Judge's Commentary

MathWorks Math Modeling Challenge 2025

Hot Button Issue: Staying Cool as the World Heats Up

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Introduction

This year's MathWorks Math Modeling Challenge focused on the growing impact of extreme heat waves. Student teams explored three questions centered around the changing temperature within a family dwelling, the peak demand for electricity, and how to compare the broader impact across multiple communities. The students' work was outstanding, marked by creativity, perseverance, and analytical rigor. We are deeply grateful for their efforts, which continue to be an inspiration, and we are especially thankful for the teachers and family members who support them throughout the process.

This report offers a broad summary of the students' submissions, including observations as well as insights into emerging trends compared to previous years. The first section is a brief statement about the overall improvement in the quality of the summaries. The sections that follow delve into observations related to the three questions in this year's problem statement. The last section provides general comments on the papers.

Summary

The quality of the summaries showed a noticeable improvement this year. A significant number were very good, and an excellent summary can make an immediate positive impression. Compared to past years, summaries more often included an overview of the problem, a general description of the models and mathematical techniques used, and a concise presentation of the team's findings.

However, one recurring issue was the formatting of the summaries. As the first part of the paper that a reader encounters, the summary should be clearly set apart from the rest of the document. Teams should assume the reader is unfamiliar with the context and the problem statement. In addition, the summary should be limited to one page and, ideally, should appear on a page by itself.

Question One

To address the first question, students had to create a model to approximate the temperature within a dwelling. Students were given temperature data for each hour of a day and were asked to determine how the temperature changes over time for different dwellings. Most teams approached the problem by modeling the exchange of energy between the interior of a dwelling and the exterior of the dwelling. It was essential for all elements—especially the units—to be consistent.

In any model, an important step is to define the state variables of the physical system. For question one, the interior and exterior temperatures were commonly used to define the state of the physical system. In this situation, the notation could be difficult to follow. For example, I read one paper in which the interior temperature at a given time, *t*, was denoted $T^{t}_{interior}$, and the exterior temperature was denoted

$T^t_{exterior}$. Identifying the appropriate variables and defining the notation was a challenging task. Teams that clearly defined their variables, used consistent notation, and chose symbols that were easy to read had an immediate advantage in effectively communicating their work to the reader.

Once the state variables and notation are defined, another important step is deciding which fundamental principles to use to establish the model. A common approach is to conserve some important quantity. For example, mass, momentum, and energy are conserved quantities. Most students decided to conserve energy in some form when developing their model to address question one.

There were essentially two ways that teams applied the principle of conservation of energy: discrete in time or continuous in time. Teams that adopted a discrete modeling approach typically based their work on the idea that the change in temperature over a given time interval is related to the total energy exchanged between the interior and exterior during that period. It was generally assumed that the energy exchanged is proportional to the difference in the external and internal temperatures. A key aspect of this approach is that the constant of proportionality should include a time scaling factor. If energy is added from the exterior over a longer time span, the resulting temperature change should be greater. Importantly, the units associated with this energy exchange should be consistent, such as Joules, to ensure correctness.

Teams that used a continuous modeling approach typically relied on the idea that the rate of change of interior temperature is proportional to the temperature difference between the interior and exterior of the dwelling. This method was most often based on Newton's Law of Cooling. In this framework, the units associated with the rate of temperature change should be units of power, such as Watts, to ensure consistency.

In either approach, the first check that a reader can perform is to verify unit consistency. Teams that clearly stated their units and maintained consistency throughout their models stood out immediately. Conversely, inconsistencies in units often signaled a fundamental flaw in the model and were quickly noticed by the judges. For example, some teams used the Stefan-Boltzmann equation to model energy exchange as a function of temperature. However, this equation assumes that temperature is measured in Kelvin. A team using this model while expressing temperatures in Celsius would immediately draw attention due to the resulting inconsistency in units.

Additional problems arose from inconsistencies with the underlying assumptions of the models. One common issue involved the use of Newton's Law of Cooling. While there is a known solution to the differential equation, it assumes a constant external temperature. Several teams used this solution even though their models included a varying external temperature. A solution derived by assuming the exterior temperature is constant is not consistent with the situation outlined in the problem statement.

Once the fundamental principles and assumptions are defined, the next step is to derive a consistent model. One hurdle was to approximate the exterior temperature as a function of time throughout the day. Student teams were provided with external temperature data at regular time intervals. The vast majority of teams used a sinusoidal function to mimic the periodic daily temperature changes, while some opted for high-order polynomials. In this instance, the approximation uses interpolation over a fixed time span, so either method can be effective if it is properly tested and justified. Simply examining the residuals of the approximation can be very helpful in supporting its validity. It should be noted that, given the nature of the data provided, linear interpolation is another valid approach. While I am only aware of a small number of teams that used this approach, it offers a simple and effective way to work with the given data.

Question Two

To address the second question, teams had to create a model to approximate the peak demand on a city's power grid. Most teams treated it as a data modeling problem to extrapolate 20 years into the future. They analyzed electricity consumption but often did not clearly define what they meant by "peak demand." Since different teams interpreted the term in various ways, it was sometimes difficult to follow a team's work and put their results in context.

Most teams assumed a relatively straightforward model and then used regression to estimate the parameters given the data that was made available to the teams. For example, one common approach was to list several variables that a team felt were important and then use multivariate regression to determine the specific relationship. This can be problematic, given that the task is to extrapolate far into the future. Given the short time and the difficulty in obtaining data, this is a good first step. Teams that provided appropriate caveats along with potential ways to improve their results tended to make a more positive impression.

