MathWorks Math Modeling Challenge 2018

High Technology High School–

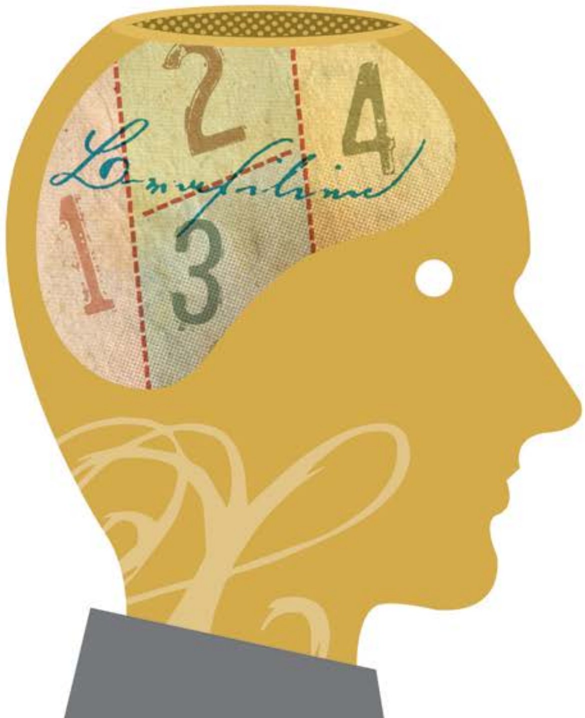
Team # 10677 Lincroft, New Jersey

Coach: Raymond Eng

Students: Eric Chai, Kyle Lui, Steven Liu, Adithya Paramasivam, Yihan Wu

MathWorks Math Modeling Challenge Finalist

$5,000 Team Prize



MathWorks Math Modeling Challenge

***Note: This cover sheet has been added by SIAM to identify the winning team after judging was completed. Any identifying information other than team # on a MathWorks Math Modeling Challenge submission is a rules violation.

***Note: This paper underwent a light edit by SIAM staff prior to posting.

Better ATE than never: Reducing wasted food

Executive Summary

One of the basic necessities in life, food, is often taken for granted by the majority of the population. Globally, approximately one third of the food produced annually in the world is lost or wasted. [1] The United States leads in food waste, with as much as 40% of the food supply gone to waste. [2]

Food losses occur at each step of the supply chain: production losses, postharvest losses, processing and packaging losses, distribution and retail losses, and consumer losses. At the consumer and retail levels, food waste might be due to aesthetic reasons, which are not actual indicators of the quality and taste of food. Although not all of the food waste can be avoided, a large proportion, especially on the part of the consumers, can be repurposed to be allocated to other populations in need.

The high amounts of wasted resources are particularly excessive when put in context with the food-insecure populations in the United States. It is estimated that 1 in 8 Americans are considered food insecure, defined as the lack of access and financial resources, at times, to enough food for an active, healthy life for all household members. [3] However, at the same time, there are many families with a great excess of food being wasted. We developed a model to determine whether the waste produced by a state, if fully repurposed, would be sufficient to cater to the needs of the food-insecure population of the state. Our model shows that the state of Texas would be able to entirely provide for the state's food-insecure population if it were to recover its wasted food.

Of course, not all families and people have identical eating habits, nor do they waste the same amount of food. We designed a model to estimate a family's annual food waste based on its members and its income, taking into consideration the amount they are expected to spend on food, the ages of the family members, and how often they eat at places out of the house. We found that, on average, we would expect a family of a single mother and toddler with an income of $20,500 to waste $739.24 annually, of two parents and two teenage children with an income of $135,000 to waste $2182.48 annually, of an elderly couple with an income of $55,000 to waste $929.00 annually, and of a single 23-year-old male with an income of $45,000 to waste $952.75 annually.

Given this information about food waste, it is prudent for local governments and other groups to ascertain the best action to take regarding this issue. A number of options for repurposing or recycling food waste exist, but they have varying efficacy for different locations and different populations. In addition, the amount that a local group can recover may fall well short of the total amount of waste that can theoretically be reclaimed. In order to determine the best strategies for reusing wasted food, we designed a model to determine the potential of profitable donations, composting, processing into biofuel, and converting into animal feed for Monmouth County, New Jersey. We found that, for a variety of reclaimed quantities of food waste, that animal feed and compost are largely unprofitable, while donations to the needy and conversions to biofuels are generally profitable.

With the ever-increasing population, more and more strains are placed on the availability of food, allowing for less and less squandered waste by the population. Whether it is with prevention methods or repurposing techniques, reducing wasted food is an important focus for sustainable developments of the future.

Contents

1 Introduction

This section outlines background information concerning the amount of food waste and the possibility of repurposing wasted food. Various components of the modeling problem are also delineated in detail. Global assumptions made during the modeling process are also summarized.

1.1 Restatement of the Problem

Our model addresses the following questions:

1. Determine whether a state's wasted food could be used to feed its food-insecure population. Apply the model to Texas.

2. Calculate the amount of annual food waste a household generates from home and outside of home based on their traits and habits.

3. Analyze and propose a repurposing strategy for a community to recycle the maximal amount of food at the minimum cost. Quantify the costs and benefits of each strategy.

1.2 Global Assumptions

- *Inflation.* As all costs will be calculated relative to each other, inflation will have the same effect on all costs, so it is treated as negligible. This study does not consider changes over large periods of time.

- *Food loss proportion independence.* The proportion of food that is lost at each step of the supply chain is independent of the proportion of food lost at each other step. Thus, they may be combined like independent probabilities.

- *Food loss vs. food waste.* Food loss and food waste sometimes have a distinction that food loss cannot be recovered while food waste is recoverable. This study only looks at food waste that can be easily recovered. By enforcing regulations, it is possible to decrease food loss as well, but its efforts are too great to be considered in this study.

2 Part I: Just eat it!

Food waste exists at every step of the supply chain from production to consumers. The amount of food wasted in each step can be important to feeding the food-insecure population within the state itself. A mathematical model was created to calculate whether each state has enough squandered food resources to meet the demands of its food-insecure population.

2.1 Assumptions

1. *Representative percentages.* Trends of food wasting in Texas mirror the national trends. No specific data is available for each state.

