M³ Challenge Sixth Place (First Honorable Mention Team Prize) - $2,500
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Moody M³ Math Challenge
The Future of High Speed Rail
Team #202

Section I: Summary

The only form of high-speed rail in America is Amtrak's Acela line, running on the Northeast Corridor between Washington D.C. and Boston. While expanding high-speed rail with the HSIPR program may alleviate the traffic on America's highways, the cost of this ambitious project and its benefit to the nation must be determined to justify its inclusion in the national budget.

First, we determined the effect of reinstituting the HSIPR program on the growth of rail travel. We utilized data from the Acela line's operation over the last ten years to deduce an exponential function with a constant growth rate. We adjusted the equation using a linear relationship accounting for the increased speed of modern rail systems versus the Acela line, which results in a faster growth rate. Finally, the calculated growth rate was implemented into an equation which related current non-high-speed ridership in each region to the potential growth of rail travel.

To determine the costs of these planned high-speed railways, a few factors had to be considered. First, initial construction costs and maintenance costs were calculated by considering the total length of the railway in each region along with the respective costs of each region. Another factor considered was the inflation rate over time, which represents an increase of the price level by the year 2032. To determine more accurate maintenance and construction costs, the inflation rate was calculated for 2032 using Consumer Price Index (CPI) values.

With the projections of usage and costs, we estimated the expenses and revenues of each of the ten proposed rail lines, finding that three of the lines had higher expenses than revenues, and that these lines would never return their initial investments. We also extrapolated that the rail system would encourage too few commuters to choose rail over automobile for the U.S. to make a significant reduction in its foreign oil imports; our calculations show that only a 0.5% decrease would occur.

To determine which line was most deserving of construction, we ranked them by their profitability and their ability to repay their initial investments. Lines that could recoup their initial expenses quickly ranked over those that could not. The California railway was found to break even the fastest and is therefore most suitable for construction of high-speed rail.

Section II: Background Situation

Rail travel is an important factor of the American economy. As a main form of transportation, railroads are necessary for the movements of goods and workers. In the past, the construction of nationwide railroads spawned the industrial revolution, bolstering the American economy and bringing it into the modern age. As the American economy slogged through another recession, Congress allocated fifty three billion dollars in funding to create a high-speed rail network modeled on the successful Amtrak Acela to fortify the economy and reduce dependence on foreign energy.

However, this November, Congress eliminated this funding, with the reasoning that the project was too expensive in the United States' current economic climate. Many members who voted for its elimination hinted that they would be more accepting of a more focused and better analyzed version of the program.

Section III: The Problem

Determine if a plan to construct new high speed rail in ten regions throughout America is worthwhile by estimating building costs, maintenance, use, and the reduction in foreign oil dependency. Rank each region considered by HSIPR so that a refocused version of the legislation can be passed.

Section IV: Assumptions

1. The Acela railway data can be accurately used to predict growth of rail travel in other regions. Maintenance rates are the same for all regions, and cover full repairs for the railways once per year.
2. Inflation rates will remain constant at current values.
3. All train users were assumed to be commuters who use the train 250 times per year.
4. All revenue that the railways brought in was from ticket sales.

Section V: Analysis of the Problem

Our analysis of HSIPR needs to include a twenty year projection of the amount of people who will use the high-speed trains to determine their usefulness. This projection needs to be broken down by region so the program can be more easily focused on which area will most benefit from the program. These projected numbers are crucial for determining the usefulness of the new high-speed railways. Next, we need to create a cost and maintenance estimation for the proposed railways and compare it to our reduction of foreign oil dependency. Finally, we will rank the HSIPR regions by how deserving they are of funding, creating a recommendation for Congressional high-speed rail decision making.

Our first model will address ridership estimations for the planned high-speed railways based on data gathered from Acela, the only current high-speed railway in America. This model will be capable of estimating ridership in each identified region of HSIPR based on the current use of low-speed trains.

