MathWorks Math Modeling Challenge 2020

High Technology High School

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M3 Challenge Technical Computing Award
HONORABLE MENTION

JUDGE COMMENTS

Specifically for Team # 13555:

This paper used technical computing for all 3 problem sections, which we loved to see. In Parts 1 and 3 it used numerical methods to fit and simulate differential equation models. In Part 2 the paper used a Python API to automatically pull in weather data: this data effects battery life and the team used that data to inform an algorithm for placing charging stations. Technical computing was also used to implement a nice queuing model for Part 2. Overall, the strength of this paper was its creativity and the breadth of ideas incorporated into the models. It would have been strengthened by more clear communication about model choices, derivations, and implementation, especially in Part 1.

Overall Judging Perspective for Technical Computing Submissions:

The use of technical computing in the final papers was judged on its effectiveness in advancing a papers' modeling, its creativity, and how it was communicated. We rewarded papers where technical computing was used in an essential way, and did not simply replace functionality which could have been implemented in a spreadsheet or on a calculator. We also rewarded clear explanations, even if the underlying algorithm was relatively simple. This year we noticed that technical computing was used in many papers to enhance presentation: some beautiful plots were used to effectively communicate student ideas and final solutions. Such uses of technical computing were also rewarded. Finally, one of the benefits of implementing a model in code is that it is very easy to change modeling choices or parameters to see how these choices effect the final problem solution. Teams which took advantage of this (through sensitivity analysis, testing multiple models, etc.) were rewarded.

MathWorks Math Modeling Challenge

***Note: This cover sheet has been added by SIAM to identify the winning team after judging was completed. Any identifying information other than team # on a MathWorks Math Modeling Challenge submission is a rules violation.

***Note: This paper underwent a light edit by SIAM staff prior to posting.

Keep on Trucking: US Big Rigs Turnover from Diesel to Electric

Executive Summary

In our increasingly interconnected nation, the transportation sector is one of its largest and most important—semi-trucks in particular carry nearly everything we buy or build. Despite the trucking industry's prevalence, it is far from efficient, accounting for over 12% of fuel purchased in the US [1]. The successful penetration of electric cars into the automobile market has underscored the trucking industry as a potential next target for this transition. With the need to analyze the necessary infrastructure for semi-truck electrification, as well as projected electric semi usage, our team seeks to provide a report with mathematically-founded insights on this issue.

Since the availability of charging infrastructure affects its usage, we first strove to analyze the electric semi's potential for market penetration. By assuming that there are no infrastructural barriers to usage, we projected the percentage of semis that will be electric over the next two decades by using a generalized Bass diffusion model, which we scaled using four external factors: cost per mile, total vehicle cost, total vehicle sales, and driving range. With constants obtained via curve-fitting to historical electric car sales, we applied our Bass model to electric semis. We found that the percentage of semis that will be electric in 2025, 2030, and 2040 are 0.16%, 0.84%, and 12.5%, respectively, and that there is a projected rapid growth period immediately after 2040.

We then determined the necessary infrastructure to support the fully electric long-haul fleet that was previously assumed. To determine the number of charging stations needed per corridor, we first placed a station at every interstate highway intersection. We then calculated stations needed between intersections by accounting for battery longevity and temperature dependence of its capacity, and single-charge electric semi range. Then, to determine chargers needed per station, we modeled each station as a M/G/k multi-server queue to determine k, the number of chargers that would yield at most a 13-minute wait time. We applied our model to five corridors: San Antonio to/from New Orleans, Minneapolis—Chicago, Boston—Harrisburg, Jacksonville—Washington, DC, and Los Angeles—San Francisco. Our model indicated that these routes would respectively require 10, 10, 11, 16, and 10 stations; with an average of 27, 24, 17, 14, and 21 chargers per station.

As suggested by the previous part, the transition to electric trucking requires infrastructure installation. We thus developed a metric to rank routes in order of when they should be developed based on projected public support of the transition around each corridor. Percentage support was modeled using economic and environmental factors, and we took a weighted sum of the percent support, construction time, and anticipated route usage. By solving a differential equation, the model found the time at which each weighted sum was greater than a specific threshold, and ranked these times least to greatest for each of the 5 corridors. Our model output suggests that the Minneapolis—Chicago and San Antonio—New Orleans corridors should be the first two targeted for development.

While the electrification of the semi-truck industry is still a relatively novel concept in 2020, recent developments by Tesla and other auto companies are making this transition a tangible reality. As the trucking industry readies itself for this economic, environmental, and infrastructural transformation, we believe the models outlined in our paper provide valuable insight into the numerous factors at play.

Contents

1 Introduction **2**
1.1 Restatement of the Problem . . . 2
1.2 Global Assumptions . . . 2

2 Part 1: Shape Up or Ship Out **2**
2.1 Assumptions . . . 3
2.2 Model Development . . . 4
2.3 Results . . . 6
2.4 Strengths and Weaknesses . . . 8

3 Part II: In It For the Long Haul **8**
3.1 Assumptions . . . 9
3.2 Model Development . . . 9
3.2.1 Determining Stations Needed Per Route . . . 9
3.2.2 Determining Chargers Needed Per Station . . . 11
3.3 Results . . . 13
3.4 Sensitivity Analysis . . . 15
3.5 Strengths and Weaknesses . . . 15

4 Part III: I Like to Move It, Move It **16**
4.1 Assumptions . . . 16
4.2 Model Development . . . 17
4.3 Results . . . 18
4.4 Sensitivity Analysis . . . 19
4.5 Strengths and Weaknesses . . . 19

5 Conclusion **20**
5.1 Further Studies . . . 20
5.2 Conclusion . . . 20

6 References **21**

7 Appendix **22**
7.1 bass.py . . . 22
7.2 stations.py . . . 25
7.3 chargers.py . . . 26
7.4 queue.py . . . 27
7.5 development.py . . . 30

1 Introduction

This section delineates the components of the modeling problem and their objectives. Global assumptions applying to the entire modeling process are also listed.

1.1 Restatement of the Problem

The problem we are tasked with addressing is as follows:

1. Assuming that all necessary electric semi infrastructure is already in place for a seamless transition to fully electric fleets, create a mathematical model to predict the percentage of semis that will be electric in 5, 10, and 20 years from 2020.

2. Create a model that determines the number of stations needed along a given route, as well as the number of chargers needed at each station, to ensure the current volume of single-driver, long-haul traffic if all trucks were electric. Demonstrate the model on the following five trucking corridors: San Antonio, TX to/from New Orleans, LA; Minneapolis, MN to/from Chicago, IL; Boston, MA to/from Harrisburg, PA; Jacksonville, FL to/from Washington, DC; Los Angeles, CA to/from San Francisco, CA.

3. Develop a ranking system to determine which trucking corridors should be targeted for development first, based on factors such as the amount of required infrastructure from Part II, community motivation for transition, cost, anticipated usage, route length, or other factors. Demonstrate the model by ranking the five trucking corridors in Part II.

1.2 Global Assumptions

1. *The transportation industry will not have any major technological advances that detracts from the appeal of electric automobiles.* It is unreasonable to assume that a new ground-breaking invention will occur.

2. *Companies that build electrical vehicles will continue to build infrastructure to support them.* It is a reasonable to assume these companies will continue to develop their technology and products, and more infrastructure needs to be built to support this.

2 Part 1: Shape Up or Ship Out

There are currently around 1.7 million semi-trucks in the United States that altogether travel an estimated 150 billion miles annually on diesel fuel. Diesel semis not only account for more than 12% of the fuel purchased in the US but also exhibit extremely poor fuel efficiency, making a transition to electric trucks an attractive prospect [1]. This section outlines a mathematical model for projecting the percentage of semis in the US that are electric in 2025, 2030, and 2040.

2.1 Assumptions

1. *All necessary infrastructure for a fully electric truck industry is already in place.* This is a given assumption in the creation of our model, and allows the model to measure the usage potential of electric semis in the absence of infrastructural barriers.

