MathWorks Math Modeling Challenge 2024

Blake School - Northrop Campus

Team #17570, Minneapolis, Minnesota

Coach: Christin Winkler

Students: Karn Kaura, Alex Wu, Curtis Ying

M3 Challenge TECHNICAL COMPUTING RUNNER UP—$2,000 Team Prize

JUDGE COMMENTS

Specifically for Team #17570—Submitted at the close of triage judging

COMMENT 1: Your executive summary is well-written and provides results and recommendations. The models chosen are creative and well-justified throughout the paper. The sensitivity analysis and analysis of strengths/weaknesses of the model are thoughtful and thorough. Overall, this was well done.

COMMENT 2: You have a nice executive summary presenting not just the problem, but also your findings. It is good practice to spell out any acronyms the first time you use them. You have some good thoughts about the model developments in the entire paper. There is a lot of depth to the work you have done.

COMMENT 3: The writeup of the work done answering Q1 is very strong. I appreciated the modeling and descriptions of the mitigation plans for the unhoused included in Q3.

COMMENT 4: Awesome! This paper contained many details in a compact report. It was also very well organized.

COMMENT 5: Your executive summary is well written and includes a nice overview of the models employed as well as a comprehensive description of your results. Your models are thoroughly described and creative.

Home-Lessening Homelessness: A Mathematical Exploration

Team #17570

Executive Summary

Dear Secretary of the U.S. Department of Housing and Urban Development,

Homelessness rates are an especially concerning issue due to skyrocketing housing costs. Although approaches from your department and nonprofit organizations have helped, predicting housing accessibility and determining how to address this issue presents a unique challenge. Thus, this report investigates long-term housing and homelessness numbers, evaluating the best overall response to this crisis.

First, we predicted the housing supply in the next 10, 20, and 50 years for Seattle, WA, and Albuquerque, NM, using population as a baseline assumption. Since both cities represent human population systems, a **logistic growth model** was used to determine the theoretical maximum number of inhabitants. Then, we converted that theoretical maximum to a theoretical maximum number of houses, assuming the supply of houses fully meets the demand. Using the new theoretical maximum in a **carrying capacity model**, we predict the number of housing units in Seattle will be 260,960 units by 2034, 265,440 by 2044, and 270,500 by 2074. In Albuquerque, we predict 402,300 units by 2034, 426,590 by 2044, and 460,390 by 2074.

In correlation, we created a model estimating the homeless population in both cities. First, we extrapolated the non-homeless population using the logistic growth model. Then, making assumptions about YOY changes, we constructed an **asymmetric Markov chain** with two absorption states. Using a corresponding recursive equation, we predict the homeless population in Seattle will be 31,029 inhabitants by 2034, 41,297 by 2044, and 56,561 by 2074. In Albuquerque, we predict 6,874 inhabitants by 2034, 9,516 by 2044, and 12,612 by 2074.

Finally, to determine the best homelessness reduction plan in Seattle, we developed a metric factoring cost, emotional benefits, success in preventing future homelessness, and waiting time. Since there were multiple criteria influencing effectiveness and efficiency, the **TOPSIS and Grey Relational Analysis (GRA) multi-criteria decision models** were combined. While TOPSIS used entropic weights calculated from each criterion's value to produce a metric and GRA did not need weights, both produced a metric between 0 (least optimal) and 1 (most optimal). Since GRA handles fuzzy data and complex relationships but assumes criteria have a linear relationship with alternative plans, and TOPSIS accommodates outliers and many alternatives but assumes independence and fixed weights, both output values were averaged to produce the final metric. Though we also included combinations of plans, our model ranked Rapid Re-Housing (RRH) as the best result, followed by Permanent Stable Housing. Finally, using a **queueing model**, we calculated that the current number of RRH units could allow approximately 1,680 unhoused people to find sustainable housing each year.

We sincerely hope that our results may be of use to you and provide insight into the current best plan for addressing homelessness. If you have any questions, please feel free to connect with us.

Best Regards,
M3 Team #17570

Table of Contents

Background

Housing is a basic human necessity. The Universal Declaration of Human Rights adopted by the United Nations declares that adequate housing is a human right necessary for a basic standard of living [1]. Yet, particularly in metropolitan areas, the increasing homeless (or unhoused) population has become a pressing issue. Homelessness has remained an unsolved problem since the inception of metropolitan areas and has been exacerbated by gentrification, budget cuts to the U.S. Department of Housing and Urban Development (including a 41% decline between 1976 and 2002 [2]), inadequate supplies of affordable housing options, and numerous other factors [3].

Recent spikes in housing prices have especially made it difficult for the economically disadvantaged to keep their homes. In large cities like Albuquerque and Seattle, the median housing cost has approximately doubled within the last decade—a much larger rate of increase than that of the annual median income [4]. In January 2024, a record half of all US renters paid over the recommended 30% of income for shelter expenses [5]. Therefore, many households and individuals have been forced to leave their homes or remain unhoused. However, being unhoused presents challenges to both individuals and communities. Twenty-one percent of individuals experiencing homelessness report serious mental illnesses and 16% report substance use disorders, leading to increased suicide rates [6]. Homelessness also disproportionally affects the LGBTQ+ community and certain racial groups, and cities with unhoused populations spend $30,000-50,000 per individual through sanitation, emergency rooms, shelters, and other services [7].

Although there is no perfect solution, the US government and numerous nonprofit organizations have already taken several approaches to address homelessness. The most notable approaches include permanent supportive housing (PSH), which provides long-term housing assistance and supportive services, rapid re-housing (RRH), which provides short-term assistance, and Section 8 (or Housing Choice) vouchers, which provide funds for low-income housing rent. However, each has benefits and drawbacks, so for the sake of helping unhoused individuals and communities thrive, it is essential to analyze current housing accessibility and homelessness rates to determine the best course of action.

Global Assumption

G-1: There will not be any significant policy shifts regarding housing or homelessness.

- **Justification:** Given the polarized nature of Congress, it is extremely difficult to predict when the government will pass certain policies to aid the housing or homelessness crises. Assuming consistent trends simplifies the situation so that it can be modeled appropriately.

1 It Was the Best of Times

1.1 Defining the Problem

Question 1 asks us to build a mathematical model that predicts changes in the amount of housing over 10, 20, and 50 years, in either two US or UK regions. We chose to model the US regions as the housing shortage's larger prevalence allowed us to gather more data easily.

1.2 Variables and Data

Symbol	Definition	Units
N	The predicted number of housing units in a given city	None
$P(t)$	The predicted population of a given city *t* years after 2010	People
K	The maximum possible housing units in a given city	Housing Units
M_P	The maximum possible population of a given city	People
k	Growth rate constant	None

Table 1: Variables for Problem 1

1.3 The Model

Inherently, the number of housing units in a region is linked to that region's population. The greater the population, the greater the demand for housing, thus leading to more houses being built. This direct correlation, however, is accompanied by an indirect one. Especially in younger cities like Seattle, as the population increases, so does the price of housing, almost always at a higher rate than the median income. Even in cities like Albuquerque, where the median age is increasing and there is little population growth, higher prices continue to increase faster than income, disincentivizing new residents, and even driving away old ones.

An impactful statistic that emphasizes this disparity is to normalize these housing prices by putting them in terms of years of income. By dividing the median price by median income, our team saw that from 2010 to 2022, each of the four regions saw one year of income representing a smaller percentage of the price of the same median house. In Albuquerque, a house used to cost 3.67 years of income, but now costs over 5. In Seattle, the median price is even higher, worth almost 6 years of income.

Thus, the short-term increase in population in any city cannot exponentially, or linearly, grow forever given these rapid price increases. Therefore, it is expected that at some time, the rate of population will slow down as it approaches an equilibrium point. Albuquerque is a real-world example of this equilibrium point, as the population from 2010 to 2022 only increased by around 30,000 people, a mere 5.66% increase over 13 years. In comparison, Seattle experienced a growth of 140,000 people, correlating to a 23.5% increase in population in the same period.

