MathWorks Math Modeling Challenge 2025

Winchester College

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M3 Challenge TECHNICAL COMPUTING WINNER—$3,000 Team Award

JUDGE COMMENTS

Specifically for Team #17781—Submitted at the close of triage judging

The judges were impressed with how this paper used technical computing in all three parts of the challenge. The students leveraged a mix of hand-coded models (e.g., a hand-coded temperature simulation for Part 1) and built-in libraries (e.g., for time series modeling in Part 2).

What stood out most was the use of technical computing in Part 3. For that problem, the students imported data from OpenStreetMaps to approximate the number of power substations within each neighborhood, which factored into their vulnerability score. They also proposed a simple but effective algorithm to place cooling centers throughout Memphis to maximize impact. We did not see any other teams use mapping libraries so effectively.

Beyond their models, the team's Python code was clear and well-commented (with minor exceptions). The judges did feel that the paper's presentation and formatting could have been improved. Plots, figures, and equations were unnecessarily spaced out, making the report overly long and more difficult to read than others. That all said, this was a great paper overall. It showcases how technical computing can be used effectively in the modeling process.

M3 Challenge 2025

Hot Button Issue:
Staying Cool as the World Heats Up

Team #17781

March 2025

1 Executive Summary

To the Administrator of the Federal Emergency Management Agency and the CEO of Memphis Light, Gas and Water:

Heat waves present an existential threat to human settlement: not only do they create direct and immediate health risks, but the economic damage caused by such events is unprecedented. Power outages, cessation of public transport and high cost of cooling not only damage economic output in Memphis, but also disproportionately affect disenfranchised communities. With the growing threat of global warming, we expect more intense and frequent heat waves. This necessitates mitigation.

Our examination of the heat waves on Memphis are two-fold. First we considered the effect of heat waves via physical mechanisms. Using Newton's Law of Cooling, universally applicable for modeling heat transfer, we analyzed the movement of heat between indoor and outdoor environments. Using the extensive Loughborough Synthetic Housing dataset, we determined precise relationships relating surface area, human heat output, and volume with heat transfer. This yielded volume as the most significant factor, furthering the disproportionate impact of heat waves on low-income communities.

In the second section of our report, we examined future power demand, such that our grid can be resilient to changes in climate change. We recognized the varying contributions of residential and commercial/industrial sectors in Memphis, modeling them using SARIMA and Holt-Winters models. We used the Dickey-Fuller test to verify the use of SARIMA. For the residential sector, we separated Summer Months and Winter Months, using two SARIMA models to model both. By combining predictions by both models, we obtained an overall forecast of power demand from residential units. For the commercial/industrial sector, we utilized a Holt-Winters forecasting model for its advantage over SARIMA models in fitting simple seasonal patterns, and implicit trend modeling. Using data from the MLGW Monthly Financial Reports, these models helped us predict the monthly change in power demand in Memphis, including the effects of seasonal patterns and general trends. We realized a potential 72.3% increase in the power demand in peak months in the next 20 years, signaling severe threats of potential power outages in the future.

To prevent the future forecased, we considered potential mitigation strategies. Aiming to evaluate the vulnerability of different neighborhoods during heatwaves, we modeled both the probability of a power-outage, as well as the economic damage. Even if a power-outage doesn't affect a neighborhood, the cost of cooling is still significant. Considering vulnerability as economic damage, we see this data as highly tangible, intuitive and thus effective to be used in public policy. Using, stochastic modeling, we determined the optimal location of emergency stations — stations designed to provide essential resources, medical assistance and backup generators — to minimize economic damage while minimizing cost of deployment.

In light of our findings, we urge immediate action to implement these mitigation strategies, including the strategic placement of emergency stations, to safeguard Memphis' residents, protect its economy, and build resilience against the escalating threat of heat waves.

Contents

2 Global Assumptions

GA-1: Each neighborhood within Memphis is affected in an equal amount by any heatwave.

Explanation and Justification: Memphis, which has an area of approximately 300 square miles [1], can be considered as a single point when affected by a heatwave, as any fluctuations in outdoor temperature across the city are negligible.

GA-2: Memphis will not be affected by a natural disaster in the time-frames mentioned.

Explanation and Justification: Natural disasters are hard to predict and there are no precedents indicating a high probability of such an event occurring within the specified time-frames. While Memphis is susceptible to certain natural hazards the likelihood of a disaster significantly disrupting operations within the given period is considered low based on available risk assessments and meteorological data.

GA-3: Climate change will follow the current predictions throughout the time-frames mentioned.

Explanation and Justification: This assumption is based on existing climate models and projections, which rely on current environmental trends. Major government interventions, while possible, are not considered due to their unpredictability and the possibility of varying degrees of implementation across different regions.

3 Q1: Hot to Go

3.1 Problem Statement

The problem requires the development of a model that predicts the variation in the indoor temperature of any dwelling during a heat wave over a 24-hour period in Memphis, Tennessee. To do this, the dataset that we use includes:

- The outdoor temperature in Memphis during the hottest day of the July 2022 heatwave.[2]
- The characteristics of each type of dwelling, including the relative room sizes and number of rooms
- A dataset of synthetically occupied test houses to receive real data on how indoor temperature varies over time

3.2 Assumptions

1A-1: The primary significant heat source at any point in time is the external temperature.

Justification: The problem states that any dwelling considered is not influenced by air conditioning or mechanical cooling systems. We do not expect standard electrical devices to significantly contribute significantly to the temperature of the room, as a primary objective of electrical appliances is energy effectiveness. ([3]) Therefore, the indoor temperature can be considered to be a function of solely the external temperature and characteristics of the building. We will therefore model the scenario as a building which is affected by heat transfer from walls, windows, roofs etc. rather than any internal cooling mechanisms.

1A-2: The building has homogeneous insulation - the building has a consistent level of thermal insulation throughout, and there are no variations in wall, roof or window insulation.

Justification: A building in a given location tends to have a relatively consistent level of insulation. While there may be slight variations due to the age of the building or the variation in materials used, the assumption is reasonable for a typical residential structure: degradation of thermal insulation lasts 15-20 years ([4]).

1A-3: All incident energy onto a building is transferred into the walls or inside the building.

Justification: Buildings in urban areas are usually made of brick, concrete and wood [5]. External radiation hitting the building's exterior is, for these materials, largely absorbed by the building [6], so it is reasonable to assume that 100% of the amount of solar radiation incident on a building at any given point affects the building's internal temperature. [7]

1A-4: Each person in the apartment has a constant heat output, which we take to be 100W ([8])

Justification: An important factor in evaluating the indoor temperature of a settlement is the number of people present in the environment. This can be estimated based on the type of dwelling, with the assumption that each person is at rest and evenly distributed throughout the household. This simplification is made due to the challenges in modeling the movement of individuals over time, as well as the impact of position on the cooling of the environment.

1A-5: Effects of windows and ventilation are not taken into account

Justification: Cooling effects of windows are largely dependent on location and type: windows, by allowing solar radiation to enter the dwelling, may increase temperature. However, effects such as window coatings (e.g. Low-E coatings) may actually contribute to cooling [9]. As the nature of windows in dwellings is not given, modeling temperature change as dependent on these variables is impossible.

Similarly ventilation allows for movement of cool or warm air in and out of the dwelling. However, the nature and effectiveness of the ventilation is not given. Thus, we will not account for ventilation either.

1A-6: Starting temperature is 5°C lower than the outside temperature

Justification: The assumption that the starting indoor temperature is 5°C lower than the outdoor temperature is based on typical building thermal behavior during the initial conditions. Particularly when the building has been unoccupied or without active heating/cooling systems for a period of time. This offset is often seen in buildings where heat retention is minimal due to factors such as: insulation and heat loss overnight.

3.3 Variables

Symbol	Definition	Units
k	Characteristic to the specific building	1/seconds, s^{-1}
T_s	Temperature of the surroundings	Kelvin, K
T_0	Initial temperature of a room	Kelvin, K
t	Time measured from the start of experiment	seconds, s
H	The amount of heat energy given off by one person in one second	Watts, W
m	The mass of air in the room	kilograms, kg
ρ	The density of air in the room	kilograms/metre^3, kgm^{-3}
m	The volume of air in the room	metres^3, m^3
c	The specific heat capacity of air	Joules / (kilogram Kelvin), Jkg^-1K^{-1}
α, β	Constants relating the volume and surface area of a room to its value of k	1 / (seconds metre^3), 1 / (seconds metre^2), s^-1m^-3, s^-1m^-2

Table 1: The variables used in the cooling model

3.4 The Model

The Model: For our model we decided to use Newton's Law of Cooling in combination with a simple linear function, representing the energy generated by humans (which we have found to be significant). Newton's Law of Cooling states that the rate of heat loss/gain is directly proportional to the difference in the temperatures between the agent and the environment.

Newton's Law of Cooling can be justified for our model as it accurately captures passive heat transfer dynamics between the external environment and the building's interior. The law states that the rate of change of temperature is proportional to the difference between internal and external temperatures, which is appropriate since the building absorbs heat primarily from the exterior (1A-1)

It is because the higher the temperature, the faster the molecules of air are moving, and therefore they hit the cool surface at a higher frequency and transfer more energy after the hit. The lower the temperature of the cool surface, the more energy it can potentially take after 1 hit.

We will also take 1 occupant of the building to generate a constant amount of heat energy per unit of time, P, which we take to be 100W/person [8]. In our model, we will add the temperature difference due to humans to the total temperature difference.

3.5 Model and Results

3.5.1 Newton's Cooling Law

We take Newton's Cooling Law to be:

$$T(t) = T_s + (T_0 - T_s)e^{kt}$$

Using the dataset for the variation in indoor and outdoor temperature for synthetically occupied test houses over time, we found a value of k for a dwelling with a given volume, insulation and surface area by solving Newton's Cooling Law for k:

$$T(t) = T_s + (T_0 - T_s)e^{kt}$$

$$T(t) - T_s = (T_0 - T_s)e^{kt}$$

$$\frac{T(t)-T_s}{T_0-T_s} = e^{kt}$$

$$\ln\left(\frac{T(t)-T_s}{T_0-T_s}\right) = kt$$

$$k = \frac{1}{t}\ln\left(\frac{T(t)-T_s}{T_0-T_s}\right)$$

3.5.2 Temperature increase due to humans

Taking the power generated by each human to be 100W, we can calculate the temperature increase of the room due to this power using the equation for specific heat capacity:

The power generated by each person inside the room is $H = 100\,\mathrm{W}$, which is equivalent to $100\,\mathrm{J/s}$.

The increase in temperature of the room, ΔT, can be calculated using the formula for the heat energy added to the air:

$$Q = mc\Delta T$$

Where Q is the heat energy added to the room, m is the mass of the air in the room, and c is the specific heat capacity of air.

Since the heat energy added by the person is Ht (where t is the time in seconds), we can write:

$$Ht = mc\Delta T$$

Next, we calculate the mass of the air in the room. The mass is given by:

$$m = \rho V$$

Where ρ is the density of air (approximately $1.225\,\mathrm{kg/m^3}$) and V is the volume of the room (in cubic meters).

Substituting $m = \rho V$ into the heat equation:

$$Ht = \rho V c \Delta T$$

Now, solving for ΔT, we get:

$$\Delta T = \frac{Ht}{\rho V c}$$

This is the formula for the increase in temperature of the room due to the heat generated by the person inside the room, which we will add to our Newton's Cooling Law temperature difference to find the total change in temperature.

3.5.3 Dataset for indoor-outdoor temperature relationship

To empirically determine k, we used a dataset from a case study conducted by Loughborough University, UK [10]. In the case study, researchers determined the indoor and outdoor temperatures over time for a single-family home. Using the derived equation for k above, we found k at each instant of time using this data and took an average (since we take the direct effect of the sun to be negligible, this is valid as k can be considered to be solely a function of t, T_0, T_s and $T(t)$). This data set is valid because physical principles are universal and the experiment was taken under the same conditions that we are modeling (e.g. no internal heating/cooling). For this case study, there were no humans present in the dwelling, so it can be considered that the only variable which affects indoor temperature is the outdoor temperature.

3.5.4 Results

With the given formula, we determine the value of k for the kitchen in the West-Wing house in the dataset. While factors such as the number of people in the apartment and the solar irradiation are considered, we aim to disentangle these variables to simplify the determination of k. This is the West-wing is completely unoccupied and has no electrical appliances running (as opposed to the East-Wing, which contains simulated occupation).

Using the formula given previously, we find values for k for each hour between 2025-05-01 and 2021-09-12 and take averages of k to determine:

$$k = 1.55 \times 10^{-6}$$

Taking averages for the value of k is motivated to decrease the effect of noisy observation and experimental error on our value of k. We take k to 3 s.f as temperature as precision is limited by time being measured to 3 s.f.