2. *The transition from diesel to electric semi-trucks can be modeled by a modified Bass diffusion model.* Bass diffusion models are typically used to represent the process of how new products, particularly technologies, are adopted in a population over time [2].

3. *The relative importance of external factors in the transition from diesel to electric semis can be approximated by their relative importances in the transition from gasoline/diesel to electric cars.* Since electric semi-trucks have yet to become widely used on roads, there is insufficient data to predict future trends. Thus, it is reasonable to assume that the adoption of electric semis will be motivated by similar factors as the adoption of electric cars, for which data is available for longer time periods.

4. *The relative cost per mile of diesel and electric vehicles impacts the number of electric vehicles purchased.* If the operational (per mile) cost of an electric vehicle is significantly greater than that of its diesel counterpart, electric vehicles are logically less likely to be adopted.

5. *The relative total vehicle cost of diesel and electric vehicles impacts the number of electric vehicles purchased.* If an electric vehicle costs significantly more (including its battery costs) than its diesel counterpart, electric vehicles are logically less likely to be adopted.

6. *The relative vehicle range of diesel and electric vehicles impacts the number of electric vehicles purchased.* If the single-charge range of an electric vehicle is significantly less than the range of its fully filled diesel counterpart, electric vehicles are logically less likely to be adopted.

7. *The total vehicle sales per year impacts the number of electric vehicles purchased.* If the total vehicle sales, including both diesel and electric vehicles, is particularly low, this typically indicates a recession in the vehicle industry, and it is reasonable to expect a decline in electric vehicles purchased.

8. *The cost per mile, total vehicle cost, and driving range of both diesel and electric vehicles are approximately constant over a 20-year period.* This assumption is reasonable for diesel vehicles because their technological development will be relatively stagnant over this time duration, and gasoline prices cannot be viably predicted. While this assumption is not technically accurate for electric vehicles, it is necessary due to the dearth of historic data on their costs and driving range.

2.2 Model Development

The analysis of how populations adopt novel products and technologies over time is a highly complex phenomenon, but can be represented by diffusion of innovation models that provide simplified mathematical representations of the adoption process' main characteristics. One such diffusion of innovation model is the Bass diffusion model, a widely-used technique in technology forecasting that is therefore applicable for modeling adoption of the technology of electric semis over time [2].

The generalized Bass model describes the fraction of adoption $f(t)$, which is defined as the number of adopters at time t out of the total number of eventual adopters. In the below equation, parameter p describes adoption of a new technology by innovators, while q describes adoption by imitators:

$$\frac{df}{dt} = (1 - f(t)) \cdot (p - q \cdot f(t)) \cdot x(t) \tag{1}$$

where $x(t)$ is a scaling function that is dependent on external factors, which are chosen to be specific to the product or technology being modeled. $x(t)$ is further defined as follows [3]:

$$x(t) = 1 + \frac{\alpha_1 \cdot x_1'(t_1)}{x_1(t_0)} + \frac{\alpha_2 \cdot x_2'(t_2)}{x_2(t_0)} + \ldots + \frac{\alpha_n \cdot x_n'(t_n)}{x_n(t_0)} \tag{2}$$

Each x_i represents an external factor. As defined in assumptions 4, 5, 6, and 7, the external factors affecting adoption of electric semi-trucks are the cost per mile, total vehicle cost, total vehicle sales, and driving range, corresponding to variables x_1 through x_4, respectively. x_1, x_2, and x_4 represent comparisons between values for diesel and electric vehicles and are thus represented by ratios of the electric vehicle value to the diesel vehicle value, as summarized in Table 2.2.1. On the other hand, x_3 is meant to track the overall state of the automobile industry and is thus not expressed as a ratio. The integrated form of the overall scaling function $x(t)$ is given below:

$$x(t) = 1 + \alpha_1 \cdot \ln\left(\frac{c_t}{c_c}\right) + \alpha_2 \cdot \ln\left(\frac{t_t}{t_c}\right) + \alpha_3 \cdot \ln(s) + \alpha_4 \cdot \ln\left(\frac{r_t}{r_c}\right) \tag{3}$$

Additionally, the integrated form of the Bass model differential equation is as follows:

$$F(t) = \frac{1 - e^{-(p+q)X(t)}}{1 + \frac{p}{q}e^{-(p+q)X(t)}} \tag{4}$$

Table 2.2.1: Definitions and Values for Constants and Variables in Modified Bass Model

Symbol	Definition	Value
p	Describes adoption due to innovators	TBD via curve fitting
q	Describes adoption due to imitators	TBD via curve fitting
α_1	Weight for fuel cost per mile	TBD via curve fitting
α_2	Weight for total vehicle cost	TBD via curve fitting
α_3	Weight for vehicle sales	TBD via curve fitting
α_4	Weight for driving range	TBD via curve fitting
c_e	Cost of electricity per mile for a electric car	\$0.04
c_d	Cost of electricity per mile for a gasoline car	\$0.37
t_e	Total cost of electric car	\$55,600
t_d	Total cost of gasoline car	\$36,718
s	Total car sales	Variable
r_e	Driving range in miles of electric car	200 miles
r_d	Driving range in miles of gasoline car	350 miles

As per assumption 3, we model the adoption of electric semi-trucks by comparing the driving factors of the transition to those of the adoption of electric personal cars. Thus, the constants were calculated using values for diesel- and electric-powered automobiles, and the proportion of electric cars out of all cars each year from 2011 to 2019 was obtained as follows [4]:

$$\%EV = \frac{Number\ of\ electric\ cars}{Number\ of\ total\ cars} \tag{5}$$

The values obtained from Equation 4 were plotted and are represented by the red dots in Figure 2.2.1. Then, Equation 1 was plotted and fit to the electric car proportion data using the curvefit function of the Python library scipy, which implements a least-squares curve-fitting procedure (bass.py). The resulting fitted curve is shown below, and had an R^2 value of 0.9975, indicating that the curve fit the data well.

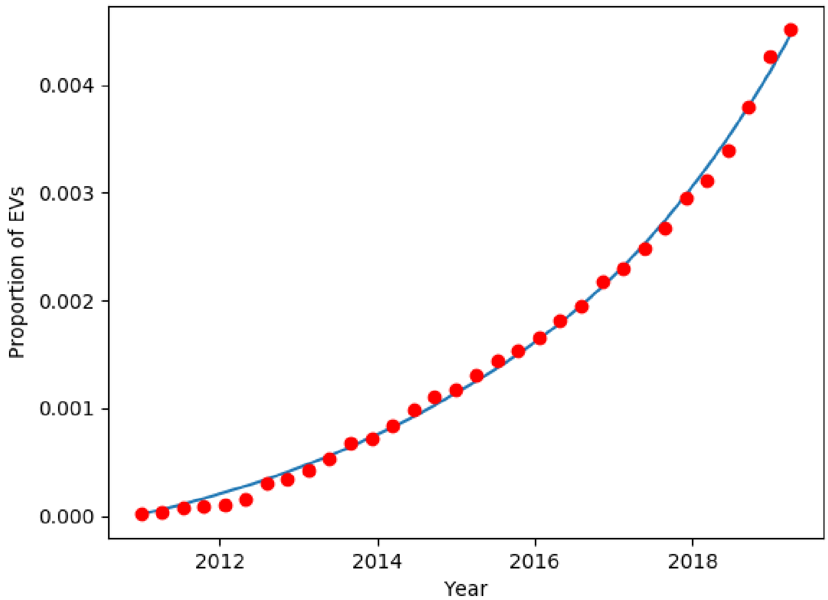

Figure 2.2.1: Bass Diffusion Model Fit to Historical Electric Car Data

Since the curve was fit to the data by changing the value of constants p, q, and α_1 through α_4, we obtained the values for these as reported in Table 2.2.2.