Our second model uses cost estimation from several other European and Asian railways to predict the cost of building each proposed section of high-speed railway. Data gathered from current railroads is used to predict maintenance costs for the new railways based on length. Finally, the projected ridership statistics will be used to calculate the reduction in cars and oil used as a result of the new railways and will be compared to the energy they consume.

Our third and final model will use profitability and the breakeven point to determine which areas are best for railway construction. Location and several other factors will be included in our overall rankings of railway regions.

Section VI: Design of the Models

VI.I: Ridership Forecast

The mathematical ridership model that we created was extrapolated from data of the Northeast Corridor. Amtrak opened the Acela railway in this corridor in December 2000. We found tables that contained the total number of riders per year before and after the Acela line was implemented.

Northeast Corridor Ridership 1988-2004 in Millions[1]

1988	1989	1990	1991	1992	1993	1994	1995	1996	1997	1998	1999	2000	2001	2002	2003	2004
11.2	11.1	11.2	10.9	10.1	10.3	11.7	11.6	11.0	11.1	11.9	12.3	12.9	13.5	13.8	13.6	14.2

Amtrak Ridership Growth FY 2000-FY 2011 in Millions[2]

2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011
8.4	8.9	9.1	9	9.5	9.6	9.4	10	10.9	9.9	10.4	10.9

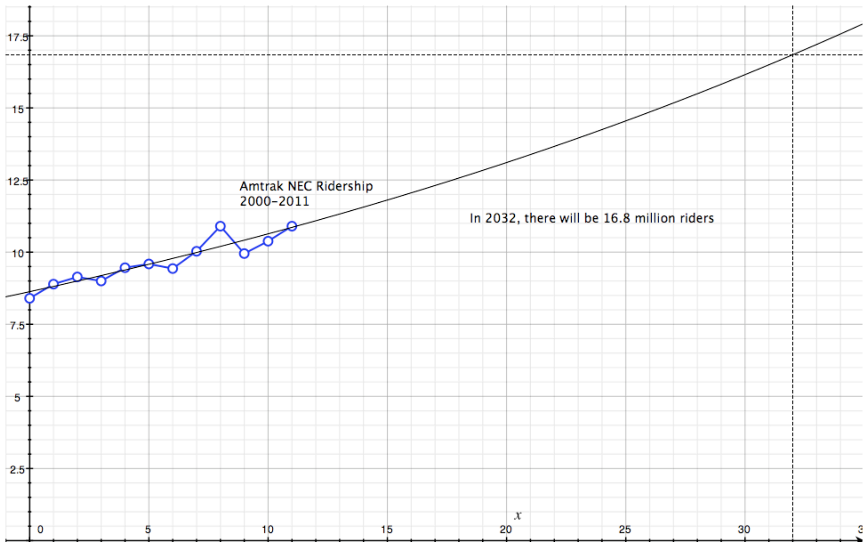
These two sources contain different data because they include different rail lines in the Northeast Corridor as ownership of Amtrak lines changed. The first source continued counting railways that had been sold before 2000 in order to keep consistency. However, the differences in the data between the two charts do not affect the calculation of the mathematical model.

From from these sources the significant change in ridership following the implementation of the Acela railway can be deduced. The ridership in years before 2000 fluctuated between ten and twelve million, but the average change is essentially zero as prices went up and down with the economy. The noticable increase of ridership in 1999 can most likely be attributed to excitement and advertising surrounding the construction of the first high-speed passenger railroad in America. After 2000, a consistent upwards growth in ridership can be attributed to the opening of Acela.