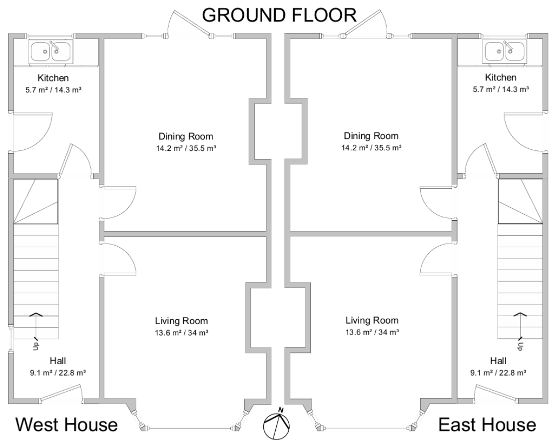

Figure 1: Floorplan of the house given in dataset. Dimensions are given in mm^2 and converted in to m^2 for consistency. Height is given separately at 1.1m

We repeat the same calculation for the living room, such that we have sufficient information to deduce α and β in

$$k = \frac{\alpha}{V} + \beta S$$

Determination of V and S are cited from the floorplan given above. For the dining room, we obtain.

$$k = 1.57 \times 10^{-6}$$

Solving simultaneous equations

$$k_1 = \frac{\alpha}{V_1} + \beta S_1$$

$$k_1 = \frac{\alpha}{V_2} + \beta S_2$$

where V_1, S_1 are the volume and surface area of the kitchen respectively and V_2, S_2 similarly for the dining room and k_1 and k_2 are the cooling constant for the kitchen and dining room respectively.

We obtain $\alpha = 8.1 \times 10^{-5}$ and $\beta = -1.2 \times 10^{-6}$.

We then get the final model:

$$T(t) = T_s + (T_0 - T_s)e^{kt} + \frac{Ht}{\rho V c}$$

For the 4 given apartments we compute 4 k values:

$$T(t) = T_s + (T_0 - T_s)e^{kt} + \frac{Ht}{\rho V c}$$

as no height is given, we use the value 18.1ft (5.51m), averaged from three separate studies [11] [12] [13].

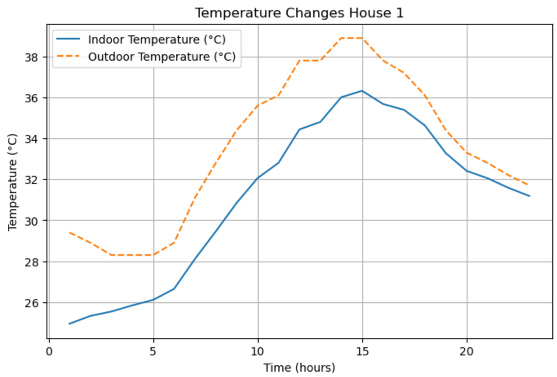

Figure 2: Temperature over 24 hours for House 1

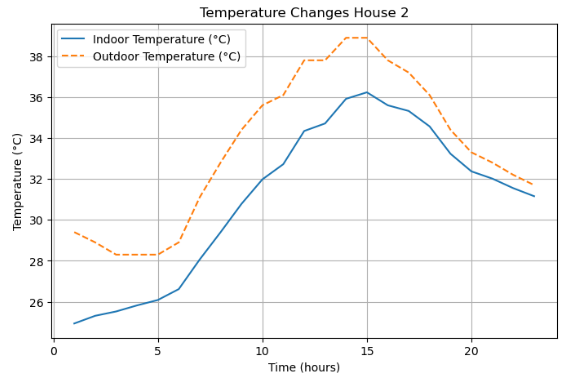

Figure 3: Temperature over 24 hours for House 2

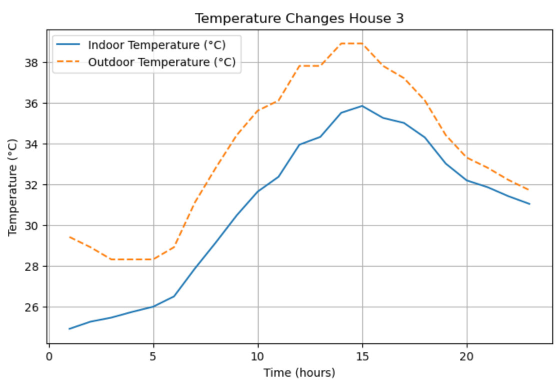

Figure 4: Temperature over 24 hours for House 3

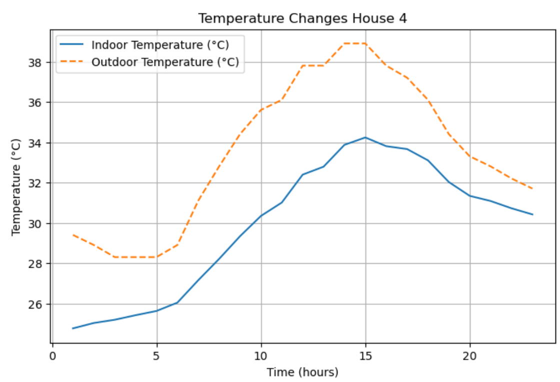

Figure 5: Temperature over 24 hours for House 4

3.6 Discussion

The generated plots for over time largely match our expectation: The house with the largest volume, is cooled much more quickly than that of other houses 3.5.4: clearly, according to our model, the larger volume and surface area of house 4 is more significant in house cooling than the number of people in the house. This is what we expect and can thus be considered as a factor of confidence in our model.

With a similar number of people in each household (e.g. house 1 and house 2), the effectiveness of cooling is very similar. The impact of human heating is largely negligible. (Figure 3.5.4)

We see the effect of our Newton's cooling model via the behavior towards the start of the day: the two values tend to an equilibrium. Similar behavior is evident towards the end.

3.7 Sensitivity Analysis

We consider the sensitivity of our model both empirically and analytically. We principally examine the sensitivity of our model to changes in observed temperature to the determination of k in terms of α and β, as extended work on sensitivity of the model to the prediction is compounded with this sensitivity

We start with the given equation:

$$k = \frac{1}{t} \ln \left(\frac{T(t) - T_s}{T_0 - T_s} \right)$$

We differentiate k with respect to T_s:

$$\frac{d}{dT_s} \left(\ln \left(\frac{T(t) - T_s}{T_0 - T_s} \right) \right)$$

Simplifying, we obtain:

$$\frac{d}{dT_s} k = \frac{1}{t} \cdot \frac{T(t) - T_0}{(T(t) - T_s)(T_0 - T_s)}$$

The sensitivity of k to changes in T_s is:

$$\left| \frac{d}{dT_s} k \right| = \frac{1}{t} \cdot \frac{|T(t) - T_0|}{|(T(t) - T_s)(T_0 - T_s)|}$$

This expression shows how sensitive k is to changes in T_s. The sensitivity depends on the values of $T(t)$, T_0, and T_s, and it decreases as T_s gets closer to $T(t)$ or T_0, since the denominator increases in such cases.

This analysis is further justified by the 0.5°C uncertainty in measuring the temperature using U-type thermistor: changes in 0.5°C is likely.

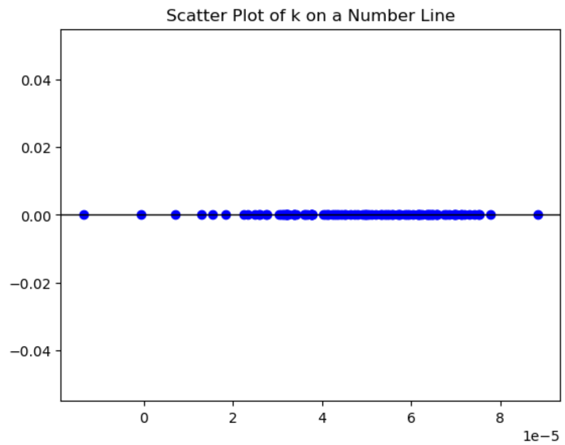

Figure 6: Values of k on a number line, after jittering each temperature by a random number sampled from a uniform distribution bounded between 0.5 and -0.5.

Empirically, (Figure 6) supports this hypothesis: k is highly sensitive to changes.

3.8 Strengths and Weaknesses

The use of Newton's Cooling Law is justified and accurate, as it takes advantage and quantifies the main thermodynamic principle behind the indoor-outdoor temperature relationship: heat flows from warmer areas to cooler areas at a determinable rate.

The model is capable of capturing gradual changes of temperature over time - when the temperature difference between inside and outside is significant (notable during a heat wave), the model is appropriate for predicting the steady-state changes that will occur in this time.

The law is accurate in terms of predicting temperature variations between rooms - in the living room and kitchen scenarios for which we calculated k, the value for the living room was lower by a factor similar to the ratio of the rooms' volumes. This is a justified property as we expect that when volume increases, the change in temperature per unit time is lower. This is because for the same amount of time, we need to heat up a larger volume of air, which therefore requires a greater energy input, so our value of k should be lower.

Furthermore, the law is useful for modeling short-term variations in temperature. Over 24 hours, the environmental value T_env will flucuate depending on the time of day, and this along with the cooling constant k will result in a similarly fluctuating indoor teperature at time t, T_t.

However, our model may not be completely accurate, as it simplifies energy transfer between the exterior and interior. The law assumes that heat transfer occurs uniformly through the surface area of the building, without consideration of heat stratification. In a real situation, the temperature distribution will not be uniform.

Another aspect to consider would be the variation in temperature across multi-story buildings - a multi-zone model would perhaps better capture heat transfer between floors, and this would require coupled differential equations to determine the relationship between two zones which we have instead considered to be negligible (this could potentially be justified by the relatively low height of buildings in Memphis [14]).

A further limitation to Newton's Law of Cooling is that it does not account for the effects of building insulation. Although this is a relatively minor oversight for short-term predictions in heatwave scenarios and the Law still provides a reasonable approximation, an analysis which incorporates insulation could improve accuracy, especially for longer-term predictions.

4 Q2: Power Hungry

4.1 Problem Statement

The problem asks us to create a model to predict the peak power demand on the Memphis power grid, and predict whether there will be any changes to the maximum demand 20 years from now. To model power demand, we split power consumption into two sectors, residential and industrial/-commercial. seasons, each of which is affected differently by the time of year, and merged two

4.2 Assumptions

2A-1: MLGW is the only power supplier for the city of Memphis.

Justification: Due to United States legislature [15], essentially only MLGW is allowed to supply electricity to the City of Memphis. As a result, using energy output data from MLGW is justifiable and reflective of the whole of Memphis' power usage. [16]

2A-2: All significant forms of energy consumption which vary periodically over the year vary seasonally.

Justification: Heating and cooling needs, lighting needs, agricultural needs, festive periods, and all other significant forms of energy usage which vary over the year, vary due to factors such as climate and lighting which line up with the seasons.

2A-3: The energy consumption will remain seasonal over the next 20 years.

Justification: Energy consumption is strongly correlated with weather conditions; During summer, it is likely that people will use cooling, and during the winter, it is likely that people will use heating. However, during spring and autumn, people are more likely to rely on natural ventilation. Analysis of past data confirms this. Air conditioning and heating are the most significant contributions to energy consumption. Thus, there is enough evidence to assume that the energy consumption will remain seasonal. [17]

2A-4: Population growth will not be significantly reduced by any carrying capacity factors over the next 20 years. We assume that technological advancements, economic growth, and innovations in fields like agriculture, medicine, and energy production will continue to provide the necessary resources to sustain the growing global population.

Justification: There is neither a reason for, nor a method of predicting, the occurrence of such a limiting factor. [18]

2A-5: The supply of electricity will always be larger than the demand for electricity, ensuring that everyone who has money for electricity will be able to get it.

Justification: In the scope of this problem, we have to model the demand for electricity and, therefore, it is sensible to assume that it is not affected by the supply of electricity. Furthermore, it is impossible to predict any major crises in energy supply, and MLGW currently also export excess energy produced to other states. [19]

2A-6: There will be no significant change in the efficiency of electricity transmission technologies, efficiency of cooling/heating units and efficiency of other technologies which might significantly change energy output without changing energy input.

Justification: Over the past few decades, advancements in technology have led to incremental improvements in the efficiency of energy transmission and conversion devices. For example, while there have been improvements in power grid infrastructure and HVAC systems, the rate of efficiency improvement has typically followed a pattern of diminishing returns.

2A-7: Majority of people will continue to use electrical ways of cooling the building.

Justification: Electrical cooling is one of the most cost-effective and available ways of cooling the building, if we assume that there will be no major breakthroughs in science it is valid to say that the most people will continue to use electrical cooling.

2A-8: People will still be sensitive towards heatwaves and will still use cooling in the future.

Justification: We do not have any reason to assume that people will develop an immunity towards higher temperatures as a result of global warming. While acclimatization can help the body adjust to warmer conditions, it does not provide immunity to the dangers of extreme heat.

