MathWorks Math Modeling Challenge 2025

North Carolina School of Science-Math

Team #17928, Durham, North Carolina

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M3 Challenge RUNNER UP—$15,000 Team Award

JUDGE COMMENTS

Specifically for Team #17928—Submitted at the close of triage judging

COMMENT 1: The team provides a very good executive summary, highlighted the model they come up with and demonstrate the result they derive from it.

In general, the paper has a well-defined structure for math model setup and analysis. It delivers clearly what model they have used and how sensitive their model is. It is exceptionally well written with variables defined, and math formula defined. It includes well-defined assumptions and justification, and provides the sensitive/strengths/weakness analysis for their model.

The team follows Newton' Law to set up the heat model for the first question. the team could use more real data to validate the model, that would make the model more sound.

The second question employs logarithmic and quadratic regression to predict the peak power demand. It needs more description on cool and not cool part for readers to understand its nature.

For the vulnerability, the team includes 5 factors with a linear formula and normalizes them. If the team could mathematically demonstrate those 5 factors are more prominent than others, that would be great!

COMMENT 2: Well stated and reasoned assumptions. Good discussion of modeling and explanations of the parameters involved. Nicely presented plots. Good discussion of strengths and weaknesses

COMMENT 3: Great job. Well-written. Highly organized.

Executive Summary

Dear Memphis City Authorities,

We understand that ensuring the safety for all citizens within your city limits is one of your top priorities. Our team conducted some extensive research and performed some intricate mathematics on how to quantitatively determine the most vulnerable neighborhoods, so that you can devote your attention and funding to them.

First, we used a mathematical formula called **Newton's Law of Heating and Cooling** to determine how heat flows between the inside and outside of a house *if the house had no HVAC system at all.* In Memphis weather, where external temperatures can easily rise above 100 degrees Fahrenheit, our research indicates that a house initially at 78 degrees Fahrenheit only takes 3 to 5 days to match the outdoor temperature, even with dropping temperatures overnight. Since living in 90 degree weather is unbearable, our finding emphasizes the importance of investing in quality HVAC systems, regardless of neighborhood affluence.

Next, we separated peak energy demand into cooling and non-cooling. We found energy demand from cooling to be a function of temperature, which is itself a function of time. For the non-cooling aspect, we separated energy demand into each sector and found the non-cooling proportion using data from the EIA. We used various **logarithmic and quadratic regressions** to forecast sector energy usage in 2045. Our model concluded the peak energy demand in Memphis, Tennessee to be 91466 and 82972 MWh in 2025 and 2045 respectively. The lower energy demand in 2045 is likely because of a decrease in industry and general increase in energy efficiency.

Lastly, we chose to develop a **vulnerability model** to assign a numeric value to how vulnerable each of Memphis's twenty-seven neighborhoods are. We considered the five largest factors—average income I, age dependency ratio E, average number of cars C, average house age Y, and average house temperature T (acquired from our first model)—and used the equation $V = \frac{EYT}{IC}$ to get a vulnerability number, where a higher number represents a more vulnerable neighborhood. Our model concludes that **South Memphis** is the most vulnerable to heat-related injuries, since it had the lowest value for V. Therefore, funding and safety efforts should be directed to South Memphis, as opposed to safer parts of the city like Collierville/Piperton.

Thank you for taking the time out of your busy day to read this letter. If you have any questions about our research, or wish to otherwise chat with us, please let us know!

Best regards,
Team #17928

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Hot Button Issue: Staying Cool as the World Heats Up

1 Q1: Hot to Go

1.1 The Problem

The first problem asks us to develop a model to predict the indoor temperature of any non-air-conditioned dwelling during a heat wave over a 24-hour period. We have selected Memphis, Tennessee as our city. Our model takes into account previous heatwave temperatures and housing data.

1.2 The Datasets

In the first dataset, we are given four example units and their descriptions [1]. For each unit, the following information is provided:

- When the unit was built in years. We will denote this as t. Note that our dataset only includes data from 1950 to the present.
- How many stories there are in the unit. We will denote this as "stories".
- How many units are in the structure (think about an apartment). We will denote this as "units".
- The dimensions of the unit in square meters. We will denote this as L.

The second dataset, which is just in another tab of the google spreadsheet, gives the approximate temperatures for a heat wave during a 24 hour period. We will use these values in our model for finding the temperature values in the house.

1.3 Assumptions and Justifications

Assumption 1.1: There are no HVAC systems in buildings.
Justification: Given in the Problem Statement.

Assumption 1.2: All units are rectangular prisms with square bases and height 9 feet (denoted in Figure (1)).
Justification: Spruce says the typical height inside of a home is 9 feet [7]. By assuming the floor plan of the unit is a square, we can calculate the area of the exposed walls easier.

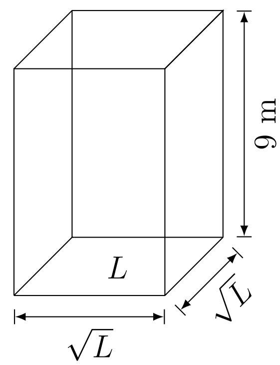

Figure 1: Layout of a unit under our assumption.

Assumption 1.3: The initial starting temperature of the dwelling is $78°C$.
Justification: This is the average temperature of a household in Memphis; that is, the temperature at 12:00 AM on the first night.

Assumption 1.4: E, a function to denote the number of exposed sides on a unit, decreases linearly as the computed value of $\frac{\text{number of units}}{\text{number of stories}}$ increases. Furthermore, each exposed side has the same surface area as the unit.
Justification: A ranch house with 3 units on one story will have more exposed area than a 3x3 cube of units (picture a Rubik's cube) where the interior unit has no exposed sides. It is not unreasonable to assume that the number of exposed sides and the number of units per story are inversely proportional.

Assumption 1.5: The number of rooms in a particular unit is irrelevant.
Justification: From a scientific standpoint, we define our system of analysis to consist of a single unit. We do not care how the interior of our system is laid out, nor what goes on within the unit.