2. *Linear trend of per-person cost.* The per-person cost is the amount of money to feed the food-insecure population. Data from the past few years show a strong linear trend.

2.2 Model Development

The team split up the model into two components of Total Waste (TW) and Total Needed (TN), which are the monetary equivalents of total wasted food and total food needed to feed the food-insecure population, respectively.

2.2.1 Total Waste

First, to calculate Total Waste, the expenditure on food by the particular state per year is calculated as a proportion of total US expenditure unless the expenditure on food is available in itself.

$$Expend_{state} = Expend_{US} \times \frac{Population_{state}}{Population_{US}} \tag{1}$$

Then, food waste by retail and consumer are considered because these are possible to be repurposed. Although food waste is also prevalent during preceding steps in the supply chain, such as in production and processing, some of these wastes are inevitable from the conditions of the machines and are therefore difficult to repurpose. Since the percent loss of each food group is not the same across the food groups, each category was calculated independently and then summed for retail loss and consumer loss, which are percentages lost of the total expenditure. The food groups were grains, fish/seafood, fruits, vegetables, dairy, eggs, meat/poultry, and nuts. The percent of wastes should be for the particular state.

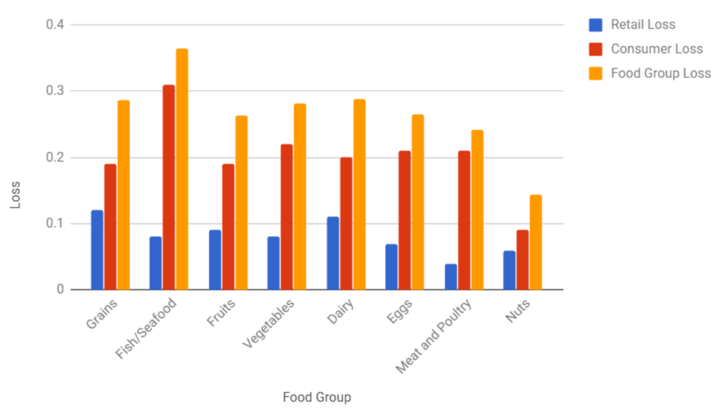


Figure 2.2.1 Loss By Food Group

$$Retail\_Loss = \sum_{fg} (\%\ retail\ loss)(\%\ of\ expenditure\ in\ food\ group) \tag{2}$$

$$Consumer\_Loss = \sum_{fg} (\%\ consumer\ loss)(\%\ of\ expenditure\ in\ food\ group) \tag{3}$$

where fg is food groups. Then, the total loss, or Recoverable_Loss, is defined to be

$$Recoverable\_Loss = 1 - (1 - Retail\_Loss)(1 - Consumer\_Loss) \tag{4}$$

$$Recoverable\_Loss = Retail\_Loss + Consumer\_Loss - Retail\_Lost \times Consumer\_Loss \tag{5}$$

which is finding total loss because retail loss and consumer loss are not disjoint probabilities.

Since the amount of expenditure is based on the amount of food available to consumers, the amount actual consumed is $Consumption_{state}$.

$$Consumption_{state} = Expend_{state} \times (1 - Consumer\_Loss) \tag{6}$$

Production is the total monetary equivalence of food available for retail.

$$Production \times (1 - Recoverable\_Loss) = Consumption \tag{7}$$

$$TW = production - consumption \tag{8}$$

$$TW = production \times Recoverable\_Loss \tag{9}$$

2.2.2 Total Needed

To calculate Total Needed, the methodology references Feeding America [4], which includes Food Budget Shortfall (FBS).

$$FBS = FI \times PPC \times 52\ weeks \times \frac{7\ months}{12\ months} \tag{10}$$

where FI is the number of people who are food-insecure and PPC is the Per Person Cost per week. FI and PPC should be specific to each state when possible.

The factor of 7 months/12 months is important because, as reported by the US Department of Agriculture, "the average household that was food insecure at some time during the year experienced this condition in 7 months of the year." [5]

2.3 Results

The above model was applied for Texas. By Assumption 1, data on percent of food lost for the whole United States from the Natural Resources Defense Council were used. [6]

PPC specific to Texas is also not available. By Assumption 1, a value for PPC was obtained following the methodology by Feeding America. Feeding America used $17.38 for

PPC, which was the value in 2015. To update this value to more current conditions, data from the latest Current Population Survey were used. Since the latest Current Population Survey was published for 2016, data were extrapolated from values from the past few years to PPC in 2017 with Assumption 2 in mind. With an R-squared value of 0.999, this extrapolation is justified because it is only 1 year in advance. It is necessary since the increase in PPC has outpaced inflation for several years. The value for 2017 was calculated to be \$18.45 using the formula of

$$P\hat{P}C = 0.55 \times year - 1090.9 \tag{11}$$

The results of applying the model on the state of Texas are summarized in the table below:

Table 2.3.1 Total Food Wasted and Needed by Food Insecure in Texas

Total Wasted	\$33,512,000,000
Amount Needed by Food Insecure	\$2,417,700,000
Excess Wasted	\$31,095,000,000

The dollar value for the total amount of food wasted in Texas makes sense because \$33,512,000,000 is approximately 1/3 of the total expenditure on food in Texas which is \$112,900,000,000. This total expenditure on food in Texas was calculated using the ratio of the population of Texas (29,366,479) [6] to the population of the United States (327,296,534) [7] multiplied by the total US expenditure as given by the M3 Food Expenditure data (\$1,258,300,000,000). [8] It was previously researched that the world wastes approximately 1/3 of all food produced which is consistent with the data. [9]

The amount of food needed by the food insecure is also consistent with the value calculated by Feeding America, which is \$2,007,143,000. [10] The increase in the amount of food needed by the food insecure accounts for inflation and increase in monetary value.

By comparing the total wasted and the total needed, it is evident that Texas is able to comfortably supply its wasted food to the food-insecure population. In fact, the amount needed is only about 7.2% of the total amount of wasted food. This might seem surprising, but it only points to the excessive amounts of food that is wasted.

2.4 Sensitivity Analysis

Table 2.4.1 shows the sensitivity analysis based on a 10% increase and decrease in retail loss and consumer loss.