Once this date of change has been established, it makes sense to switch to the second data source which extends consistently until the year 2011. Using this data, we performed an exponential regression to create a function which models the growth of use of trains in the Northeast, $R(t)$, where

$$R(t) = 8.6 \times 10^6 (1.02117)^t,$$

R is the total ridership, and t is the time in years elapsed since 2000.



$$y = a \cdot b^x$$
$$a = 8.6 \qquad b = 1.02117$$
$$y = \text{Ridership, millions}$$
$$x = \text{Fiscal years since 2000}$$

Expressing this equation as an exponential function of the form

$$A(t) = A_0(1 + r)^t$$

we find that the rate of growth of ridership in the Northeast expressed as a percent is 2.117% per year. Since the growth of Northeast ridership can be attributed to the introduction of the high-speed rail line Acela, we know that other high-speed rail lines under the same conditions will also create this rate of growth.

However, the new rail planned has a goal speed of 354 km/hr, while the Acela line travels at 241 km/hr.[3] The important impact of increased speed can be evidenced in the growth of ridership following Acela's implementation. The 2.117% increase in ridership is directly attributed to the increased speed. The speed of new rail lines will be significantly faster than Acela, so the increase in usage will also be greater. In order to model this increase in growth rate, we created a relationship using the points (127,0) and (241, .02117), where the first parameter is the speed in miles per hour and the second is the growth percent of the ridership with this speed. These coordinates come from the speeds and growth rates before and after the introduction of the Acela. Since only two points exist, the only relationship we can accurately constuct is linear, where

$$R(v) = .0001857v - .02358,$$

R is the ridership growth rate, and v is the velocity of the rail in km/hr. Using this relationship, we calculated that the growth rate when the projected speed of 354 km/hr is reached is 4.204%.

Next, we can insert this value into the exponential formula that we are using to model future ridership. Thus we have

$$R(t) = R_0(1 + .04204)^t$$

where t is the time in years elapsed since 2011 and R_0 is the total ridership of the regional railways in 2011. Just as an equation could be used to model the growth after Acela was opened, this new equation can be used with the non-high-speed rail ridership to project high-speed ridership. The second source for the Northeast rail also had a breakdown of the ridership across the country, so we added the lines together to find the base ridership of each region identified for HSIPR improvements.[2]

Location	2011 Ridership
Southeast	1,166,808
California	5,563,031
Pacific Northwest	852,269
South Central	84,039
Gulf Coast	34,600
Chicago Hub	2,830,264
Florida	2,232,000
Keystone	1,342,507
Empire	1,637,406
Northern New England	772,138

By plugging these initial values into the equation and setting $t = 20$, the projected ridership figures for 2032 are as follows:

Location	2011 Ridership
Southeast	2,641,512
California	12,594,029
Pacific Northwest	1,929,433
South Central	190,254
Gulf Coast	78,330
Chicago Hub	6,407,375
Florida	5,052,977
Keystone	3,039,273
Empire	3,706,889
Northern New England	1,748,027

Since the original data with which we calculated these values was ridership, it includes all of the factors that have affected it in the past. For example, rising oil prices are bound to impact train use. As gas prices rise, commuters will tend to use more public transportation, such as trains. Yet, the growth of oil prices occurred during the domain of our initial data. Increased ridership from rising oil prices is accounted for in our overall growth percent that was calculated with these numbers. Other factors such as population growth are accounted for in the same manner.

VI.II.I: Inflation

Inflation is a necessary consideration when dealing with the financial implications the future may have on our high-speed rail model. Over time, as inflation indicates, price levels rise due to various factors. Due to the unpredictability of the future state of the economy, the potential variation in the rate of inflation cannot be definitively accounted for in our model. In order to project inflation as accurately as possible, we must therefore extrapolate past trends into the future to give a base for other inflation calculations. The average rate of inflation for the past 20 years had to be calculated from historic inflation rates.[4] This value is 2.87%, and this will serve as our constant rate of inflation for the model. Inflation rate is a basic calculation using a percentage change in values, which come from the Consumer Price Index (CPI). The CPI represents the value of a market basket of goods for the average consumer, which tends to increase with time. A reasonable percent estimate for the rate for inflation at time $t = n$ can be determined by calculating the percent change in the CPI over a period of one year, where