4.3 Variables

4.3.1 Residential

Symbol	Definition	Units
y_t	Observed value (power demand) at time t	kWh
μ	Mean of the series (constant)	kWh
p	Order of non-seasonal autoregressive (AR) terms	N/A
d	Degree of non-seasonal differencing	N/A
q	Order of non-seasonal moving average (MA) terms	N/A
P	Order of seasonal autoregressive (SAR) terms	N/A
D	Degree of seasonal differencing	N/A
Q	Order of seasonal moving average (SMA) terms	N/A
s	Length of the seasonal cycle (e.g., 12 for monthly data with yearly seasonality)	N/A
ϕ_i	Parameters of the non-seasonal autoregressive component	N/A
θ_j	Parameters of the non-seasonal moving average component	N/A
Φ_k	Parameters of the seasonal autoregressive component	N/A
Θ_l	Parameters of the seasonal moving average component	N/A
ϵ_t	White noise error term (residual)	kWh

Table 2: Description of variables used in the SARIMA Models

4.3.2 Industrial/Commercial

Symbol	Definition	Units
L_0	Initial level of the time series that is computed using the average of the first seasonal period	kWh
T_0	Initial trend of the time series and the ratio between the average values of the first and second seasonal periods	N/A
S_0	The initial seasonal factors which adjust the level of the time series based on known seasonal variations	N/A
m	The number of periods in a season cycle (12) to represent the months in the year)	N/A
h	The forecast horizon is the number of periods for which	Months
y_{t+h}	The forecasted value at the time t+h	kWh

Table 3: Description of variables used in the Holt-Winters model

4.4 The Model

The Model: We decided to model energy usage by splitting it up into two sectors: residential and industrial/commercial, based on the unique characteristics and fluctuations in usage due to the different scenarios of building use.

4.4.1 Dataset

As MGLW is the main state power supplier in Memphis, they provided a monthly financial report from 2022 to 2024. The data included the monthly total energy sales in Memphis and a breakdown of MGLW's sector sales [20]. However, it did not include a breakdown of Memphis' sector sales. We plotted the portion of MGLW's power that is Memphis throughout time. Unsurprisingly, this was relatively constant at around 0.75.

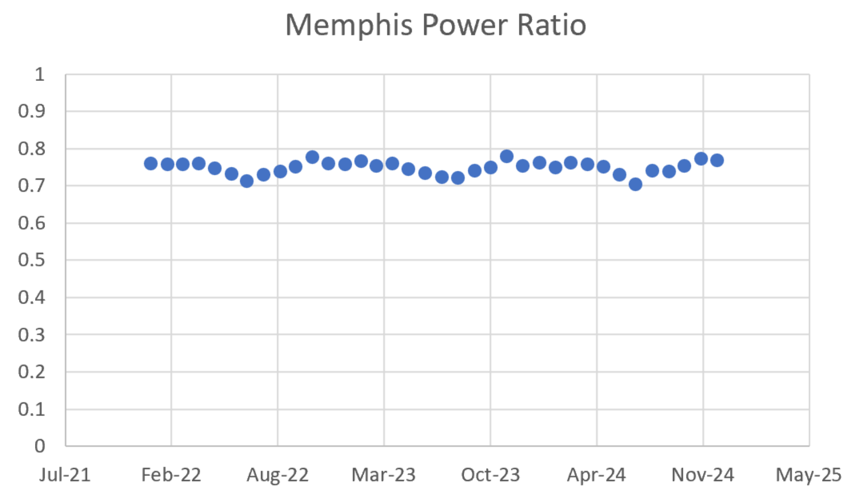

Figure 7: Proportion of MGLW's Monthly Sales that is Memphis

Therefore, we assumed that the breakdown of MGLW will be the breakdown of Memphis and tackled the sectors separately as they had different shapes when plotted.

4.4.2 Residential energy usage

Energy usage, especially in residential buildings where climate control is on 24/7, is likely to be significantly affected by both heating and cooling mechanisms during the year. Therefore, we decided to split our data into two 'seasons' of the year, the 'summer' of which being the period when heating is dominant and the 'winter' being the period when which cooling is dominant. To model the change in energy consumption, therefore, we used a double SARIMA model to fit residential energy usage, treating alternating regions separately as shown on the graph.

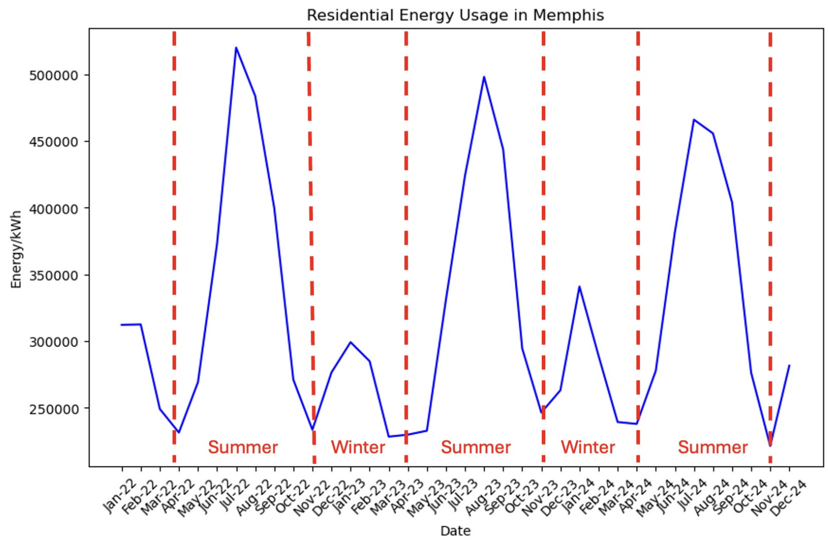

Figure 8: Splits of Residential Energy Usage in Memphis

Using a SARIMA model in this context is appropriate for two main reasons. The first is that power demand in a city is known to follow a seasonal pattern, including daily, weekly, and yearly cycles. SARIMA models explicitly incorporate seasonal components, making them well-suited for capturing these recurring patterns. The second reason is that power demand likely follows a general trend, being heavily dictated by factors such as population size and efficiency of electrical appliancies. The "Integrated" (I) component in SARIMA allows the model to handle non-stationary data by differencing, which removes trends and makes the data stationary. This makes SARIMA especially well suited to accounting for both seasonal patterns and general trends in data. Splitting up the two seasons, we are effectively modeling two phases in the year, within both of which is also found non-climate-control energy consumption. Some of this non-climate-control energy consumption can be considered to be continuous over the year, and other sectors will vary seasonally in accordance with 2A-2. Therefore, there are no overlaps between the red lines and so conducting two SARIMA models is valid. Since summer cooling appliance usage may follow a different trend over time to winter heating appliance usage, it is more appropriate to use this 'bi-seasonal' SARIMA method than fitting the data using one SARIMA model. We decided to split the seasons based on the two times of the year at which temperature is most 'comfortable'; i.e. when both heating appliance use and cooling appliance use are at their minimum. Taking a 'comfortable' temperature to be 18°C, we establish April and October/November to be these two 'comfortable' points [21].

To setup the SARIMA models, we must first tune the parameters of the models to improve their fit on the data provided. A SARIMA model takes 7 parameters, the process of determining each of these will be outlined below.

$$SARIMA(p, d, q)(P, D, Q)_s$$

$$y_t = \mu + \sum_{i=1}^{p} \phi_i y_{t-i} + \sum_{j=1}^{q} \theta_j \epsilon_{t-j} + \sum_{k=1}^{P} \Phi_k y_{t-ks} + \sum_{l=1}^{Q} \Theta_l \epsilon_{t-ls} + \epsilon_t$$

To start off, we set s = 12 to reflect the fact that we were working with monthly data of power demand. Following this, we ran Augmented Dickey-Fuller (ADF) tests on our Summer and Winter datasets to determine how many differences we had to take before the ADF test returned a p-value lower than our chosen significance level of 5%. Both Summer and Winter datasets reached this threshold immediately, with the ADF test returning 1.67e-07 for our Summer dataset and 1.17e-07 for our Winter dataset, both without running finite differences. We therefore set d = 0 for both SARIMA models. Next, we determined p. This was done by considering the Partial Autocorrelation Function (PACF) of each dataset, and checking how many lag values are significant before the first insignificant lag value.

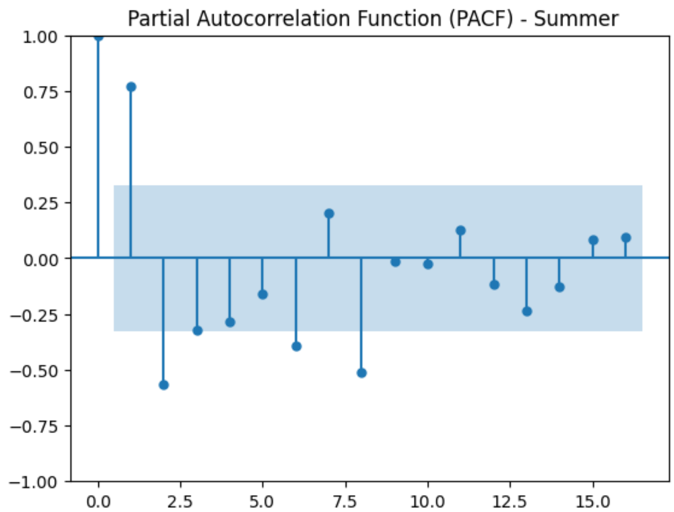

Figure 9: Partial Autocorrelation Function (PACF) Plot for the Summer Dataset

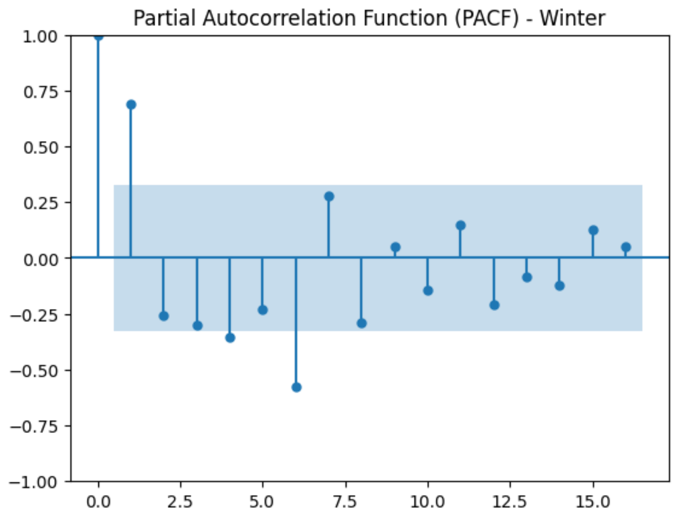

Figure 10: Partial Autocorrelation Function (PACF) Plot for the Winter Dataset

By inspection, for the Summer dataset, the first three lag values of the PACF were significant, and the fourth lag value was insignificant. Therefore, we set p = 4 for the Summer SARIMA model. However, only the first two lag values were significant for the Winter PACF. Therefore, we set p = 2 for the Winter SARIMA model.

A similar process was carried out on the Autocorrelation Function (ACF) to determine q.

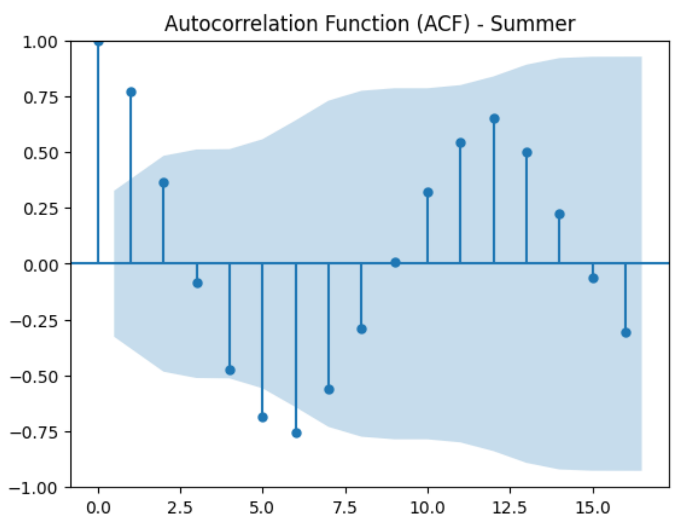

Figure 11: Autocorrelation Function (ACF) Plot for the Summer Dataset

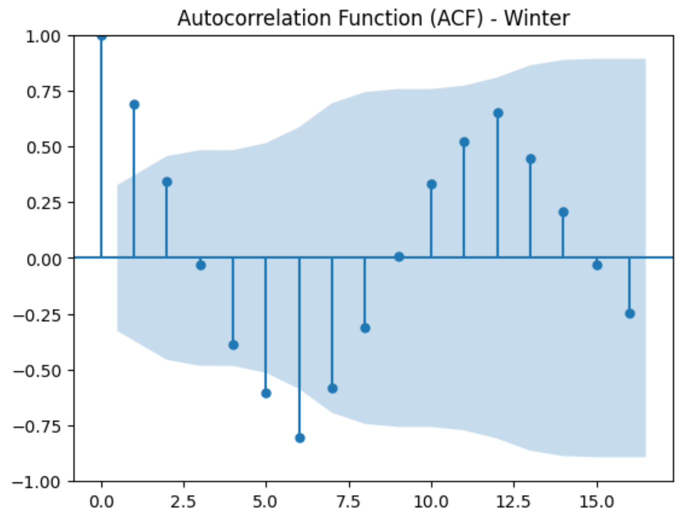

Figure 12: Autocorrelation Function (ACF) Plot for the Winter Dataset

By inspection, both datasets had only 2 autocorrelation values larger than the standard threshold value of 0.3. Therefore, we set p = 2 for both SARIMA models.