Assumption 1.6: Our scale of analysis for our houses is 1950 to 2024 (74 years). In 1950, all houses were made of purely brick. In 2024, all houses were made of purely wood. The thermal conductivity and specific heat capacity of these two materials changes linearly with respect to time.
Justification: We chose the timescale 1950 to 2024 because our data primarily fell within these 74 years. In the 1950s, many Memphis homes were built using solid brick due to the availability of clay in the region [11]. Over time, houses gradually modernized to their current form of being nearly composed entirely of wood [13].

While it may sound absurd to work with a constantly changing thermal conductivity and specific heat capacity, this is an average consideration of all houses across the city (some houses will be newer than others—and with our definition, "newer" means they consist more of wood.) It's also important to recognize that finding exact data on what houses were made of at any specific year in Memphis is exceptionally difficult.

Lastly, please note that we used Fahrenheit for all of our temperature values in our computational data. Memphis is in America, after all!

1.4 The Model

We begin by considering the simplest form of of heat interactions between the inside and outside of a building. We can do this by using Newton's Law of Heating and Cooling:

$$T(t) = T_{outside} + (T_{initial\ inside} - T_{outside})\, e^{-kt}, \tag{1}$$

where $T(t)$ denotes the temperature in Fahrenheit of the inside of the unit; $T_{outside}$ denotes the temperature outside (variable with time of day); $T_{initial\ inside}$ denotes the temperature initially inside the unit; and k is the heating/cooling constant. First, we must find k before we plug given values in and find how the temperature in the house fluctuates during a 24 hour period.

1.4.1 Finding k

Definition 1 (HC Constant)**.** Define k in Equation (1) to be the **HC constant**. According to Worsnop, Flint, et. al [22], the HC constant can be equivalently expressed as

$$k = \frac{KA}{msd} \tag{2}$$

where

- K is the thermal conductivity of the material used to build the house in W/mK (The Kelvin unit in the denominator is not relevant; the thermal conductivity of a material operates independent of the temperature scale.) See Figure 2.
- A is the exposed surface area of the unit
- m is the mass of the house
- s is the specific heat capacity of the material used to build the house
- d is the thickness in m of the exterior walls of the house

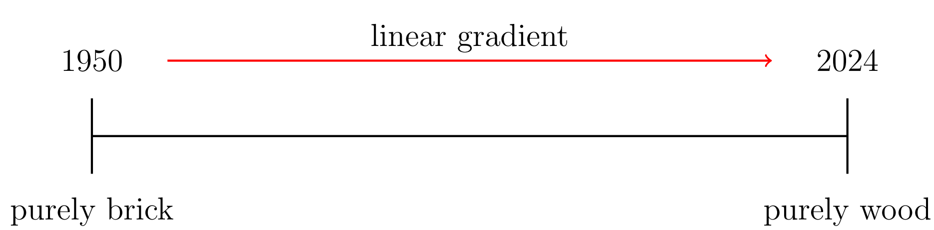

Figure 2: Demonstration of house composition changes over time under our assumption.

We now embark on a scavenger hunt to find these values.

First, we shall consider the thermal conductivity K of our building materials. The thermal conductivity of a brick is $1\frac{W}{mK}$ [9], and the thermal conductivity of wood is $0.1\frac{W}{mK}$ [14]. So, linearizing the two data points $(1950, 1)$ and $(2024, 0.1)$ and extrapolating the middle gives us:

$$K = 1 + (0.1 - 1) \cdot \frac{1}{74} \cdot (t - 1950),$$

where t is the time in years.

Next, we consider the surface area exposed by the unit. As per our assumption, we assumed an inverse relationship between E and $\frac{\text{number of units}}{\text{number of stories}}$, where E, again, is a function to denote the number of exposed sides on a unit. So, for simplicity, set E as $\frac{1}{\frac{\text{number of units}}{\text{number of stories}}}$. Each of these exposed sides has the same surface area as the measurement of the unit given in the dataset, which we denoted as L. Plugging this in, we obtain

$$A = E \cdot L = \frac{1}{\frac{\text{number of units}}{\text{number of stories}}} \cdot L \tag{3}$$

The next parameter we shall consider is the mass of the unit. This is simply equal to the density of the air times the volume it occupies:

$$m = \rho_{\text{air}} \cdot V = 9L\rho_{\text{air}}, \tag{4}$$

where L is the area of the floor plan and 9 is the height of the unit per our assumption.

Calculated similarly to the thermal conductivity, a formula for the specific heat capacity s can be extrapolated by considering the endpoints in the years 1950 and 2024. The specific heat capacity of a brick is $840\frac{J}{g^\circ C}$ [6], and the thermal conductivity of wood is $1760\frac{J}{g^\circ C}$ [12]. So, linearizing the two data points $(1950, 840)$ and $(2024, 1760)$ and extrapolating the middle gives us

$$s = 840 + (1760 - 840) \cdot \frac{1}{74} \cdot (t - 1950).$$

1.4.2 Using Newton's Law of Heating and Cooling

Now that we have found a symbolic solution for the k value, we can use Python to iterate through the 24 hour period and find the house temperature every hour. This requires us to use our assumption that the initial temperature in the house at midnight is around $78°F$. Then, we perform our first iteration through the equation obtained from Newton's Law of Cooling, which is $T(t) = T_{outside} + (T_{initial\ inside} - T_{outside})\, e^{-kt}$. We keep on repeating this process, plugging in the newly obtained value for the temperature of the house at hour $x - 1$ to find the temperature of the house at hour x. Notice that k stays the same during the entire 24 hour time period but changes depending on which house/unit we are talking about.

1.5 Results

Finally, we can test Equation (1) for the four examples provided in our dataset. We did this computationally[1]; the blue curve models the temperature inside of the unit, while the orange curve models the same outdoor temperature dataset over five days (which we obtained from the dataset [1]).