$$\frac{di}{dt} = \frac{\left(\text{CPI}_{t_n} - \text{CPI}_{t_{n-1}}\right)}{\text{CPI}_{t_{n-1}}} \times 100\%$$

and $\frac{di}{dt}$ is the percentage rate of inflation. The CPI in 2010 was 218.056.[5] None of the necessary data for the inflation model precedes 2010, so this will serve as our base year at time $t = 0$, with time expressed in years elapsed since 2010. The rate at which the CPI changes varies directly with its current value.[6] Thus we have

$$\frac{dp}{dt} = .0287\,p,\ p_0 = 218.056$$

where p is the value of the CPI at t years after 2010, and the value of p_0 is the CPI in 2010. When solved, the differential equation gives us $p(t)$, where

$$p(t) = 218.056e^{.0287t}$$

and predicts the value of the CPI at time t. This model shall be used to predict the future maintenance costs and revenue.

VI.II.II: Initial Costs

The initial building of the railways represents a hefty cost in itself. Because this is a fixed cost or a one-time cost, the numbers used do not need to be adjusted for inflation for the year 2032. In order to determine the initial costs, we averaged the costs of building high-speed railways per kilometer, and multiplied that dollar value by the total distance, in kilometers, of each geographic region. Obviously, the initial costs per kilometer will be a fixed value, since the same type of railway is being built throughout the country. The only variable is the distance of the proposed railway in each region. As the total distance of the high-speed railway fluctuates, the initial cost of building the respective railways in the regions will vary as well.

Initial Construction Costs by Region

Geographic Region	Total Distance (Kilometers)	Construction Cost (Per Kilometer)	Total Construction (Initial) Cost
Southeast	876.5	$53,529,956.5	$46,900,000,000.0
California	≅1,300.0	$53,529,956.5	$69,600,000,000.0
Pacific Northwest	750.0	$53,529,956.5	$40,100,000,000.0
South Central	1,599.6	$53,529,956.5	$85,600,000,000.0
Gulf Coast	1,644.7	$53,529,956.5	$88,000,000,000.0
Chicago Hub	5,000.0	$53,529,956.5	$268,000,000,000.0
Florida	1,046.0	$53,529,956.5	$56,000,000,000.0
Keystone	562.0	$53,529,956.5	$30,100,000,000.0
Empire	740.0	$53,529,956.5	$39,600,000,000.0
Northern New England	786.9	$53,529,956.5	$42,100,000,000.0

[7] [8] [9] [10] [11] [12] [13] [14] [15] [16] [17] [18] [19] [20]

VI.II.III: Maintenance Costs

In considering the costs, specifically the maintenance costs, it is also imperative to take into account the time period. From a macroeconomic standpoint, costs will continue to rise, on average, every year due to the inflation rate. In order to formulate more accurate maintenance costs for each geographic region, we recalculated the maintenance costs for the year 2032, which represents a more accurate perspective on the costs of building such high-speed railways. In initially calculating the maintenance costs, which include the costs of keeping the quality of tracks high and safe, installing rails and anchors, tightening bolts, and other similar costs, the average cost of maintenance per kilometer ($154,160.02), found in the year 2010, was multiplied by the total amount of kilometers for each high-speed railway in the respective geographic regions. This, in turn, yielded the total maintenance cost, unadjusted for inflation. From this number, we theorized that the maintenance costs, unadjusted for inflation, would remain constant for each year, since maintenance represents work that must be completed every year no matter what. The rate is determined by calculating the percent change in the CPI over a period of time, with one year being the base year. Hence, 2031 represents the base year, and 2032 is the year for which we are trying to calculate the inflation rate. In order to calculate the inflation rate for the year 2032, the CPI of 2031 had to be subtracted from the CPI of 2032, and the difference was divided by the CPI of 2031. By using the function as formulated before, the calculation yielded an inflation rate of 2.91%, or expressed as a decimal, 0.0291. With the inflation rate now calculated, in order to calculate the adjusted maintenance costs in the year 2032, each year’s maintenance cost must be multiplied by 1.0291 (1+0.0291), and this will produce the adjusted maintenance costs for the year 2032 for each geographic region.