To best handle a wide variety of population growth and predict it over time, a logistical model was chosen. Often used to model populations, this model also makes sense intuitively, as it is defined as having the growth (or derivative) increase and then decrease exponentially as the population gets higher. The equation for the model is

$$P(t) = \frac{M_P}{1+e^{(-a*(t-b))}} \tag{1}$$

where P(t) is the population, t is the time in years, M_P is the maximum population, and a and b are constants. Using Matlab's inbuilt curve-fitter, we found that the maximum populations for Seattle and Albuquerque are 942,900 and 562,400 respectively. Given Pearson Correlation Coefficients of 0.9497 and 0.9656, our confidence in these values is high, even if the capacity for Albuquerque suggests that it has already reached its maximum population

Knowing the maximum population in either city, we can now determine the maximum number of households. To do so, we divide the population each year by the number of housing units. The equation for such is shown below,

$$ANP_i = \frac{Population_i}{Number\ of\ Housing\ Units_i} \tag{2}$$

where ANP_i is the average number of people per household in year i. For Seattle, the ANP_i values were consistently 2, leading us to a maximum number of housing units in Seattle of

$$\frac{942{,}900\ people}{2\ people\ per\ housing\ unit} = 471,450\ units.$$

For Albuquerque, the ANP_i values demonstrated a logarithmically decreasing trend after 2015. Hence, after fitting a logarithmic graph to the data, we determined that the ANP will reach a minimum of 2.07, leading us maximum number of housing units in Albuquerque of

$$\frac{562{,}400\ people}{2\ people\ per\ housing\ unit} = 271,691\ units.$$

Given these values, we constructed an alternative carrying capacity model reliant on Equation 3,

$$\frac{dN}{dt} = kN(1 - \frac{N}{K}), \tag{3}$$

where N is the number of housing units, k is the growth constant, and K is the maximum number of housing units. To solve this equation, we utilized the Matlab 'ode45' function, which solves nonstiff differential equations. This allowed us to manually enter the K-value, and manipulate the k-value until our Pearson Correlation Coefficient was maximized between 0 and 1. MATLAB is preferable because the common calculator requires immense computational power to solve such differential equations. Streamlining the computation saved us a plethora of time in our modeling process.

1.4 Results

Utilizing the carrying capacity model as determined above, the curve was fitted to both the Albuquerque and Seattle population data. The two plots, scaled down by 100,000, and their corresponding predictions, are shown below in Figures 1 and 2.

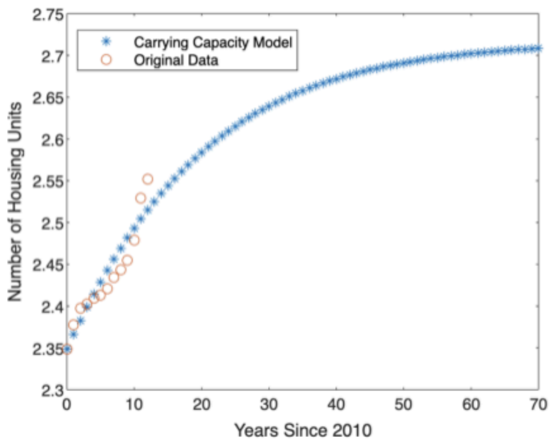


Figure 1: Graphed Model of Households In Albuquerque since 2010

Year	2034	2044	2074
Number of Housing Units	260,960	265,440	270,500

Table 2: 10-year, 20-year, and 50-year Predictions in Seattle

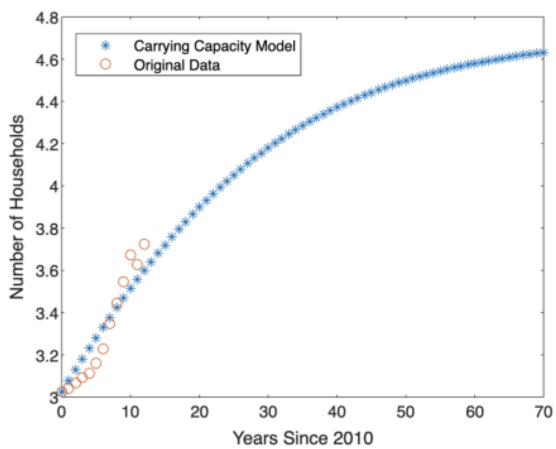


Figure 2: Graphed Model of Households In Seattle since 2010

Year	2034	2044	2074
Number of Housing Units	402,300	426,590	460,390

Table 3: 10-year, 20-year, and 50-year Predictions in Seattle

1.5 Discussion

Looking closer at the data, Figure 1 depicts the predicted number of housing units in Albuquerque, with the maximum number of units proportional to the maximum population. Our model predicts 260,960 units by 2034, 265,440 units by 2044, and 270,500 units by 2074.

In comparison, Figure 2 illustrates Seattle's growth. Similar to Figure 1, Figure 2 models the number of housing units, which continues to rise with Seattle's population until it reaches the equilibrium point. Our team predicts Seattle will have 402,300 units by 2034, 426,590 units by 2044, and 460,390 units by 2074.

In summary, a multitude of factors were used to predict changes in the number of housing units in two unique regions of the United States. Taking into account the distinct influence of rising housing prices, income level, and population growth in both Albuquerque and Seattle, a carrying capacity model was made to determine the change in housing units over both the short and long term. While Albuquerque is predicted to grow 5000 units at each benchmark, Seattle grows almost 5 times that, demonstrating how higher population growth leads to more housing units.

As housing prices continue to rise, and the shortage continues, our model demonstrates that remedies to this issue must be viable in long-term. Ultimately, the housing supply is dependent on population growth, which itself is dependent on economic, infrastructural, and financial factors.

1.6 Sensitivity Analysis

For this question, we performed sensitivity analysis on the housing units carrying capacity model by changing both K, the maximum number of housing units, and k, the growth constant. For the maximum number of housing units, we chose two arbitrary numbers: one above the carrying capacity and one below. For the growth rate, we calculated a new k-value using the following equation, which estimates the Compound Annual Growth Rate (CAGR) for Seattle and Albuquerque:

$$CAGR_{Albuquerque} = \left(\frac{housing\ units\ in\ 2022}{housing\ units\ in\ 2010}\right)^{\left(\frac{1}{2022-2010}\right)} - 1 = \left(\frac{255178}{234891}\right)^{\frac{1}{12}} - 1 = 0.0069$$

$$CAGR_{Seattle} = \left(\frac{housing\ units\ in\ 2022}{housing\ units\ in\ 2010}\right)^{\left(\frac{1}{2022-2010}\right)} - 1 = \left(\frac{372436}{302465}\right)^{\frac{1}{12}} - 1 = 0.0175.$$

In addition, we used an arbitrarily high k-value. The results of the sensitivity analysis are shown below.

	Number of Housing Units			
Year	K = 260,000; k = constant	K = 300,000; k = constant	K = constant; k = 0.0069	K = constant; k = 0.09
2034	252900	279640	239850	266880
2044	255880	287990	241740	269720
2074	259220	297660	246830	271570

Table 4: Albuquerque Sensitivity Analysis

	Number of Housing Units			
Year	K = 400,000; k = constant	K = 500,000; k = constant	K = constant; k = 0.0175	K = constant; k = 0.09
2034	363890	416320	344870	442950
2044	377100	445230	306410	459460
2074	394510	486290	398780	470670

Table 5: Seattle Sensitivity Analysis

The sensitivity analysis corroborates our expectations, confirming the validity of our model. When the carrying capacity is shifted below the calculated carrying capacity, the model is heavily restricted, causing limited growth in housing. Similarly, using the small compound annual growth rate limits the change, because the growth rates fail to account for changes in the second derivative of the data. In contrast, a higher carrying capacity allows the model to grow more rapidly, resulting in higher predicted values. Changing the carrying capacity appears to have a larger effect than the k-values, implying that our precise calculations of the carrying capacities from the general population play a pivotal role in our ultimate model.

1.7 Strengths & Weaknesses

1.7.1 Strengths

- **Simplicity.** An important strength of our model is the transparency it allows the user, and the high accessibility and clarity it presents. Having an unambiguous model allows us to have greater trust in our prediction, as opposed to more complicated equations that we may not fully understand.
- **Multiple Factors.** Much of our research in Problem 1 dealt with the individuality of each region we would model. By taking into consideration relative housing prices, increasing or stagnant population, and more, our team had a tailored, customized model rather than a general curve fit.
- **Highly Intuitive.** By using a logistic model for each set of data, instead of a linear, exponential, or polynomial, the model allows for a rational short-term prediction and a long-term equilibrium. This type of model ensures that for any time t, the value returned will align with empirical evidence.