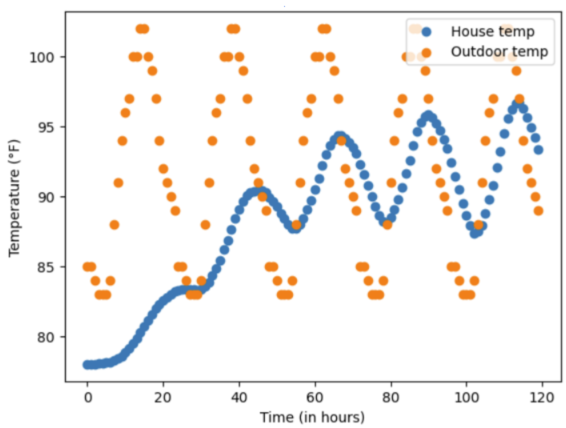

Figure 3: Home 1 over 5 days.
$t = 1953$, units/stories $= 1/1$, $L = 88$

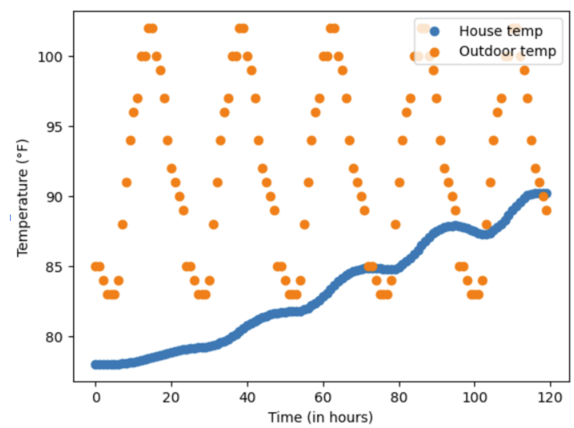

Figure 4: Home 2 over 5 days.
$t = 1967$, units/stories $= 8/2$, $L = 63$

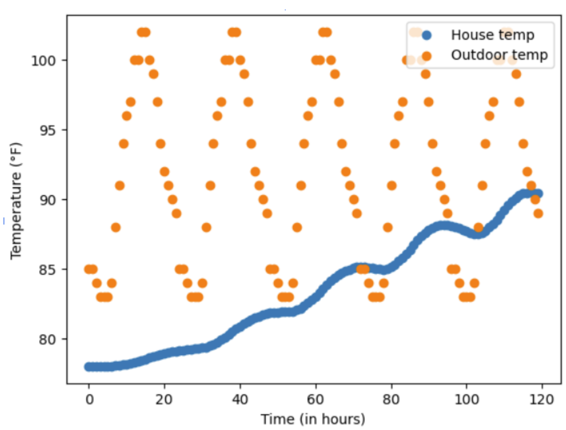

Figure 5: Home 3 over 5 days.
$t = 2003$, units/stories $= 30/25$, $L = 74$

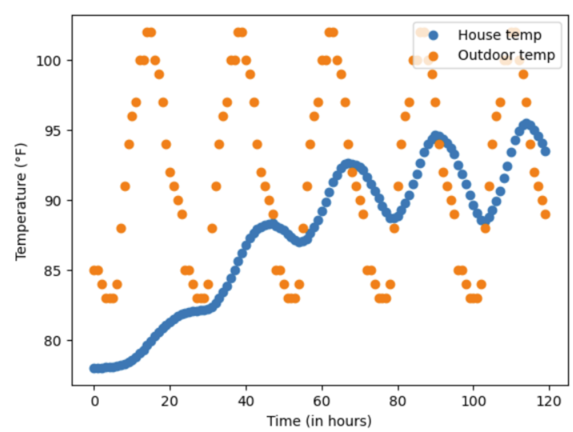

Figure 6: Home 4 over 5 days.
$t = 1990$, units/stories $= 1/2$, $L = 278$

1.6 Discussion

As one can notice from all four graphics, the temperature of the unit begins to rise when it is lower than its surrounding temperature, and begins to fall when it is higher than its surrounding temperature (which occurs rarely). Over time, the temperature in all four homes approaches the average temperature outside, which makes sense, because there is no HVAC system to keep the unit cool. Our model only stresses the importance of having a functioning HVAC system in areas with extreme temperatures.

[1]The raw source code may be located in Appendix A.1.

1.7 Sensitivity Analysis

We shall proceed by varying Home 1's parameters, one at a time—that is, the number of stories, units, years, and its size.

Below are the graphs we get when we increase to 10 stories (top left), build it 50 years later (top right), increase to 10 units (bottom left), and increase to 1000 sq. ft (bottom right; we also tried decreasing to 10 sq. ft. but didn't discern any noticeable difference).

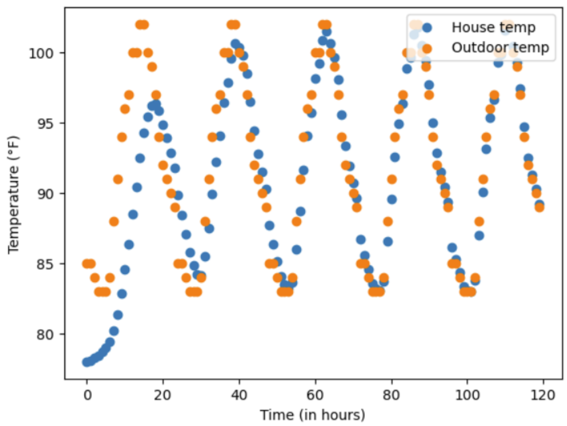

Figure 7: Home 1 increased to 10 stories.

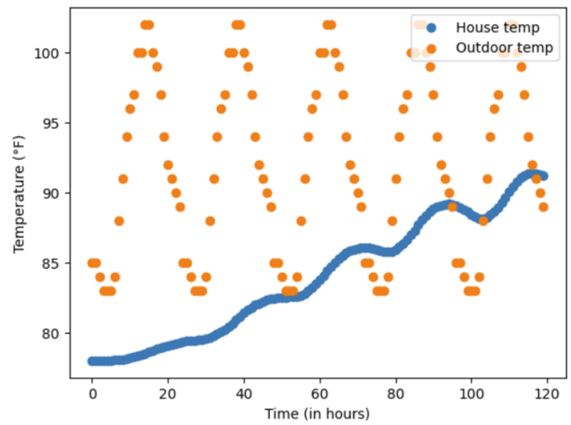

Figure 8: Home 1 built 50 years later.