Maintenance Costs by Region

Geographic Region	Total Distance (Kilometers)	Maintenance Cost (Per Kilometer)	Total Maintenance Cost (Unadjusted)	Total Maintenance Cost in 2032 (Adjusted for Inflation)
Southeast	876.5	$154,160.02	$135,121,257.5	$139,053,286.1
California	≅1,300.0	$154,160.02	$200,408,026.0	$206,239,899.6
Pacific Northwest	750.0	$154,160.02	$115,620,015.0	$118,984,557.4
South Central	1,599.6	$154,160.02	$246,594,368.0	$253,770,264.1
Gulf Coast	1,644.7	$154,160.02	$253,546,984.9	$260,925,202.2
Chicago Hub	5,000.0	$154,160.02	$770,800,100.0	$793,230,382.9
Florida	1,046.0	$154,160.02	$161,251,380.9	$165,943,796.1
Keystone	562.0	$154,160.02	$86,637,931.2	$89,159,095.0
Empire	740.0	$154,160.02	$114,078,414.8	$117,398,096.7
Northern New England	786.9	$154,160.02	$121,308,519.7	$124,838,597.6

[21]

VI.II.IV: Energy vs. Cost

Now that we have a cost estimate, we can compare it to money saved on energy. Generally, use of high-speed rail is more energy-efficient than automobile use. Amtrak cites the Department of Energy Oak Ridge National Laboratory in claiming that rail travel uses 31% less energy per passenger mile than travel by car.[22] This suggests that the construction of a high-speed rail network would decrease energy use in the United States by replacing the car as the mode of transportation for many commuters. From Amtrak's data, we can calculate the overall effect of travelers choosing rail over automobile on US oil imports. The conservation in BTU per passenger-mile[22] is

$$3{,}538\frac{\text{BTU}}{r\times\text{mi}} - 2{,}435\frac{\text{BTU}}{r\times\text{mi}} = 1{,}103\frac{\text{BTU}}{r\times\text{mi}}$$

where r is the number of passengers. In order to estimate the total barrels of oil saved, we must find the average distance of Acela trips so that our calculations may continue. This number was found by averaging the ticket cost per mile of a random sampling of 10 simulated Acela trips,[23] the value of such being $.61/mi. If Acela's total revenue comes exclusively from ticket sales, we divide the values to give

$$\$491{,}654{,}117\$ \div 0.61\frac{\$}{\text{mi}} = 8.06\times10^{8}\,\text{mi}$$

which is the total distance traveled by Acela trains in 2011. The ridership, which is equivalent to the total number of trips, is the last value we need to calculate the average trip. Once again, we divide to give

$$8.06 \times 10^{8}\ \text{mi} \div 3{,}379{,}126\ \text{trips} = 238.5\ \frac{\text{mi}}{\text{trip}}$$

which is the average mileage per trip. Returning to our energy calculations, we multiply our estimated distance of 238.5 mi by the conservation per $r \times \text{mi}$ to find that the total energy saved per person is

$$1{,}103\ \frac{\text{BTU}}{r \times \text{mi}} \times 238.5\ \text{mi} = 263{,}000\ \frac{\text{BTU}}{r}.$$

Then we multiply by our projection of ridership in the year 2032 to find the energy consumed by the rail system

$$263{,}000\ \frac{\text{BTU}}{r} \times 37{,}400{,}000\ \frac{r}{\text{year}} = 9.84 \times 10^{12}\ \frac{\text{BTU}}{\text{year}}.$$