Table 2.2.2: Constant Values for Bass Model

Constants	Value
α_1	5.12831681
α_2	15.1822397
α_3	1.29455132
α_4	0.695236647
p	-0.000125231319
q	0.355918396

2.3 Results

To apply the values of EV (electric vehicle) proportions obtained from curve-fitting to the adoption of electric semi-trucks beginning in 2020, we shift the starting time of the graph in Figure 2.2.1 (2011) to 2020, since electric semis are not currently in use and our all-infrastructure-available assumption 1 dictates that the transition to electric trucks begins today. We then plot the Bass diffusion model to predict the proportion of electric semis from 2020 (where the proportion is equal to 0) to 2040 (in 20 years—the endpoint of our analysis). This graph is shown in Figure 2.3.1. We also obtain specific prediction values of the percentage of semis that will be electric in 2025, 2030, and 2040, which are reported in Table 2.3.1.

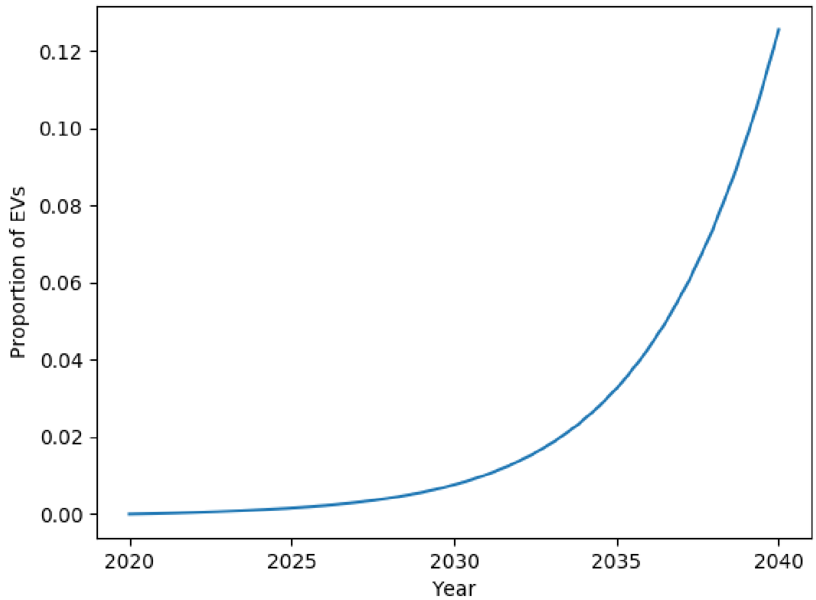

Figure 2.3.1: Electric Semi Proportions Projected by Bass Model, 2020 to 2040

Table 2.3.1: Percentage of Electric Semis in Five, Ten, and Twenty Years

Year	Percentage of Semis that are Electric
2025	0.1658
2030	0.8379
2040	12.53

The projected percentages of electric semi-trucks from our modified Bass diffusion model are relatively low. However, given how our model was constructed—based on real historical data on the adoption of electric cars—the results are expected, since in 8 years the percentage of electric cars only rose to about 4%. Additionally, in context, the low levels of adoption predicted by our model suggest that the benefits of adopting electric semis most likely are unable to outweigh the benefits of continuing to use diesel trucks.

Since our curve in Figure 2.3.1 follows an apparent exponential trend, we wanted to investigate the end behavior of the function given more time to see if the trend persisted. Thus, we graphed the function until 2080, as shown in Figure 2.3.2. From this graph, we can form the additional conclusion that the values in Table 2.3.1 are low because they just precede the period of rapid exponential growth from approximately 2040 to 2060. As the Bass diffusion model is essentially a modified logistic growth model, the shape of Figure 2.3.1 is very logical because the proportion of electric semis initially rises slowly before growing rapidly and leveling off at a maximum of 1. It is also reasonable in context because the initial slow-growth period represents innovators adopting the new technology of electric semis, while the middle rapid-growth period represents imitators rapidly adopting a popular technology, while the final slow-growth period represents laggards eventually adopting the technology as well.

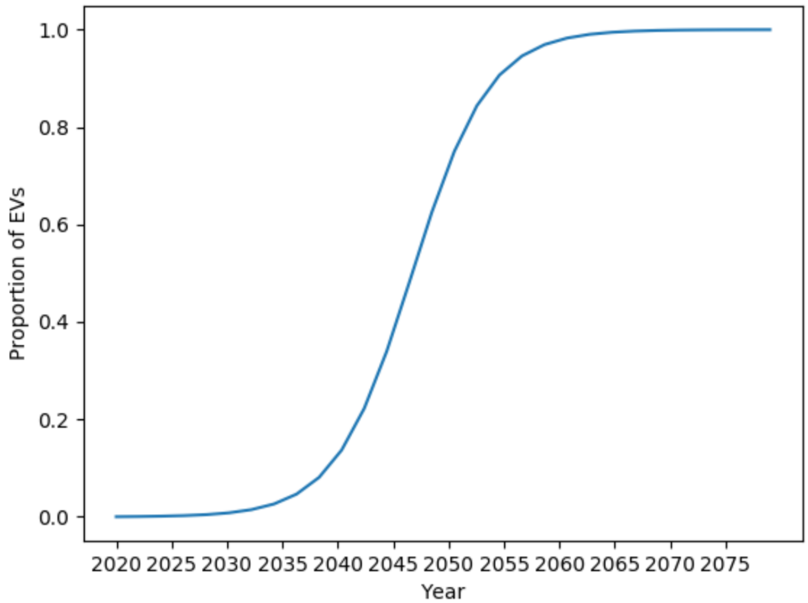

Figure 2.3.2: End Behavior of Bass Diffusion Model

2.4 Strengths and Weaknesses

Our model's strength lies in its implementation of the Bass model, which is a well-known model for diffusion of innovations that has been demonstrated to model the phenomenon well even when generic constants are used. It also is able to incorporate historical data because of its assumption that the adoption of electric semis can be compared to the adoption of electric cars—a particularly important strength given that data for electric semis currently are essentially unavailable. In addition, the curve-fitting procedure yielded a high R^2 value of 0.9975 and the final Bass curve has a logical, intuitive shape. Our model can also easily be expanded to incorporate a wider variety of factors; similarly, each of the external factors considered in the scaling function $x(t)$ can be made functions of time once more years of historical data eventually become available.

However, a lot of values in our model are held constant when they would perhaps be more aptly modeled as functions of time. The assumptions of constancy were necessary due to the shortage of data involving both electric cars and electric trucks. In addition, since companies are the purchasers of semi-trucks and individuals are typically the purchasers of automobiles, the adoption patterns of the two products may not be closely comparable. Overall, given the overarching data limitations of the technology in question, our model is built logically and outputs sensible results.

3 Part II: In It For the Long Haul

Development and installation of charging infrastructure is necessary for sustainability and expansion of the electric truck industry. In this section, we formulate a model to determine the sufficient number of stations, and chargers per station, along a given electric truck corridor, and apply the model to five corridors: San Antonio to New Orleans, Minneapolis to Chicago, Boston to Harrisburg, Jacksonville to Washington, DC, and Los Angeles to San Francisco.

3.1 Assumptions

1. *A charging station will be placed at every interstate highway intersection.* This is an efficient placement method given that these intersections will generally be the locations with the highest volume of traffic flow, allowing charging stations to service the maximum number of vehicles.

2. *Interstate highways are linear between intersections.* Logically, highways do not contain any significant turns, so the difference between a straight line between two intersections and the actual highway path should not be very large.

3. *Truck batteries are charged to 80% of capacity and are charged at 20%.* Electric vehicle batteries are typically charged up to 80% of capacity to ensure battery longevity, so we assume long-haul electric semis will be charged to this maximum recommended value. We also assume that the trucks are charged at 20% battery to not only preserve battery quality but more importantly to provide a safety net against a vehicle's battery dying before reaching the next charging station [1].

4. *Battery degradation of long-haul semis is negligible over a 5-year period.* While the battery capacity of long-haul semis do degrade over time, we assume that this change will not be significant because companies typically change out their long-haul trucks after 5 years to be repurposed as short-haul or regional-haul semis, which are not considered in our model [1].