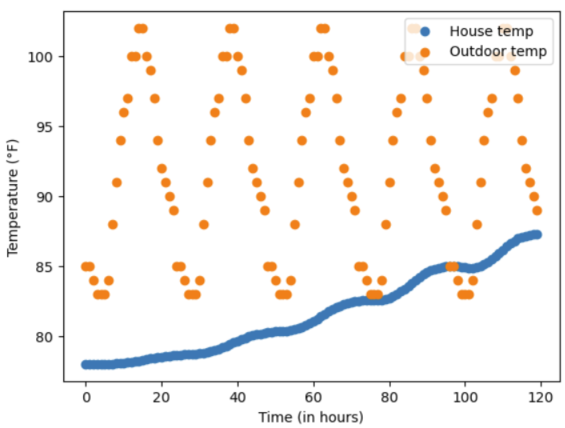

Figure 9: Home 1 increased to 10 units.

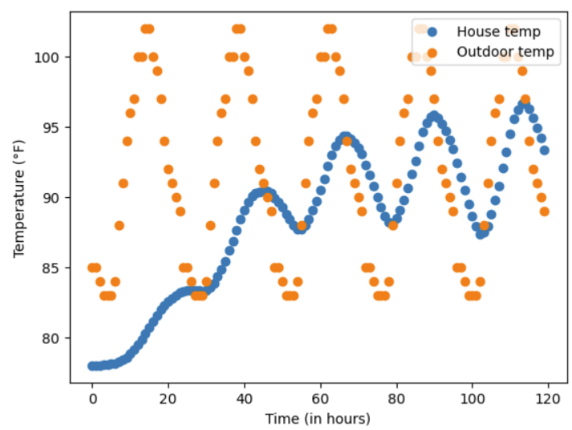

Figure 10: Home 1 increased to a size of 1000 sq. ft. Decreasing to 10 sq. ft. produces a similar curve.

These graphs show that when you increase the number of stories in the unit, the indoor temperature rapidly approaches the trends of the outdoor temperatures. On the other hand, if a house was built later, or if the number of units in the house was increased, then the house temperatures started to drastically decrease and there was also much less variability in the temperature values.

Additionally, our model is very robust when it comes to modifying the size of the unit since the house temperatures remained consistent regardless of if we drastically increased or decreased the dimensions of the unit.

1.8 Strengths and Weaknesses

1.8.1 Strengths

One large strength of our model was its visible oscillatory temperature over time, given the variable external temperatures. Visually plotting the two temperatures on the same axis was an excellent choice, as it allowed us to compare the relative temperatures for day and night over multiple days.

Another strength is that after around 5 days, the average indoor temperature is around the average outdoor temperature, which matches expectations, since we expect the temperatures to reach equilibrium.

1.8.2 Weaknesses

One of the main weaknesses of our model is that we couldn't find sufficient data on what materials are used to build the houses in Memphis, Tennessee, specifically during the time period 1950 to 2024. So, we only considered the endpoints and extrapolated two of the variables, thermal conductivity and specific heat, from them. With better data, our K and s variables would be more accurate and might change the temperature graphs for each 24 hour period.

Another weakness is that we weren't sure what the mass $= m$ exactly means in our equation for the HC Constant. In our code, we just assumed that it was the mass of the air inside the house, but it could also include the mass of the house in general, from the walls and furniture to even the people.

Finally, although we assumed that the starting temperature inside the house is $78°F$ at midnight, the blue curve doesn't return to that value at midnight of the next day. This causes the temperature to always gradually increase. The problem is a little ambiguous about when the 24 hour period starts, but if we had more time, we would research more on what a more suitable initial temperature for the house would be.

2 Q2: Power Hungry

2.1 The Problem

The second problem asks us to create a model that predicts the peak demand that our city's power grid should be prepared to handle during the summer months and any changes in the maximum energy demand 20 years from now.

2.2 Assumptions and Justifications

Assumption 2.1: Our selected sectors – residential, commercial, and industrial — encompass all electricity consumption. Transportation does not contribute to electricity consumption.
Justification: Our data from provides the total energy consumption along with a breakdown for each sector [5]. Since the sum of each sector's energy consumption is precisely equal to the total, there are no other sectors that contribute to energy consumption. Transportation was disregarded since the majority of that sector's energy is consumed through gasoline or diesel, which does not draw from the power grid. We disregarded electric vehicles because of how little there are in Tennessee, especially relative to other sectors [19].

Assumption 2.2: Our regression models are unaffected by outside factors.
Justification: It is highly likely for largely unpredictable factors such as accelerated climate change, policy changes, or technological development to affect our values for energy consumption or temperature. However, given how difficult these are to predict, we cannot assume any to be true and thus rely on a regression.

Assumption 2.3: Tennessee total electricity consumption is proportional to Memphis total electricity consumption.
Justification: Given our limited breadth of data on specifically Memphis electricity consumption, we generalized Tennessee's to be a rough proportion of Memphis based on population.

Assumption 2.4: The percentage of residential, commercial, and industrial energy consumption used for cooling in the United States is the same as Tennessee and therefore Memphis.
Justification: Due to a lack of data regarding specifically Tennessee and Memphis, we have to generalize US data to be applicable to Memphis. Since percentage of energy consumption for cooling is largely reliant on temperature, and Tennessee is central, geographically speaking, it is not unreasonable to assume that the Tennessee cooling percentage is similar to the average US cooling percentage.

Assumption 2.5: The proportion of population in Memphis is around 0.0851 times the total Tennessee population
Justification: The population of Tennessee is around 7204000 and the population of Memphis is around 613110. Dividing this, we get a population ratio of around 0.0851.