Many teams noticed that the consumption of electricity did not increase for later dates of the Memphis data and adjusted their models appropriately. This is an important observation and demonstrates the value of exploring data and trying to gain insights from the information available. Teams that were able to explain this behavior and further refine their models demonstrated an even deeper understanding of the interplay between modeling and data exploration. For example, some teams noted that the introduction of LED lighting and other efficiency gains might represent a potential shift in electricity usage patterns, supporting the use of a logistic model for electricity demand.

In addition to regression methods, many teams used autoregressive integrated moving average (ARIMA) or seasonal autoregressive integrated moving average (SARIMA) to approximate future electricity use. While these approaches may provide a reasonable approximation of future trends, their use and appropriateness should be clearly justified. For example, some teams discussed the underlying assumptions associated with the method, and some teams implemented tests to determine if the data was consistent with the assumptions.

Additionally, these methods have several parameters that must be defined before implementing them. The parameters should be discussed, including a rationale for how and why the team determined their values. Once implemented, the results should be examined closely and checked to verify they are consistent with the data. Finally, enough information should be provided in the narrative to ensure that a reader could repeat and verify the stated results.

The use of SARIMA may be appropriate and can provide a reasonable approximation of future trends. However, the model itself does not necessarily offer good insights into which aspects of the system most influence those trends. When these methods are used, it becomes even more important to carefully examine and scrutinize the results. The team bears an additional responsibility to identify the parameters with the greatest impact and to explore the implications of the results in greater detail.

Question Three

To address the third question, teams had to create a “vulnerability score” to help policymakers better allocate resources in response to extreme heat-related events. This is a difficult question to address and is open to interpretation. Similar questions have been asked in past M3 Challenge events, but this year the teams did a much better job of justifying their choices.

Most teams used a linear combination of various factors, which is a common approach that has been used in past events. Teams often described several factors deemed important, and then a model was defined as a linear combination of the different factors. Unlike past years, though, a larger number of teams took a much more structured approach to determine the values of the coefficients in their final model.

For example, more teams normalized each of the factors they explored. The most common normalization was to define a linear function for each factor to map the values of the factor to the interval [0,1]. This is an important step, and it highlights the need to balance the factors so that one factor does not dominate the final expression simply because it has a larger range of values.

Once the normalization is defined, a set of coefficients must be determined to be used in the final linear combination. Many teams explored the relationships between the factors. Some teams used linear regression and made the coefficients in their final model consistent with the parameters from the regression. Other teams examined a correlation matrix to compare the different factors and made their coefficients proportional to the values found in the correlation matrix. A small number of teams used principal component analysis. Each of the approaches can be good, and teams that clearly described an appropriate methodology made a positive impression. It is important to note that for each approach, a team has the burden to explicitly state the relationships that are assumed and to state how their final result provides a robust relationship between the stated variables.

The increasing use of such methods to guide the assignment of coefficients in the final model is a notable trend. Using a linear model is a straightforward, common, and effective strategy, but determining appropriate coefficients is a challenge. A structured, robust approach that aligns the final model with the data helps ensure that the results are consistent and result in a reliable measurement.

Finally, in addition to a linear combination of relevant factors, a small number of teams opted to use a Cobb-Douglas production model. This is another potentially good option. The same methods discussed above can then be used to determine the coefficients associated with the model.

General Modeling Issues

A couple of general observations about a few larger trends are provided here. First, the responses to the three questions tended to be independent of one another. This was surprising because the three questions are closely related. For example, the teams that used their response to the first question to help determine the peak electricity demand tended to make a more positive impression. The ability to interpret a model and then expand on it is an important skill that demonstrates a team understands the model and the implications of the model.

Secondly, teams that adopted an existing method while recognizing and accounting for its assumptions also tended to make a stronger impression. Adapting existing work is valuable, but it is important to remember that every technique is designed with a specific context in mind. It is important to explicitly acknowledge that context and then demonstrate that the current situation is consistent with the method's requirements. This includes clearly identifying the assumptions associated with a given method and verifying that the necessary conditions are met.

Finally, discussions of a model's sensitivity tended to be better compared to previous years. Every year, many teams discuss the sensitivity of their models. Those discussions do not always provide explicit insights into which inputs have the greatest impact on the outputs of the model. This year, however, teams more commonly provided a better and more structured discussion of the sensitivity of their models. A straightforward discussion showing how small changes to the model or data affect the results offers valuable insight into the model's robustness. It is encouraging to see this growing trend toward more insightful discussions of this aspect of the teams' models.

Conclusion

Overall, there were encouraging trends in the papers submitted in this year's event. The models were relatively straightforward and motivated by the underlying physical principles dictated by the physical context. Students did a good job of justifying their approaches. Also, the level of analysis of the models is improving.

Two issues that stood out to the judges were the consistency of units in the development of the models and the alignment of the context with the underlying assumptions of the models and techniques employed by the student teams.

With respect to the full submissions, there were improvements in the summaries as well as the analysis of the final models. Students are getting better at preparing for M3 Challenge and are offering a more comprehensive analysis of good, basic models. In doing so, students are offering more insight into the phenomena they have studied.

The continual improvements and better preparation are made possible by the dedication of those who support our students. Teachers, parents, and others who give them the time, encouragement, and space to challenge themselves are doing something special. We are all grateful for your efforts and recognize the difference you are making in these students' lives. Thank you.

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Finally, I am deeply grateful for the continued support of MathWorks. Beyond providing resources, they are a direct and invaluable partner in this work.