Now that the p, d, q, and s parameters were set, we tuned the P, D, and Q values to improve the fit of our model. We ended up choosing P, D, Q = (2, 1, 5) and (1, 1, 0) for the Summer and Winter models respectively. We then ran both models to predict 240 months (20 years) into the future. The results of individual models can be seen below.

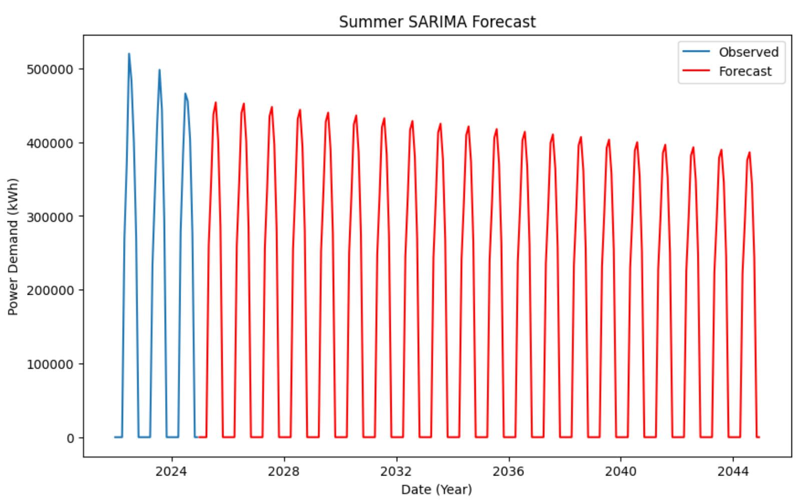

Figure 13: SARIMA Forecast - Summer

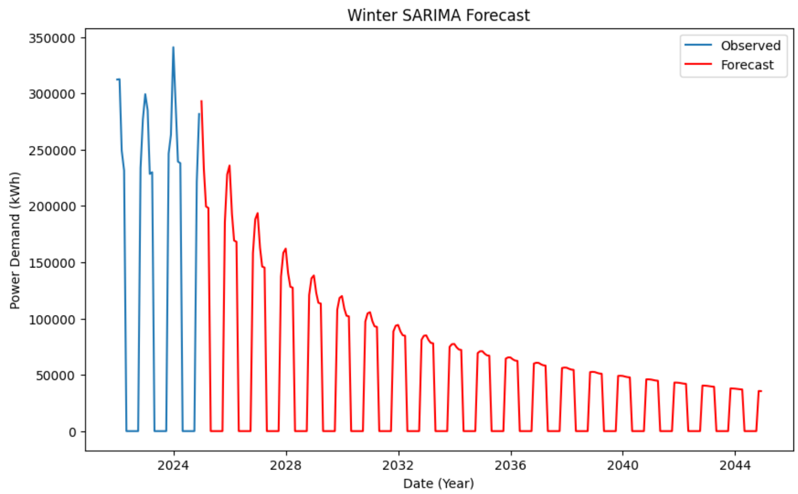

Figure 14: SARIMA Forecast - Winter

4.4.3 Industrial/Commercial energy usage

We decided to use the Holt-Winters Model to predict power usage within the industrial and commercial sectors. Holt-Winters is a good model for this case because it takes into account both seasonal fluctuation (which follows from the seasonal nature of energy consumption through cooling/heating). Unlike the residential case, the cooling/heating does not have to be turned on 24/7 in commercial and industrial buildings, so the trends will be less affected by seasonal cooling/heating fluctuations (unlike in residential buildings). This makes Holt-Winters model a more suitable choice in this case compared to our double SARIMA for the residential model.

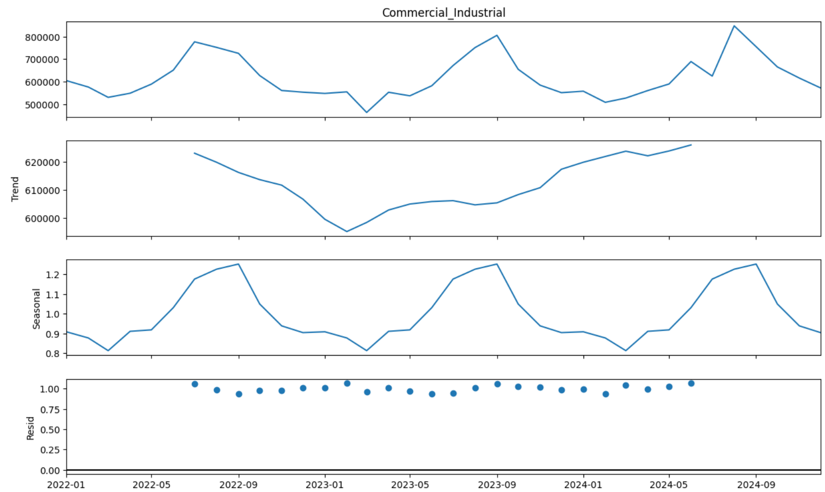

Figure 15: Seasonal Decomposition of the MGLW dataset

The Holt-Winters model would include the overall trend. This trend in the data can be explained by the growth/decline in the population, which results in more workers who require electricity (cooling and equipment) to work; by the growth in the average temperature throughout the world (workers will need more cooling).

In particular, we chose to utilise the multiplicative form of Holt-Winters instead of additive as it can represent the growth scaling with seasonal fluctuations instead of constant seasonal variations.

The Holt-Winters model is represented by the following equations:

1. **Level Update Equation:**

$$L_t = \alpha \frac{y_t}{S_{t-m}} + (1-\alpha)(L_{t-1} \times T_{t-1})$$

   The level L_t is updated by a weighted combination of the observed value y_t adjusted by the seasonal factor S_{t-m} (for the previous season) and the previous period's level L_{t-1} multiplied by the trend component T_{t-1}.

2. **Trend Update Equation:**

$$T_t = \beta \frac{L_t}{L_{t-1}} + (1-\beta)T_{t-1}$$

   The trend T_t is calculated as a weighted combination of the ratio of the current level L_t to the previous level L_{t-1}, and the previous trend component T_{t-1}.

3. **Seasonal Update Equation:**

$$S_t = \gamma \frac{y_t}{L_t} + (1-\gamma)S_{t-m}$$

The seasonal component S_t is updated by a weighted combination of the ratio of the observed value y_t to the current level L_t and the seasonal factor from the same period of the previous season, S_{t-m}.

4. **Forecast Equation:**

$$\hat{y}_{t+h} = (L_t \times T_t^h) \times S_{t+h-m(k+1)}$$

The forecast for future periods $t+h$ is given by multiplying the current level L_t by the trend component T_t raised to the power of the forecast horizon, and adjusting this by the seasonal factor $S_{t+h-m(k+1)}$ corresponding to the future time step. The forecast incorporates both the trend and seasonality, adjusting for any growth in the series over time.

The values of the smoothing parameters α, β, γ are set by default by the statsmodels library [22]. We then tuned the parameters of the model such that it fits the dataset.

4.5 Results

4.5.1 Residential

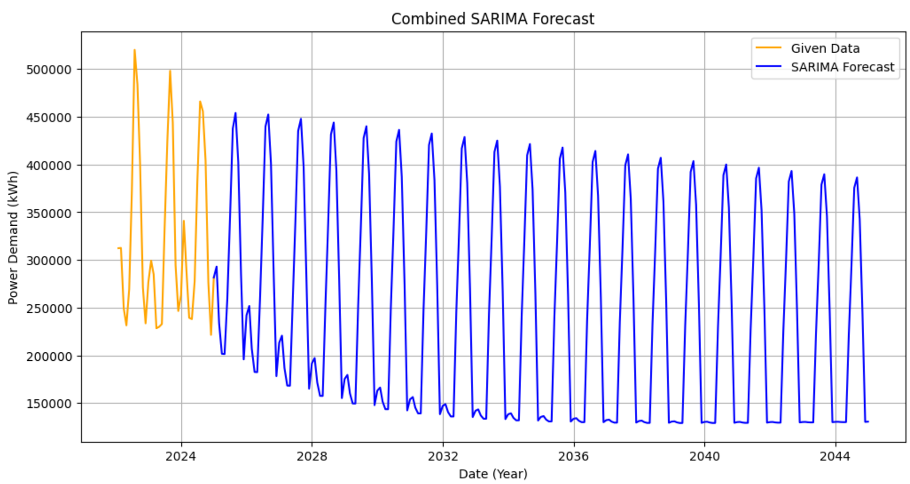

Figure 16: Final Power Demand Forecast by Combined SARIMA model

4.5.2 Industrial/Commercial

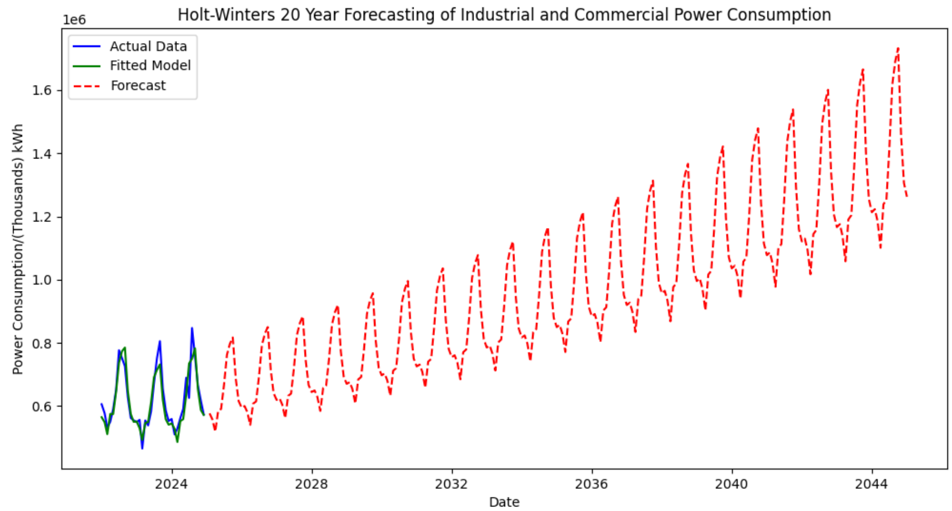

Figure 17: Holt-Winters Model predicting the next 20 years

Our Holt-Winters Model has forecasted the power consumption of the commercial and industrial sectors in 2044 to have a peak of 1732641 Thousand kWh and a lowest value of 1100794 Thousand kWh.

4.5.3 Final Forecast

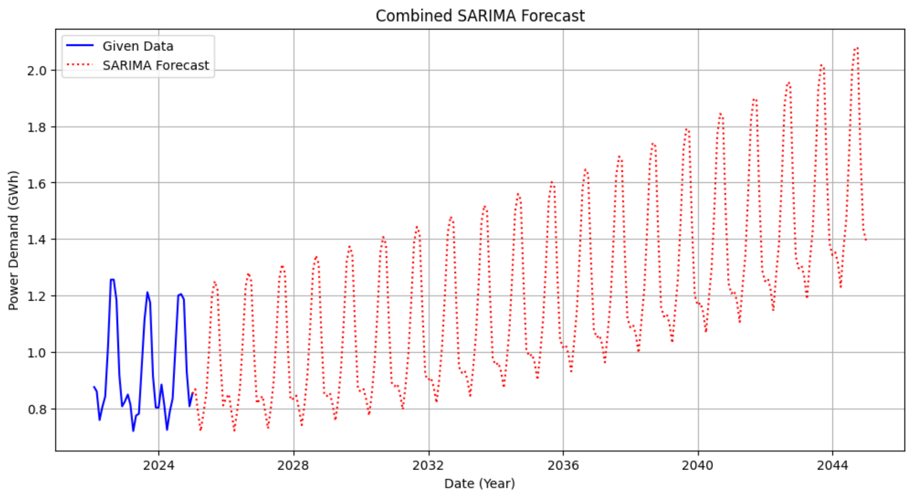

Figure 18: Final Combined Forecast of Power Demand

The final combined forecast is obtained by simply adding the forecasted power demands of the residential and commercial/industrial sectors of the City of Memphis. Selected results of peak power demand during the Summer months are shown below (August of each year consistently has the highest power demand; the values shown below are therefore taken from August of each year).

Year	Power Demand (GWh)	% Increase from 2024
2024	1.2056	N/A
2029	1.3739	14.0%
2034	1.5596	29.4%
2039	1.7907	48.5%
2044	2.0771	72.3%

Table 4: Forecasted Values of Peak Power Demand

4.6 Discussion and Analysis

4.6.1 Residential

Our SARIMA models show that Memphis City's residential power demand will follow a decreasing trend in the next 20 years. This is true of both Summer and Winter peaks — both end up plateauing at around 0.38GWh and 75MWh respectively. This result makes intuitive sense. The general downwards trend in residential power demand is likely driven by Memphis' decreasing population — Memphis is a city that experiences mass outwards migration. As people leave the city and houses are left uninhabited, it is expected that the residential power demand decreases accordingly. Another factor to consider is the increase in efficiency of electrical appliances such as air conditioning and chargers. This decreases the consumption per capita and contributes to the downwards trend. These two factors likely counteract the effects of global warming, which would increase the overall temperature of the city, meaning that more power would've been needed to cool residences in the Summer. However, the effects of Global Warming are still present, as it also means that Winters become warmer, thus decreasing the power required to warm residences during the Winter. This is likely why our model shows a quicker decrease in Winter peaks compared to Summer peaks, as the effects of Global Warming accelerate the move away from using heaters during Winter.