Table 2.4.1 Sensitivity Analysis for Total Food Wasted

Independent Variable	Percent Change in Independent Variable	Percent Change in Total Wasted
Retail Loss	+20%	+6.87%
Retail Loss	+10%	+3.40%
Retail Loss	-10%	-3.34%
Retail Loss	-20%	-6.61%
Consumer Loss	+20%	+13.86%
Consumer Loss	+10%	+6.93%
Consumer Loss	-10%	-6.93%
Consumer Loss	-20%	-13.86%

The sensitivity analysis is consistent with what was expected. Because retail loss generally contributes to less wasted food as compared to consumer loss, it is logical that a change to retail loss would lead to a lesser change to the total wasted food as compared to a similar change to consumer loss.

2.5 Strengths and Weaknesses

This model is strong and outputs sensible results. The results are easily confirmed with outside research. The sensitivity analyses are favorable to changes in the parameters as well. Changes to retail loss and consumer loss are consistent with what is expected. Furthermore, various food groups such as fruits, vegetables, and grains were investigated rather than tackling food as a whole. Thus, a more comprehensive model was created that accounts for varying monetary values for each food group. In addition, the model extrapolates an increase in PPC or the amount of money that food-insecure people need to become secure. This accounts for the increase in monetary value over time, and because the r-squared value is 0.9999, the linear extrapolation is appropriate.

The model is weak because it does not account for food waste that occurs before retail. This was largely due to the fact that wastes at this point is a result of accidents and other unpredictable events that could not be sensibly modeled. Furthermore, food wastes during harvesting and manufacturing would be difficult to reclaim and repurpose towards the food insecure.

Another weakness of the model is not accounting for obesity and overeating. However, this can be justifiably overlooked because based on the model, the state of Texas wastes and excessive amount of food that more than provides for the food insecure. In fact, because the world wastes a large amount of food on a global scale, this statement holds true for most areas that the model can be applied to.

3 Part II: Food foolish?

When analyzing individual food wastes, many factors, such as traits and habits, play a role in personal choices and lifestyles. Food wastes can also occur at many settings, such as at home and outside home. A mathematical model was created to determine the amount of food wasted by different households, and then the model was applied to four test cases.

3.1 Assumptions

1. *Grocery stores.* Grocery store wastes are assumed to be at-home wastes. It is assumed that the food purchased at grocery stores will be brought home, and if any is uneaten, it would be included in wastes at home.

2. *Food waste by institutions.* The excess amount of food wasted at institutions is the responsibility of the institutions themselves and not on individuals. Therefore, only the food wasted by individuals while eating at these institutions are included.

3. *Food plans.* All members of the same household would be in the same US Department of Agriculture food plans. [11] This is logical as the quality and quantity of the food for each family member would be similar.

4. *Genders of test cases.* The genders of the individuals in the test cases would be important for considering their food consumption and waste. The single parent with toddler is assumed to be a single mother, since single mothers with a child are more prevalent. The family of four is assumed to be a father, a mother, a teenage son, and a teenage daughter. The single 23-year-old is assumed to be a male. These assumptions can be easily changed to reflect different genders.

5. *Percent of waste.* The percent of waste is assumed to be uniformly distributed across a certain age interval. For example, the individuals in the same age group would waste the same proportion at an restaurant regardless of their wealth. All of the test cases are within the same wealth statuses.

6. *Food waste patterns in a household.* It is assumed that children will waste the same percent of food as their parents. This is logical as children might be raised with the same lifestyle habits. Food waste ratios are also not available for the generation of the children, who are Generation Z.

3.2 Model Development

The two areas of food waste for an individual is at home and outside of home, which includes school cafeterias and restaurants. As per Assumption 1, grocery store wastes are included in the at home wastes.

3.2.1 At-Home Wastes

To calculate the at-home wastes, individuals first need to be categorized into 1 of 4 Official United States Department of Agriculture food plans: thrifty plan, low-cost plan, moderate-cost plan, and liberal plan. Households are split up by their annual individual incomes. In addition to annual incomes, the number of people in the household affects the amount of money allocated to food. However, this number of people needs to be an adjusted person based on calorie and nutrient needs. To find the adjusted person factor, the cost of food is summed over each plan for an age interval and then scaled. It was also noted that 57% of the workforce is male and 43% is female.

The income per adjusted person (IAP) is

$$IAP = \frac{Household\ Income}{Adjusted\ Person} \tag{12}$$

Using individual income brackets from the American Community Survey, the four food plans are divided for each quartile. [12] For example, the first quartile of income would be on the thrifty plan, the second quartile on the low-cost plan, the third quartile on the moderate-cost plan, and the last quartile on the liberal plan. The quartiles are used instead of 1 standard deviation or 2 standard deviation away from the mean because the income data for the US is not normally distributed and is heavily skewed right.

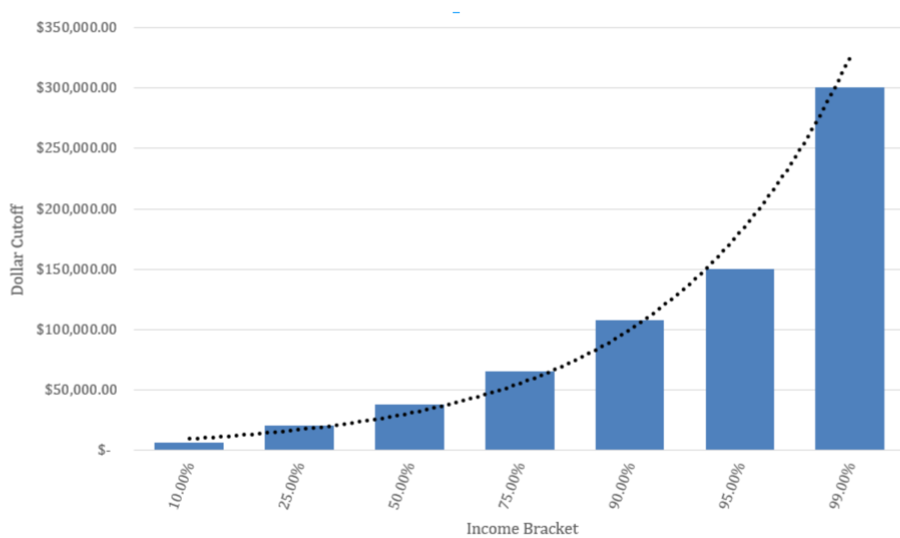


Figure 3.2.1 Individual income bracket distribution of the US [12]