5. *Maximum battery capacity decreases linearly as temperature decreases.* Approximate linearity is apparent from graphs of battery capacity at different temperatures [5], so perfect linearity is reasonably assumed in our model to make the process of calculating required distance between charging stations more efficient.

6. *The minimum range of an electric semi per charge is 200 miles.* This is the lower end of the current claims for the range of electric semis (Daimler electric e-Cascadias) [1]. Since technological innovation in the electric vehicle industry is expected to increase this range over time, it is reasonable to assume that 200 miles will be less than or equal to the ranges of approximately all electric semis.

3.2 Model Development

3.2.1 Determining Stations Needed Per Route

Perhaps the most crucial consideration in placing charging stations is assuring that vehicles will not be left "stranded" between two stations. For this reason, several worst-case scenario accommodations, as described below, are necessary to account for approximately all electric semis.

As per assumption 1, we assume the placement of a charging station at each interstate highway intersection (at a rest area) along a trucking route. However, additional charging stations will need to be placed along the route between those points because the distance between major intersections may be too large for an electric semi to travel on a single charge. Also, it is expected that electric semis will neither be charged to full capacity nor charged only when the battery is completely depleted; assumption 2 states that the vehicles are charged to about 80% and charged at about 20%, according to research.

In addition, an electric vehicle's battery capacity depends upon the temperature in which it operates; maximum capacity decreases approximately linearly as temperature decreases (see assumption 5). Consequently, charging stations will need to be placed at increasingly smaller intervals as a truck travels from a warmer region to a colder region, particularly for longer routes that pass through a significant temperature gradient. We calculate the degradation of driving range as a function of temperature from a 2019 AAA study that found that at an ambient temperature of 20°F, electric vehicles can only travel 88% of the distance that they can at 75°F [6]. As per assumption 5, we perform a linear regression to yield the below equation for the battery capacity as a proportion out of 1, where T is the ambient temperature in degrees Fahrenheit:

$$Cap(T) = 0.00218T + 0.836 \tag{6}$$

The distance between charging stations is also dependent on the range of the electric semi. Our worst-case scenario truck range is given by assumption 6 as 200 miles on a single charge (0% to 100%). Finally, to determine the maximum required distance R between consecutive charging stations, we combine the above factors into the equation below:

$$R(T) = 0.6 \cdot Cap(T) \cdot 200 \tag{7}$$

Since charging infrastructure will be placed along major trucking routes, we also place charging stations at each endpoint (which often coincide with interstate intersections). We wrote a Python program (stations.py) to decide the total stations needed by first calculating T and R. In order to account for the worst-case scenario (maximum degradation of battery capacity), we decided to calculate T based on the coldest days of the year. To this end, we obtained data from the Python API *forecastio*; however, the API limited the number of daily queries per user. Since we found that according to the National Oceanic and Atmospheric Administration, the coldest day of the year lies in January for a majority of the country [7], we decided it would be appropriate to limit our search to only January in order to estimate the coldest temperatures. The values of T are used to calculate R for each station.

Starting at the colder endpoint of each trucking corridor, the program then iterates through every segment between highway intersections (which is equal to $Stations_{intersections} - 1$ segments) until the warmer corridor endpoint. If the distance between the endpoints of a segment exceeds R, the program places a new charging station a distance R towards the other endpoint and repeats the process: calculate R, compare R to the distance remaining, and place a charging station if necessary. During this process, $Stations_{between}$ is the total number of stations added by the program for the entire route. Thus, the total number of stations along a route is calculated as follows:

$$Stations_{total} = Stations_{intersections} + Stations_{between} \tag{8}$$

Note that the number of in-between stations calculated by starting at the colder endpoint and moving toward the warmer endpoint will usually, but not always, be equal to the number of in-between stations calculated in the opposite direction. We chose the former direction for our algorithm because the R values are slightly shorter when moving from colder to warmer temperatures than vice versa and we wanted to account for the worst-case scenario. In this case, our value for $Stations_{between}$ may occasionally be higher than the opposite-direction calculation for corridors with particularly sparse highway intersections.

Our model is demonstrated on five sample corridors in Section 3.3.

3.2.2 Determining Chargers Needed Per Station

To determine the number of chargers needed per station, we used queuing theory to analyze the wait times at each station based on the rate of trucks entering the station and charging at the station. We used the M/G/k multi-server queuing model, which describes a queue where the trucks entering are Markovian because we assume that the arrival rate is constant (i.e., we use a homogeneous Poisson process), the truck charging time has a general distribution for which we found data, and there are k chargers as opposed to the M/G/1 model that only has one charger. With this queue, pictured below, we can determine the average waiting time and the average number of trucks waiting to charge at a certain station.

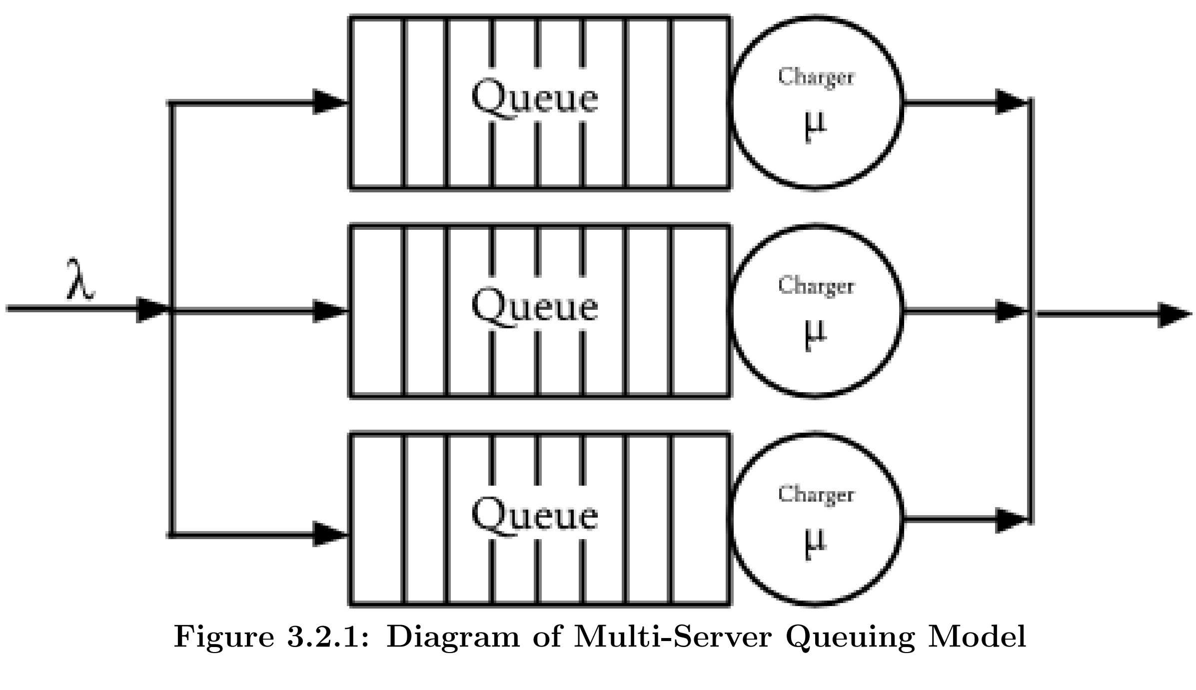

Figure 3.2.1: Diagram of Multi-Server Queuing Model

We define three variables that are inputted into the model. First, we define λ as the number of trucks that arrive at the charging station per hour or the mean rate of arrival. This data was retrieved from the corridor data dataset given to us in the M3 information set. We obtained the value of truck traffic at each of the locations of our charging stations detailed in Section 3.2.1. Where the truck traffic was not available, we used the average of the car/truck proportion along the corridor to convert the car traffic into truck traffic. Then, we obtained the average charging time and its standard deviation and found the reciprocals μ and σ [8]. The value k represents the number of chargers inputted into the model.

There is no way to directly obtain the average waiting time, but one effective method is to approximate mean waiting time using an M/M/c queue, which is a queue where charging times have an exponential distribution. This method is called the Kingman's law of congestion [9].