1.7.2 Weaknesses

- **Thoroughness of Factors.** Given more time, our team could have investigated the median income's relationship with housing supply more deeply. Realistically, as prices continue to rise faster than income, fewer people will be fiscally able to afford a house. Quantifying this percentage of the population would likely have strengthened our results.

2 It Was the Worst of Times

2.1 Problem Restatement

Question 2 asks us to predict the homeless population in Albuquerque and Seattle in the next 10, 20, and 50 years.

2.2 Assumptions and Justifications

2-1: The population model created in question 1 appropriately predicts the population of both cities for the next 50 years.

- **Justification:** To understand and predict the homeless population for each city, the general population trend must be known. Thus, we assume the population model in Problem 1 is correct, allowing us to narrow our lens to this one specific population.

2-2: A given person who is classified as homeless one year has a 95% of being homeless the next year.

- **Justification:** Since 2017, homelessness has been on the rise, with an increase of around 6% [8]. As more people are becoming homeless than leaving homelessness, coupled with the high mortality rate [9], it is clear that it is hard to escape unstable housing. Thus, our team decided to assume that 95% of homeless people stay unhoused. This percentage demonstrates the high stagnation rate, yet accounts for the few that reach stable housing.

2-3: The per-capita homeless population in Seattle and Albuquerque accurately reflects the percentage of people who become homeless each year.

- **Justification:** Although more people are not recorded as losing their homes, it is nearly impossible to find exact data on this percentage. The per-capita statistic quantifies a general proportion for each state and allows us to derive a rate of homelessness.

2.3 Variables

Symbol	Definition	Units
$h(i)$	The population of homeless people in a given year.	people
$p(i)$	The predicted population of non-homeless people in a given year.	people
SH	The percentage of homeless people that stay homeless the next year	%
CH	The percentage of non-homeless people that become homeless the next year.	%

Table 6: Variables for Problem 2

2.4 The Model

For this model, we constructed an asymmetric Markov chain with two absorption states. A graphic representing the model is shown below:

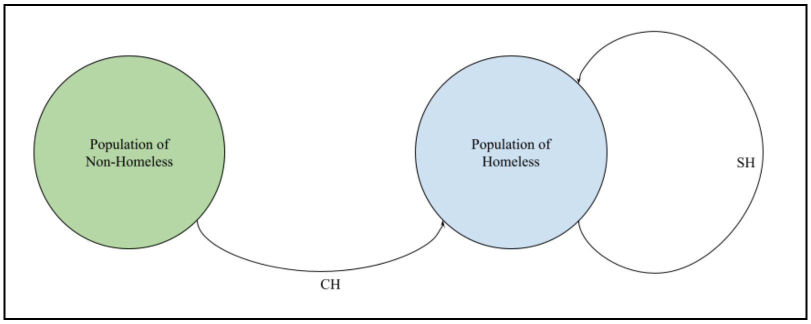


Figure 3: Asymmetric Markov Chain for Homelessness

This Markov Chain depends on known values for the population of the non-homeless, which we derived by subtracting the homeless population from the total population from 2010 to 2022. Then, we used the logistic model in equation 1 to predict future values for the population of non-homeless individuals. The values for the extrapolated population of non-homeless are shown in the Appendix. With this data, we used the following recursive equation to derive our predictions of the homeless population, shown in Equation 4:

$$h(i + 1) = SH * h(i) + CH * p(i) \tag{4}$$

This model is preferable to alternatives because it considers the carrying capacity of various cities without limiting the homeless population. As housing prices skyrocket, we predict that the homeless population will increase at a faster rate than the non-homeless population because more people will not be able to afford housing. To achieve results, we coded the recursive equation into MATLAB, set the variables as parameters, and iterated over the length of the non-homeless population array to achieve results for the homeless population. MATLAB helped expedite the computing process over 50 years, as well as cleanly displayed the results in an accessible format.

2.4 Results

For both Seattle and Albuquerque, we used the 2022 homeless population as the initial, or h(0) population since it was the most recent available data. The asymmetric Markov Chain produced the results shown in Table 7 below.

	2034	2044	2074
Seattle	31,029	41,297	56,541
Albuquerque	6,874	9,516	12,612

Table 7: Predicted Homeless Populations in Seattle and Albuquerque for 2034, 44, and 74.

2.5 Sensitivity Analysis

To perform sensitivity analysis on the model, we varied the assumptions for SH and CH by 5% in both directions. The results of our sensitivity analysis are shown in Tables 8 through 11.

	2034	2044	2074
Seattle	22,827.73	26,856.00	31,077.33
Albuquerque	5,257.96	6,312.39	6,874.51

Table 8: Lowering SH by 5%

	2034	2044	2074
Seattle	43,423.36	70,277.46	152,487.63
Albuquerque	9,200.93	15,627.34	33,969.16

Table 9: Raising SH by 5%

	2034	2044	2074
Seattle	29,836.69	39,448.44	53,760.71
Albuquerque	6,564.73	9,060.79	11,985.90

Table 10: Lowering CH by 5%

	2034	2044	2074
Seattle	32,217.02	43,145.64	59,322.02
Albuquerque	7,183.12	9,971.06	13,238.24

Table 11: Raising CH by 5%

The results from the sensitivity analysis confirm our expectations. Increasing either the SH or CH percentages scales the predictions of homeless individuals up and decreasing either percentage scales the predictions downward. When the SH percentage is altered, the fluctuations in the predictions are greater because the h(i+1) depends more heavily on the h(i) state than the p(i) state. In contrast, variations of the CH percentage deviated far less from the original results due to the low percentage of people transitioning from the non-homeless state to the homeless every year.

2.6 Discussion

For question 2, we developed an asymmetric Markov chain to model unhoused populations 10, 20, and 50 years into the future for Seattle and Albuquerque. We first found the population of non-homeless people from 2010-2022 and used a recursion function to forecast the unhoused population. Our results showed that in 2034, 2044, and 2074, Seattle would have 31,029, 41,297, and 56,541 homeless people while Albuquerque would have 6,874, 9,516, and 12,612, respectively. Our sensitivity analysis shows that our model is robust when the proportion of housed people that stay housed fluctuates but less so when the proportion of homeless people that stay homeless fluctuates.

2.7 Strengths & Weaknesses

2.7.1 Strengths

- **Adaptability.** Our model that was used to predict homeless populations can be applied to other sets of data. In addition, it can be easily adapted to a dynamic Markov Chain if the populations of non-homeless people were not restricted.
- **Connection To Question 1.** The model used drew upon techniques created and justified in question 1. Cohesion between models helps to make a more complete understanding of the problem.

2.7.2 Weaknesses

- **Oversimplification.** While this model is simple, it oversimplifies the homeless population by only using the change in the non-homeless population, which does not account for other factors like costs and median income. With more time, this could be implemented to strengthen our model.
- **Volatility in the "Stay Homeless" Population.** As seen in our sensitivity analysis, a small change in the proportion of unhoused people that remain unhoused drastically impacts our resulting forecasts. Therefore, this model requires precise data, which may not always exist.

3 Rising from This Abyss

3.1 Problem Restatement

Question 3 asks us to develop a model that tackles long-term homelessness in at least one of the cities we explored (we chose Seattle). We are tasked with considering both the results for questions 1 and 2, as well as our model's adaptability to unforeseen circumstances such as natural disasters, economic recessions, or increased migrant populations.

3.2 Assumptions and Justifications

3-1: Participants in Rapid Re-Housing will leave immediately when they find a sustainable housing unit.

- **Justification:** The Rapid Re-Housing program provides short-term housing for up to three months [10]. However, it makes sense for the US government to end support early if the participant no longer needs the service, given that other unhoused people are waiting for a spot.

3.3 Introduction to Two-Part Model

Our model is twofold. Firstly, we evaluate pre-existing homelessness prevention systems and policies through a new, Multi-Criteria Decision Making (MCDM) metric. Our metric relies on two MCDM models: Grey Relational Analysis and Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS). This helps us determine the optimal solution for tackling homelessness. Then, using given homelessness data, we developed a queuing network to evaluate the efficacy of our model based on a set of criteria. The pre-existing homelessness prevention systems we considered were Permanent Stable Housing, Section 8 Vouchers, Rapid Rehabilitation Housing, as well as the combinations of Permanent Stable Housing and Section 8, as well as Rapid Rehabilitation and Section 8. We chose these solutions because they have an empirical track record of success in Seattle, our chosen city for question 3, and have readily available data.

