M³ Challenge Fourth Place, Meritorious Team Prize of $7,500

Academy for Advancement of Science and Technology, Team #25
Hackensack, NJ

Coach: Ken Mayers

Students: Jared Landsman, Ian Osborn, Pavel Panchekha, Mark Lipa Velednitsky, Sherry Wu

Moody's Math Challenge

Making Sense of the 2010 Census

Team 25

March 7th, 2010

Summary

Our major claim is that the use of undercount correction by statistical methods, in particular through the Post-Enumeration Survey (PES), has a margin of error significantly greater than the undercount it corrects for. Furthermore, we believe that the errors in this undercount correction are intrinsic to a wide range of possible correction algorithms; in other words, there is no simple fix for these biases. Since these errors seem potent enough to cause an overcount greater than the original undercount, we recommend that no undercount correction take place. Finally, we suggest various clerical changes to the overall Census process and in particular to the imputation methodology that will eliminate the undercount caused by human error, an improvement that, while not eliminating undercount completely, will improve the results of the Census visibly. Finally, we suggest expanding the time the Census has from nine to twelve months, which would allow enough time for the Census to use more thorough matching and investigative methods, further increasing Census accuracy.

Our next section discusses the current method for apportioning the House of Representatives. We begin by arguing that the fixed limit of 435 Representatives is both arbitrary and damaging, and that it must be expunged. Next, we propose a method of apportionment that causes the number of Representatives to safely and smoothly fluctuate near 500. This “Growth Factor Method” incorporates both population and population growth rate. We employ multiple tests to ensure that our method is both accurate and consistent. We conclude by comparing the current apportionment method with our Growth Factor Method in an attempt to show the latter's superiority and stability.

In our third section, we explore the relative effectiveness of several methods that already exist to draw congressional districts in states. We measure these methods using five criteria set out in a popular dissertation concerning congressional districts. Rather than completely endorse either method, we develop our own method which combines the best aspects of both. While perhaps the method could be improved further, it already produces demonstrably better and fairer districting than current approaches. Further, the districts are produced in a manner that reduces the effects of political battles ("gerrymandering"); and this method is also significantly faster than current approaches (on a modern home computer, even California can be partitioned in a few hours).

Part I: Should the Census figures be adjusted for the undercount?

We will attempt to prove that the errors implicit in the PES and other undercount correction techniques lead to greater error in the Census than the original numbers, and so the Census should not be adjusted for overcount.

In examining the 1990 Census, Darga (cite Darga) found that the PES-based undercount correction method had several significant sources of systematic bias (that is, errors that will not cancel out in the final survey). Let us examine these sources.

Errors in Undercount Correction

First, we give an overview of the current undercounting correction technique. After the official Census is taken, the Census Bureau sends enumerators to selected regions of the United States. These enumerators are tasked with conducting small-scale censuses of their own, mostly distributing special PES forms or conducting interviews. Afterward, these forms are sent to the Census Bureau, where they are matched with the records of the official Census. Any persons who are not matched are thus either overcounted or undercounted; persons participating in the PES but not in the Census would be undercounts, whereas those counted by the Census but not by the PES are overcounts. From this, overall overcount and undercount statistics are determined and corrected for. It should be noted that these statistics are not taken overall, but across thousands of strata; for example, there is a separate undercount correction for African-American males between 20 and 30 living in the South. This approach would, in the ideal, work; however, there are several practicalities that lead to gross errors.

The first source of error, matching error, is a core feature of the undercounting problem. During the PES, new reports must be matched to original Census records. The goal is to match PES results to people who participated in the actual Census. In theory, unmatched records are records of those who were not counted. However, the matching procedure is intrinsically complex. While approximately 80% of matches can be made by computer (because of overwhelming evidence linking a PES report and a Census form), the remaining 20% of "difficult cases" are matched by professional human matchers. In a study conducted by the Census, the Census distributed identical records to two groups of independent trained matchers (cite P8). Though the groups used the same guidelines, differences of several percentage points could be found. While 2-3% error may seem small, one must remember that the effect corrected for will be on the order of 2%. Thus, the matching process itself injects as much as 100% error. For example, a matching error of only 104 persons in the 1990 Census resulted in an overall decrease of 250,000 persons in the undercount estimate, a 5% differential (cite Hogan). Similar effects should be kept in mind when dealing with the PES in general; small effects are magnified due to the small size of the PES in comparison with the Census as a whole.