After determining which food plan the household falls into, the cost of food per week can then be calculated for the entire household. The food plan costs are based on a 4-person family, so if the given household does not have 4 people, a weighting factor is required as per the US Department of Agriculture. "For individuals in other size families, the following adjustments are suggested: 1-person—add 20 percent; 2-person—add 10 percent; 3-person—add 5 percent; 4-person—no adjustment; 5- or 6-person—subtract 5 percent; 7- (or more) person—subtract 10 percent. " [11] The total household food cost is then

$$Food\_Cost = \sum_{hm}(Cost\ of\ Good \times Weighting\ Factor) \tag{13}$$

where hm is household member.
With data from the US Department of Agriculture, the percent of food waste at home, which is a combination of on groceries and on meals, is extracted. The total loss is calculated as before as

$$Total\_Loss = 1 - (1 - Grocery\_Loss)(1 - Meal\_Loss) \tag{14}$$

$$Total\_Loss = Grocery\_Loss + Meal\_Loss - Grocery\_Loss \times Meal\_Loss \tag{15}$$

Table 3.2.1 Percent Grocery and Meal Waste Based On Generation

Age Interval	Age Group	Grocery Loss(%)	Meal Loss (%)	Total Loss (%)
18 - 36	Millennial	7.4	5.6	12
36-50	Generation X	9.5	7.1	15
51+	Baby Boomer	10	6.7	16

The total food waste at home is at last calculated to be

$$FW_{home} = Total\ Loss \times Food\ Cost \tag{16}$$

3.2.2 Outside-Home Wastes

Outside-home wastes are split into two major categories: food wasted in restaurants and food wasted in schools. Before wastes can be calculated, it is necessary to first determine the amount of food that is consumed at each site by each member of the family.

For the purposes of this model, eating at restaurants is defined as eating at both restaurants and fast food venues. Using the M3 Food Table provided from the National Health and Nutrition Examination Survey (NHANES), the amount of food intake at home versus the amount of food intake at are used to calculate a ratio by which the Weekly_Cost_At-Home for each family member can be multiplied to achieve the Weekly_Cost_Restaurant. This must be done for adults and children separately because of the varying amounts of food consumption at each location.

Similarly, the data for the amount of food intake at home versus the amount of food intake at school were used to calculate a ratio to determine the School_Weekly_Cost. Because adults are assumed to not have to eat at schools, that ratio does not need to be calculated.

With the ratios calculated, the weekly cost of food at restaurants and schools can be calculated from the weekly cost of food at home.

$$Weekly\_Cost\_Restaurants_{Children} = Weekly\_Cost\_At\_Home \times \frac{\sum Food\_Restaurants_{Children}}{\sum Food\_At\_Home_{Children}} \tag{17}$$

$$Weekly\_Cost\_Restaurants_{Adults} = Weekly\_Cost\_At\_Home \times \frac{\sum Food\_Restaurants_{Adults}}{\sum Food\_At\_Home_{Adults}} \tag{18}$$

$$Weekly\_Cost\_Schools = Weekly\_Cost\_At\_Home \times \frac{\sum Food\_Schools}{\sum Food\_At\_Home} \tag{19}$$

All the weekly cost at restaurants for each family member are then summed into one final weekly cost at restaurants. This weekly cost at restaurants is multiplied by a constant 10%, the average amount of food wasted at restaurants. [13] Likewise, the weekly cost at schools for all teenagers (those in middle school and high school) and all children (those in elementary school, kindergarten, and pre-kindergarten) are then summed. Teenagers and children are required to remain in separate sums because children are estimated to waste 45% of food in school cafeterias [14] while teenagers are estimated to waste 26.1% of food in school cafeterias. [13]

$$Weekly\_Wasted\_Restaurants = Weekly\_Cost\_Restaurants \times 10\% \tag{20}$$

$$Weekly\_Wasted\_School_{Teenagers} = Weekly\_Cost\_School_{Teenagers} \times 45\% \tag{21}$$

$$Weekly\_Wasted\_School_{Children} = Weekly\_Cost\_School_{Children} \times 26.1\% \tag{22}$$

Finally, the weekly amount wasted at restaurants and schools for each member of the family is added together to achieve a weekly amount wasted outside of home. This amount is summed with the weekly amount wasted at home and then multiplied by 52 to determine the annual amount of food wasted by the household in dollars.

$$Annual\_Wasted = 52 \times \sum Weekly\ Waste \tag{23}$$

3.3 Results

In calculating the adjusted persons for each household, it was found that the single parent with a toddler is 1.49 adjusted persons, the family of four is 3.91 adjusted persons, the elderly couple is 1.86 adjusted persons, and the single male is 1.06 adjusted persons.

With this adjusted persons scaling, the single parent with a toddler and household income of $20,500 was set to follow the thrifty food plan. The family was then determined to waste $739.24 annually. The family of four with a household income of $135,000 followed a low-income food plan and wasted $2182.48 annually. The elderly couple with a household income of $55,000 followed the low-income food plan and wasted $929.00 annually. The single 23-year old man with an income of $45,000 followed the moderate-income food plan and wasted $952.75 annually. The results are summarized in the data table below:

Table 3.3.1 Family Household Incomes, Food Plan, and Amount of Food Wasted Annually

Family	Household Incomes	Food Plan	Amount Wasted Annually
Single Parent with Toddler	$20,500	Thrifty	$739.24
Elderly Couple	$55,000	Low-Income	$929.00
Single Man	$45,000	Moderate-Income	$952.75

The results of the model are consistent with what was expected. The single parent with a toddler experiences the lowest income per adjusted person. As a result, they must follow a thrifty food plan and must remain more cautious than other families of how much food they waste. The family of four and elderly couple have roughly the same income per adjusted person, both following a low-income food plan. However, because the family of four is twice as large and includes teenagers, who generally waste more food than adults, the amount of food wasted by the family of four is much greater than the amount wasted by the elderly couple. The single man experiences the greatest income capita and is able to afford a moderate-income meal plan. Thus, he tends to waste the greatest amount of food. These wasted amounts are logical and consistent with the results of Section 2. From Section 2, dividing the total amount wasted by the population of Texas gives approximately $1142 wasted per year per individual. This is in line with the values presented in the above table.