2.3 The Model

We begin by defining our peak demand as a function of the year (t). The peak demand is the same day as the highest temperature day, and we can break down demand into the following

$$D_{peak} = D_{cool} + D_{not\ cool} \tag{5}$$

Breaking it down further,

$$D_{peak}(t) = E_{cool}(T(t)) + \sum e_{sector} \cdot k_{sector} \tag{6}$$

where

- t is the time, given by the current year
- $D_{peak}(t)$ is the peak demand in a day for a given year in MWh
- $T(t)$ is the peak temperature for a given year, measured in °F.
- $E_{cool}(T(t))$ is the energy from cooling as a function of $T(t)$, in MWh.
- e_i is the total electricity usage of a sector, in MWh
- k_i is a constant representing the percentage of a sector's power consumption that is not dedicated to cooling

2.3.1 Determining e_{sector} and k_{sector}

Using a dataset from the EIA [3], we found each sector's energy consumption in Tennessee. We then found k_{sector} for each sector using more information from the EIA [2]. From this, we fit a regression to each sector's energy consumption.

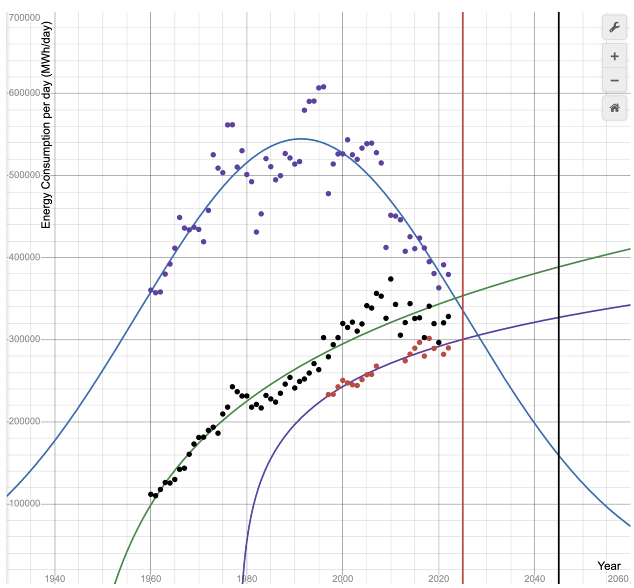

Figure 11: Regressions for each sector's $e_{sector} \cdot k_{sector}$ vs t. Residential sector is logarithmic in green. Industrial is an inverted bell curve in blue. Commercial is logarithmic in purple.

We decided on these regressions because we know that each sector's energy consumption must approach some asymptotic limit (i.e. they cannot keep decreasing or increasing forever). Since residential and commercial are increasing, logarithmic fits them well, and since industrial increases then decreases, we fit an inverted bell curve to it. In addition, for commercial we only considered years after 1997 due to a large dip in energy usage.

Sector	**Regression Function**	R^2
Residential	$160188.651\ln(56.97447x - 110722.112) - 999192.157$	0.9307
Industrial	$544637.027e^{-(0.0206626x-41.14538)^2}$	0.7464
Commercial	$75114.7374\ln(80.02597x - 158309.322) - 317591.096$	0.9117

2.3.2 Determining $T(t)$

We fit a regression to data from Memphis Weather [21] and Weather Underground [20] to project the peak temperature for any given month t. It is sinusoidal because of temperature's seasonal nature. We added a linear aspect to roughly account for climate change.

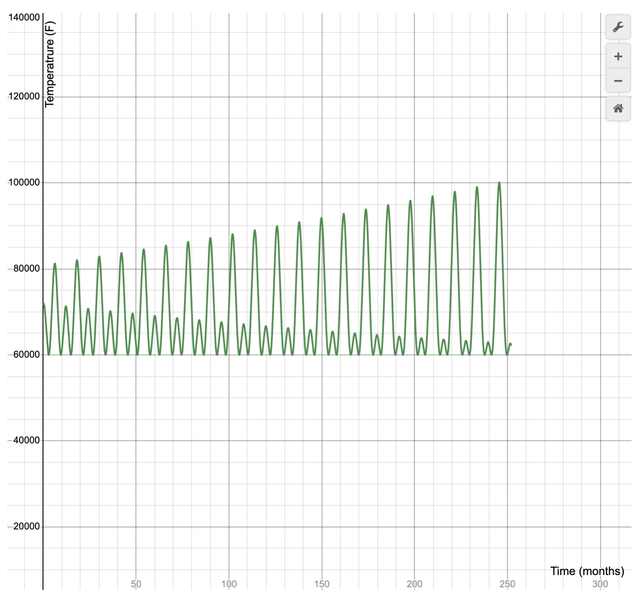

Figure 12: Sinusoidal Temperature vs time regression of Memphis, Tennessee

Regression function	R^2
$19.28\sin(0.525x - 1.682) + 64.14936 + 0.0345725x$	0.9528

2.3.3 Determining $E_{cool}(T(t))$

Using Equation (6), we can solve for E_{cool} as follows

$$E_{cool}(T(t)) = D_{peak}(t) - \sum e_{sector} \cdot k_{sector} \tag{7}$$

This can also be rewritten as

$$E_{cool}(T(t)) = D(t) \cdot (1 - k_{total}) \tag{8}$$

Here, $(1 - k_{total})$ represents the percentage of total demand that is from cooling. Next, we find $E_{cool}(T(t))$ to be

$$E_{cool}(T(t)) = D(t) \cdot (1 - k_{total}) \tag{9}$$

Using datasets from the EIA [4] and Weather Underground [20], we temperature over time and total electricity consumption over time.

We estimated $1 - k_{total}$ to be 0.157 using our datasets determining e_{sector} and k_{sector}, where we added the total electricity consumption from HVAC and divded by the total electricity consumption to get the value 0.157.