First, we define ρ as the probability of the chargers being used. It should be noted that k needs to be increased if $\rho \geq 1$ because it will yield an unstable curve. At a minimum, k should be large enough that $\rho < 1$ for the model to be able to predict waiting times.

$$\rho = \frac{\lambda}{k\mu} \tag{9}$$

Then, Little's law provides the following results given L_q as the mean number of trucks in the queue and W_q as the mean waiting time in the queue [10]:

$$L_q = \lambda W_q \tag{10}$$

According to the M/G/k queue model, we can define the probability of there being 0 trucks in the entire system for a M/M/c, that is, waiting or charging, as follows:

$$P_0 = \left(\sum_{m=0}^{k-1} \left(\frac{(k\rho)^m}{m!}\right) + \frac{(k\rho)^k}{k!(1-\rho)} \right)^{-1} \tag{11}$$

Using that, we can find the mean number of trucks in the M/M/c queue using the following:

$$L_q = \frac{P_0(\frac{\lambda}{\mu})^k \rho}{k!(1-\rho)^2} \tag{12}$$

We then find the average waiting time for the M/M/c queue using Little's law:

$$W_q^{M/M/c} = L_q/\lambda \tag{13}$$

Finally, we can write the following to use Kingman's law of congestion to approximate the average of the M/G/k that we want to analyze:

$$W_q^{M/G/k} = \frac{C_s^2 + 1}{2} W^{M/M/k} \tag{14}$$

where C_s is defined as the coefficient of variation of the charging time distribution:

$$C_s^2 = \frac{\sigma_s^2}{(1/\mu)^2} \tag{15}$$

Market surveys indicate that customers are only willing to wait a maximum of 13 minutes to start charging their cars [11]. Therefore, we found the minimum k that yielded a wait time of less than 13 minutes. The resulting average chargers per station are shown in Table 3.3.2 in Section 3.3.

3.3 Results

To demonstrate the functionality of our model, we apply it to the following five trucking corridors:

1. San Antonio, TX to/from New Orleans, LA
2. Minneapolis, MN to/from Chicago, IL
3. Boston, MA to/from Harrisburg, PA
4. Jacksonville, FL to/from Washington, DC
5. Los Angeles, CA to/from San Francisco, CA

The values for the number of interstate highway intersections, additional stations needed, and total stations along a route are given in Table 3.3.1. The number of interstate highway intersections along a route was obtained manually from Google Maps. We also provided the length of each route in miles, which is not required for our model but is included in the table for reference.

Table 3.3.1: Charging Station Breakdown for Five Sample Trucking Corridors

Corridor	Total Mileage	Intersection Stations	In-between Stations	Total Stations
San Antonio, TX to New Orleans, LA	543	6	4	10
Minneapolis, MN to Chicago, IL	408	6	4	10
Boston, MA to Harrisburg, PA	390	10	1	11
Washington, DC to Jacksonville, FL	706	10	6	16
San Francisco, CA to Los Angeles, CA	382	6	4	10

The numbers of total required stations outputted by our model are logical. Notably, a greater total mileage for a route does not necessarily correspond to a greater number of total stations. This is because our model, by placing a charging station at each interstate highway intersection, places more charging stations around more urban regions, since these regions will also have more tighter-knit networks of highways. In addition, since the distance between interstate intersections is smaller around more urban regions (particularly for the Boston to/from Harrisburg corridor), the Python program also logically places a fewer number of in-between stations (only 1 for that same corridor).

Then, to determine the number of chargers required per station, we implemented our M/G/k multi-server queuing model developed in Section 3.2.2. We obtained graphs of the daily average queuing time per truck while waiting for a charger and the average number of trucks in the queue as a function of the number of chargers at the station for one sample charging station (in San Francisco, CA); these are displayed in Figure 3.3.1. Since drivers are only willing to wait 13 minutes in line (or 0.217 hours), this sample station would need 26 chargers. This procedure was repeated for every station (defined in Section 3.2.1) along the five corridors, and the average number of chargers per station and total number needed for each corridor are reported in Table 3.3.1.

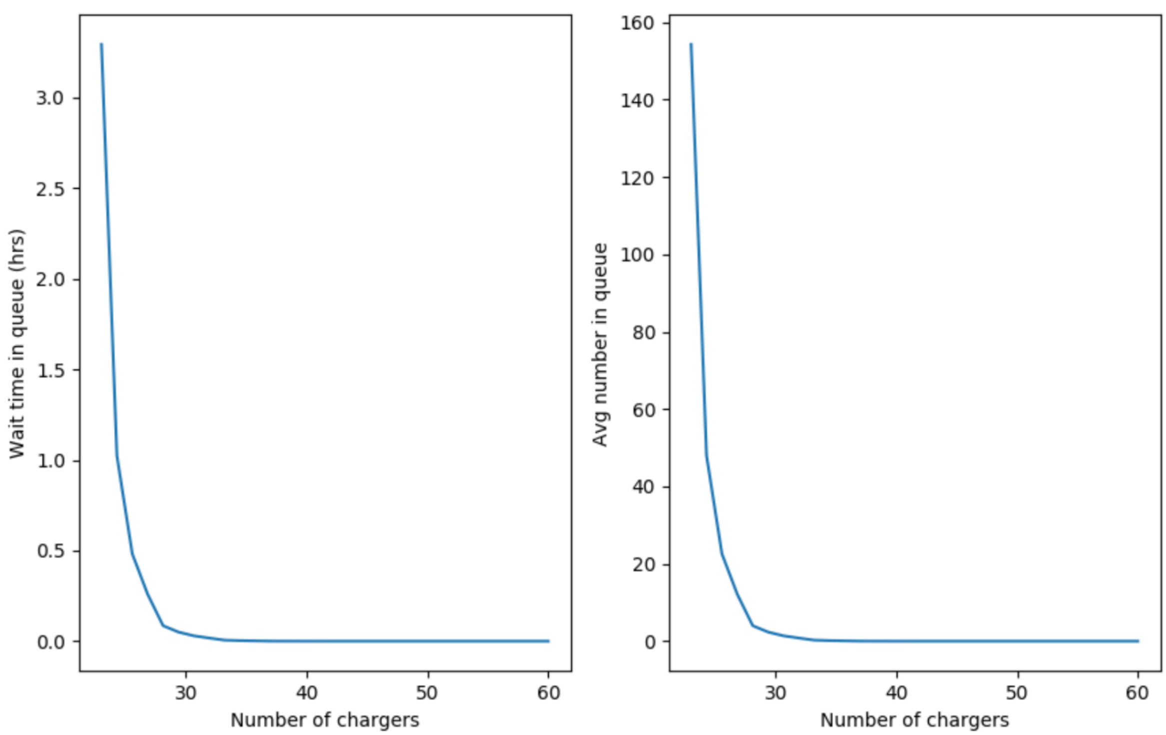

Figure 3.3.1: Average Wait Time and Queue Size for Sample San Francisco Charging Station

Table 3.3.2: Charging Station Breakdown for Five Sample Trucking Corridors

Corridor	Average Number of Chargers	Total Number of Chargers
San Antonio, TX to New Orleans, LA	27.4	274
Minneapolis, MN to Chicago, IL	23.6	236
Boston, MA to Harrisburg, PA	17.1	188
Washington, DC to Jacksonville, FL	14	224
San Francisco, CA to Los Angeles, CA	21.4	214

The shape of the curves in Figure 3.3.1 are logical because with a very small number of chargers, the wait time and number of trucks in queue will both be very large because electric vehicles require relatively long periods of time to charge from 20% to 80%. This value rapidly decreases as more chargers become available; however, the queue time quickly levels out to 0 because if additional chargers are added to a station where there are already enough stations for the traffic volume to be serviced, queue time will no longer decrease (there are no more vehicles waiting to use those chargers). Furthermore, the number of chargers per station projected in the table are also feasible for installation.