4.6.2 Commercial and Industrial

Our model's results follows our reasoning that the industrial and commercial power consumption will increase over time as we expect the consequences of heat waves to intensify assuming a lack of major intervention in climate change prevention. Despite the decreasing population in Memphis, we believe that the commercial and industry sectors will continue to grow. The growth in electrical consumption reflects this and the prediction that it will develop to 59% is a believable figure.

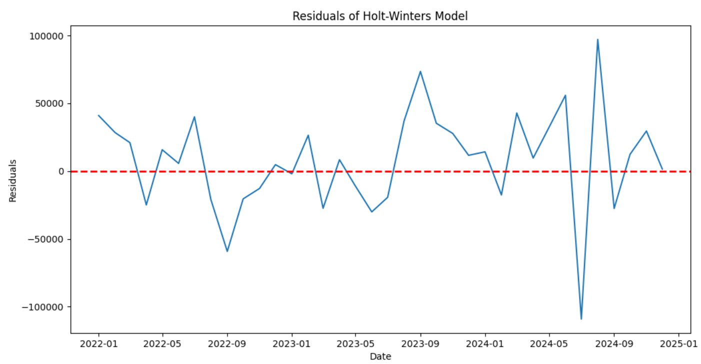

Figure 19: The residuals of the model over the forecast horizon

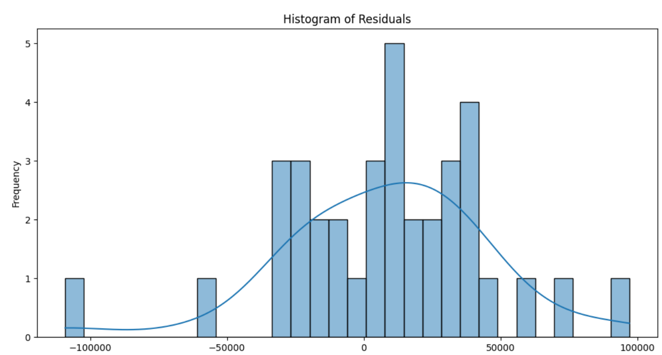

Figure 20: Histogram of residuals

To test the validity, we plotted the residuals of the Holt-Winters model 19, illustrating the difference between given and fitted values over time. The red dotted line at zero serves as a baseline for visual aid to identify any systematic patterns or biases in the model's errors. The residuals appear to be randomly distributed both in magnitude and direction which indicates that it is accurate throughout the forecasting horizon.

20 Moreover, the histogram of residuals displays the distribution of the errors from the Holt-Winters model. The overlaid kernel density estimate (KDE) visualises the shape of the residuals, providing insight into their distribution and any potential deviations from normality. As the distribution is centerd around 0, this suggests that the residuals are random and do not overestimate or underestimate.

4.6.3 Overall

Our models show that Memphis' power demand will steadily increase. This is driven by Memphis' growing economy and growing need for power in commercial and industrial contexts. Although Memphis' decreasing population and global increases in energy efficiency mean that Memphis' residential power demand follows a decreasing trend, this is dominated by Memphis' growing importance in manufacturing and transportation industries, which increases the city's power consumption. Leading tech companies have recently chosen Memphis as the location for constructing huge manufacturing facilities. We expect that this will only increase in the future, and growing popularity of Memphis as an industrial hub will no doubt be the main driver for Memphis' power demand. Therefore, we suggest the Memphis City officials be prepared for what could be a 72.3% increase in overall energy consumption in the next 20 years.

4.7 Holt-Winter Sensitivity Analysis

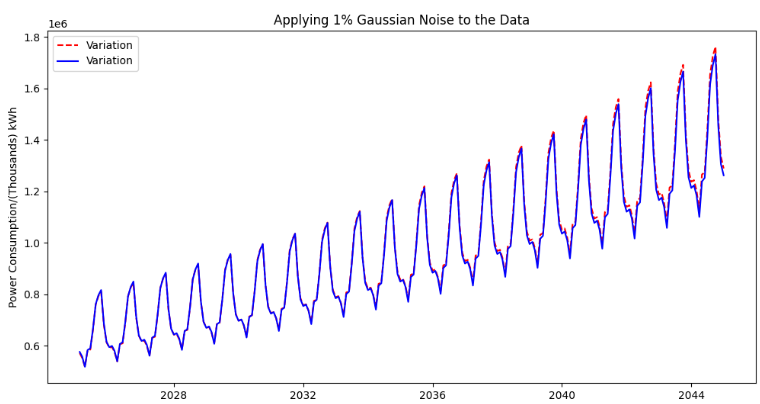

Figure 21: Original predictions overlayed with jittered predictions

Figure 22: Percentage change of predicted value with jitter and predicted value with no jitter

We applied ±1% Gaussian noise to the initial dataset from 2022 to 2024 and forecasted 20 years of data with these jittered values. We found that the $R^2 = 0.9975$ which demonstrates that the forecast remains highly robust despite small perturbations in the input data, preserving the overall trend and seasonality. The extremely high R^2 value gives us great confidence in the accuracy of the model in the short-term. However, we do not have a large enough dataset to confirm its accuracy in the long-term.

4.8 Strengths and Weaknesses

4.8.1 Residential

The double-SARIMA model we implemented for residential power usage proved to be justifiable in that it splits the two types of climate control usage into two independent models, as there is no data to suggest they will follow the same trend. Global warming could have an opposite effect on the usage of cooling appliances compared to its effect on the usage of heating appliances, so the splitting of this data is reasonable. By splitting the data, we are also less likely to over-fit to irrelevant patterns from the off-season.

However, there may be very slight inaccuracies in the months which fall on the transition between seasons, as it is not possible to fit the transition line at the perfect time. In addition, very long-term predictions using the model (e.g. 50+ years ahead) could become less well-informed, as there is a potential risk of large-scale environmental or policy changes occurring, for which our model cannot account.

4.8.2 Commercial/Industrial

Holt-Winters reflects the overall growth in energy consumption trend caused by heat waves. Especially, the rise in both intensity and frequency of heat waves spurred by global warming would push for more cooling energy consumption in commercial properties as well as necessitating additional energy for compensatory measures in industry due to failed machinery. This is further exacerbated by the potential increase in exports from Memphis. What's more, it retains the strong periodicity of the city's energy consumption and is highly accurate for short-term predictions where seasonality is the dominant factor. As such, it is reliable for planning purposes.

However, in the long term Holt-Winters model is less effective because the further we go in time the less accurate it is. It relies on a large number of assumptions such as stationary in the seasonal effect and does not take into account any other factors, such as economic growth, weather anomalies and policy changes which undoubtedly have a strong relationship with the power consumption in the commercial and industrial sectors.

The Holt-Winters model fundamentally is completely reliant on past data to make forecasts. The limited amount of data to fit the curve makes the prediction synthetic. and will have a lack of adaptability to structural changes.

Another property of the model is its smoothing function, which may smooth out important noise. Irregular spikes are smoothed out and may not portray the all of the relationships.

5 Q3: Beat the Heat

5.1 Problem Statement

Question 3 requires us to develop a measure of vulnerability for different region. The ambiguity of vulnerability within this question introduces difficulty in quantifying meaning values of "vulnerability". For the most interpretable output, we consider vulnerability to be the economic damage caused by a heat wave.

This calculation involves two distinct components: the probability that the given neighborhood will be affected by a power outage and the damage that will be caused if the neighborhood experiences a power outage.

After determining these values, we then determine the optimal placement of emergency centers under principle of maximal utility.

5.2 Assumptions

3A-1: The probability that a neighborhood experiences a power outage is dependent on the number substations within a n meter radius

Justification: Substations are critical within urban energy grids: performing step-down and step-up voltage transformation via transformers, routing and controlling the flow of electricity and detection of faults. The presence of multiple substations significantly reduce the probability of power outages: with the failure of one substation, other substations are able to route energy to households, commerce and industry [23].

3A-2: The income that any resident of Memphis receives from their job is uniform across the year.

Justification: Since we cannot predict when employees will receive bonuses and employment changes, we will take their income to be a gradual stream of money which flows at a constant rate throughout the year. The majority of people will be in stable industries and would have low variability in their pay.

3A-3: There will be no significant changes in electricity distribution throughout the city.

Justification: Buildings and the facilities will grow at the same rate in each neighborhood (in terms of electricity consumption), no new neighborhood will appear.

3A-4: If a heatwave occurs in Memphis, it will hit each neighborhood with the same severity.

Justification: Heat waves are meteorological effects that span a wide region. Memphis is a geographically compact city which covers an area of around 300 square miles. Thus, it is sensible to assume uniformity. [1]

3A-5: Linear Relationship Between Substations and Reliability of the grid in a neighborhood.

Justification: The more substations there are, the more reliable the area is as there is a probability that a substation will fail(therefore, the more substations there are, the less the probability of the whole system failing).

3A-6: Duration of outage proportional to number of substations.

Justification: It takes time to fix the substation in order for the outage to end. If there are multiple substations which form the network, we have to fix all of them in order to get the electricity going. In the worst case scenario, they cannot be fixed at the same time, and therefore, the time to fix the whole network is directly proportional to the number of substations.

3A-7: The environment temperature is the same throughout the city.

Justification: Memphis is a city that is relatively flat urban area and microclimates will have negligible temperature difference to the general forecasts of the entire area. [1]

3A-8: The Earth is a sphere

Justification: The Earth is approximately spherical and distance from points on the surface to the center range from 6,353 km to 6,384 km which is about a 0.3% difference [24]. This is a negligible distance and therefore it is a sufficient approximation for the Earth to be a sphere.

3A-9: Memphis is small enough to neglect the curvature of the earth.

Justification: Memphis is around 48 km from East to West, and around 32 km from North to South. This is a tiny region compared to the surface area of the Earth, so the curvature of the Earth can be neglected for this problem.

3A-10: We take a power outage to last, on average, 6 hours.

Justification: Power outages are dependent on their cause but from historical data, there are common causes and response mechanisms. To simplify the variability, we can assume outage durations are averaged to 6 hours to deal with the general case. [**?**]

5.3 Variables

Symbol	Definition	Units
P_i	Probability of outage in neighborhood i	Unitless (probability)
W_i	Loss of wages in neighborhood i	USD
T_i	Cost of cooling in neighborhood i	USD
E_i	Expected economic damage in neighborhood i	USD
lat_{min}	Minimum latitude defining Memphis	Degrees
lat_{max}	Maximum latitude defining Memphis	Degrees
lon_{min}	Minimum longitude defining Memphis	Degrees
lon_{max}	Maximum longitude defining Memphis	Degrees
(lat_p, lon_p)	Latitude and longitude substations	Degrees
Q	Cooling rate of air conditioning	Watts
T_c	Threshold temperature	Degrees Celsius
P	Power consumption of air conditioning	Watts
k	Thermal decay constant	Unitless
ρ	Density of air	kg/m^3
v	Volume of the house	m^3
c	Specific heat capacity of air	J/(kg·K)
$T_{\text{out}}(t)$	Outdoor temperature at time t	Degrees Celsius
$T_{\text{in}}(t)$	Indoor temperature at time t	Degrees Celsius
$T_{\text{in},0}$	Initial indoor temperature	Degrees Celsius
R	Radius of effectiveness for emergency centers	Meters
C	Overlapping circles in the center placement	Unitless
S	Economic damage of a heatwave in a neighborhood	USD

Table 5: Definition of variables used in the model.

5.4 The Model

The Model:

We construct a model that considers the expected economic damage of a heatwave to a given neighborhood

$$E_i = P_i \times W_i + (1 - P_i)T_i \tag{1}$$

where P_i is the probability of a heatwave power outage, W_i is the the loss of wages as a result of the heatwave and T_i is the cost of keeping houses in the neighborhood at a comfortable temperature.

The use of expected value is appropriate in this context because it provides a comprehensive measure of risk by combining the probability of an event (e.g., a heatwave) with the economic damage that would result if the event occurs. This approach is particularly useful for decision-making under uncertainty, as it allows us to quantify the potential impact of rare but high-consequence events like heatwaves.

5.4.1 Computing the probability of power outage

Using OpenStreetData [25] , we obtain the location of substations within the Memphis Area. As the Memphis border is irregular, we use the bounding box, defined by the maximum and minimum latitudes and longitudes. In the case of the emphasis, we can contain the entirety of Memphis within a bounding box with minimum latitude $lat_{min} = 35.0$, maximum latitude $lat_max = 35.2$, minimum longitude $lon_{min} = -90.2$ and maximum longitude $lon_{max} = -89.9$.