Another significant source of error is the invention of fictitious data. Darga found that fabrication of data occurs often, and that those employees who do fabricate data will fabricate significant amounts of their data---usually on the order of 30% of all those they surveyed. Given the tiny effect of even small numbers of reports (see the effect of 104 persons above), fabricated data represents a huge portion of the false undercount; Darga estimates its effect as causing approximately 8% of the overall estimate of undercount, and this is only for the cases of fabrication that are actually found out; West (cite West) determined that approximately 39% of fabrication is noticed, leading to a final estimate of 20% error due to fabrication. While not as large as the error due to matching problems, it is nonetheless significant, and also hard to combat. Due to the lack of time for training and the effects of having field employees, the fabrication rate seems hard to change. Finally, fabrications lean heavily toward reporting undercount, as it is significantly more difficult for an enumerator to report false counting than false omission. This means that the 20% error will indeed mostly act to inflate the undercount statistic.

Another error is due to the ambiguities present in the Census's notion of “usual residence.” For example, college students commonly report their “usual address” as their parents' home while the PES is conducted, but claim their “usual address” as their college dormitory while the Census itself is conducted.[1] This means that many college students represent both an undercount and an overcount; however, due to the aforementioned difficulties in reporting overcount, there is a bias toward undercount. This ignores the further issue of local statistics, which are in fact far more important for many uses than national-level statistics. For example, the national distribution of college-age students will shift dramatically toward suburban areas due to the misreporting examined above. Overall, the errors due to this seem to be on the order of a 5% increase in undercount, but the damage done is significantly larger than this, due to the corruption of local statistics that an undercount correction would entail---statistics used for more in-depth analysis than pure population counting. Since Census statistics are used for ten years by the government to inform policy decisions, the rectitude of these local statistics is at least as important as the accuracy of the total population count; we will give an example later, when examining the undercount correction method as a whole.

More errors are caused by the assignment of PES reports to the incorrect Census block (blocks designated by the Census subdividing the US into approximately 8 million pieces). Each PES report must be located in a specific Census block. However, reports

1

are often miscategorized, with errors leading to approximately 100% error.[2] To combat this, the Census matchers actually search for reports within a radius of two blocks. However, while this recovers many reports, an even larger search radius would enable yet better matches. This expansion of radius would, however, be infeasible due to the time constraints on the Census; meanwhile, significantly better training would be required to lower the number of misattributed reports.

The number of unresolved cases represents a pernicious thorn in the side of the undercount correction. For the 1990 Census, approximately 1.6% of households were marked as unresolved. While the small size of this number is a testament to the ingenuity of Census enumerators, it is nonetheless rather close to the overall undercount size and could swing the undercount anywhere between 10 million undercounted and 1 million overcounted (that is, if all unresolved cases were undercounts, the result would be 10 million undercounts overall, whereas if all were overcounts, a total overcount of 1 million persons would be found)[3][4]; in other words, it could create as much as 100% error. This is another error implicit in and intrinsic to the undercount correction process.

Finally, to discuss errors related to the Census questionnaire and data gathering, some Census terminology must be clarified. On the Census form, three housing unit statuses were available: Occupied, Vacant, and Delete. Forms were classified as Vacant if the self-response return had no names on the roster and the respondent-reported household size was 0. Forms that had no names on the roster or blank respondent-reported household sizes were classified as Unresolved Occupied/Vacant.

In determining the expected household size, a programming error affected the status resolution for some Vacant enumerator returns. The Interview Summary Population of 0 was recorded as blank, which may have caused up to 133,438 Vacant returns as Deletes and up to 258,963 Vacant returns as Unresolved Occupied/Vacant. Additionally, up to 145,367 (or 75.78%) of 191,826 housing units that had their occupancy status imputed may have been affected by the error. Reducing these minor errors will increase the accuracy of the estimate of the nation's population.[5]

---

[2] Parmer, Randall (1991). P-11 Report: Balancing Error Evaluation. U.S. Bureau of the Census, 1990 Post-Enumeration Survey Evaluation Project, Series #M-2.