3.4 Strengths and Weaknesses

One strength of the model is the accounting for adjusted persons in the family. This step is important because it can not be assumed that every person in the family eats an equal amount of food and spends an equal amount of money on food. Males generally eat more than females, and adults tend to eat more than children. Thus, the scaling for adjusted persons appropriately scales everybody's weekly expenditures on food to the same scale.

One weakness of the model is the lack of rigorous categorization of households into the four meal plan categories. This was mainly due to the lack of necessary data points to construct the distribution curve. Currently, the model uses quartiles to distinguish the four households. This approach is more appropriate than using standard deviations from the mean because it is known that the frequency of household incomes does not create a normal distribution.

4 Part III: Hunger Game Plan?

Communities at various levels, such as schools, towns and counties, are beginning to take advantage of opportunities in repurposing potentially wasted food. The food recovery hierarchy focuses on prevention, recovery, and recycling. Here, only recovery and recycling apply, as the goal is to repurpose already wasted food. The main strategies identified are donations to food-insecure population, composting, biofuel technology, and animal feeds. A model was developed to quantify the costs and benefits associated with these strategies and a combination of the strategies. Then, it was applied to the local county, Monmouth County of New Jersey, to provide insight on which strategies can repurpose the maximal amounts of food at the minimum cost.

4.1 Assumptions

1. *Anaerobic digestion of food waste.* Another common recycling strategy of anaerobic digestion is assumed to be included in biofuel technologies.
2. *Gasoline usage.* The consumption of gasoline is assumed to be proportional to population between New Jersey and Monmouth County.
3. *Animal feed consumption.* The vast majority of animal feed for livestock is assumed to be consumed by cattle.
4. *Biofuel production.* Biofuel produced from waste is assumed to be processed by Covanta Union, Inc., 32 miles away from Monmouth County.
5. *Food donation.* Food donations are assumed to not require a centralized location for storage. That is, people wishing to donate food are assumed to deliver it directly to those who need food.
6. *Cost linearity.* Modeling the cost of each strategy is assumed to be linear. As more food is processed in each program, more money has to be put in to maintain the program.

4.2 Model Development

For the four strategies identified, each is defined to be a function of the monetary equivalence of the food wasted for each strategy:

$$Net\_Benefit(x) = a(1 - e^{-dx}) - (bx + c) \tag{24}$$

The first term in the function is the benefits of the strategy. It is an exponential model to reflect the law of diminishing returns. The law of diminishing returns states that the output benefit will eventually plateau and even decrease past a certain point. This is reasonable because in the case of donations, after all of the food-insecure population is fed, the act of donation does not yield as much benefits. The cost function, which is the second term of the function, is linear as per Assumption 6.

The constants in the function all have physical significance. The constant a is the maximum benefit drawn out of the strategy. The constant d is how quickly the function reaches the maximum benefits. The product ad, by taking a derivative, is the amount of benefit from one dollar increase in food waste. The constant b is the cost of the strategy per dollar of food. The constant c is the startup cost and any set yearly costs for the program. The task then is to determine the value of each constant for the four strategies.

4.2.1 Donation

Donations are giving food that would be wasted to local food-insecure populations. The constant a would be the food budget to feed all of the food-insecure people in the region. This is calculated with the same formula for Total Need as in Section 2. The constant d should be the inverse of a because as explained above, the product ad should equal 1 for donations, since $1 of donated food should give that same $1 of food to families in need.

The constant b for donations is the cost, which is purely transportation of the food to appropriate locations. To get the distance of transport, the county is taken to be a circle, and the average distance between any two points inside the circle is calculated. To calculate this, the probability density, p(l) , for a circle of radius r is given by [15]

$$p(l) = \frac{4l}{\pi r^2} \arccos \frac{l}{2r} - \frac{2l^2}{\pi r^4} \sqrt{r^2 - \frac{l^2}{4}} \tag{25}$$

Then the average distance is given by

$$\int_0^{2r} lp(l)dl = \frac{128r}{45\pi} \tag{26}$$

With the average distance, the constant b, or the transport cost, is calculated as

$$b = average\ distance \times \frac{cost\ of\ transport}{dollar\ equivalence\ carried\ with\ each\ transport} \tag{27}$$

The constant c for donations is 0 since there is no start-up costs.

4.2.2 Composting

Composting is putting possibly wasted food to creating fertilizers. The constant a is the dollar amounts of fertilizers that is saved with using wasted food.

$$a = area\ of\ farmland\ in\ acres \times dollar\ of\ fertilizer\ applied\ per\ acre \tag{28}$$

To calculate d, the amount of compost that can be acquired from 1 dollar equivalence is food is found to be 0.0326 dollars of compost per dollar of food. [16] Then d is calculated to be 0.0326 divided by a.

The constant b for composting is the cost per dollar equivalence of food. Data was found from the Environmental Protection Agency, and dimensional analysis was done to convert it into dollar of cost per dollar equivalence of food. The constant c for composting is the start-up cost to set up an in-vessel compost and the yearly maintenance cost as cited by the Environmental Protection Agency. [17]

4.2.3 Biofuel

The strategy of investing in biofuel is using wasted food to regenerate energy. The constant a for biofuel is the dollar amount of oil from the wasted food.

$$a = Price\ per\ gallon \times \frac{population_{county}}{population_{NJ}} \times Oil\ Use_{NJ} \tag{29}$$

To calculate the constant d, note that the dollar amount of biofuel outputted from one dollar equivalence of food is 0.106. Then d is calculated to be 0.106 divided by a.

The constant b for biofuel is

$$b = Distance_{Biofuel\ Plant} \times \frac{Cost\ Per\ Mile}{Truck\ Capacity_{dollars}} \tag{30}$$

The constant c for biofuel is \$0, because an existing biofuel plant is used. It is reasonable that the county will not invest in its own biofuel plant as per Assumption 4.