3.4 Sensitivity Analysis

Table 3.4.1: Sensitivity Analysis for Average Number of Chargers per Station

Constant	-10% Change in Constant	+10% Change in Constant
λ	-9.489%	+8.759%
μ	+9.489%	-6.569%
σ	-1.460%	+0.730%

We conducted a sensitivity analysis on the three variables we used in our model to determine the average number of chargers per station, by changing each constant by +10% and -10% and calculating the resulting change in the average number of chargers predicted. The small percent changes in the average number of chargers, each in the appropriate direction, shows that our model is robust and resilient to small alterations in its parameters.

3.5 Strengths and Weaknesses

One of the greatest strengths of our first model—for calculating the required number of stations—is its focus on the worst-case scenario of nearly all of its parameters. This allows our calculation of stations to cover essentially all electric semi-trucks by assuming the minimum estimated single-charge range and the minimum ambient temperatures as well as calculating the number of in-between stations from the colder to the warmer endpoint of corridors. It is also comprehensive, for it takes into account factors such as the degradation of battery life due to temperature drops along a route. On the other hand, our model's "generous" estimate most likely renders the cost of installing the proposed infrastructure more costly than absolutely necessary; particularly as technology for electric vehicles is in development, the single-charge range can probably be expected to rise within the decade.

Our second model is sound because it uses an established method, queuing theory, for determining the number of chargers to be placed at each station. The resulting figures align with intuition, and a sensitivity analysis suggests that the model is resilient to small changes. It also relies on relatively few, easily obtainable variables so that the model can easily be applied to other trucking corridors. However, the most prominent drawback of the queuing model is that the number of chargers needed is calculated based on average daily traffic data, and so at rush hour or particularly busy travel days, truck drivers would most likely have to queue for more than the desired amount of 13 minutes.

4 Part III: I Like to Move It, Move It

The transition to electric trucking is motivated by a variety of factors—economic, environmental, or otherwise. Considering these factors, we developed a metric to rank the desirability of developing trucking corridors and used it to rank the five routes in Part II.

4.1 Assumptions

1. *The only infrastructure required for electrification of the trucking industry are the charging stations.* While maintenance centers, among other structures, might also be necessary, we make this assumption in order to use our infrastructure models from Part II in making our metric.

2. *The decision of whether to develop infrastructure depends on community motivation, time necessary for development, and its anticipated usage.* Support from the community is essential to develop infrastructure. Infrastructure that takes less time to create should be created first because it also has less cost. In addition, chargers in higher-frequency routes should be created first to suit the demands of the population.

3. *Any constants that are factored into the calculation of every corridor are arbitrary because they will not affect the final ordering of which corridors should be built first.* The proportionalities and variable data is what will determine this order. The constants are there to ensure the values make sense numerically. This assumption is further justified by the results of our sensitivity analysis.

4. *Each charger costs the same amount of money and takes the same amount of time to construct.* As chargers are identical, their installation should be similar.

4.2 Model Development

Table 4.2.1: Variables Used in Differential Equations

Constants	**Meaning**	**Value**
α	Proportionality constant of people convinced by others who want route	0.01
β	Proportionality constant of people changing their belief due to economic or environmental reasons	0.02
γ	Proportionality constant for people initially wanting route due to economic reasons	0.004
E	Fixed initial percentage of people who want to route due to environmental reasons	0.2
λ_1	Weight on the time it takes to construct	-0.0007
λ_2	Weight on the anticipated usage	$7.7 \cdot 10^{-6}$

The proportion of r_i denotes how many people support the building of electric charging stations. Future supporters will either be influenced by their own desires (increased convenience or wanting to save the environment) or by people who are already in favor of the construction of electric charging stations. To account for this, the change in the proportion of people who support building more charging stations is the sum of two factors as detailed below:

$$\frac{dr_i}{dt} = \alpha r_i(1 - r_i) + \beta(1 - r_i)$$

Initially, there is a fixed number of people who support the development for environmental reasons, which is denoted by E. There are also a number of people who support the development due to economic reasons. The revenue that is produced is proportional to both the traffic and the number of chargers available, so it is proportional to $n_i T_i$. Additionally, individuals who earn less are more likely to support the development due to economic reasons, so this component of r_i is proportional to $\frac{n_i T_i}{I_i}$. γ has been chosen as the constant of proportionality for this term. The formula for r_i is below.

Table 4.2.2: Average Median Household Income and Daily Truck Traffic Along Each Route [12][13]

Route #	Average Median Household Income	Avg Daily Truck Traffic
1	$\$54,756$	14,288
2	$\$63,732$	16,022
3	$\$71,523$	9,293
4	$\$56,788$	9,515
5	$\$71,228$	13,975

$$r_i(0) = \frac{\gamma n_i T_i}{I_i} + E \tag{16}$$

We ran our model with the values in Table 4.2.1. As α, β, γ, and E do not change with location, as per assumption 3, they are arbitrary and The choice to begin development is based on the general percentage of the population who agree with it, the time it takes to construct, and the anticipated usage. The time it takes to construct is proportional to the number of chargers because each charger takes the same amount of time to construct. The anticipated usage is proportional to the traffic of trucks through the route. The minimum threshold such that development should begin is when this expression is $\frac{1}{2}$ because a majority of people agree with the development in the limiting case that $n_i = 0$ and $T_i = 0$. The values of λ_1 and λ_2 serve as arbitrary weighting factors to ensure that they are of the same magnitude. :

$$r_i + \lambda_1 n_i + \lambda_2 T_i \geq \frac{1}{2} \tag{17}$$

4.3 Results

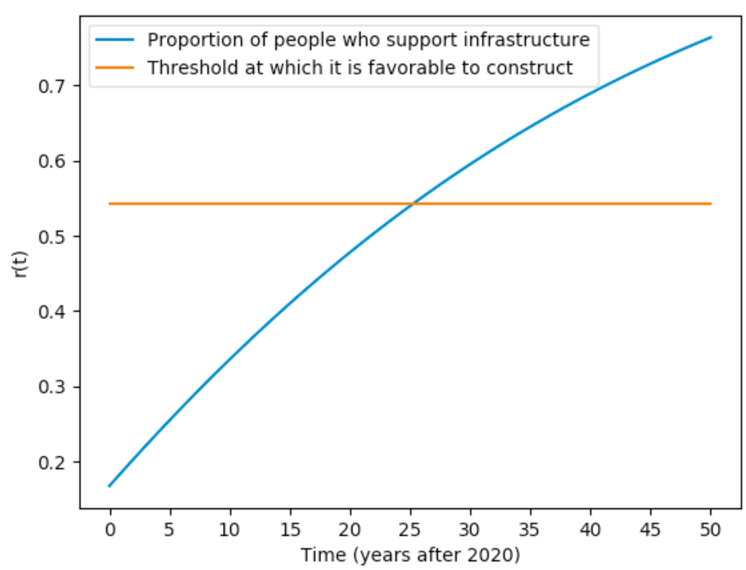

Figure 4.3.1: Proportion of People who Support Infrastructure over Time in the Los Angeles, San Francisco Area

We used development.py to calculate the time in years it takes for the proportion of people to exceed the threshold. As seen in Figure 4.3.1, in 25.3 years, the proportion of people who support the infrastructure will exceed the threshold at which it is favorable to build it. Doing similar for the other corridors, we arrived at the results below in Table 4.3.1.

Table 4.3.1: Number of Years Until it is Favorable to Build Electric Semi Infrastructure

Route Number	Years Until Route Becomes Favorable	Ranking
San Antonio-New Orleans	21.9	2
Minneapolis-Chicago	21.2	1
Boston-Harrisburg	30.3	5
Jacksonville-Washington, DC	30.0	4
Los Angeles-San Francisco	25.3	3

As shown by the above table, the route from Minneapolis to/from Chicago should be targeted for development first, followed closely by the San Antonio-New Orleans corridor. The remaining ranks of favorability are the LA-San Francisco, Jacksonville-DC, and Boston-Harrisburg corridors. The values in the second column suggest that our model is reasonable because the numbers all fall within a small range; since each corridor spans several hundred miles, many regional differences are "averaged out," resulting in the number of years until favorability being relatively constant across routes.