Figure 23: The map highlights an approximate border of Memphis, shaded in red. This visualization is useful for understanding the spatial extent of Memphis and its surrounding regions.

we then identify all of the power stations with position (lat_p, lon_p) if (lat_p, lon_p) is within the bounding box. Combined with a plot with the latitude and longitude of the neighborhoods given in the provided dataset [2], we obtain (Figure 24).

We see that several of the neighborhoods (e.g. Oakland, Frayser) are outside of the radius, thus we remove these neighborhoods from the area.

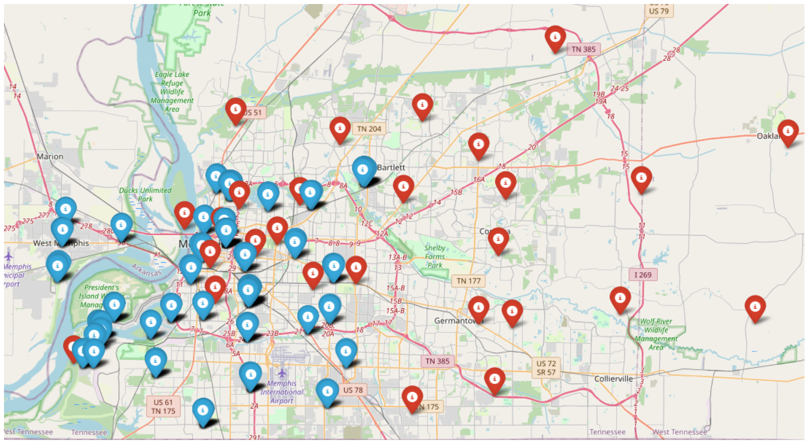

Figure 24: The map highlights the neighborhoods with red pins, and substations with blue pins. We will take some to be out of range, as they are not within the borders we have defined.

Then, using the operating distance of a substation [26], we consider a circle of radius 2000m around each neighborhood, to see how many substations are available. Specifically, we substitute the latitude and longitude into the circle equation with radius 2000m and center at the substation.

To convert this value into a meaningful probability, we normalise the neighborhood data between 0 and 0.3. The value of 0.3 was chosen based on the July 19th 2023 power outage, in which an estimated up to 30% of households lost MLGW power [27]. As such we evaluate the following P_i value for the neighborhoods:

Neighborhood	P_i
Downtown / South Main Arts District / South Bluffs	0.0476
East Midtown / Central Gardens / Cooper Young	0.0714
Uptown / Pinch District	0.301
South Memphis	0.0476
North Memphis / Snowden / New Chicago	0.190
Hollywood / Hyde Park / Nutbush	0.0952
Coro Lake / White Haven	0.143
East Memphis – Colonial Yorkshire	0.0714
Midtown / Evergreen / Overton Square	0.0595
East Memphis	0.0238
South Forum / Washington Heights	0.0714

5.4.2 Computing the wages lost as a result of power outage

In the case of a power outage a household, we consider the wages lost a result of power outage.

5.4.3 Computing the cost of cooling

In the case of no power outage in a household, we consider the cost of cooling the room to the comfortable temperature of 18°C [28]. 20-25% is used for energy spending, so clearly the cost of cooling is significant.

In each relevant neighborhood, to use our proposed heating equation in question one, we must use a value for the size of the house. To do this, we take a weighted average of townhouse and apartment, and use the average house size in Tenessee to get these values. The "Other" category is disregarded to significant differences in the size of the houses (e.g. boathouse and mobile houses have vastly different sizes) and unique physical factors may make our model invalid. https://hndrealty.com/making-the-choice-townhomes-for-rent-vs-single-family-https://www.bankrate.com/real-estate/average-home-size/ https://www.rentcafe.com/average-rent-market-trends/us/tn/memphis/: :text=What

Then, we adapt our original formulation of temperature to include a cooling factor via air conditioning. We model air conditioning as having a cooling rate $Q = 2400$ [29], as being activated when the temperature is below threshold temperature $T_c = 27$ [30] and consuming $P = 2400$ watts of energy.

The cooling effect included in our model

$$\frac{Q \cdot \text{time}}{\rho \cdot v \cdot c}$$

is similarly derived to that of the heating effect of humans. Including this term in our formula gives

$$T_{\text{in}}(t) = \begin{cases} T_{\text{out}}(t) + (T_{\text{in},0} - T_{\text{out}}(t))e^{kt\cdot} + \frac{100p}{\rho vc}, & \text{if } T_{\text{in}}(t) \leq 27 + 273 \text{ (No cooling)} \\ T_{\text{out}}(t) + (T_{\text{in},0} - T_{\text{out}}(t))e^{kt\cdot} + \frac{100p}{\rho vc} - \frac{Q \cdot t}{\rho vc}, & \text{if } T_{\text{in}}(t) > 27 + 273 \text{ (Cooling ON)} \end{cases}$$

We then run this formulation on the computed k values computed using the average size of the house. As in Q1, we multiply our area by the average height of 5.8m. Over a 24 hour period, we obtain the cost per house in a 24 hour period (Figure 25). Using the average length of a heat wave [31] of 3 days, we then multiply this value by the number of households in the area as well as the number of days during an average heatwave.

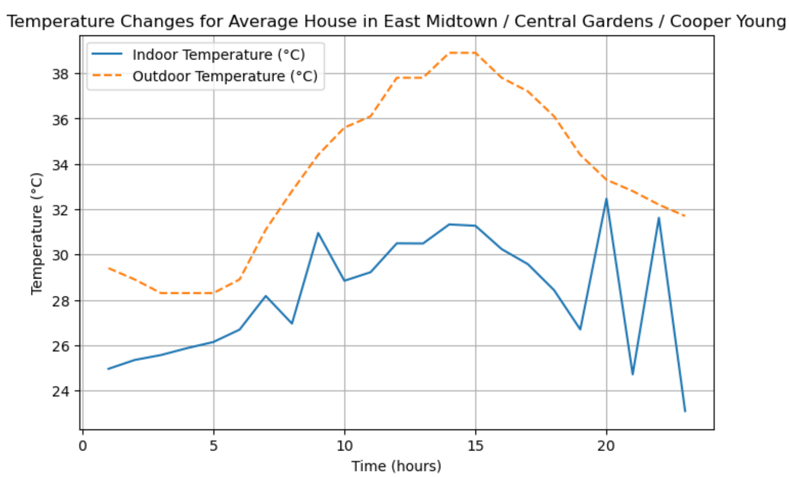

Figure 25: The variation of indoor temperature over time, accounting for air conditioning

We then get the following costs for cooling for each neighborhood:

Neighborhood	**Cooling Cost / USD**
Downtown / South Main Arts District / South Bluffs	326007
East Midtown / Central Gardens / Cooper Young	597313
Uptown / Pinch District	161655
South Memphis	628020
North Memphis / Snowden / New Chicago	380371
Hollywood / Hyde Park / Nutbush	488622
Coro Lake / White Haven	811883
East Memphis – Colonial Yorkshire	843528
Midtown / Evergreen / Overton Square	378675
East Memphis	686565
South Forum / Washington Heights	172780

5.4.4 Optimal emergency center placement

The Objective

We will take each neighborhood to be a specific pair of coordinates in latitude-longitude form. Our objective is to find the best possible placement of a fixed number of 'emergency centers', such that the E_i of each neighborhood within a set radius of an emergency center will decrease by a set 'gain factor'. We will use 0.3 as our gain factor, a reasonable value of the ratio of original power operation to power operation running on emergency reserves [32].

The Algorithm

We are given the positions of neighborhoods (which each have a score S), in coordinates, a fixed radius R away from which the emergency centers are effective, and a number of emergency centers we can place.

Drawing a circle of radius R around each town, we put one point in each region of overlap between two or more circles - let the number of circles they are in be C. We order these points in a list from highest to lowest C.

For every neighborhood, we also place a point randomly in a place in its circle which doesn't overlap with any other circles (so C is 1 here).

Then, taking each point, we simulate placing one emergency center at that point, and reduce the score S of the neighborhoods in the circles the emergency center is in by 1/5 of their original score S (the actual value of this constant is not important as in the end

We compute the total score of the neighborhoods, S, for each different place we can put the emergency center, and determine in which point the emergency center must be placed to minimize the total value S.

We repeat this process for each emergency center to be placed, updating the neighborhoods' scores for each iteration.

5.5 Results

We assigned each neighborhood a score using the formula for the expected economic damage of a heatwave to this neighborhood as defined earlier:

$$E_i = P_i \times W_i + (1 - P_i)T_i \quad (2)$$

To calculate W_i, we took the average wage for each neighborhood, and divided these values by 365.25*4 to obtain the average money lost over a 6 hour period:

Neighborhood	W_i **for 6 hours**
Downtown / South Main Arts District / South Bluffs	612741.69
East Midtown / Central Gardens / Cooper Young	854739.69
Uptown / Pinch District	99545.98
South Memphis	442922.82
North Memphis / Snowden / New Chicago	348760.02
Hollywood / Hyde Park / Nutbush	446878.76
Coro Lake / White Haven	1103191.53
East Memphis – Colonial Yorkshire	1520241.73
Midtown / Evergreen / Overton Square	544296.63
East Memphis	1683818.96
South Forum / Washington Heights	115113.76

Table 6: Neighborhoods and their values of W_i for 6 hours

Our final scores were:

Neighborhood	**Score**
Downtown / South Main Arts District / South Bluffs	338300.0199
East Midtown / Central Gardens / Cooper Young	630927.2954
Uptown / Pinch District	143015.6369
South Memphis	620084.4295
North Memphis / Snowden / New Chicago	374950.0392
Hollywood / Hyde Park / Nutbush	485042.7301
Coro Lake / White Haven	849350.3351
East Memphis – Colonial Yorkshire	887046.5678
Midtown / Evergreen / Overton Square	387550.7571
East Memphis	707942.3625
South Forum / Washington Heights	169071.56

Table 7: Neighborhoods and their scores

Using the algorithm defined in section 5.4.4, we then plotted the optimal points of emergency centers. Our final results, for 15 emergency centers across the 11 neighborhoods, are:

Emergency center	**Latitude**	**Longitude**
Emergency Center 1	35.14029	-89.99170
Emergency Center 2	35.12101	-89.94999
Emergency Center 3	35.05788	-90.16713
Emergency Center 4	35.11358	-90.04242
Emergency Center 5	35.12746	-89.91268
Emergency Center 6	35.16238	-90.02596
Emergency Center 7	35.18117	-89.97649
Emergency Center 8	35.15455	-90.05155
Emergency Center 9	35.14029	-89.99170
Emergency Center 10	35.12717	-89.94313
Emergency Center 11	35.04112	-90.18361
Emergency Center 12	35.11358	-90.04242
Emergency Center 13	35.09932	-89.90194
Emergency Center 14	35.18001	-89.97438
Emergency Center 15	35.16238	-90.02596

Table 8: Coordinates for Emergency Centers

5.6 Sensitivity Analysis

By using the noise function to jitter the values of E_i, and using a noise function from 0-5 percent, we obtained a new value for the final percentage decrease in total E_i score. The percentage change relative to the initial percentage decrease in E_i was 2.59, which is is negligible, so our model is at an acceptable sensitivity level.

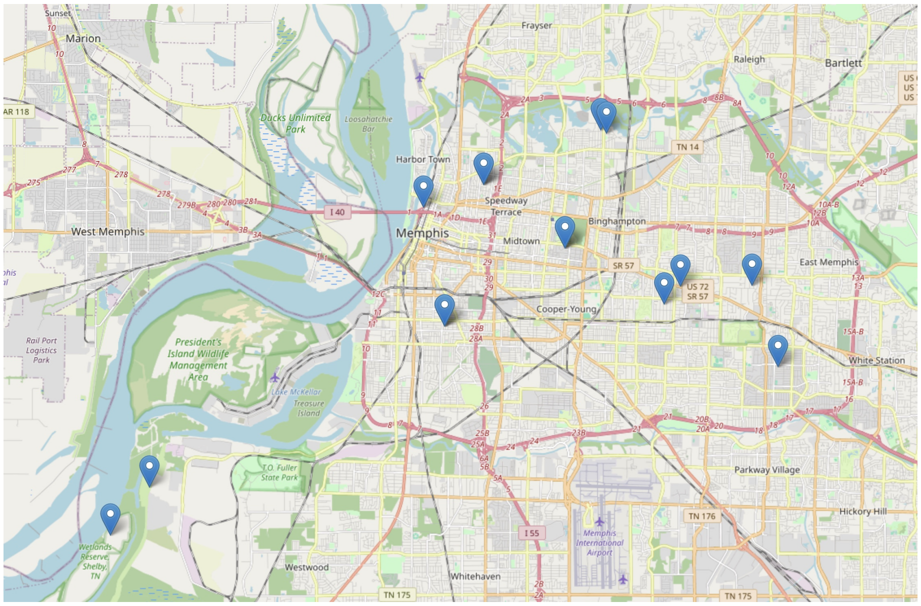

Figure 26: Proposed Emergency center locations (some of which are at the same location)

med

We also jittered the input parameters modeling cooling, examining how the cost varied over time. Overall, the effect is largely negligible (0.23% average change in cost). This is likely as the order of magnitude of input parameters are significant.