4.2.4 Animal Feed

The maximum benefit for animal feed is replacing all animal feed with repurposed wasted food. According to assumption 3, cattle consume the vast majority of feed provided to all livestock. The average yearly cost to feeding a cow is \$300. [18] Thus, the formula for the constant a becomes:

$$a = 300 \times Number_{Cows} \tag{31}$$

The cost per dollar of food for animal food accounts for the transportation of food to farms. This was calculated in a similar fashion to the cost per dollar of food for donations.

$$b = average\ distance \times \frac{cost\ of\ transport}{dollar\ equivalence\ carried\ with\ each\ transport} \tag{32}$$

The start up cost for animal feed is setting up the distribution center. Thus c is a constant \$2,300, the start up price for the distribution center. [17]

To determine the undiminished benefit, daily caloric intake for humans and cows was used as a baseline to appropriately compare the value of cow feed versus the value of human food. This is a reasonable comparison because food does not change caloric value based on what organism consumes it. The daily caloric intake for humans was found to be 2,500 kcal, while the daily caloric intake for cows was found to be 430,000 kcal. [19] The daily cost for cow feed was found previously when calculating constant a for animal feed. The daily cost for human was three times the national average meal price of \$2.94. [10]

$$d = \frac{\frac{Daily\_Cost\_Cow}{Daily\_Calorie\_Cow} \times \frac{Daily\_Calorie\_Human}{Daily\_Cost\_Human}}{a} \tag{33}$$

4.3 Results

To apply this model to Monmouth County, NJ, first, the radius of Monmouth County as an ideal circle is calculated by equating the area of the county to a circle. The radius was found to be about 11 miles. Then, each constant was calculated and summarized in the data table below.

Table 4.3.1 Family Household Incomes, Food Plan, and Amount of Food Wasted

	Donating	Composting	Biofuels	Animal Feed
a	\$31,446,733.50	\$6,160,000.00	\$567,026.36	\$235,800.00
b	\$0.000765	\$0.054645	\$0.002227	\$0.000765
c	\$0	\$39,370	\$0	\$2,300
d	0.0000000318	0.0000000053	0.0000001864	0.0000000023

To obtain the broadest range of results, the model was performed on Monmouth County based on the percentage of waste Monmouth County would be willing to recover and repurpose. For example, if Monmouth County sought to repurpose 10% of food waste, the model demonstrates how best to invest food in various strategies. The results are summarized in the table and graph below.

Table 4.3.2 Amount of Food Ideally Invested in Each Strategy Based on Percent Waste Recovered in Monmouth County

Percent of Waste Recovered	Donating	Composting	Biofuels	Animal Feed
10%	\$71,656,000	\$0	\$92,000	\$0
20%	\$133,323,000	\$0	\$10,172,000	\$0
30%	\$196,767,000	\$0	\$18,475,000	\$0
40%	\$225,632,000	\$0	\$20,712,000	\$0
50%	\$225,632,000	\$0	\$20,712,000	\$0
60%	\$225,632,000	\$0	\$20,712,000	\$0
70%	\$225,632,000	\$0	\$20,712,000	\$0
80%	\$225,632,000	\$0	\$20,712,000	\$0
90%	\$225,632,000	\$0	\$20,712,000	\$0
100%	\$225,632,000	\$0	\$20,712,000	\$0

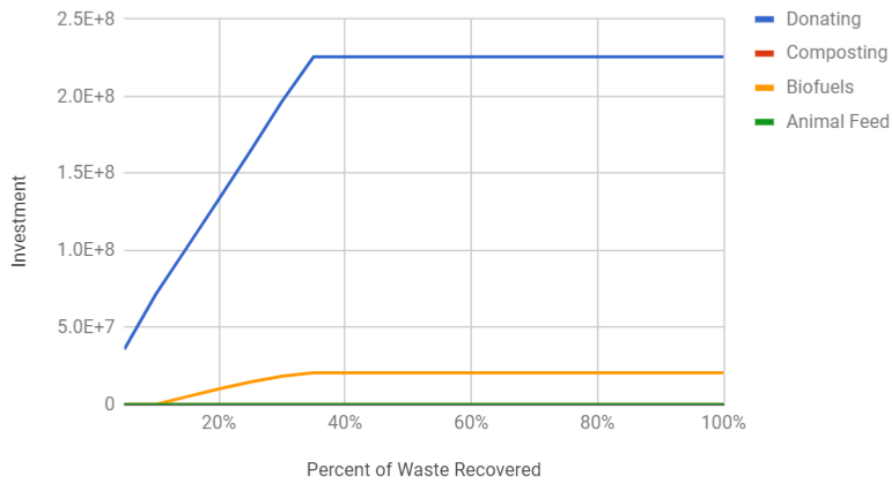


Figure 4.3.1 The Amount of Food Ideally Invested in Each Strategy based on Waste Recovered in Monmouth County

The results of this model were not immediately obvious or intuitive. To confirm the data, each strategy was further explored. Based on the amount of food invested to each strategy, the return value was calculated. The results are shown in Figure 4.3.2.

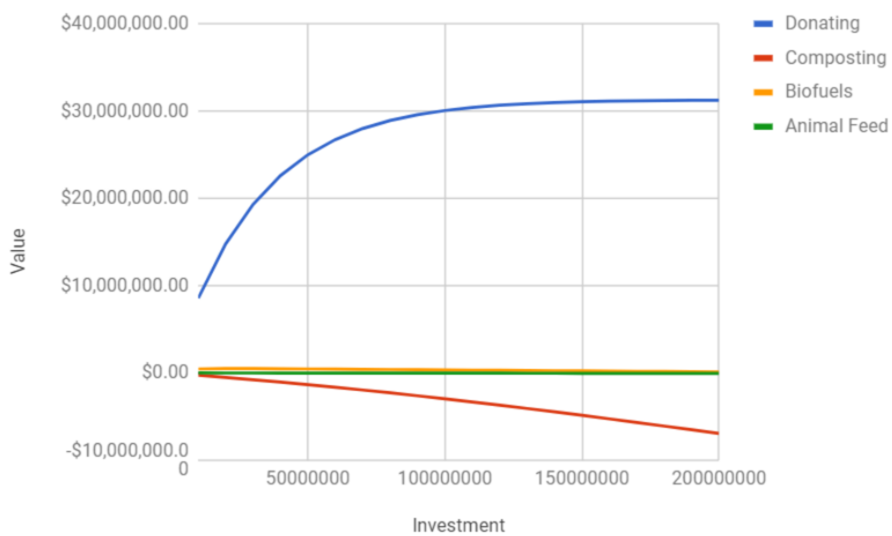


Figure 4.3.2 The Amount of Food Invested in Each Strategy Versus the Amount of Return in Monmouth County