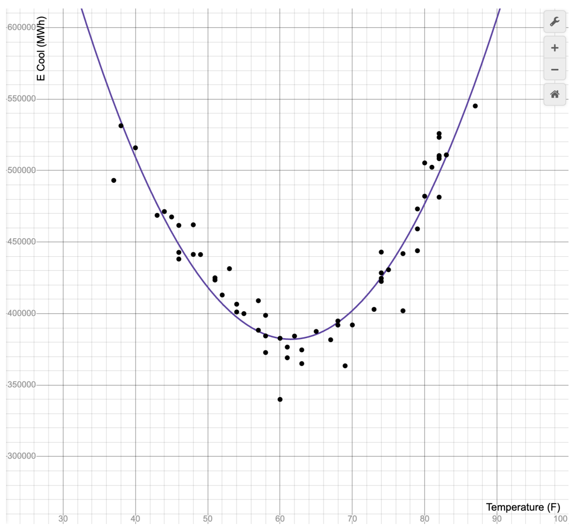

Figure 13: Quadratic regression of E_{cool} vs T

Regression function	R^2
$276.11x^2 - 33948.4769x + 1425563.98$	0.8905

We then multiplied this function by the cooling coefficient of 0.157 so that we only accounted for cooling and not total electricity consumption. Then, we can substitute the $T(t)$ from the regression equation into x to get $E_{cool}(T(t))$ as a function of time.

2.4 Results

Much of our data are based on the entirety of Tennessee. To convert to Memphis, we multiply by the proportion of population in Memphis [16] to the population in Tennessee [17], which we found to be approximately 0.0865. Using our values obtained through regression, we find that the following values for peak energy consumption.

	Peak Demand (MWh)
2025	91466
2045	82972

2.5 Discussion

The peak demand for any given year usually occurs in July since it is the hottest month of the year, therefore cooling is on overtime. So, the peak demand for 2025 and 2045 is expected to occur in July.

Interestingly, we found that the peak demand for energy consumption in 2045 was lower than 2025. This is due to a lower industry power consumption, which is a clear trend shown in our data from the EIA. We are unsure of why industry is declining in power usage in Memphis, but it could be a combination of more energy efficient technologies and a gradual transition from industry to commercial businesses.

2.6 Sensitivity Analysis

From our temperature vs time regression, we predicted that the average temperature in July 2025 will be around 85 degrees Fahrenheit and the average temperature in July 2024 will be around 91.9 degrees Fahrenheit.

We measured the resulting effects of changing the temperature by 1 degree Fahrenheit, starting from 82 F and going to 94 F, on the cooling energy demand.

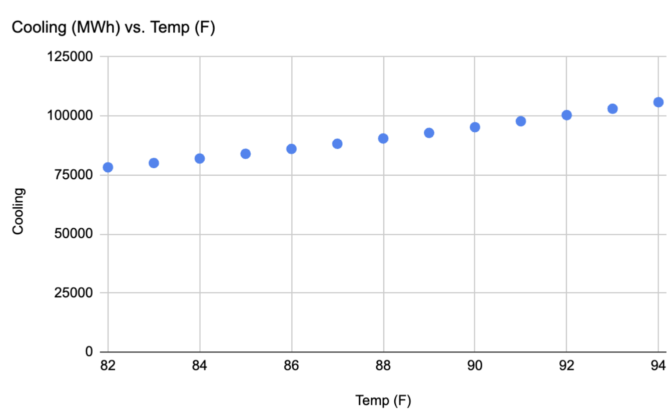

Figure 14: Sensitivity analysis of D_{peak} for varying temperatures, 2025

Temp (F)	Cooling	%change
82	78241	n/a
83	80064	0.02329980445
84	81975	0.02386840528
85	83969	0.02432448917
86	86052	0.02480677393
87	88221	0.02520568958
88	90476	0.02556080752
89	92821	0.02591847562
90	95250	0.02616864718
91	97766	0.02641469816
92	100369	0.02662479799
93	103059	0.02680110393
94	105836	0.02694573012

We can see the cooling to be not very sensitive to temperature changes, meaning our model is robust. If added to the total energy consumption, the percent change would be even less.

2.7 Strengths and Weaknesses

2.7.1 Strengths

Our model takes into account each and every sector when finding peak energy use, accounting for how they change over time.

Our model's data are almost entirely sourced from the EIA, meaning there will be a more consistent measuring and testing across data gathering.

2.7.2 Weaknesses

One of our major weaknesses was our reliance on regressions, particularly troublesome for forecasting the various sector energy usages. While regressions are mathematically sound and reliable, they require assumptions about the changing geopolitical and economic landscape of our world. For example, a policy change in industrial energy efficiency requirements or residential building codes could throw our estimates off balance.

Another weakness is our temperature regression, which is very crude. Since max temperature is often highly variable, it is hard to completely predict, especially 20 years into the future.

3 Q3: Beat the Heat

3.1 The Problem

The third problem asks us to create a vulnerability score V for various neighborhoods so city officials can equitably allocate resources for minimizing the effects heatwave or power grid failure.

3.2 Assumptions and Justifications

Assumption 3.1: Data provided in the MathWorks data sheet for each neighborhood is representative of the entire neighborhood.
Justification: An accurate model would be impossible to create if the data provided consisted of anomalies or was inconsistently scaled across different regions.

Assumption 3.2: When given a range between which a house is constructed, we assume that it was built during the middle year.

3.3 The Model

We begin by defining the scale of our vulnerability score V. Our team decided to scale V linearly within the range [0, 1]: a small value for V means the community is less at risk and doesn't need to be prioritized, while a large value for V means the community is at significant risk and should be significantly prioritized.