The energy density of gasoline[24] is $47{,}300\ \text{J/g}$ or $44.8\ \text{BTU/g}$, so dividing by this we find that gasoline is consumed at a rate of

$$9.84 \times 10^{12}\ \frac{\text{BTU}}{\text{year}} \div 44.8\ \frac{\text{BTU}}{\text{g}} = 2.20 \times 10^{11}\ \frac{\text{g}_{\text{gasoline}}}{\text{year}}.$$

Because approximately 46% of crude oil that is refined produces crude oil,[25] the crude oil needed to produce this amount of gasoline is

$$2.20 \times 10^{11}\ \frac{\text{g}_{\text{gasoline}}}{\text{year}} \div 0.46 = 4.77 \times 10^{11}\ \frac{\text{g}_{\text{oil}}}{\text{year}}.$$

Not all of this oil would be imported; 42% would come from domestic sources, while 58% is imported.[26] The amount that is imported is

$$4.77 \times 10^{11}\ \frac{\text{g}_{\text{oil}}}{\text{year}} \times 0.58 = 2.77 \times 10^{11}\ \frac{\text{g}_{\text{foreign oil}}}{\text{year}}$$

The United States imports crude oil at varying densities, with the approximate average being an API gravity of 28.5°[27], or $884\ \text{kg/m}^3$. Thus, the volume of imported oil is

$$2.77 \times 10^{11}\ \frac{\text{g}_{\text{foreign oil}}}{\text{year}} \times \frac{1\text{kg}}{1000\text{g}} \times \frac{1\text{m}^3}{884\text{kg}} \times \frac{264.2\text{gal}}{\text{m}^3} \times \frac{1\text{bbl}}{42\text{gal}} = 19{,}700{,}000\ \frac{\text{bbl}_{\text{foreign oil}}}{\text{year}}.$$

The rate of national oil consumption at a 2010 estimate was 6.97 billion barrels/year,[28] or more than two orders of magnitude greater. If this rate is maintained into the future, then the introduction of the rail system would decrease both foreign and domestic oil consumption in the U.S. by 0.5%. This decrease is marginal and not worth the cost to build and maintain calculated earlier.

VI.III.I: Projected Revenue for 2032

Our criteria for ranking the regions have several components. The amount that each region is deserving of high-speed rail is heavily based off of location and existing state of railways, both of which play into economic advantage. However, while these factors are important, congressional spending on railways should be considered as an investment, making the financial self-sufficiency of each region the most important factor in determining which region most deserves aid.

Similar to the ridership statistics that were extrapolated from the Acela railway, profit estimates can be created the same way. In 2011, the Acela railway generated $983,452,555 in revenue from 1,166,808 trips.[29] This means that each trip on the high-speed railway resulted in a $117 revenue per ride. By multiplying this value into the estimated ride numbers for 2032, we found projected revenue for each region of the proposed high speed railway.

These values must be adjusted for inflation for the year 2032, as there would exist discrepancies between the unadjusted and adjusted values. As previously calculated, the inflation for the year 2032 is expected to be around 2.91%. Similar to the adjusted maintenance costs, each projected revenue value is multiplied by 1.0291 (1 + .0291), and this yields the projected revenues for each geographic region, adjusted for inflation.

Projected Revenue 2032 by Region

Geographic Region	**Projected Revenue 2032 (Unadjusted)**	**Projected Revenue (Adjusted for Inflation)**
California	$1,474,382,982.0	$1,517,287,526.8
Chicago Hub	$750,111,418.7	$771,939,661.0
Florida	$591,552,126.0	$608,766,292.9
Empire	$433,965,502.0	$446,593,898.1
Keystone	$355,807,737.5	$366,161,742.7
Southeast	$309,241,824.9	$318,240,762.0
Pacific Northwest	$225,878,825.7	$232,451,899.5
Northern New England	$204,641,521.3	$210,596,589.6
South Central	$22,273,050.7	$22,921,196.5
Gulf Coast	$9,170,118.1	$9,436,968.5

VI.III.II: Net Profit

However, these values cannot be reasonably compared because each region of track has different upkeep and initial construction costs. To factor these values in, we subtracted the maintenance costs generated in part B, adjusted for inflation, to find net profit of the new railways. Operating costs can be overlooked because each new railway has essentially the same operating costs. The length of the railway has little effect on how much it costs to operate. This cost is more a function of the number of cars on each set of tracks, which is similar for each railway. Since the operating costs are similar to each other, they will have little effect on the profit rankings, since it will affect them all equally. Therefore, we found the profit ranking of each railway by subtracting the maintenance from the revenue.