3.4 MCDM Model

3.4.1 Grey Relational Analysis Model

The first part of the metric we used to evaluate pre-existing solutions was a Grey Relational Analysis (GRA) Model. GRA is a Multi-Criteria Decision Making (MCDM) technique designed to evaluate correlations and similarities between given options and an ideal alternative. It outputs a Grey Relational Grade (GRG), which is a statistical measure from 0 to 1 that determines the similarity between a given alternative with the ideal solution. To perform GRA, a set of given criteria is necessary, for which we used the following 6 criteria in Table 12.

Criteria	Variable	Beneficial or Non-Beneficial Attribute	Explanation
Minimum Cost	MC	Non-Beneficial	The minimum estimated cost to implement the solution.
Maximum Cost	MaC	Non-Beneficial	The maximum estimated cost to implement the solution.
Emotional Benefit	EB	Beneficial	The emotional benefit derived from each option, relative to the other options.
Success in	SPH	Beneficial	The percentage of people obtaining

Preventing Homelessness			stable homes through the solution.
Wait Time	*WT*	Non-Beneficial	The time it takes for a homeless person to obtain the solution after applying.

Table 12: Criteria used in GRA, Classified as Beneficial and Non-Beneficial

Using these criteria, we created the following evaluation matrix y_{ij} with *m rows* and *n* columns, based on the solutions and the number of criteria, respectively.

$$y_{ij} = \begin{bmatrix} MC_1 & MaC_1 & . & . & . & D_1 \\ MC_2 & . & & & & \\ . & & . & & & \\ . & & & . & & \\ . & & & & . & \\ MC_m & . & . & . & . & WT_m \end{bmatrix}$$

With the matrix, GRA can now be performed using the detailed, step-by-step instructions shown in the Appendix.

3.4.2 TOPSIS Decision Model

The second method we used to rank the program options was the Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) Model, which utilizes Euclidean distances. TOPSIS is based on the concept that the best choice will have the shortest geometric distance from the positive ideal solution (PIS) and the longest geometric distance from the negative ideal solution (NIS). The result of the TOPSIS method is a performance score from 0 to 1 that demonstrates its distance from the NIS. The closer the value is to 0, the less optimal that choice is.

The same 5 criteria used in the GRA model above are used in this model. To determine the weights that each of these criteria should receive, we performed entropy, which is an algorithmic procedure to determine the relative weights of criteria based on their distributions. In entropy, the evaluation matrix is represented by the variable X_{ij}, where *i* and *j* represent an intersection in the matrix. The steps to calculate the weights based on entropy and perform TOPSIS are shown in detail in the Appendix.

3.4.3 Combined Model

Both GRA and TOPSIS have benefits and downsides. For example, GRA tends to be better at handling data with uncertain information (fuzzy data) and can deal with complex relationships between the data, but it assumes the criteria have a linear relationship with the alternatives. In contrast, TOPSIS remains unaffected by outliers and can accommodate a variety of alternatives, but it assumes independence and fixed weights. As such, our metric combines both MCDM models into a system that weights them equally. Using γ_i and P_i, which represents the GRG from GRA and performance score from TOPSIS respectively, our metric, λ, is calculated as

$$\lambda_i = (\frac{1}{2})(\frac{\gamma_i}{\sum_1^m \gamma_i} + \frac{P_i}{\sum_1^m P_i}) \tag{5}$$

where *m* is the number of solutions. Using this metric, each of the solutions can be ranked.

3.4.4 Applying the Metric

To apply the combined GRA and TOPSIS metric, we used the following data shown below for the city of Seattle.

	MC ($)	MaC ($)	EB	SPH (%)	WT (years)
Permanent	16,000 [7]	22,000 [7]	4	96 [7]	0.25 [11]
Section 8	4, 831 [12,13]	5, 797 [12,13]	1	69 [14]	2.5 [15]
Rapid	7,531 [16]	10,560 [10]	2	70 [10]	0.083 [17]
Permanent + Section 8	20831	27797	5	98.76	2.5
Rapid + Section 8	12182	16357	3	90.7	2.5

Table 13: Data used in the Combined Metric Calculation

For emotional benefits, we ranked the plans from 1 (worst) to 5 (best) based on how much stability the plan would give recipients. While emotional help services are important, we ranked housing as the most important factor since it would fully provide for physical needs (which take priority over emotional needs), followed by money, which partially covers physical needs. Therefore, permanent housing and section 8 funds would provide the most benefit, followed by permanent housing alone, funds and rapid re-housing (since re-housing only provides temporary housing), rapid re-housing, and section 8 funds. For combinations of plans, the costs were added together since receiving both benefits means both must be paid for. For the success rate at preventing homelessness, the combined percentage was calculated as the probability of either plan succeeding, or the union of the two. Finally, the combined wait time was calculated as the maximum wait time for either program, since an unhoused person could wait for both at the same time. The results of the metric calculation are shown below.

	GRG	TOPSIS Score	Calculated Metric (λ)	Rank
Permanent	0.6421002697	0.74184	0.5392078539	2

Section 8	0.6	0.29787	0.3312530388	3
Rapid	0.6372943687	0.80553	0.5654032549	1
Permanent + Section 8	0.6	0.22831	0.3008831956	4
Rapid + Section 8	0.5026649503	0.21688	0.2632526568	5

Table 14: Results of the Combined Metric Calculation

Using our metric, we determined that Rapid Rehabilitation Housing is the most effective solution to homelessness, with permanent stable housing falling behind as a close second. Both GRA and TOPSIS put a heavy weight on the wait time category, which agrees with our expectations because the accessibility to homeless shelters or financial assistance is often the largest barrier to solving homelessness.

3.5 Queuing Model

Given that Rapid Rehabilitation Housing proved to be the most effective, pre-existing solution to homelessness, we developed a simple queuing model to predict its effect on the homeless population in Seattle.

3.5.1 Developing the Model

Ordinarily, RRH's success rate of approximately 70% [10] and maximum duration of 3 months [10] would permit a simple model that assumes a constant number of successful participants each 3-month time frame. However, entering and leaving RRH locations is a continuous process and there is a wait time of approximately one month to get RRH [17]. Additionally, the unhoused population forms a line (or queue) that updates each time someone in the program finds stable housing and leaves early or is forced to leave. Furthermore, the number of people finding sustainable housing each month is not set given the inherently inconsistent process. Therefore, a queueing model with fixed arrival times (meaning that once someone leaves another takes the same spot in a negligible amount of time) and repeated random iterations best represent the situation. To code this network, we used nested for-loops in MATLAB to go through 3-month chunks of time. Within each iteration, the states of homeless people and homeless people who found stable housing were tracked and consistently updated. This process was preferable to simple calculations because it allowed us to run numerous iterations and perform quick sensitivity analyses. In addition, similar to the carrying capacity model, defining our variables parameters makes the code easy to update in the event we want to track another population, such as not homeless but qualified for RRH

3.5.2 Executing the Model

To simulate the optimal impact of the RRH queue over several years, each year was broken into 12 months to account for early departures. Then, since the program's success rate at preventing future homelessness is 70% for 3 months, the probability of failing to find sustainable housing 3 months in a row would equal the program's overall failure rate, or

$$(1 - P_m)^3 = 1 - P_{3m} \tag{6}$$

where P_{3m} is the program's overall success of 70% and P_m is the success rate for one month, which was calculated to be about 33%.

To account for the participants leaving earlier than expected, the participants were grouped into stages based on whether they were in Month 1, Month 2, or Month 3 of the program. Each month, since each participant has a random (since the city is an uncontrolled environment) 33% chance to leave on average, a Monte Carlo simulation was performed to determine the number of successful participants. Those with more months were transferred to the next month's group and those unable to find sustainable housing would return to being unhoused, as visually shown below in Figure 4.