Of the several parameters that were provided to us in the MathWorks data sheet, our team decided that it ultimately boiled down to five factors that determined the vulnerability of a neighborhood. These are as follows:

1. The average income of all working residents in dollars, I.
   - Justification: Affluence is primarily correlated with quality of life. For example, neighborhoods with a lot of money invest in more reliable sources of energy, and can also pay money to escape danger. [15]
2. The dependency ratio, E, which is calculated by dividing the number of people above 65 and under 18 by the total population.
   - Justification: A higher dependency ratio means that those who can work are expected to provide more for their families. Areas where demographics primarily consist of the elderly and children are more vulnerable, as they are less able. As a result, others may need to take care of the vulnerable family members amidst heat waves, further reducing the size of the workforce.
3. The percentage of adults with cars, C.

- Justification: If a large proportion of adults can drive, they can escape to cooler parts of the city quicker (like a grocery store or mall). Also, cars have air conditioning, so they can be stayed in if it gets too hot in a building and/or outside.

4. The average age of the houses, Y.

   - Justification: Usually, older houses deteriorate in terms of insulation and heat retention quality, so older houses are more vulnerable than newer ones [10]. However, we also found out from Model 1 that some newer houses heat faster than older houses, depending on other factors, like the type of house, which we will touch on next.

5. The average temperature in the house, T (based on the type of house, which is based on the number of units/story)

   - Justification: Here, our data distinguishes four types of houses: detached whole homes, townhouses, apartments, and mobile homes. From Model 1, we can determine the average temperature based on the number of stories and units in a house. For simplification purposes, we assumed:
     - Detached home: 1 unit, 2 stories, size 200 sq. m.
     - Townhouse: 5 units, 2 stories [8], size 175 sq. m.
     - Apartment: 10 units, 6 stories [18], size 84 sq. m.
     - Mobile home: 1 unit, 1 story, size 30 sq. m.

     This way, we can use the average age for a house in each neighborhood (Y) combined with Model 1 (by plugging in the above "units" and "stories" values) to determine the average temperature (T) in a particular type of house in a given neighborhood.

Now, to write a formula that takes these parameters into account, we need to consider the relationship between each of the five factors and the overall vulnerability. Increasing the average income will lead to less vulnerability, so $V \propto 1/I$. A larger dependency ratio leads to higher vulnerability, so $V \propto E$. A community with more widespread access to cars will be less vulnerable, so $V \propto 1/C$. Lastly, the older houses are, the more vulnerable the house, so $V \propto Y$.

In summary: we have

$$V \propto E, \quad V \propto Y, \quad V \propto T, \quad V \propto 1/I, \quad V \propto 1/C$$

$$\implies V = \frac{EYT}{IC}. \tag{10}$$

This is the equation we will use to compute vulnerability scores. However, we have one more step. The last step of our model involves normalizing our parameters to ensure they are comparable; that is, a vulnerability score as large as 50,000 can't easily be compared to 2. Normalizing the data also ensures we get a consistent scale of vulnerability on the range [0, 1], where 1 represents the most vulnerable and 0 represents the least vulnerable. To normalize, we apply Minimum-Maximum Scaling to each individual data point per neighborhood:

dataPointNormalized = (dataPoint - minValue) / (maxValue - minValue).

Now that we have a quantitative formula, we can finally plug in our data values given to us in the MathWorks spreadsheet. The normalized vulnerability numbers are calculated and displayed in the table in the next section.

3.4 Results

Neighborhood	Vulnerability Score
Downtown / South Main Arts District / South Bluffs	0.020603
Lakeland / Arlington / Brunswick	0.014736
Collierville / Piperton	0.0
Cordova, Zipcode 1	0.046872
Cordova, Zipcode 2	0.029642
Hickory Withe	0.005269
Oakland	0.045878
Rossville	0.048527
East Midtown / Central Gardens / Cooper Young	0.379575
Uptown / Pinch District	0.806439
South Memphis	1.0
North Memphis / Snowden / New Chicago	0.673773
Hollywood / Hyde Park / Nutbush	0.797985
Coro Lake / White Haven	0.690060
East Memphis – Colonial Yorkshire	0.386531
Midtown / Evergreen / Overton Square	0.520133
East Memphis	0.186879
Windyke / Southwind	0.024216
South Forum / Washington Heights	0.868059
Frayser	0.577245
Egypt / Raleigh	0.373415
Bartlett, Zipcode 1	0.063664
Bartlett, Zipcode 2	0.193546
Bartlett, Zipcode 3	0.057776
Germantown, Zipcode 1	0.083342
Germantown, Zipcode 2	0.007964
South Riverdale	0.136263

Above are the vulnerability scores for all twenty-seven neighborhoods, computed using Equation (10).[2]

3.5 Discussion

Notice that the vulnerability score is the lowest for **Collierville/Piperton**, which means that they are the least vulnerable and need the least resources. However, the vulnerability score is the highest for **South Memphis**, which means that they are the most vulnerable and need the most resources. Therefore, we propose that the city of Memphis, Tennessee should allocate money proportionally to the vulnerability score V. For example, if Memphis had a total of X dollars to allocate for heat waves treatment, then the neighborhood Egypt / Raleigh should receive $X \cdot \frac{0.378205}{\sum V} = X \cdot \frac{0.378205}{8.09599} = 0.0467X$ dollars.

3.6 Strengths and Weaknesses

3.6.1 Strengths

One strength of our model was its basis for comparison, and how our model integrates the meaning of each of our variables. When we multiply the parameters in Equation (10), we only truly care about the *relative* vulnerability, and normalizing takes care of setting the basis of comparison. Normalizing the data also eliminates the need to weight any one factor more than another.

3.6.2 Weaknesses

One large weakness of our model lies in how simple our formula ended up. The MathWorks data spreadsheet listed over ten parameters for use; yet, we determined that all of the parameters fell into one of five main categories[3]. Homogenizing the different data sets into five averages eliminated a lot of the nuance that likely played into developing an accurate model.

[2] The raw source code may be located in Appendix A.2.

[3] For example, we did not consider the percentage of people who had a bachelor's degree, because education level directly correlates with income.