Now, the results of the model begin to make sense. Donations experience the greatest return value because of the vast number of food insecure people who could be fed by wasted meals. However, this value levels off as more and more people are fed. Biofuels demonstrate the second greatest return value as the value for selling biofuels outweighs the cost of repurposing wasted food into biofuels. However, animal feed demonstrates minimal return because livestock food is much cheaper compared to human food. Similarly, composting demonstrates a negative return because the cost of converting wasted food to compost and fertilizer is never paid off by the profits from fertilizer. Thus, it becomes sensible to invest the most food into donations and, once strategy peaks in return, to invest in biofuels. The animal feed and composting returns become negligible and insignificant.

4.4 Strengths and Weaknesses

A major strength of the model is that it considers four different strategies for reusing and recovering food waste. This gives multiple options for the studied area, and depending on internal factors, the most beneficial strategies can be chosen based on the specific local situation. Furthermore, the model considers the viability of using a combination of strategies to get the most out of the food waste. In addition, the model is also able to find the optimal combination of strategies even if not all of the food waste is recovered.

A weakness of the model is that all of the distances used were approximate for the county. Specifically, for intra-county travel, locations were considered to be evenly randomly distributed across the county, which is likely not the case. It was also assumed that the nearest biofuel plant is located exactly 32 miles from all locations in the county, not taking into account differences in location within the county.

5 Conclusion

5.1 Further Studies

A number of assumptions were made to construct our models in lieu of adequate data. In the first model, a lack of state-specific data necessitated the use of national waste data as a substitute, making the results less applicable to Texas on its own. The second model made a number of assumptions that could be eliminated with analysis of raw census data on the sizes and incomes of families, as well as detailed information about the typical eating habits of families of varying size, income and composition. Finally, the third model would likely benefit from more detailed economic analysis of the strategies considered.

5.2 Summary

The first model focuses on determining whether a state's food-insecure population could by sufficiently fed by the state's wasted food. The amount of food was first standardized to a dollar amount for comparison purposes. To calculate the amount of wasted food, the amount of food loss at retail and loss by consumers were studied and added in turn. The amount of food necessary to feed the food-insecure population was calculated by first determining the needs of one person and then multiplying by the entire population. The model was then applied to Texas, and it was determined that Texas can more than feed the food-insecure population with the amount of wasted food. The results were readily confirmed with outside research.

The second model focuses on determining annual household food waste based on income and members of the family. For this model, food waste was split into that at home and that outside of home. An important aspect to this model was scaling each family member to adjusted persons because each member of the family consumed a different amount of food. Once this was complete, the family was placed into one of four meal-plans based on income: thrifty, low-income, moderate-income, liberal. This allowed for appropriate measure of how much money is spent on food annually. With this value, the percent of wasted food can be taken based on the age and generation of each family member. When applying this model to the four families given, it was found that the single parent with a toddler wasted the least amount of food. The family of four wasted the greatest total amount of food. The single man wasted the greatest amount of food per person.

The third and final model focuses on determining appropriate food repurposing strategies for a specific community. The model seeks to maximize food repurposed and minimize the cost of repurposing. Food donations, biofuels, composting, and animal feeds were four strategies investigated in this model. The model takes in the percent of food the specified community seeks to repurpose as input. To quantify the benefits of each strategy, an exponential curve was used because the return value was expected level off more was invested in the strategy. To quantify the costs of each strategy, a linear equation was used because a set start-up price was expected along with a constant maintenance price. After applying the model to Monmouth County, it was determined that only food donations and biofuels are appropriate strategies to implement.

References

[1] Key facts on food loss and waste you should know! (n.d.). Retrieved March 04, 2018, from http://www.fao.org/save-food/resources/keyfindings/en/

[2] U.S. Food Waste Challenge FAQ's. (n.d.). Retrieved March 04, 2018, from https://www.usda.gov/oce/foodwaste/faqs.htm

[3] Key facts on food loss and waste you should know! (n.d.). Retrieved March 04, 2018, from http://www.fao.org/save-food/resources/keyfindings/en/

[4] How We Got the Map Data. (n.d.). Retrieved March 04, 2018, from http://www.feedingamerica.org/research/map-the-meal-gap/how-we-got-the-map-data.html

[5] Coleman-Jensen, A., Rabbitt, M. P., Gregory, C. A., Singh, A. (2017, September). Household Food Security in the United States in 2016. Retrieved March 04, 2018, from https://www.ers.usda.gov/publications/pub-details/?pubid=84972

[6] Texas Population, 2018 (Projections). (2014, December 16). Retrieved March 04, 2018, from Gunders, D. (2017, August 16). Wasted: How America Is Losing Up to 40 Percent of Its Food from Farm to Fork to Landfill. Retrieved March 04, 2018, from https://www.nrdc.org/resources/wasted-how-america-losing-40-percent-its-food-farm-fork-landfill

[7] U.S. and World Population Clock. (2018, March 04). Retrieved March 04, 2018, from https://www.census.gov/popclock/

[8] Food Expenditures. (2016, January 26). Retrieved March 04, 2018, from https://www.ers.usda.gov/data-products/food-expenditures/food-expenditures/Food

[9] SAVE FOOD: Global Initiative on Food Loss and Waste Reduction. (n.d.). Retrieved March 04, 2018, from http://www.fao.org/save-food/resources/keyfindings/en/

[10] Map the Meal Gap. (n.d.). Retrieved March 04, 2018, from http://map.feedingamerica.org/

[11] USDA. (2018, January). Official USDA Food Plans: Cost of Food at Home at Four Levels,. Retrieved March 4, 2018, from https://www.cnpp.usda.gov/sites/default/files/CostofFoodJan2018.pdf