[3] Parmer, Randall (1991). P-11 Report: Balancing Error Evaluation. U.S. Bureau of the Census, 1990 Post-Enumeration Survey Evaluation Project, Series #M-2.

[4] Breiman, Leo (1994). The 1991 Census Adjustment: Undercount or Bad Data. *Statistical Science*, 9(4):458-537.

[5] Rosenthal, Miriam. (2003). Operation Analysis of the Decennial Response File Linking and Setting of Housing Unit Status and Expected Household Size. Census 2000 Evaluation L.2.

More data gathering ambiguity was caused by confusion of the enumerators. The unresolved enumerator returns, which constitute 1.34% of the 38,796,478 enumerator returns, were a result of contradictory and missing responses on the Census form. The irrelevance of the Interview Summary Section in some unusual situations was a source of contradictions. Another source of unresolved enumerator returns could have been caused by enumerators' lack of a thorough review of the forms.[6]

Overall Effects of Errors

Overall, we see that various errors in the undercount correction process could swing the undercount estimate by as much as 250%. Thus, the undercount estimation procedures currently in use are completely unreliable. Darga, for example, finds that the undercount estimate of 5 million Americans for the 1990 Census transforms into an estimate of 1.5 million after correcting for the errors above. Unfortunately, it would be difficult, if not impossible, to correct these errors without the years of analysis available to Darga and others; meanwhile, the Census must be ready within nine months. Furthermore, as discussed above, the errors we have identified are intrinsic to the undercount correction problem and thus present no simple fixes. This is not a case of flawed methodology but rather of an impossible task. But the inaccuracies in the undercount correction pale in comparison to the errors created in local data. Most of the errors discussed affect certain strata of the population disproportionately. Thus, the undercount correction procedure will cause great errors in demographic data. Darga provides an example based on the effect undercount correction would have had on the 1990 Census:

> The counts of White, Native American, and Asian/Pacific renters in Detroit and Chicago would be decreased by 5% in 2000, but they would be inflated by 11% in 2010. Thus, there would seem to be a dramatic increase in renters and a shift away from home ownership in these cities relative to the actual trend. In contrast, other central cities in these same metropolitan areas would have their counts for these demographic categories inflated by 21% in 2000 and by only 4% in 2010. The faulty adjustment factors would therefore make it appear that huge numbers of white renters had moved from Detroit and Chicago to other nearby central cities before 2000, but that they moved back in the next decade.

Of course, there was no such actual migration to and fro.

[6] Rosenthal, Miriam. (2003). Operation Analysis of the Decennial Response File Linking and Setting of Housing Unit Status and Expected Household Size. Census 2000 Evaluation L.2.

It should be clear that any type of undercount correction is a flawed technique and that its use in the 2010 Census would cause more harm than good. It is thus our recommendation to the Census Bureau and the House of Representatives that no undercount correction be done for the 2010 Census. However, we propose several alternative suggestions that may improve the results of the Census, bringing higher accuracy (and perhaps even lowering undercount) without endangering local statistics or increasing bias.

Modifications to Current Census Techniques

First, we recommend an expansion of the time allotted to the Census from nine to twelve months. This will give time to enable both the better training of staff (due to budgetary reasons, it isn't possible to train these employees during the years between Censuses) and the more specific matching of records. To determine the effects of this, we examine the major sources of imputations for the 2000 Census. Imputation is the method by which the Census Bureau assigns household data to households whose occupancy status or occupant number is unknown. Thus, it represents a sort of "educated guessing." According to reports by the Census Bureau,[7] many of the imputations were caused by time constraints and enumerator error. Thus, an increased amount of time allotted to the Census would decrease the number of imputations in two ways. Furthermore, improved training would increase the accuracy of reports and may also decrease the amount of fabricated data (as noted by West,[8] experienced enumerators fabricate at a much lower rate than inexperienced enumerators).

The extra time allotted can be used to check the program used for data collection to make sure that small bugs are squashed before the production version. Additionally, the questionnaire can be enhanced in several ways. More comprehensive instructions for unusual cases would be beneficial. A redesigned Interview Summary Section would improve the uniformity of responses.