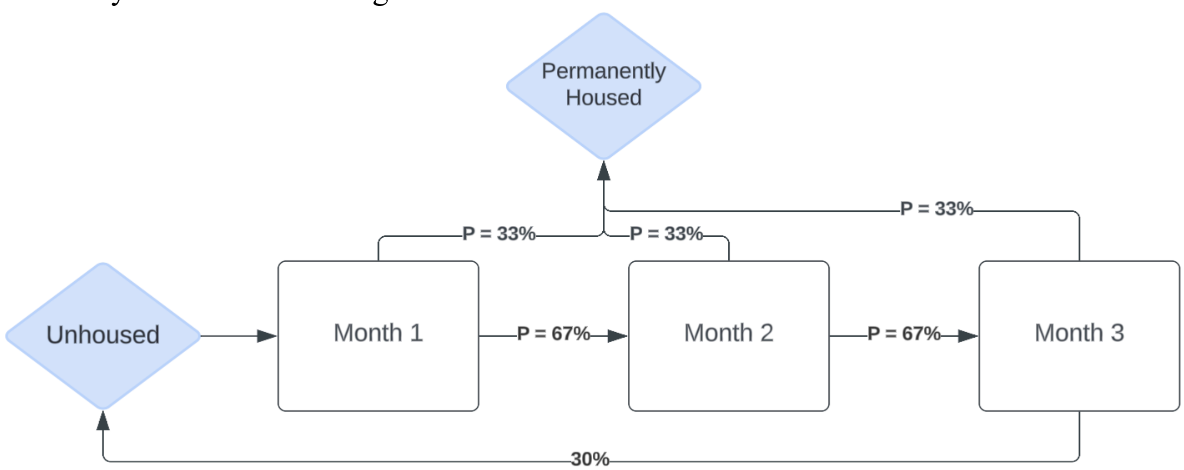


Figure 4: Queueing Model Diagram

3.5.3 Results

Using the model above, the average number of successful participants over 1 year and 5 years was calculated using 100 simulations. The results are shown in Table 15 below.

Number of Years	People in Stable Housing	Standard Deviation
1	1687.57	33.40
5	8431.83	63.55

Table 15: Results of Queuing Model

Therefore, the current number of RHH units can optimally help an average of about 1680 people per year.

3.6 Sensitivity Analysis

To test the model's robustness, the RHH units' capacity and the rate of finding sustainable housing were varied by 10% in separate trials. The results of the Monte Carlo simulation of 100 5-year-long trials are shown below in Table 16.

	Capacity + 10%	Capacity - 10%	Success Rate + 10%	Success Rate - 10%
Number Moved to Stable Housing	9,252.97	7,570.28	9,278.38	7,578.44
Standard Deviation	81.87	77.60	65.69	62.00

Table 16: Sensitivity Analysis based on raising and lowering Capacity and Success Rate

Therefore, the model is somewhat robust, as a 10% change in either variable leads to an approximately proportional change in the number of people moved to stable housing. However, the success rate appears to preserve the standard deviation of the original model, which contrasts with the standard deviations in the capacity systemically. Therefore, the model overall is more robust when dealing with a change in success rate than a change in capacity.

3.7 Discussion & Adaptation

For question 3, we developed a metric that combined Grey Relational Analysis and TOPSIS, two Multi-Criterion Decision Making Models, to rank the best response program for homelessness in Seattle. For both MCDM techniques, we considered 5 independent criteria, which were minimum cost, maximum cost, emotional benefit, success rate, and wait time. After applying our metric to data we found from research, the decision models returned Rapid Rehabilitation Housing to be the most optimal, with Permanent Stable Housing coming in a close second.

Next, our team developed a queuing model to predict its effectiveness. By simulating the entering and exiting of unhoused people in the program, we could quantify its success. Its ability to optimally move around 1680 unhoused people each year is, at least in the short term, a viable strategy for reducing homelessness in Seattle, which corroborates results from our earlier metric.

We believe our model is highly adaptable to unforeseen circumstances. At the bare minimum, each of the solutions presented can be qualitatively evaluated. For example, Section 8 Vouchers would be poor in the face of economic recession, as the value of the dollar would decrease. However, they can be very useful during natural disasters, when the money can be used to find shelter and protection from storms, hurricanes, etc. In contrast, Permanent Stable Housing would fare poorly during natural disasters due to its susceptibility to destruction, but would likely retain its property value during economic recession. To account for these factors, we would add a “resistance” criteria in our matrix that ranks and/or quantitatively justifies scores for each of the options. Then, when our top option is identified, we would add noise to the Queuing Model to disrupt the natural flow of homeless populations between sheltered states.

3.8 Strengths & Weaknesses

3.8.1 Strengths

- **Multiple Methods.** By using both the GRA and TOPSIS decision models, we could compare and contrast our results from two differing methods. This was especially strong because our results correlated with each other, reflecting the breadth and depth of our considerations. In addition, attaching a queuing network model enabled both a cause-and-effect investigation of the question.
- **Modeling Depth.** We took into account multiple societal factors that the programs impact. Our model can therefore better represent the situation and not oversimplify it. We also did not include excessive factors that would overly complicate the evaluation metric.
- **Objectivity.** Since TOPSIS and Grey Relational Analysis, both determine weights based on the data and not from subjective input, the metric will not be biased.

3.8.2 Weaknesses

- **Limited Data Considerations.** In addition to not considering solutions to homelessness in Albuquerque, NM, we also lacked important parameters in our queuing network (ie. wait time) which reduced its complexity.
- **Non-Standardized Metric.** Since TOPSIS and Grey Relational Analysis produce results based on the differences in data and the number of options, the metric is not standardized. That means that even if one option has the same data in two different tests, its metric value will be different based on the data of other options.

4 Conclusion

In this paper, we began by predicting the number of housing units in 2034, 2044, and 2074 in Seattle, Washington, and Albuquerque, New Mexico. We explored both a logistic growth model and a carrying capacity model, analyzing the effect that population growth would have on the demand, and therefore supply, of housing units. Then, we transitioned to predicting the homeless population in 2034, 2044, and 2074 in Seattle and Albuquerque. For this section, we drew upon techniques from the first question 1 and used an asymmetric Markov chain to model future homeless populations in both cities. Finally, we evaluated solutions to homelessness in Seattle by a) determining the best solution using MCDM models and b) evaluating the best solution by discovering its impact on Seattle's homeless population through a queuing model.

We believe that the housing crisis and homelessness is a rapidly evolving issue that requires careful consideration by policymakers, mathematicians, and economists to fix. Our paper takes the first step in doing so by demonstrating the power of mathematical models to understand the danger of leaving these problems unattended.

5 References

[1] *The human right to adequate housing.* (n.d.). United Nations. Retrieved March 2, 2024, from https://www.ohchr.org/en/special-procedures/sr-housing/human-right-adequate-housing

[2] Dolbeare, C. N., & Crowley, S. (2002). *Changing Priorities: The Federal Budget and Housing Assistance* [Pamphlet]. National Low Income Housing Coalition. https://nlihc.org/sites/default/files/Changing-Priorities-Report_August-2002.pdf

[3] National Academies Press. (2018). *Permanent supportive housing*. National Academies Press. https://doi.org/10.17226/25133

[4] A Tale of Two Crises, MathWorks Math Modeling Challenge 2024, curated data, https://m3challenge.siam.org/kdfrldh/.