5.7 Strengths and Weaknesses

Our method considers both power outage states and non-power outage states when accounting for the net money lost/spent. Through our model, we have found that the power-outage effect of heat waves is significant, but even if there is no power outage, the cost of cooling is still largely significant.

Our economic measure (E_i) is very interpretable and can be applied again to improve on the model. This means that this result can be well understood by non-technical personell, and can therefore be used to inform policy. Additionally, if presented to the public, these figures are understandable and intuitive.

Our model is also prescriptive. After predicting the potential damage to the economy, we prescribed potential solutions. Our proposed stochastic sampling algorithm, although not necessarily optimal, presents a solution that considers the cost of emergency centers. This is necessary as the opportunity cost of emergency centers may be high, which means that funds could be spent elsewhere.

However, our model does not account for changes in climate, such as global warming. We do concede that this is important, and will consider this effect in future works.

Also, we do not have enough information about emergency stations, so it is difficult to say whether the values we chose for the number of emergency stations. In theory it can be higher or lower depending on the budget. Although the output for coordinates looks valid, we have not proven that this is necessarily the best approach to solving the problem. Although data regarding the cost of the emergency centers and the government's budget is unavailable, the fast-execution time of our algorithm makes it easy to adapt to policy.

Our model is also highly adaptable to different regions. By replacing Memphis in different states, we may be able to consider cross-state grid problems, which may be significant.

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6 Code Appendix

6.1 Simulate Temperature

```python
import pandas as pd

t_out = pd.read_csv("test.csv")["temp"].to_numpy() +273
t_out

import math

def get_k(v,s):
    a = 8.1 * 10**(-5)
    b = -1.2 * 10**(-6)
```

```

    k = a/v + b*s
    return k

m1 = 88
m2 = 63
m3 = 74
m4 = 278

k1 = get_k(m1*5.8,math.sqrt(m1)*4)
k2 = get_k(m2*5.8,math.sqrt(m1)*4)
k3 = get_k(m3*5.8,math.sqrt(m3)*4)
k4 = get_k(m4*5.8,math.sqrt(m4)*4)

import math
from matplotlib import pyplot as plt
import numpy as np

hours = np.arange(1, len(t_out))  # Time in hour

def get(k,v,p):
    t_in = t_out[0] - 5
    ts = []
    for i in range(1,len(t_out)):
        time = 3600*i
        # Temperature model with using model stated, with human term
        t = t_out[i] + (t_in - t_out[i])*math.e**(k*time)+ (100 * p) / (1.225 * v * 1005)
             * 3600
        # Get data in celcius for interpretability
        ts.append(t - 273)
    return ts

temp_values = get(k2, m2 * 5.8, 3)

plt.figure(figsize=(8, 5))
plt.plot(hours, temp_values, label="Indoor Temperature ( C )")
plt.plot(hours, t_out[1:] - 273, label="Outdoor Temperature ( C )", linestyle="dashed")

# Labels and legend
plt.xlabel("Time (hours)")
plt.ylabel("Temperature ( C )")
plt.title("Temperature Variation Over Time")
plt.legend()
plt.title("Temperature Changes House 2")
plt.grid(True)

# Show the plot
plt.show()
```

6.2 SARIMA

```
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
import math
from statsmodels.tsa.statespace.sarimax import SARIMAX
from statsmodels.graphics.tsaplots import plot_acf, plot_pacf
from statsmodels.tsa.seasonal import seasonal_decompose
```

```python
from sklearn.metrics import mean_squared_error

# Load the dataset
data = pd.read_csv('sample_data/Residential_Summer.csv')
data['Date'] = pd.to_datetime(data['Date'], format='%b-%y')
data.set_index('Date', inplace=True)
# Plot the time series
plt.plot(data['Residential'])
plt.title('Residential')
plt.xlabel('Date')
plt.ylabel('Residential Power Use')
plt.show()

# Run Dickey-Fuller Test
from statsmodels.tsa.stattools import adfuller

def ad_fuller(timeseries):
    print ('Dickey-Fuller Test indicates:')
    df_test = adfuller(timeseries, regression='ct', autolag='AIC')
    output = pd.Series(df_test[0:4], index=['Test Statistic','p-value','#Lags Used','
        Number of Observations Used'])
    print(output)

print(ad_fuller(data['Residential']))

# lot PACF and ACF

from statsmodels.graphics.tsaplots import plot_acf, plot_pacf

plot_acf(data['Residential'])
plt.title('Autocorrelation Function (ACF) - Summer')
plt.show()
plot_pacf(data['Residential'])
plt.title(' Partial Autocorrelation Function (PACF) - Summer')
plt.show()

# Automatically find the best SARIMA parameters
model = auto_arima(data['Residential'], seasonal=True, m=12, d=0, stepwise=True, trace=
    True)
print(model.summary())

# Define SARIMA Parameters
# SARIMA(p, d, q)(P, D, Q, s)
# p: AR order, d: Differencing, q: MA order
# P: Seasonal AR, D: Seasonal differencing, Q: Seasonal MA, s: Seasonality (12 for
    monthly data)
order = (4, 0, 2)  # Non-seasonal parameters (p, d, q)
seasonal_order = (2, 1, 5, 12)  # Seasonal parameters (P, D, Q, s)

# Fit SARIMA Model
model = SARIMAX(winter_data, order=order, seasonal_order=seasonal_order)
results = model.fit(disp=False)
print(results.summary())

# Forecast for the next 12 months
forecast_steps = 240
forecast = results.get_forecast(steps=forecast_steps)
forecast_mean = forecast.predicted_mean
```

```
confidence_intervals = forecast.conf_int()

# Plot the forecast
plt.figure(figsize=(10, 6))
plt.plot(winter_data, label='Observed')
plt.plot(forecast_mean, label='Forecast', color='red')
# plt.fill_between(confidence_intervals.index, confidence_intervals.iloc[:, 0],
    confidence_intervals.iloc[:, 1], color='pink', alpha=0.3)
plt.title('Summer SARIMA Forecast')
plt.xlabel('Date (Year)')
plt.ylabel('Power Demand (kWh)')
plt.legend()
plt.show()
```

6.3 Holt-Winters

```
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
import seaborn as sns
from statsmodels.tsa.holtwinters import ExponentialSmoothing
from statsmodels.tsa.seasonal import seasonal_decompose
from sklearn.metrics import r2_score

# Load the dataset from an Excel file
df = pd.read_excel('Q2/Spreadsheets/memphis_electricity.xlsx', parse_dates=["Date"])

# Set 'Date' column as the index and sort the DataFrame by date
df.set_index("Date", inplace=True)
df = df.sort_index()

# Plot the time series data for visualization
plt.figure(figsize=(12, 6))
sns.lineplot(x=df.index, y=df["Commercial_Industrial"])
plt.title("Time Series Data")
plt.xlabel("Date")
plt.ylabel("Commercial_Industrial")
plt.show()

# Decompose the time series to identify seasonal components
# Assuming monthly data with yearly seasonality (period=12)
decomposition = seasonal_decompose(df["Commercial_Industrial"], model="multiplicative",
    period=12)
decomposition.plot()
plt.show()

# Define the seasonal period (e.g., 12 for monthly data with yearly seasonality)
m = 12

# Calculate initial level (L0) as the average of the first season
L0 = df["Commercial_Industrial"].iloc[:m].mean()

# Calculate initial trend (T0) as the ratio of the average values of the first two
    seasonal cycles
T0 = (df["Commercial_Industrial"].iloc[m:2*m].values / df["Commercial_Industrial"].iloc[:
    m].values).mean()

# Calculate initial seasonal factors (S0) for the multiplicative model
```

```python
S0 = df["Commercial_Industrial"].iloc[:m] / L0
initial_seasonality = list(S0)

# Print initial parameters for verification
print(f"Initial Level (L0): {L0:.2f}")
print(f"Initial Trend (T0): {T0:.2f}")
print(f"Initial Seasonality: {initial_seasonality}")

# Fit the Holt-Winters Exponential Smoothing model with multiplicative trend and seasonality
model = ExponentialSmoothing(
    df["Commercial_Industrial"],
    trend="mul",
    seasonal="mul",
    seasonal_periods=m
).fit()

# Forecast future values for the next 240 months
forecast_horizon = 240
predictions = model.forecast(forecast_horizon)

# Create future dates for the forecast
future_dates = pd.date_range(df.index[-1], periods=forecast_horizon + 1, freq="M")[1:]

# Convert predictions to a DataFrame for easier handling
forecast_df = pd.DataFrame({"Date": future_dates, "Forecast": predictions})
forecast_df.set_index("Date", inplace=True)

# Plot the actual data, fitted model, and forecasted values
plt.figure(figsize=(12, 6))
plt.plot(df.index, df["Commercial_Industrial"], label="Actual Data", color="blue")
plt.plot(df.index, model.fittedvalues, label="Fitted Model", color="green")
plt.plot(forecast_df.index, forecast_df["Forecast"], label="Forecast", color="red", linestyle="dashed")
plt.legend()
plt.title("Holt-Winters 20 Year Forecasting of Industrial and Commercial Power Consumption")
plt.xlabel("Date")
plt.ylabel("Power Consumption/(Thousands) kWh")
plt.show()

# Save the forecast results to an Excel file
output_file = "Q2/Spreadsheets/forecast_results.xlsx"
forecast_df.to_excel(output_file)
print(f"Forecast results saved to {output_file}")

# Calculate residuals (difference between actual data and fitted values)
residuals = df["Commercial_Industrial"] - model.fittedvalues

# Plot the residuals
plt.figure(figsize=(12, 6))
sns.lineplot(x=df.index, y=residuals)
plt.axhline(y=0, color='red', linestyle='--', linewidth=2)
plt.title("Residuals of Holt-Winters Model")
plt.xlabel("Date")
plt.ylabel("Residuals")
plt.show()

```

```
# Plot the histogram of residuals to visualize their distribution
plt.figure(figsize=(12, 6))
sns.histplot(residuals, bins=30, kde=True)
plt.title("Histogram of Residuals")
plt.xlabel("Residuals")
plt.ylabel("Frequency")
plt.show()

# Create a copy of the original DataFrame to add noise
df_noisy = df.copy()

# Define a noise factor (e.g., 1% of each data point)
noise_factor = 0.01

# Generate Gaussian noise scaled to each data point
gaussian_noise = np.random.normal(0, noise_factor, len(df)) * df["Commercial_Industrial"]

# Add noise proportionally to the data
df_noisy["Commercial_Industrial"] = df["Commercial_Industrial"] + gaussian_noise

# Fit the Holt-Winters model to the noisy data
model = ExponentialSmoothing(
    df_noisy["Commercial_Industrial"],
    trend="mul",
    seasonal="mul",
    seasonal_periods=m
).fit()

# Forecast future values for the noisy data
new_predictions = model.forecast(forecast_horizon)
new_forecast_df = pd.DataFrame({"Date": future_dates, "Forecast": new_predictions})
new_forecast_df.set_index("Date", inplace=True)

# Calculate the variation between the original forecast and the noisy forecast
variation = 100 * (new_forecast_df["Forecast"].sub(forecast_df["Forecast"])).div(
    forecast_df["Forecast"])

# Plot the original and noisy forecasts
plt.figure(figsize=(12, 6))
plt.plot(forecast_df.index, new_forecast_df["Forecast"], label="Gaussian", color="red",
    linestyle="dashed")
plt.plot(forecast_df.index, forecast_df["Forecast"], label="No Gaussian", color="blue")
plt.legend()
plt.title("Applying 1% Gaussian Noise to the Data")
plt.xlabel("Date")
plt.ylabel("Power Consumption/(Thousands) kWh")
plt.show()

# Plot the variation due to noise
plt.figure(figsize=(12, 6))
plt.plot(forecast_df.index, variation, label="Variation", color="red", linestyle="dashed"
    )
plt.legend()
plt.title("Jitter Testing")
plt.xlabel("Date")
plt.ylabel("Difference (%)")
plt.show()
```

```
# Compute R  value between original forecast and noisy forecast
r2 = r2_score(forecast_df["Forecast"], new_forecast_df["Forecast"])
print(f"R  Value: {r2:.4f}")
```