Net Profit By Region

Location	**2032 Net Profit**
California	1,311,047,627.20
Florida	442,822,496.80
Empire	329,195,801.40
Keystone	277,002,647.70
Southeast	179,187,475.90
Pacific Northwest	113,467,342.10
Northern New England	85,757,992.00
Chicago Hub	-21,290,721.90
South Central	-230,849,067.60
Gulf Coast	-251,488,233.70

VI.III.III: Constant for Comparison

Next, we needed to compare each region's profits to the original amount invested in them. By performing this operation,

$$\frac{\text{original building cost}}{\text{profit ranking}} = k,$$

we get a value that can be reasonably compared between the different regional railways. The *k* value takes into account revenue, maintenance costs, and original investment, which determine the financial benefits of each railway. The generated *k* value represents the time it will take for each railway to pay for itself. While no absolute numbers can be generated due to inflation, a smaller positive *k* value corresponds to a faster break-even point. Theoretically, a substantial net profit in the denominator of our *k* expression and a small initial construction cost will result in a *k* value less than one. Contrarily, large initial costs and miniscule profits produce greater *k* values in an exponential manner. Therefore, the rail lines with the smallest positive *k* value are more financially deserving of congressional investment. Negative *k* values mean that the railway is losing money (there is never a positive profit). With this *k* value, we can reasonably compare the worthiness of each region of rail line to rank their merit for funding.

Constant of Comparison

Location	**Construction Cost/ 2032 Net Profit (k)**
California	53.09
Keystone	108.66
Empire	120.29
Florida	126.46
Southeast	261.74
Pacific Northwest	353.41
Northern New England	490.92
Gulf Coast	-349.92
South Central	-370.81
Chicago Hub	-12,587.64

This table represents a profitability ranking for all of the proposed high-speed railways. California is the best investment. The Chicago Hub area ranks the worst. Even though railways have a large economic benefit around cities, the increased growth of ridership in Chicago resulting from a high-speed network does not merit the construction of 5000 kilometers of new track, more than three times the length of the next highest region. High positive *k* values, such as Northern New England represent a slim margin of profit that will eventually pay for itself. However, since our net profit calculations did not include operating costs, there is a high likelihood that these railways will also be losing money. Yet, the operating costs, which are equal for each railway, do not affect the overall ranking of investment worthiness.

Therefore it can be concluded that the basis for worthiness of investment is the financial gain generated from the proposed railway. Using our system calculations, we created a method for comparing financial worthiness by dividing the original cost by the profit to find a new value, *k*, which could be compared between regions. Through this method, we ranked the ten different regions as shown in the above chart. The most deserving region of funding has the lowest positive *k* value, whereas poor investments are negative. By comparing the financial constants, *k*, we can accurately compare which rail line is most deserving of funding: California.

Section VII: Recommendations

Since high-speed rail in the United States is relatively new, testing our models with current data is impossible. We used the data from the only high-speed track in America to create our models; therefore, we cannot cross-reference our equation because the only other high-speed railways are in different countries facing severely different economic and population situations. The best way test our model would be to build a preliminary, short test track in one of the regions and record whether train ridership values matched over the course of a year or two. However, if our predictions are correct, the implementation of high-speed rail will be inefficient. Even the most effective high-speed rail in California will require over fifty years to pay for itself; and this estimation does not even include operating costs for the railways. We find that the economic and transportation gains which result from the HSIPR program do not merit the cost of implementation.

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