[5] Ludden, J. (Host). (2024, January 25). Housing is now unaffordable for a record half of all U.S. renters, study finds [Audio podcast episode]. In *Morning edition*. NPR. https://www.npr.org/2024/01/25/1225957874/housing-unaffordable-for-record-half-all-u-s-renters-study-finds

[6] Saldua, M. (2023, November 15). Addressing Social Determinants of Health Among Individuals Experiencing Homelessness. *SAMHSA*. https://www.samhsa.gov/blog/addressing-social-determinants-health-among-individuals-experiencing-homelessness

[7] *Supportive Housing Solution*. (n.d.). The Citizens Campaign. Retrieved March 2, 2024, from https://thecitizenscampaign.org/nssp/solutions/supportive-housing-solution/

[8] *State of Homelessness: 2023 Edition*. (2023). National Alliance to End Homelessness. Retrieved March 2, 2024, from https://endhomelessness.org/homelessness-in-america/homelessness-statistics/state-of-homelessness/

[9] NAEH. (2021, December 20). Not One More: Honoring Those Who Died Homeless. *National Alliance to End Homelessness*. https://endhomelessness.org/blog/not-one-more-honoring-those-who-died-homeless/

[10] *Report Finds Rapid Re-housing Is an Effective Low-Cost Intervention to Homelessness*. (2018, October 22). National Low Income Housing Coalition. Retrieved March 2, 2024, from https://nlihc.org/resource/report-finds-rapid-re-housing-effective-low-cost-intervention-homelessness

[11] *Getting Placed in Supportive Housing*. (n.d.). coalition for the homeless. Retrieved March 2, 2024, from https://www.coalitionforthehomeless.org/get-help/im-in-need-of-housing/getting-placed-in-supportive-housing/

[12] *Section 8 Vouchers.* (n.d.). Tenants Union of Washington State. Retrieved March 2, 2024, from https://tenantsunion.org/rights/section-8-vouchers

[13] *Seattle, WA Low Income Housing.* (n.d.). Low Income Housing. Retrieved March 2, 2024, from https://www.lowincomehousing.us/WA/seattle.html

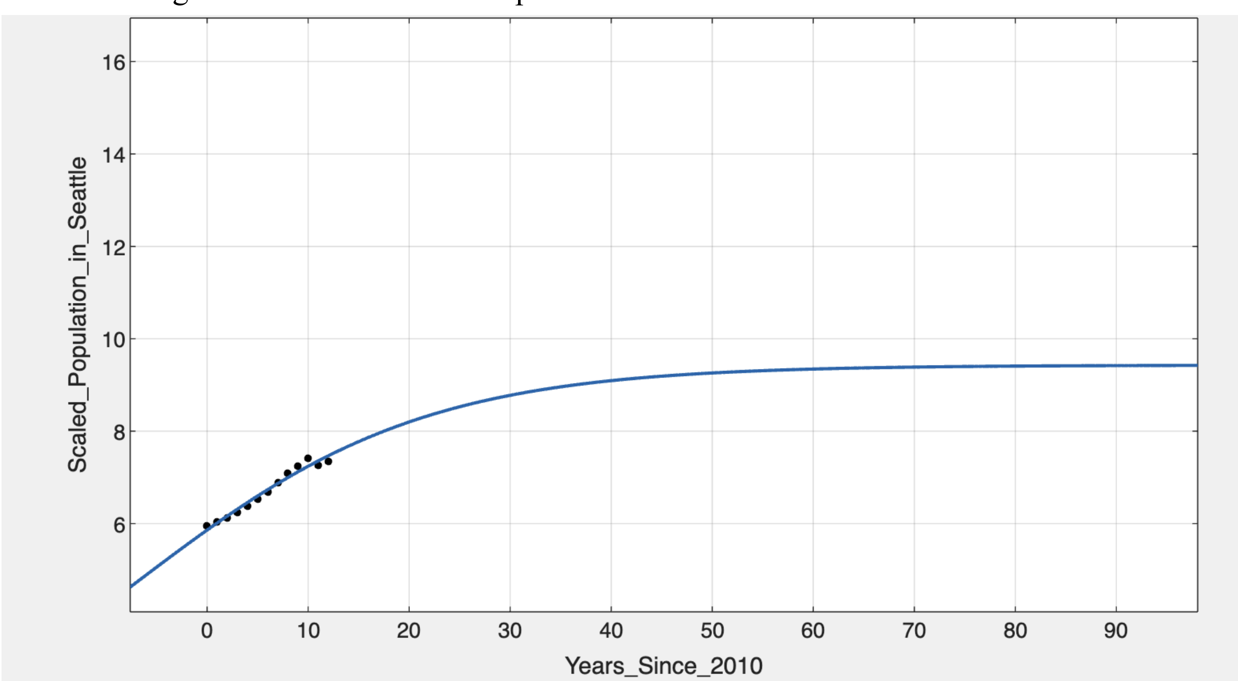
[14] *Strengths and Weaknesses of the Housing Voucher Program: Hearings before the Subcommittee on Housing and Community Opportunity* (2003) (testimony of Margery Austin Turner).
https://www.urban.org/sites/default/files/publication/64536/900635-Strengths-and-Weaknesses-of-the-Housing-Voucher-Program.pdf
[15] Eligibility Team. (2017, December 13). *Section 8 Housing Openings in Washington.* Eligibility. Retrieved March 2, 2024, from
https://eligibility.com/section-8/washington-wa-section-8-benefits
[16] Greenstone, S. (2018, September 17). What would it cost to house and provide treatment for Seattle's homeless? *Seattle Times.*
https://www.seattletimes.com/seattle-news/homeless/what-is-the-cost-to-house-and-provide-treatment-for-seattles-homeless/
[17] HUD. (2014, July). *Rapid Re-Housing Brief.* HUD Exchange. Retrieved March 2, 2024, from https://www.hudexchange.info/resource/3891/rapid-re-housing-brief/

6 Appendix

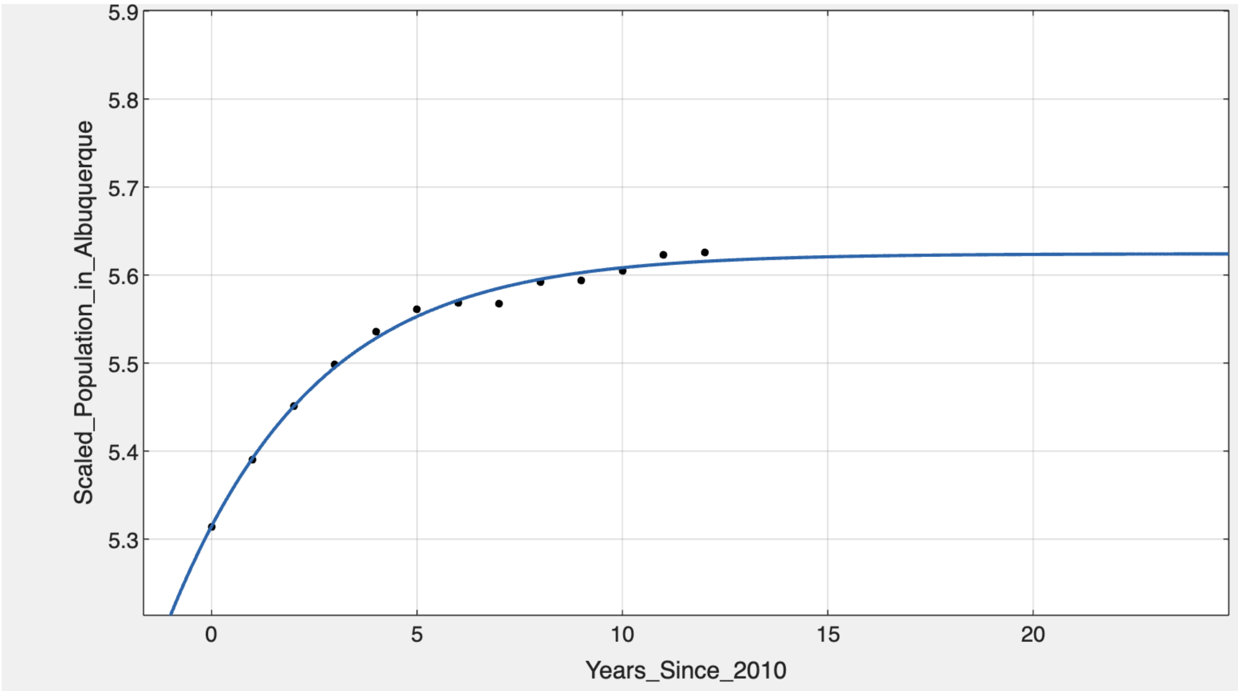
6.1 Question 1

6.1.1 Graphs

MATLAB Logistic Model for Seattle Population



MATLAB Logistical Model for Albuquerque Population



6.1.2 MATLAB Code

Carrying Capacity Model for Seattle Housing Units

```
% Coefficient for Growth Rate

k = .0491;

% Maximum number of Houses

K = 4.715;

% Time to run the code through

T = 0:1:70;

% Defining the differential Equation

h = @(t,y)[k.*y(1).*(1-(y(1)/K))]

% Utitilize the ode45 function to solve

[T za] = ode45(h,T,[3.02465])

% Graphing the Logistics Function

plot(T,za(:,1),'*')
```

```
hold on

% Data for the Seattle Housing Units

x = [0 1 2 3 4 5 6 7 8 9 10 11 12];

y = [3.02465, 3.04164, 3.06694, 3.09205, 3.11286, 3.15950, 3.22795, 3.34739,
3.44503, 3.54475, 3.67337, 3.62809, 3.72436];

% Graphing Original Data on the same plot

plot(x,y,'o')

hold off

% Graph labels

xlabel('Years Since 2010');

ylabel('Number of Housing Units');

legend('Carrying Capacity Model','Original Data');

legend("Position", [0.15191,0.80076,0.36272,0.088462])

% Calculating values for specific years

% Values for years 24, 34, and 64

years = [24, 34, 64];

values = interp1(T, za(:,1), years);

% Displaying the values

disp('Values for specific years:');

disp(['Year 24: ', num2str(values(1))]);

disp(['Year 34: ', num2str(values(2))]);

disp(['Year 64: ', num2str(values(3))]);
```