6.4 Stochastic Circles Algorithm

```
import random
from shapely.geometry import Point
import shapely.ops
import numpy as np


def random_point_in_polygon(poly, attempts=1000):
    """Return a random point inside the given polygon (if possible)."""
    minx, miny, maxx, maxy = poly.bounds
    for _ in range(attempts):
        p = Point(random.uniform(minx, maxx), random.uniform(miny, maxy))
        if poly.contains(p):
            return p
    return None

def get_candidate_points(towns, circles, R):
    candidate_points = []
    n = len(circles)

    for i in range(n):
        for j in range(i+1, n):
            inter = circles[i].intersection(circles[j])
            if not inter.is_empty:

                if inter.geom_type == 'Point':
                    candidate_points.append(inter)
                elif inter.geom_type == 'MultiPoint':
                    candidate_points.extend(list(inter.geoms))

                elif inter.geom_type in ['LineString', 'LinearRing']:
                    candidate_points.append(inter.interpolate(0.5, normalized=True))

                elif inter.geom_type == 'Polygon':
                    candidate_points.append(inter.representative_point())
                elif inter.geom_type == 'MultiPolygon':
                    for poly in inter.geoms:
                        candidate_points.append(poly.representative_point())

    unique_candidates = []
    tolerance = R * 1e-3
    for pt in candidate_points:
        if not any(existing.distance(pt) < tolerance for existing in unique_candidates):
            unique_candidates.append(pt)
    candidate_points = unique_candidates

    for i, circle in enumerate(circles):

        other_circles = [circles[j] for j in range(n) if j != i]
        union_others = shapely.ops.unary_union(other_circles)

        non_overlap = circle.difference(union_others)
        if not non_overlap.is_empty:
```

```
            if non_overlap.geom_type == 'MultiPolygon':
                poly = random.choice(list(non_overlap.geoms))
            elif non_overlap.geom_type == 'Polygon':
                poly = non_overlap
            else:
                poly = non_overlap
            rp = random_point_in_polygon(poly)
            if rp:
                candidate_points.append(rp)

    return candidate_points

def count_overlapping(candidate, circles):
    """Count how many circles (town regions) contain the candidate point."""
    count = 0
    for circle in circles:
        if circle.contains(candidate):
            count += 1
    return count

def evaluate_candidate(candidate, towns, circles):
    """
    For each town, if the candidate lies within its circle,
    reduce its score by 70% (multiply by 0.3) and sum the scores.
    """
    total = 0.0
    for i, town in enumerate(towns):
        if circles[i].contains(candidate):
            total += 0.3 * town['score']
        else:
            total += town['score']
    return total

def main():
    random.seed(0)
    Total_initial_score = 2633957984


    towns= [


    {'A': 'Downtown / South Main Arts District / South Bluffs', 'x': 35.15684, 'y': -90.06845, 'score': 338300.0199},
    {'A': 'East Midtown / Central Gardens / Cooper Young', 'x': 35.13562, 'y': -90.00188, 'score': 630927.2954},
    {'A': 'Uptown / Pinch District', 'x': 35.15225, 'y': -90.03465, 'score': 143015.6369},
    {'A': 'South Memphis', 'x': 35.09976, 'y': -90.04015, 'score': 620084.4295},
    {'A': 'North Memphis / Snowden / New Chicago', 'x': 35.17251, 'y': -90.01727, 'score': 374950.0392},
    {'A': 'Hollywood / Hyde Park / Nutbush', 'x': 35.17534, 'y': -89.9602, 'score': 485042.7301},
    {'A': 'Coro Lake / White Haven', 'x': 35.05383, 'y': -90.17276, 'score': 849350.3351},
    {'A': 'East Memphis     Colonial Yorkshire', 'x': 35.11012, 'y': -89.94812, 'score': 887046.5678},
    {'A': 'Midtown / Evergreen / Overton Square', 'x': 35.14497, 'y': -89.98153, 'score':
```

```
        387550.7571},
    {'A': 'East Memphis', 'x': 35.11505, 'y': -89.90701, 'score': 707942.3625},
    {'A': 'South Forum / Washington Heights', 'x': 35.1274, 'y': -90.04469, 'score':
        169071.56}



]

    #SENSITIVITY ANALYSIS
    # mean = 0
    # std_dev = 5
    # noise = np.random.normal(mean, std_dev, size=11)
    # counter = 0
    # for town in towns:
    #     town['score'] = town['score'] +town['score']*noise[counter]/100
    #     counter+=1



    R = 0.0179662235
    #iterations value corresponds to the number of emergency stations we place - it has
        to be determined by the government and inputted in the system
    iterations = 15
    for iter_num in range(iterations):
        print(f"\n--- Iteration {iter_num+1} ---")

        circles = [Point(t['x'], t['y']).buffer(R) for t in towns]

        candidate_points = get_candidate_points(towns, circles, R)

        candidate_data = []
        for pt in candidate_points:
            C = count_overlapping(pt, circles)
            total_score = evaluate_candidate(pt, towns, circles)
            candidate_data.append((pt, C, total_score))

        candidate_data.sort(key=lambda x: x[1], reverse=True)

        best_candidate = min(candidate_data, key=lambda x: x[2])

        print("Candidate Points (ordered by descending overlap count):")
        for cand in candidate_data:
            print(f"Point: ({cand[0].x:.5f}, {cand[0].y:.5f}), Overlap Count: {cand[1]},
                Total Score: {cand[2]:.5f}")

        print("\nBest candidate point for emergency centre placement:")
        print(f"Point: ({best_candidate[0].x:.5f}, {best_candidate[0].y:.5f}), Overlap
            Count: {best_candidate[1]}, Total Score: {best_candidate[2]:.5f}")

        for i, town in enumerate(towns):
            if circles[i].contains(best_candidate[0]):
                original_score = town['score']
                town['score'] *= 0.3

        total_current_score = sum(t['score'] for t in towns)
        print(f"Total score after iteration {iter_num+1}: {total_current_score:.5f}")
        if iter_num==iterations-1:
```

```
            print(f"Final score: {total_current_score:.5f}")


    print("\nFinal percentage:")
    finalvalue = sum(t['score'] for t in towns)
    print(100*finalvalue/Total_initial_score)

if __name__ == '__main__':
    main()
```

6.5 Cooling Simulation

```
import pandas as pd

t_out = pd.read_csv("test.csv")["temp"].to_numpy() +273

def get_k(v,s):
    a = 8.1 * 10**(-5)
    b = -1.2 * 10**(-6)

    k = a/v + b*s
    return k

# !
neighborhood = "Uptown / Pinch District"
area = 90.79519795
k = get_k(area**5.8,math.sqrt(area)*4)

hours = np.arange(1, len(t_out))  # Time in hour

rho = 1.225  # Density of air (kg/m )
c = 1005  # Specific heat capacity of air (J/kg K)
H = 100  # Heat generated by the person (W)
Q_cool_max = 70  # Maximum cooling rate (W)
P_cool = 2400  # Power consumption of cooling system (W)
C_elec = 0.12  # Cost of electricity ($/kWh)

def get(k,v,p):
    t_in = t_out[0] - 5
    ts = []
    t_on = 0
    cooling_input = 0
    for i in range(1,len(t_out)):
        time = 3600*i
        print(cooling_input)
        t = t_out[i] + (t_in - t_out[i])*math.e**(k*time)+ (100 * p) / (1.225 * v * 1005)*3600 - cooling_input
        if t > 27+273:
            cooling_input = (Q_cool_max * time) / (rho * v * c)
            t_on += 3600
        else:
            cooling_input = 0

        # print((100 * p) / (1.225 * v * 1005) * -10000)
        # print(t)
        # print((100 * p) / (1.225 * v * 1005))
        ts.append(t - 273)
    cost = (P_cool * (t_on / 3600)) / 1000 * C_elec  # Convert time to hours and power to
```

```
        kW
    print(f"Cost of cooling: ${cost:.2f}")
    return ts

temp_values = get(k, area * 5.8, 4)

plt.figure(figsize=(8, 5))
plt.plot(hours, temp_values, label="Indoor Temperature ( C )")
plt.plot(hours, t_out[1:] - 273, label="Outdoor Temperature ( C )", linestyle="dashed")

plt.xlabel("Time (hours)")
plt.ylabel("Temperature ( C )")
plt.title("Temperature Variation Over Time")
plt.legend()
plt.title("Temperature Changes for Average House in {}".format(neighborhood))
plt.grid(True)

# Show the plot
plt.show()
```

6.6 Determination of K

```
import pandas as pd
import numpy as np

ZERO = 273

df = pd.read_csv("housing/Weather_hourly.csv")

df["Date/Time"] = pd.to_datetime(df["Date/Time"], format = "mixed")

# Define the start and end date
start_date = pd.to_datetime("2021-05-01")
end_date = pd.to_datetime("2021-09-12")

filtered_df = df[(df["Date/Time"] >= start_date) & (df["Date/Time"] <= end_date)]

# Extract the temperature and hours
temp = filtered_df["Dry Bulb (degC)"]+ZERO  # Adding ZERO if needed
hours = filtered_df["Date/Time"].dt.strftime("%H:%M")  # Getting hours in HH:MM format

out_temp = filtered_df[['Date/Time', 'Dry Bulb (degC)']]
out_temp

room = "U07_dining"

df = pd.read_csv("housing/West_AT_hourly.csv")

# Convert the "Date/Time" column to datetime format if it's not already
df["date"] = pd.to_datetime(df["date"]).dt.tz_localize(None)

start_date = pd.to_datetime("2021-05-01")
end_date = pd.to_datetime("2021-09-12")

filtered_df = df[(df["date"] >= start_date) & (df["date"] <= end_date)]

in_temp = filtered_df[['date', room]]
in_temp
```

```python

import math
t_in = in_temp[room].to_numpy()+ZERO
t_out = out_temp["Dry Bulb (degC)"].to_numpy()+ZERO


t_in_0 = t_in[0]

ks = []
for i in range(1,len(t_in)):
    t = i*3600
    try:
        k = (math.log((t_in[i]-t_out[i])/(t_in-t_out[i])))
    except:
        pass
    ks.append(k)
```

6.7 Determining Substations and Substation Radius

```python
import folium
import pandas as pd
import requests
from geopy.distance import geodesic

# Load neighborhood data from CSV
neighborhoods_df = pd.read_csv("neighborhoods.csv")  # Ensure the CSV has 'Lat', 'Lon',
    and 'Neighborhood' columns

# Define the bounding box for Memphis, Tennessee (south, west, north, east)
bbox = (35.0, -90.2, 35.2, -89.9)  # (south, west, north, east)

# Overpass API endpoint
overpass_url = "http://overpass-api.de/api/interpreter"

# Create a map centered on Memphis
map_center = [35.1495, -90.0490]  # Approximate center of Memphis
combined_map = folium.Map(location=map_center, zoom_start=12)

# Initialize a list to store substation locations
substations = []

# Overpass query to fetch power substations
overpass_query = f"""
[out:json][timeout:25];
(
  node["power"="substation"]({bbox[0]},{bbox[1]},{bbox[2]},{bbox[3]});
  way["power"="substation"]({bbox[0]},{bbox[1]},{bbox[2]},{bbox[3]});
  relation["power"="substation"]({bbox[0]},{bbox[1]},{bbox[2]},{bbox[3]});
);
out body;
>;
out skel qt;
"""

# Send the request to the Overpass API
response = requests.get(overpass_url, params={'data': overpass_query})

# Check if the request was successful
```

```python
if response.status_code == 200:
    data = response.json()  # Parse the JSON response

    # Process and plot the substations
    for element in data['elements']:
        if element['type'] == 'node':
            lat, lon = element['lat'], element['lon']
            # Check if the substation is within the bounding box
            if bbox[0] <= lat <= bbox[2] and bbox[1] <= lon <= bbox[3]:
                substations.append((lat, lon))  # Add substation location to the list
                folium.Marker(
                    location=[lat, lon],
                    popup=f"Substation Node {element['id']}",
                    icon=folium.Icon(color="red")  # Use red for substations
                ).add_to(combined_map)
        elif element['type'] == 'way':
            if 'center' in element:
                lat, lon = element['center']['lat'], element['center']['lon']
                # Check if the substation is within the bounding box
                if bbox[0] <= lat <= bbox[2] and bbox[1] <= lon <= bbox[3]:
                    substations.append((lat, lon))  # Add substation location to the list
                    folium.Marker(
                        location=[lat, lon],
                        popup=f"Substation Way {element['id']}",
                        icon=folium.Icon(color="green")  # Use green for ways
                    ).add_to(combined_map)
else:
    print(f"Error: Unable to fetch substation data. Status code: {response.status_code}")

# Calculate the number of substations within 2000 meters for each neighborhood
neighborhoods_df['substation_count'] = 0  # Initialize count column

for index, row in neighborhoods_df.iterrows():
    neighborhood_location = (row['Lat'], row['Lon'])
    for substation_location in substations:
        distance = geodesic(neighborhood_location, substation_location).meters
        if distance <= 2000:
            neighborhoods_df.at[index, 'substation_count'] += 1

# Print the results
print(neighborhoods_df[['Neighborhood', 'Lat', 'Lon', 'substation_count']])

# Add a circle around each neighborhood with the substation count
for index, row in neighborhoods_df.iterrows():
    folium.Circle(
        location=[row['Lat'], row['Lon']],  # Center of the circle
        radius=2000,  # Radius in meters
        color="blue",  # Border color of the circle
        fill=True,  # Fill the circle
        fill_color="blue",  # Fill color
        fill_opacity=0.2,  # Transparency of the fill
        popup=f"Neighborhood: {row['Neighborhood']}<br>Substations within 2000m: {row['substation_count']}",  # Popup text
    ).add_to(combined_map)

# Save the map to an HTML file
combined_map.save("neighborhoods_and_substations_with_count.html")
print("Map saved to 'neighborhoods_and_substations_with_count.html'. Open this file in
```

```
    your browser to view the map.")
```