A Code Appendix

A.1 Q1: Hot to Go

```
#data given
import matplotlib.pyplot as plt
import pandas as pd
import numpy as np
import math

temps = pd.read_csv("temps.csv")
print(temps)

year=input("Year built")
units=input("# of total units")
stories=input("# of stories")
size=input("sq ft / meters")

#assuming that in 1950, houses were made of brick, and in 2024,
    houses are made of wood
#Thermal conductivity values: brick (1), wood (0.1)
K=1-0.9*(1/74)*(year-1950) #linear decrease in thermal
   conductivity of material from 1950-2024
A=(1/(units/stories))*size #m^2, surface area exposed
m=3*1.3*size #kg, mass of house, ignoring furniture, assuming
   typical ceiling height is 9m
#Specific heat values: brick (840), wood (1760)
s=840+920*(1/74)*(year-1950) #linear increase in specific heat
   of material from 1950-2024
d=0.2 #m, thickness of walls, assuming it's constant regardless
    of material

k=-(K*A)/(m*s*d) #cooling constant
days=5 #how many days we want to show in the graph of house
   temperature
print(k)
#assuming that the initial temperature of the house is equal to
    the initial outdoor temperature
t=temps["Temperature (degrees F)"] #list of outdoor
   temperatures for 24 hours
#H=[t[0]] #F, list of house temperatures
H=[78]
time=list(range(24*days)) #list of hours passed
for x in range(1,24*days):
    T=t[x%24] #the outdoor temperature cycles every 24 hours
```

```
    H.append((T + (H[x-1]-T)*math.e**(k*x)).tolist()) #
       appending calculated H value from Newton's Law of
       Cooling to list
plt.scatter(np.array(time), np.array(H), label="House temp") #
   graph of house temperature
x=np.tile(np.array(t),days) #list of given outdoor temperatures
    for a certain number of days
plt.scatter(np.array(time), x, label="Outdoor temp") #graph of
   outdoor temperature
plt.xlabel("Time (in hours)")
plt.ylabel("Temperature (degrees F)")
#plt.title("House and Outdoors Temperatures vs. Time")
plt.legend(loc="upper right")
```

A.2 Q3: Beat the Heat

```
import matplotlib.pyplot as plt
import pandas as pd
import numpy as np
import math

temps = pd.read_csv("temps.csv")
print(temps)

data = pd.read_csv("part3.csv") #from MathWorks spreadsheet
#print(data)

def avgtemp(year, units, stories, size): #function that
   calculates average temperature from graph
    #assuming that in 1950, houses are made of brick, and in
       2024, houses are made of wood
    #Thermal conductivity values: brick (1), wood (0.1)
    K=1-0.9*(1/74)*(year-1950) #linear decrease in thermal
       conductivity of material from 1950-2024
    A=(1/(units/stories))*size #m^2, surface area exposed
    m=3*1.3*size #kg, mass of house, ignoring furniture,
       assuming typical ceiling height is 9m
    #Specific heat values: brick (840), wood (1760)
    s=840+920*(1/74)*(year-1950) #linear increase in specific
       heat of material from 1950-2024
    d=0.2 #m, thickness of walls, assuming it's constant
       regardless of material

    k=-(K*A)/(m*s*d) #cooling constant
    days=5 #how many days we want to show in the graph of house
        temperature
    #print(k)
    #assuming that the initial temperature of the house is
       equal to the initial outdoor temperature
    t=temps["Temperature (degrees F)"] #list of outdoor
       temperatures for 24 hours
    #H=[t[0]] #F, list of house temperatures
    H=[78]
    time=list(range(24*days)) #list of hours passed
    for x in range(1,24*days):
        T=t[x%24] #the outdoor temperature cycles every 24
           hours
        H.append((T + (H[x-1]-T)*math.e**(k*x)).tolist()) #
           appending calculated H value from Newton's Law of
```

```
            Cooling to list
    return sum(H)/len(H)

houses=(data["Homes built 2010 or later"]+data["Homes built
   1990 to 2009"]+data["Homes built 1970 to 1989"]+data["Homes
   built 1950 to 1969"]+data["Homes built 1950 or earlier"]) #
   total number of houses, which is surprisingly not equal to
   the number of households!
data1=pd.DataFrame([data["Neighborhood"],data["income"],(data["
   pop 65+"]+data["pop 18-"])/data["Population"],(data["
   households w/ vehicles"]/data["# households"]),2024-(2017*
   data["Homes built 2010 or later"]+2000*data["Homes built
   1990 to 2009"]+1980*data["Homes built 1970 to 1989"]+1960*
   data["Homes built 1950 to 1969"]+1940*data["Homes built 1950
    or earlier"])/houses]).T #converting the columns into the
   five factors that we will consider in our vulnerability
   score
data1=data1.rename(columns={'Unnamed 0': '% old/young', '
   Unnamed 1': '% vehicles', 'Unnamed 2': 'House Age', "Unnamed
    3": "House Temp"}) #renaming some columns :)
data1["House Temp"]=data1["income"]
data1.head()
for x in range(len(data1)):
    year=2024-data1["House Age"][x] #finds the year the average
        house in the neighborhood was built based on the
       average age
    data1.loc[x,"House Temp"]=(avgtemp(year,1,2,200)*data["
       Detached whole house"][x]+avgtemp(year,1,1,30)*data["
       Mobile Homes/Other"][x]+avgtemp(year,5,2,175)*data["
       Townhouse"][x]+avgtemp(year,10,6,84)*data["Apartments"][
       x])/houses[x] #finds the average house temperature based
        on the average year that the house was included and
       average stories/units/size values for that certain house
        type
data1.head()

data_n=data1.copy() #this will be the normalized dataframe
data_n["V"]=1.85*data_n["% old/young"]*data_n["House Age
   "]*1.45*data_n["House Temp"]/(1.1*data_n["income"]*data_n["%
    vehicles"]) #calculating the vulnerability score
data_n["V"] = (data_n["V"]-min(data_n["V"]))/(max(data_n["V"])-
   min(data_n["V"])) #normalizing the vulnerability score
#sum(data_n["V"]) #used to figure out how much money to
   allocate
data_n
```

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