Carrying Capacity Model for Albuquerque Housing Units

```
% Coefficient for Growth Rate

k = .0557;

% Maximum number of Houses

K = 2.717;
```

```
% Time to run the code through

T = 0:1:70;

% Defining the differential Equation

h = @(t,y)[k.*y(1).*(1-(y(1)/K))];

% Utitilize the ode45 function to solve

[T za] = ode45(h,T,[2.34891])

% Graphing the Logistics Function

plot(T,za(:,1),'*')

hold on

% Data for the Albuquerque Housing Units

x = [0 1 2 3 4 5 6 7 8 9 10 11 12];

y = [2.34891, 2.37735, 2.39718, 2.40277, 2.40961, 2.41326, 2.42070, 2.43402,
2.44382, 2.45476,2.47926, 2.52924, 2.55178];

% Graphing Original Data on the same plot

plot(x,y,'o')

hold off

% Graph labels

xlabel('Years Since 2010');

ylabel('Number of Housing Units');

legend(['Carrying Capacity Model' ...

   ''],'Original Data');

legend("Position", [0.14631,0.80162,0.36272,0.088462])

% Calculating values for specific years

% Values for years 24, 34, and 64

years = [24, 34, 64];

values = interp1(T, za(:,1), years);

% Displaying the values

disp('Values for specific years:');
```

```
disp(['Year 24: ', num2str(values(1))]);

disp(['Year 34: ', num2str(values(2))]);

disp(['Year 64: ', num2str(values(3))]);
```

6.2 Question 2

6.2.1 Data Tables

2010	586218
2011	594202
2012	604017
2013	615575
2014	628901
2015	642895
2016	658119
2017	676602
2018	696711
2019	713106
2020	729500
2021	720871
2022	736330
2023	747200
2024	757670
2025	767740
2026	777410
2027	786690
2028	795580
2029	804090
2030	812220
2031	819980
2032	827380
2033	834430
2034	841140

2035	847510
2036	853570
2037	859330
2038	864780
2039	869950
2040	874850
2041	879480
2042	883860
2043	888010
2044	891920
2045	895610
2046	899100
2047	902390
2048	905490
2049	908410
2050	911160
2051	913750
2052	916190
2053	918490
2054	920650
2055	922680
2056	924590
2057	926380
2058	928070
2059	929650
2060	931140
2061	932530
2062	933850
2063	935080
2064	936230
2065	937320

2066	938330
2067	939290
2068	940180
2069	941020
2070	941810
2071	942550
2072	943240
2073	943890
2074	944500

Extrapolated Non-Homeless Population in Seattle

2010	529401
2011	537361
2012	543652
2013	548641
2014	552322
2015	554805
2016	555637
2017	555400
2018	557862
2019	557850
2020	558861
2021	560769
2022	560020
2023	560220
2024	560370
2025	560480
2026	560550
2027	560610
2028	560650
2029	560680

2030	560700
2031	560720
2032	560730
2033	560740
2034	560740
2035	560750
2036	560750
2037	560750
2038	560760
2039	560760
2040	560760
2041	560760
2042	560760
2043	560760
2044	560760
2045	560760
2046	560760
2047	560760
2048	560760
2049	560760
2050	560760
2051	560760
2052	560760
2053	560760
2054	560760
2055	560760
2056	560760
2057	560760
2058	560760
2059	560760
2060	560760

2061	560760
2062	560760
2063	560760
2064	560760
2065	560760
2066	560760
2067	560760
2068	560760
2069	560760
2070	560760
2071	560760
2072	560760
2073	560760
2074	560760

Extrapolated Non-Homeless Population in Albuquerque

6.2.2 Code For Question 2

```
% Define constants

SH = 0.95;  % Value of SH constant

CH = 0.00326;  % Value of CH constant

% Example values of p(i) (replace with your own data)

p = [736330, 747200];  % Sample values of p(i)

% Initial value of h

h_initial = 13368;  % Initial value of h

% Compute h(i+1) using the model

h = zeros(size(p));  % Initialize h array

h(1) = SH * h_initial + CH * p(1)  % Compute h(2) based on the initial value

% Compute h(i+1) for i > 1

for i = 2:numel(p)

   h(i) = SH * h(i-1) + CH * p(i);
```

```
end
% Display the results
disp('Values of h(i+1):');
disp(h);
```

6.3 Question 3

6.3.1 Explanation of GRA

To perform GRA, we can follow the following steps:

1. Use these categorizations to process the data using the following equations:

   Beneficial Attribute:

   $$y_i^*(k) = \frac{y_i^o(k) - min\, y_i^o(k)}{max\, y_i^o(k) - min\, y_i^o(k)} \quad (7)$$

   Non-Beneficial Attribute:

   $$y_i^*(k) = \frac{max\, y_i^o(k) - y_i^o(k)}{max\, y_i^o(k) - min\, y_i^o(k)} \quad (8)$$

   where k represents the criteria, $y_i^o(k)$ represents the data value of the program option for the kth criteria, $max\, y_i^o(k)$ is the largest value of $y_i^o(k)$ for the kth criteria, and $min\, y_i^o(k)$ is the smallest value of the $y_i^o(k)$ for the kth criteria. The result of this step is an equal-sized matrix as y_{ij} with normalized values between 0 and 1, known as the normalized matrix.
2. Calculate the grey relational coefficient, $\xi_i(k)$, using the following equation

   $$\xi_i(k) = \frac{\Delta_{min} + \varsigma\Delta_{max}}{\Delta_{oi}(k) + \varsigma\Delta_{max}} \quad (9)$$

   where $\Delta_{oi}(k)$ represents the deviation sequence for each of the criteria and ς represents the distinguished coefficient. In GRA, the distinguished coefficient is a parameter that is used to adjust the weight of the deviation sequence. A value of 0.5 for the distinguished coefficient is commonly used, although it can be adjusted. The equations for the deviation sequence, Δ_{min}, and Δ_{max} are shown below.

   $$\Delta_{0i}(k) = \left\|y_0^*(k) - y_i^*(k)\right\| \quad (10)$$

$$\Delta_{max} = \max_{\forall j \in i} \max_{\forall k} \left\| y_0^*(k) - y_j^*(k) \right\| \tag{11}$$

$$\Delta_{min} = \min_{\forall j \in i} \min_{\forall k} \left\| y_0^*(k) - y_j^*(k) \right\| \tag{12}$$

In this set of equations, the deviation sequence can be explained as the absolute value of the difference between the maximum value in the normalized matrix (the ideal value of 1) and each of the values in the normalized matrix. The delta minimum and maximum are the smallest and largest values of the entire deviation sequence, respectively. The output of this step is another matrix of the same size as y_{ij} that has the grey relational coefficient in each of its elements.

3. Calculate the Grey Relational Grade (GRG), γ_i, using the following equation

$$\gamma_i = \frac{1}{n} \sum_{k=1}^{n} \xi_i(k) \tag{13}$$

where *n* is the number of criteria (in this case 5). The result of this step is a single column with individual GRGs for each option.