[12] United States Income Brackets And Percentiles In 2017. (2017, September 27). Retrieved March 04, 2018, from https://seekingalpha.com/article/4109731-united-states-income-brackets-percentiles-2017

[13] Barclay, E. (2012, November 27). For Restaurants, Food Waste Is Seen As Low Priority. Retrieved March 04, 2018, from https://www.npr.org/sections/thesalt/2012/11/27/165907972/for-restaurants-food-waste-is-seen-as-low-priority

[14] Shanks, C. B., Banna, J., Serrano, E. L. (2017). Food Waste in the National School Lunch Program 1978-2015: A Systematic Review. Journal of the Academy of Nutrition

and Dietetics, 117(11), 1792-1807. doi:10.1016/j.jand.2017.06.008

[15] García-Pelayo, R. (2005). Distribution of distance in the spheroid. Journal of Physics A: Mathematical and General, 38(16), 3475-3482. doi:10.1088/0305-4470/38/16/001

[16] Fertilizer consumption (kilograms per hectare of arable land). (2015). Retrieved March 04, 2018, from https://data.worldbank.org/indicator/AG.CON.FERT.ZS

[17] Food Waste Management Cost Calculator. (2009, September). Retrieved March 04, 2018, from https://www.epa.gov/sites/production/files/2017-01/foodcost3.xls

[18] Purdue Forage Information. (n.d.). Retrieved March 04, 2018, from https://www.agry.purdue.edu/ext/forages/

[19] Fitzgerald, A. (2012, November 03). How much energy is in a cow? Retrieved March 04, 2018, from http://www.lifebygeek.com/2012/11/03/how-much-energy-is-in-a-cow/

6 Appendix

6.1 Part II: Food foolish?

```python
import pandas as pd

def get_plan(unit_income):
    if unit_income < 20000:
        return 'thrift'
    if unit_income < 37610:
        return 'low'
    if unit_income < 65240:
        return 'moderate'
    else:
        return 'liberal'

def get_unit_income(hh_income, weight):
    return hh_income/weight

def get_family_weight(plan_options, family):
    weight = 0
    for member in family:
        weight += plan_options['weight'][plan_options['group'] == member].item()
    return weight

def get_at_home(plan_options, family, plan):
    weekly = {}
    for member in family:
        weekly[member] = plan_options[plan][plan_options['group'] == member].item()
    return weekly

def get_restaurant(food_constants, at_home):
    weekly = {}
    for key, value in at_home.iteritems():
        var = plan_options['age'][plan_options['group'] == key].item()
        if var == 'baby boomer' or var == 'millenial':
            weekly[key] = at_home[key] * food_constants['restaurant_adult']
        else:
            weekly[key] = at_home[key] * food_constants['restaurant_child']
    return weekly
```

```python
def get_school(food_constants, at_home):
    weekly = {}
    for key, value in at_home.iteritems():
        var = plan_options['age'][plan_options['group'] == key].item()
        if var != 'millenial' and var != 'baby boomer':
            weekly[key] = at_home[key] * food_constants['school_child']
    return weekly

def get_waste(at_home_weekly_cost, restaurant_weekly_cost, school_weekly_cost):
    total_waste = 0
    restaurant_waste = sum(restaurant_weekly_cost.values()) * waste_constants['restaurant']
    school_waste = 0
    for key, value in school_weekly_cost.iteritems():
        var = plan_options['age'][plan_options['group'] == key].item()
        if var == 'child' or var == 'toddler':
            school_waste += school_weekly_cost[key] * waste_constants[var]
    parent = 'millenial'
    for key, value in at_home_weekly_cost.iteritems():
        if plan_options['age'][plan_options['group'] == key].item() == 'baby_boomer':
            parent = 'baby_boomer'
    at_home_waste = sum(at_home_weekly_cost.values()) * waste_constants[parent]
    return (at_home_waste + school_waste + restaurant_waste) * 52

plan_options = pd.read_excel('At_Home_Food_Expenditures.xlsx')
food_constants = {'restaurant_adult': 0.2796387411,
                  'restaurant_child': 0.2284839082,
                  'school_child': 0.1340035795}
waste_constants = {'baby_boomer': 0.125856,
                   'millenial': 0.1603,
                   'restaurant': 0.1,
                   'child': 0.261,
                   'toddler': 0.45}

family = ['Female: 19-50', 'Male: 19-50', 'Female: 14-18', 'Male: 13-18']
family = ['Female: 19-50', 'Child: 1-3']
household_income = 135000

if __name__ == '__main__':
    weight = get_family_weight(plan_options, family)
    unit_income = get_unit_income(household_income, weight)
    plan = get_plan(unit_income)
    at_home_weekly_cost = get_at_home(plan_options, family, plan)

    restaurant_weekly_cost = get_restaurant(food_constants, at_home_weekly_cost)
    school_weekly_cost = get_school(food_constants, at_home_weekly_cost)
    print at_home_weekly_cost
    print sum(at_home_weekly_cost.values()), sum(restaurant_weekly_cost.values()), sum(school_weekly_cost.values())
    total = get_waste(at_home_weekly_cost, restaurant_weekly_cost, school_weekly_cost)
    print total
```

6.2 Part III: Hunger Game Plan?

```
import math

food = 717475465.9
steps = 1000000

# variable vectors
# a: maximum benefit
# b: cost per unit of food
# c: start-up cost
# d: undiminished benefit
# invest: amount invested in each so far
# equations are of the format a(1-e^(-dx))-(bx+c)
a = [31446733.5,6160000,567026.36,235800]
b = [0.000765448907,0.05464,0.002226760457,0.000765448907]
c = [0,39370,0,2300]
d = [0.0000000317998,0.0000000052864,0.0000001863588,0.0000000022887]
invest = [0,0,0,0]

# incremental benefit of option i
db = lambda i: a[i]*d[i]*math.e**(-d[i]*invest[i])-b[i]

for i in range(steps):
    benefits = [db(x) for x in range(len(invest))]
    if max(benefits) <= 0: break
    opp = benefits.index(max(benefits))
    invest[opp] += food/steps
print invest
print sum(invest)
```