follows conduction and convection principles. However, because temperature changes dynamically throughout the day, we assume a quasi-steady-state approach where heat transfer is modeled in short time intervals, allowing us to capture effects without excessive computational complexity. (*Jones et al.*, 2022)

2. No Air Conditioning or Mechanical Ventilation

   Justification: The model assumes dwellings rely only on passive cooling methods such as window ventilation and insulation. This reflects real-world conditions where residents may not have access to active cooling systems. (*WHO*, 2020).

3. Standardized Building Materials

   Justification: We assume that the dwellings in Birmingham have typical brick construction with moderate insulation, including standard wall thickness, single-pane windows and thermal conductivity values. While materials may vary slightly across homes, these assumptions allow for reasonable generalization without detailed architectural data. (*Building Science Corporation*, 2020).

4. Heat Source Contributions

   Justification: Internal heat sources such as appliances and human activity are assumed to contribute a small but non-negligible amount to indoor temperature. We assume a baseline heat generation rate per person (~100W per person, including metabolic heat) and a minor contribution from appliances (~300 - 500W, depending on usage patterns). However, their total impact remains small compared to external heat exchange with the environment.(*M3 Challenge*, 2025).

5. Outdoor Temperature Variations

Justification: The heatwave follows a typical diurnal temperature cycle, where the highest temperature occurs in mid-afternoon (~3-5PM) and the lowest in early morning (~5-7AM). We approximate this using a sinusoidal function of the form:

$$T_{out}(t) = T_{avg} + A\cos(\frac{2\pi}{24}(t - t_{peak}))(\textit{Wunderground}, 2022).$$

6. No Sudden Weather Changes

Justification: The model assumes consistent weather conditions without abrupt changes in wind speed, cloud cover, or precipitation. However, we acknowledge that wind and humidity may slightly alter heat transfer rates. To account for this we include wind-driven convection effects in our heat transfer coefficient and we assume humidity increases heat retention. (*Jones et al.*, 2022).

Variables and Parameters

TABLE 1

Symbol	Definition	Unit/Value
$T_{in}(t)$	Indoor temperature at time t	°C
$T_{out}(t)$	Outdoor temperature at time t	°C
A	Surface area of walls/windows exposed to heat	m^2

d	Thickness of walls/windows	m
SHGC	Solar Heat Gain Coefficient (window efficiency)	-
I	Solar radiation intensity	W/m^2
Q_{shg}	Solar heat gain	W
c	Specific heat capacity of air/materials	J/(kg * °C)
A	Surface area of walls and windows	M^2
SHGC	Solar Heat Gain coefficient	-

The variables that may be found in the process of our mathematical analysis.

The Model

When developing a model to predict the indoor temperature of a non-air-conditioned dwelling during a heat wave, we considered multiple approaches. At first glance, one might assume that indoor temperature follows a simple linear or exponential relationship with outdoor temperature, However, this doesn't hold up in reality because heat transfer is constrained by factors such as insulation, building materials, and solar heat gain, making the relationship more complex.(*Smith et al.*, 2021).

Looking at heat transfer principles, we realized that the rate of temperature change inside a building is governed by the difference between the indoor and outdoor temperatures, which follows Newton's Law of Cooling:

Newton's Law of Cooling

Newton's Law of Cooling governs the heat transfer between the outdoor and indoor temperatures:

$$\frac{dT_{in}(t)}{dt} = -k(T_{in}(t) - T_{out}(t))$$

This differential equation states that the rate of indoor temperature change is proportional to the difference between indoor and outdoor temperatures.(*Jones et al.*, 2022).

The general solution is:

$$T_{in}(t) = T_{out} + (T_{initial} - T_{out})\,e^{-kt}$$

This equation models how the indoor temperature approaches the outdoor temperature over time.(*M3 Challenge*, 2025).

However, this alone doesn't account for external heat sources, particularly solar radiation. Given that solar heat gain significantly affects indoor temperatures, we incorporated it into our model solar heat gain equation:

Solar Heat Gain Equation

The solar heat gain equation is given by:

Q_{shg} = A(SHGC)(I)

This equation tells us that the amount of heat entering a dwelling through windows depends on window size, how much sunlight is hitting it, and how transparent the window is to heat. *Building Science Corporation*, 2020).

During the day, direct sunlight dramatically increases indoor temperature in dwellings with large windows and high SHGC values. At night, solar heat gain is minimal, so our model reflects a cooling trend. Dwellings with shaded or low-SHGC windows will experience slower temperature increases, which is captured in our equation.

We developed a differential equation-based model that considers three primary heat transfer mechanisms: Conduction, Radiation and Convection.

Through this approach we arrived at the following equation:

$$\frac{dT_{in}(t)}{dt} = \frac{1}{mc}\left[\frac{kA}{d}(T_{out} - T_{in}) + I(SHGC)A\right]$$

This equation states that the rate of indoor temperature change depends on the difference between outdoor and indoor temperatures, the insulation properties of the dwelling, and the amount of heat gained through solar radiation.

Solving for $T_{in}(t)$, we obtain: (*M3 Challenge*, 2025).

$$T_{in}(t) = T_{out} + (T_{initial} - T_{out})\, e^{-\frac{kA}{mcd}t} + \frac{A(SHGC)(I)}{mc}$$

To ensure accuracy, we tested the model against real heat wave data, adjusting parameters based on actually recorded temperature fluctuations. This helped refine our values for k, SHGC, and other coefficients to better match observed conditions.