4.4 Sensitivity Analysis

We conducted a sensitivity analysis on all the constants in our model, inflating and deflating each by 10% to see how they affected our results. These changes in the constants did not affect the ordering in which the charging stations should be built. This shows that the constants in our model have arbitrary values and that the results rise from the differing income distribution, traffic, and cost for each corridor.

4.5 Strengths and Weaknesses

Our model is resilient because it is unaffected by many confounding constants, as shown in our sensitivity analysis. The model is comprehensive, as it considers both the economic and environmental factors that may persuade people to support the building of electric charging stations. A weakness of our model is that averages data across wide swaths of geography. While the socioeconomic status and geography may differ greatly between the ends of each corridor, our model averages them out to come to one value for each corridor. Overall, our model is solid in predicting the order in which charging stations should be developed along each of the corridors.

5 Conclusion

5.1 Further Studies

Our first model relies upon comparing the adoption of electric trucks starting in 2020 to the adoption of electric cars starting in 2011. This was necessary to create our Bass diffusion model because there is currently no data on electric semi usage. Evidently, further studies could be done with a greater degree of confidence in the results, and a wider variety of factors, once the necessary data is available. Our infrastructure model could be improved in the future if more accurate values are obtained for the single-charge value of an electric semi's range.

5.2 Conclusion

In Part 1, we used the Bass diffusion model with our own scaling factor based on data about electric vehicles so that we could predict the proportion of electric trucks that would be in use by 2025, 2030, and 2040. We produced a curve by solving this differential equation to determine that there would be 0.1658%, 0.8379% and 12.53% adoption in the trucking industry by 2025, 2030, and 2040, respectively.

In Part 2, we determined the number of charging stations that we would need by placing a station at every traffic intersection on interstate highways, and then calculating the amount of stations needed in between, based on battery range as a function of temperature. Then, we determined the number of chargers that would be required at each station by modeling each station as a M/G/k queuing system to determine k, the number of chargers that would be required to reduce the average wait time to charge to be less than 13 minutes. We applied this to five corridors: San Antonio to New Orleans, Minneapolis to Chicago, Boston to Harrisburg, Jacksonville to Washington, DC, and Los Angeles to San Francisco.

In Part 3, we found the proportion of individuals who wanted the development as a function of time. We computed a weighted sum of this percentage, the time it takes to construct the infrastructure, and the anticipated route usage. Using a differential equation, we found the time at which each weighted sum was greater than a specific threshold value, and we ranked these times from least to greatest for the same five corridors studied previously. The model indicated that Minneapolis-Chicago and San Antonio-New Orleans corridors be targeted first for development.

6 References

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7 Appendix

7.1 bass.py

```python
import numpy as np
from scipy.integrate import odeint
from scipy.optimize import curve_fit
import matplotlib.pyplot as plt
import sys
import pandas as pd


def read_vehicle_sales(): # Read in database on historic vehicle sales
    def dateparse(x): return pd.datetime.strptime(
        x, '%Y-%m-%d') # Convert string to date
    df = pd.read_csv('TOTALSA.csv', parse_dates=[
                     'DATE'], date_parser=dateparse)
    df = df.groupby(df['DATE'].dt.year).mean() # Get means so its only by year
    return np.array(df['TOTALSA'].to_list()) # Return a list


sales = read_vehicle_sales()


def fuel_cost(t): # Our constant ratio of fuel cost
    return 0.04/0.37


def vehicle_sales(t): # Retreive data from database
    if t > 2020:
        return sales[-1] # Constant afterwards

    return sales[int(t)-1976]


def truck_range(t): # Const range ratio
    return 200/350


def truck_cost(t): # Constant cost
    return 55600/36718


total_cars = 263.6 * 1e6 # Total cars, relatively constant

hist_data = np.array([ # List of electric vehicles data
    4000,
```

```
    10000,
    18000,
    22000,
    28000,
    40000,
    80000,
    90000,
    110000,
    140000,
    180000,
    190000,
    220000,
    260000,
    290000,
    310000,
    345000,
    380000,
    405000,
    435000,
    480000,
    515000,
    575000,
    605000,
    655000,
    705000,
    780000,
    820000,
    895000,
    1000000,
    1125000,
    1190000,
]) / total_cars

# Starts at 2011, goes to end of Q1 2019
hist_t = np.linspace(2011, 2019.25, len(hist_data))


def fit_func(t, a1, a2, a3, a4, p, q): # Function to fit the parameters
    def bass(f, t):
        x = 1 + a1*np.log(fuel_cost(t)) + a2*np.log(truck_cost(t)) + a3 * \
            np.log(vehicle_sales(t)) + a4 * \
            np.log(truck_range(t)) # Our general Bass parameter

        dfdt = (1-f) * (p - q*f) * x # Bass differential
        return dfdt

    f0 = hist_data[0] # Run on historical data
```

```
    sol = odeint(bass, f0, t)
    return sol[:, 0]


print("Starting fitting")

print(hist_t)
print(hist_data)

popt, kcov = curve_fit(fit_func, hist_t, hist_data, # Fir curves, with our initial guesses
                       method='lm', maxfev=100000000, p0=[2.5, 1, 1, 0.4, 0.06, 0.37])

# # print(hist_t)
print(popt)
# print(kcov)
# sys.exit(0)

hist_func = fit_func(hist_t, *popt) # Get the actual historical values
residuals = hist_data - hist_func
ss_res = np.sum(residuals**2)
ss_tot = np.sum((hist_func-np.mean(hist_func))**2)
r_squared = 1 - (ss_res / ss_tot) # Calculate r^2 manually

print("RSQUARED", r_squared)

#popt = [1, 1, 1, 1, 1]

t1 = np.linspace(2011, 2070, 30) # Our predicted times
#t1 = hist_t

print(t1)

sha_func = fit_func(t1, *popt)

# # time pointse(0, 20)

# t2 = np.linspac
# # solve ODE
# f_sharif = odeint(sharif, y0, t1)
# f_bass = odeint(bass, y0, t2)

# plot results
plt.plot(t1+9, sha_func) # Shift by 9 years
#plt.plot(hist_t, hist_data, 'ro')
plt.xlabel('Year')
plt.xticks(np.arange(min(t1+9), max(t1+9)+1, 5)) # Make x labels better
plt.ylabel('Proportion of EVs')
```

```
plt.show()
```

7.2 stations.py

```
import forecastio
import datetime
from time import sleep
from haversine import haversine, Unit

# Query API to get the temperature at a coordinate


def findTemperature(cord):
    lat = cord[0]
    lng = cord[1]
    temperature = 0
    for day in range(1, 32):
        current_time = datetime.datetime(2019, 1, day, 0, 0, 0)
        data = forecastio.load_forecast(
            api_key, lat, lng, time=current_time)
        forecast = data.hourly()
        temp = 0
        for hour in forecast.data:
            temp += hour.temperature
        temp /= len(forecast.data)
        # print(str(1)+"-"+str(day)+": " + str(temp))
        temperature += temp
        sleep(0.1)
    temperature /= 31
    # print(str(lat)+','+str(lng)+": " + str(temperature))
    return temperature


api_key = "d5c793a085e1e881b6fe6df6476e8832"

# san antonio to new orleans
# cords = [(29.417665, -98.491482), (29.777760, -95.360726), (30.272725, -92.012239),
# (30.436903, -91.162000), (30.160854, -90.428748), (29.972942, -90.075752)]

# minneapolis to chicago
# cords = [(44.977757, -93.259357), (44.002900, -90.427642), (43.491266, -89.476002),
# (42.527031, -88.968241), (42.252348, -88.958323), (41.879750, -87.652840)]

# boston to harrisburg
# cords = [(42.359472, -71.059326), (42.110469, -72.031678), (41.745425, -72.642173),
# (41.014642, -73.693406), (41.117336, -74.159782), (40.871334, -74.446501),
```

```
# (40.664688, −74.649443), (40.565243, −75.557297), (40.433047, −76.519679),
# (40.272597, −76.886925)]

# DC to Jacksonville
# cords = [(38.902646, −77.058297), (37.571089, −77.439774), (37.205219, −77.396232),
# (35.978383, −77.895145), (35.460325, −78.505470), (34.598341, −79.143567),
# (34.190401, −79.832836), (33.323715, −80.553616), (32.112230, −81.234370),
# (30.328741, −81.661589)]

# San Francisco to LA
# cords = [(37.766656, −122.427427), (37.690650, −121.922926), (37.744831, −121.553756),
# (34.336156, −118.499355), (34.153521, −118.386955), (34.054378, −118.242518)]


stations = []
for index in range(len(cords)−1): # Go through each coordinate
    print('new intersection station')
    travelled = 0
    station = 0
    position = cords[index]
    destination = cords[index+1]
    # Get the distance between stops
    distance = haversine(position, destination)
    print("distance: "+str(distance))
    while 1: # Keep on going until we run out of energy
        temperature = findTemperature(position)
        capacity = 0.00218*temperature+0.836
        step = 200*0.6*capacity
        print("step: "+str(step))
        travelled += step
        if travelled > distance:
            break
        station += 1
        position = ((cords[index][0]+cords[index+1][0])/2,
                    (cords[index][1]+cords[index+1][1])/2)
    stations.append(station)

print(stations)
```