6.3.2 Explanation of Entropy Weighting and TOPSIS

To derive entropic weights, we can follow the following steps:

1. Normalize the evaluation matrix to create the project outcomes p_{ij}, where

$$p_j = \frac{X_{ij}}{\sum_{i=1}^{m} X_{ij}} \tag{14}$$

2. Compute the entropy measure, E_{ij}, of the project outcomes.

$$E_j = \frac{-1}{ln(m)} \sum_{i=1}^{m} (p_{ij})(ln(p_{ij})) \tag{15}$$

3. Define the objective weight, w_j, based on the entropy measure.

$$w_j = \frac{1-E_j}{\sum_{j=1}^{n} (1-E_j)} \tag{16}$$

Note that entropy requires values greater than zero because the natural log of zero does not exist. Hence, we replaced zeros with ones when performing entropy, which will have a negligible impact on the data. TOPSIS can now be performed with the following steps:

1. The same evaluation matrix as in entropy was reconstructed with *m* rows and *n* columns, based on the number of programs and the number of criteria, respectively. This matrix is represented by the variable $(X_{ij})_{m \times n}$, where each i and j represent an intersection in the matrix.

2. Next, the matrix was normalized through Normalization Under Root Summation, as shown in the equation below

$$\overline{X}_{ij} = \frac{X_{ij}}{\sqrt{\sum_{j=1}^{m} X_{ij}^2}} \tag{17}$$

where each of the values in the matrix is normalized to a value between 0 and 1. This is done by dividing each value by the square root of the sum of the squares of all values in each column. This is crucial as many of the criteria are not measured on the same scale. If the matrix was not normalized, values such as MV and F would be considered far more important than factors like CC or WA, which are on a much smaller numerical scale. The bigger a normalized value is, the bigger it is relative to all the other values in that criteria.

3. The normalized matrix is then properly weighted, using the entropic weights explained above. The normalized matrix, $\overline{X}_{ij}$ is multiplied by the weight matrix w_j, creating the weighted normalized matrix V_{ij}.

$$V_{ij} = w_j \cdot \overline{X}_{ij} \tag{18}$$

4. The best and worst values for each criterion can then be determined by picking the lowest and highest values from each column. Since there is a multitude of factors in play, each program may have drawbacks. Finding the worst and best of all the choices in each criterion constructs a metric to compare the rest of the options. These “best” and “worst” values are expressed in row matrices V^+_j and V^-_j respectively, each the same length as the weighted matrix V_{ij}.

5. After finding the best and worst values for each criterion, the total Euclidian distance from the PIS and NIS to each of the programs be calculated using the equations below

$$S^+_i = \sqrt{\sum_{j=1}^{n} (V_{ij} - V^+_j)^2} \tag{19}$$

$$S^-_i = \sqrt{\sum_{j=1}^{n} (V_{ij} - V^-_j)^2} \tag{20}$$

where S^+_i and S^-_i are the distances from the PIS and NIS for each program. Essentially, using the standard distance formula for a geometrically Euclidean plane, the total distance from the optimal PIS and NIS for each criterion is determined and added together for each land use. Graphically, the coordinate point $(V^+_1, V^+_2, \cdot \quad \cdot \quad \cdot V^+_{n-1}, V^+_n)$ and $(V^-_1, V^-_2, \cdot \quad \cdot \quad \cdot V^-_{n-1}, V^-_n)$ represents the most and least ideal points in the *n*th dimensional plane since each criterion adds another plane in which distance is measured. We then measure the distance from each of the two points to the *m* number of choices listed, each represented by its own coordinate point.

The higher the S value, the farther it is away from the respective point. To find the most optimal of all choices given to us, we look for the smallest S^+_i value and the largest S^-_i. If there was one option that had the PIS for all of its criteria, its S^+_i would equal zero, showing that it contains all the best values from all rows, and therefore has a distance of zero to the PIS. Inversely, a point that shows a S^-_i of zero shows that it contains all the NIS values in all criteria, making it the worst choice possible.

6. However, these S values almost always return a number between 0 and 1. To find the choice close to S^+_i but still far from S^-_i, we determine the performance score P_i, represented by the equation

$$P_i = \frac{S^-_i}{S^+_i + S^-_i} \tag{21}$$

resulting in a normalized value between 0 and 1. This expression is unique as it takes into account not only the proximity to the best choice but also the distance from the worst. As we want the distance to the NIS to be large, the larger the performance score is, the better it performs.

6.3.3 TOPSIS and Entropy Weighting

```
%Shannon Entropy: Define Data: Each row is an option, each column is a criteria

X = [16000 22000 4      96     0.25

4831 5797   1      69     2.5

7351 10560  2      70     0.083

20831      27797  5      98.76  2.5

12182      16357  3      90.7   2.5];

% Normalize the Values in Matrix

[n, m] = size(X);

normalized_data = zeros(n, m);

for i = 1:m

   column_sum = sum(X(:,i));

   normalized_data(:,i) = X(:,i) ./ column_sum;
```

```
end
% Calculate sum of X*lnX
log_normalized = normalized_data .* log(normalized_data);
new_sum = sum(log_normalized);
%Finding Ej: Normalizing Log Values
Ej = -1/log(n) .* new_sum;
%Finding Objective Weight for data
Ej_comp = 1 - Ej;
Ej_sum = sum(Ej_comp);
W = Ej_comp ./ Ej_sum
```

```
% TOPSIS Method
Wcriteria = [0,0,1,1,0];
% 0 means non-beneficial (i.e. smaller is better), 1 is beneficial (larger is better)
% Different normalization technique
Xval=length(X(:,1));
Y = zeros([Xval,length(W)]);
for j=1:length(W)
   for i=1:Xval
   Y(i,j)=X(i,j)/sqrt(sum((X(:,j).^2)));
   end
end
Normalized_Matrix = num2str([Y]);
% calculating the weighted normalized matrix
for j=1:length(W)
   for i=1:Xval
       Yw(i,j)=Y(i,j).*W(j);
```

```
   end
end
Weighted_Normalized_Matrix = num2str([Yw]);
% calculating the positive and negative best
for j=1:length(W)
   if Wcriteria(1,j)== 0
       Vp(1,j)= min(Yw(:,j));
       Vn(1,j)= max(Yw(:,j));
   else
       Vp(1,j)= max(Yw(:,j));
       Vn(1,j)= min(Yw(:,j));
   end
end
Positive_best = num2str([Vp])
Negative_best = num2str([Vn])
% Euclidean distance from Ideal Best and Worst
for j=1:length(W)
   for i=1:Xval
       Sp(i,j)=((Yw(i,j)-Vp(j)).^2);
       Sn(i,j)=((Yw(i,j)-Vn(j)).^2);
   end
end
for i=1:Xval
   Splus(i)=sqrt(sum(Sp(i,:)));
   Snegative(i)=sqrt(sum(Sn(i,:)));
end
% calculating the performance score
```

```
P=zeros(Xval,1);

for i=1:Xval

   P(i)=Snegative(i)/(Splus(i)+Snegative(i));

end

Performance_Score =  num2str([P])
```

6.3.4 Queuing Code

```
% Set variables for current homeless population (useful in case of simulating
many years), duration of RRH, success probability, capacity of RRH units, and
number of years

num_homeless = 13368;

rehab_duration = 3; % in months

prob_leaving_each_month = 1 - (0.3)^(1/3)

rehab_capacity = 425;

simulation_duration_years = 1;

% Initialize variables for initial state (proportional distribution of month 1
to month 2, meaning month 2 has 67% of month 1, and so on…)

num_stable_housing = 0;

month_1_rehab = 201;

month_2_rehab = 134;

month_3_rehab = 90;

% Simulation loop

for year = 1:simulation_duration_years

% For each year and month

  for month = 1:12

      % Track individuals exiting rehab using Monte Carlo simulation

      num_out_of_rehab = round(prob_leaving_each_month * (month_1_rehab +
month_2_rehab + month_3_rehab));
```

```
      num_stable_housing = num_stable_housing + num_out_of_rehab;

      num_homeless = num_homeless + (1 - prob_leaving_each_month) *
month_3_rehab;

      num_out_of_rehab

      % Update non-successful individuals as progressing to next month

      month_3_rehab = month_2_rehab * (1 - prob_leaving_each_month);

      month_2_rehab = month_1_rehab * (1 - prob_leaving_each_month);


      % Getting new rehab participants

      month_1_rehab = rehab_capacity - month_2_rehab - month_3_rehab;

      num_homeless = num_homeless - month_1_rehab;

  end

end

fprintf('Number of homeless after %d years: %d\n', simulation_duration_years,
num_homeless);

fprintf('Number in stable housing after %d years: %d\n',
simulation_duration_years, num_stable_housing);
```