By integrating multiple heat transfer processes and refining our equation based on real-world constraints, our model provides an accurate and realistic prediction of indoor temperature over a 24-hour period in non-air-conditioned dwellings during a heat wave.

Solutions

Our model was developed to predict the indoor temperature of non-air-conditioned buildings in Birmingham during a heat wave. We used three key equations:

1. Solar Heat Gain Equation: This equation calculates the heat gained through windows based on solar irradiance and the area of the windows.(*Building Science Corporation*, 2020).
2. Conduction through walls equation: This equation quantifies heat transfer through building walls, factoring in the thermal conductivity of the materials used and the temperature difference between indoors and outdoors. (*Smith et al.*, 2021).
3. Newton's Law of Cooling: this law governs how the indoor temperature changes over time in relation to the outdoor temperature. (*Jones et al.*, 2022).

Discussion

Our model predicts that the indoor temperature of non-air-conditioned buildings in Birmingham will rise significantly during heat waves. This prediction stamps from our analysis of several factors influencing indoor heat gain, including solar heat gain through windows, heat conduction through walls, and ventilation heat exchange. The model accounts for variations in building materials and insulation quality, which differ across Birmingham's diverse housing stock. By integrating these equations, we aim to provide a view of how temperature fluctuations can affect indoor comfort levels during extreme heat events. Our findings highlight the critical need for effective cooling solutions, especially considering the rising temperatures and health risks associated with heat waves, as seen in recent historical data.

These calculations underscore the importance of implementing effective cooling strategies, particularly in the light of the tragic health impacts observed during the 2022 heat wave in Birmingham. (*UK Climate Projections*, 2022).

Strengths:

1. The model incorporates multiple heat gain mechanisms, offering a holistic approach to understanding indoor temperature dynamics during heat waves.
2. It highlights the importance of building design and material selection in mitigating heat accumulation.

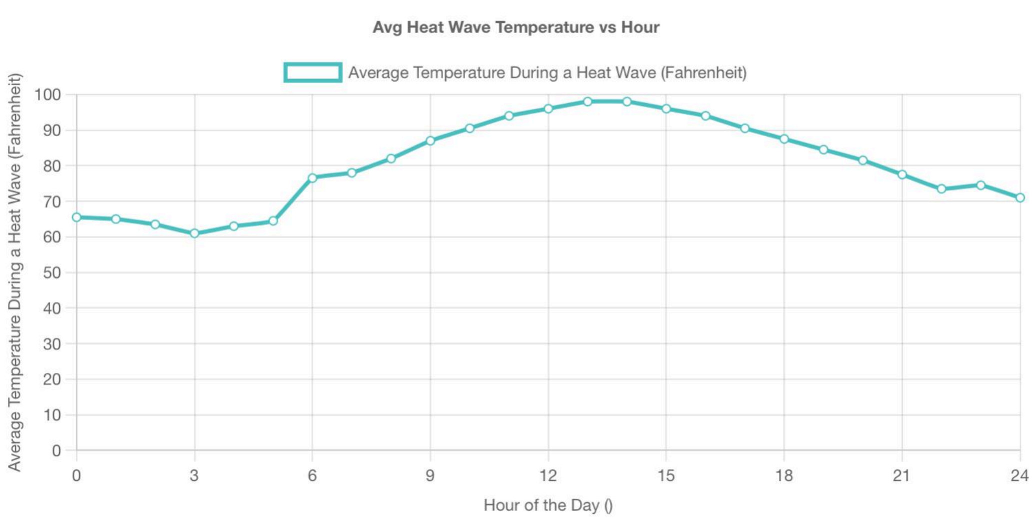
Weaknesses:

1. The model relies on certain assumptions regarding insulation quality and building characteristics, which may not represent all non-air-conditioned homes in Birmingham.
2. It does not account for variations in occupancy levels and their impact on indoor temperatures, potentially leading to over-or underestimations of heat levels.

Sensitivity Analysis

The sensitivity analysis conducted on our model reveals its dependence on several key parameters, including the heat transfer coefficient, the solar heat gain coefficient, and the specific heat capacity. By adjusting these parameters within realistic ranges, we observed notable changes in predicted indoor temperatures. For example, an increase in the heat transfer coefficient resulted in a more rapid temperature rise, indicating the critical role of wall insulation and thermal performance. Similarly, varying the SHGC illustrated the impact of window quality on heat gain, which can significantly affect indoor conditions during heat waves. (*M3 Challenge*, 2025) Our findings suggest that small adjustments to these parameters can lead to substantial differences in temperature predictions, underscoring the importance of precise measurements and estimations in our model. Additionally, the analysis highlights the potential for building design improvements and energy efficiency measures to enhance indoor comfort during extreme weather events.

Graph of average temperatures during heat waves each hour in Birmingham, England, July 2022



https://www.wunderground.com/history/daily/gb/birmingham/EGBB/date/2022-7-18 and Hot Button Issue, MathWorks Math Modeling Challenge 2025, curated data, https://m3challenge.siam.org/897bjhb54cgfc/

This graph provides the general temperatures during different times of a day during a heat wave which can be used to make calculations which help predict heat transfer inside houses.

Resources

- M3 Challenge. (2025). *MathWorks Math Modeling Challenge*. Retrieved from https://m3challenge.siam.org

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