7.3 chargers.py

```
from haversine import haversine, Unit

# san antonio to new orleans
cords1 = [(29.417665, −98.491482), (29.777760, −95.360726), (30.272725, −92.012239),
          (30.436903, −91.162000), (30.160854, −90.428748), (29.972942, −90.075752)]
```

```
# minneapolis to chicago
cords2 = [(44.977757, −93.259357), (44.002900, −90.427642), (43.491266, −89.476002),
          (42.527031, −88.968241), (42.252348, −88.958323), (41.879750, −87.652840)]

# boston to harrisburg
cords3 = [(42.359472, −71.059326), (42.110469, −72.031678), (41.745425, −72.642173),
          (41.014642, −73.693406), (41.117336, −74.159782), (40.871334, −74.446501),
          (40.664688, −74.649443), (40.565243, −75.557297), (40.433047, −76.519679),
          (40.272597, −76.886925)]

# DC to Jacksonville
cords4 = [(38.902646, −77.058297), (37.571089, −77.439774), (37.205219, −77.396232),
          (35.978383, −77.895145), (35.460325, −78.505470), (34.598341, −79.143567),
          (34.190401, −79.832836), (33.323715, −80.553616), (32.112230, −81.234370),
          (30.328741, −81.661589)]

# San Francisco to LA
cords5 = [(37.766656, −122.427427), (37.690650, −121.922926), (37.744831, −121.553756),
          (34.336156, −118.499355), (34.153521, −118.386955), (34.054378, −118.242518)]

total = 0
for i in range(len(cords5)−1):
    # Get the total distance in miles for each route
    total += 0.6214*(haversine(cords5[i], cords5[i+1]))
    print(total)
```

7.4 queue.py

```
import numpy as np
import matplotlib.pyplot as plt
from scipy.special import factorial
import sys

# Amount of traffic to chargers per route
path1 = [19818, 16500.77339, 17648.38488, 8377.002754, 27139.56221]
path2 = [6009.516888, 33000, 12600, 13100, 11900]
path3 = [4338, 19191, 60935, 9840, 7721, 10301, 6787, 13821, 14065, 7030]
path4 = [11726, 12082, 5583.716502, 3924.404523,
         5017.443366, 8480, 5990, 5513, 13944]
path5 = [16519, 20023, 8779, 11397, 10490]



# lmbda = 1/10
# mu = 1/15
# sigma = 8.33**0.5
```

```
# Our M/G/k queue function
def get_W_mgk(k, lmbda, mu, sigma):
    Ca2 = 1
    Cs2 = mu**2 * sigma**2
    p = lmbda/(k*mu)

    # print(p)

    P0_sigma = 0
    for m in range(k):
        P0_sigma += ((k*p)**m)/factorial(m, exact=False)
        # print(m, P0_sigma)

    # print("fac", (factorial(k, exact=False)))
    P0_sigma += ((k*p)**k)/(factorial(k, exact=False) * (1-p))

    # print(P0_sigma)

    P0 = 1/P0_sigma

    # print(P0)

    L = (P0*(lmbda/mu)**k * p) / (factorial(k, exact=False) * (1-p)**2)

    # print(L)

    W_mmc = L / lmbda

    # print(W_mmc, Cs2)

    W_mgk = ((Ca2 + Cs2)/2) * W_mmc

    return W_mgk


# print(get_W_mgk(2))
# sys.exit(0)
char = []
chargers = []

a = 2.1*(500/55 + 2.1)/365/24 # Our lambda
b = 2.1 # Mu
c = 1.533 # Sigma

change_percent = 0.1 # Change percentage for sensitivity analysis
```

```
sensitivity = [ # List of sensitivity comparisons
    (a, b, c),
    (a*(1 + change_percent), b, c),
    (a*(1-change_percent), b, c),
    (a, b*(1 + change_percent), c),
    (a, b*(1-change_percent), c),
    (a, b, c*(1 + change_percent)),
    (a, b, c*(1-change_percent))
]

for variables in sensitivity: # Do each set of variables
    l = variables[0]
    mu_l = variables[1]
    sigma_l = variables[2]
    #print(l, mu_l, sigma_l)
    for traffic in path1: # Do each path
        lmbda_l = l*traffic

        k_init = 5

        k = list(range(k_init, 60))
        W = []
        for k_c in k: # Run each charger

            W.append(get_W_mgk(int(k_c), lmbda_l, mu_l, sigma_l))
        # print(W)
        for w_i, w in enumerate(W):
            #print(w_i, w)
            if w < 13/60 and w > 0: # Get the one that is just beloe 13 min wait time
                #print("MIN NUMBER OF CHARGERS", w_i + k_init)
                char.append(w_i + k_init)
                break
        # print(char)
    chargers.append(char)
    char = []
# print(chargers)
avgs = [sum(x)/len(x) for x in chargers] # Get averages of paths
changes = [100*(avg-avgs[0])/avgs[0]
           for avg in avgs] # Get percent change from avgs
print(changes)
# fig, axs = plt.subplots(1, 2)
# axs[0].plot(k, W)
# axs[0].set_xlabel("Number of chargers")
# axs[0].set_ylabel("Wait time in queue (hrs)")
# axs[1].plot(k, np.array(W)*lmbda)
# axs[1].set_xlabel("Number of chargers")
```

```
# axs[1].set_ylabel("Avg number in queue")
# plt.show(block=True)
```

7.5 development.py

```
import numpy as np
from scipy.integrate import odeint
from scipy.optimize import curve_fit
import matplotlib.pyplot as plt
import sys
import pandas as pd

I = [54756, 63732, 71523, 56788, 71228]
n = [274, 236, 188, 224, 214]
T = [14288, 16022, 9293, 9515, 13975]
E = 0

gamma = 0.004


def model(r, t):
    alpha = 0.01
    beta = 0.02
    drdt = alpha*r*(1-r) + beta*(1-r)
    return drdt


for i in range(5):
    # initial condition dependent on population and income
    r0 = gamma*n[i]*T[i]/I[i] + E

    # time points
    t = np.linspace(0, 50)

    # solve ODE
    r = odeint(model, r0, t)

    lmbda_1 = -0.0007
    lmbda_2 = 7.7*1e-6
    y = 1/2 - lmbda_1*n[i] - lmbda_2 * T[i]

    # plot results
    rate = plt.plot(
        t, r, label='Proportion of people who support infrastructure')
    threshold = plt.plot(
        t, y+t-t, label='Threshold at which it is favorable to construct')
```

```
plt.xlabel('Time (years after 2020)')
plt.xticks(np.arange(min(t), max(t)+1, 5))
plt.ylabel('r(t)')

plt.legend()
plt.show()
```