Finally, to complement the time now allowed to the Census, we suggest increasing the Census budget to pay for enumerator training and to increase enumerator salaries. The benefits of better training have already been amply discussed; larger salaries will increase the status of the enumerator position and hopefully bring employees to carry out their work with more precision and care, both increasing accuracy and decreasing undercount. We estimate the overall effectiveness of these methods from the Census report on

[7] http://www.census.gove/dmd/www/pdf/Report21.PDF

[8] West, Kirsten K. (1991). P-6 Report: Fabrication in the P-Sample: Interviewer Effect. U.S. Bureau of the Census, 1990 Post-Enumeration Survey Evaluation Project, Series #G-2.

imputations.[9] According to the report, 93% of occupancy imputations were caused by enumerator error (in other words, enumerator inexperience forced almost all of the educated guessing of occupancy status that the Census Bureau had to do), as were some status imputations and some household size imputations. These together can be estimated to amount to approximately 500,000 people.

Furthermore, some large families were not included in the 2000 Census due to a change from seven to six individual slots on the Census forms. However, using the longer allowed time and greater enumerator training, these families could be contacted directly by the enumerator, thus including them in the Census. Even for an extremely conservative estimate, where this would help only 10% of large families, this would reduce the number of imputations by another 220,000. Overall, the mechanisms suggested herein should decrease Census error and provide higher accuracy without the damaging effects of undercount correction on local statistics and without the large errors implicit in current undercount correction techniques.

Part II. What method should Congress select for apportioning the House of Representatives?

In this section, we will discuss the optimal distribution of House seats given the population of each state. In addressing this issue, it will be crucial to discuss the actual number of House seats available. We will first present mathematical and social reasons that the current number of Representatives (i.e., 435) is nonoptimal and argue for an increase of this size to 500 members. Next, we will discuss an algorithm for the apportioning of House seats that will be more precise and fairer than current methods. Making use of an exponential model of population growth, we will produce a more stable distribution of seats.

Number of Representatives

The first point of discussion is the number of Representatives in the House. Since 1911, the number of representatives in the House has remained constant at 435 due to Public Law 62-5. The question remains: Whence did this number come? According to Ralph Lozier, a Missouri Representative from 1923 to 1935 and Chair of the Census Committee from 1931 to 1935, it would seem as though 435 is completely arbitrary: "There is absolutely no reason, philosophy, or common sense in arbitrarily fixing the membership of the House at 435 or at any other number." This leads to another question:

[9] http://www.census.gove/dmd/www/pdf/Report21.PDF

Is 435 truly the best number of Representatives for the House to hold? We shall show that there are inherent flaws in the current system.

The House is violating the one-person-one-vote principle. The number of Representatives is too small to accurately account for every person in the United States, with district sizes ranging from about 500,000 (Wyoming) to 900,000 (Montana). The average size, 650,000 is almost 22 times the population per district recommended by our founding fathers (about 30,000). Albeit the Constitution's elastic clause allows for larger district sizes if necessary, but having only 435 Representatives is a hassle for other reasons as well. The average years of service for members of the 108th congress was 10.2. Representatives who ran for re-election won 97% of the time. With such high incumbency rates, even mediocre Representatives can keep their jobs. Additionally, the small number of Representatives in the House can lead to unethical collusion between Representatives, a tactic that can often be seen in the economics of oligopolies and is illegal among firms in the United States. Although it is not illegal for representatives to collude, it can be very dangerous and lead to an imbalance of power. Finally, a small number of Representatives lead to large district populations. Currently the average population of a district is 647,000, whereas the Constitution clearly states that the population of a district should never exceed 30,000. Of course, the Constitution's elastic clause allows for larger district sizes if necessary, but it is important to note the growing number of people that Representatives are expected to represent.

We have argued that the number of House Representatives must increase from its current 435. We will call our method of apportionment of the House of Representatives the “Growth Factor Method.” This method requires determining an original constant, setting a fluctuation value, and incorporating the demographic of population growth. A logical approach would be to make the set number of House Representatives fluctuate near a multiple of 50, the number of states in the nation.

The major reason for using a multiple of 50 is that of numerical stability. Briefly, consider a United States with 50 states of almost equal population. If the number of Representatives were 500, each state would have 10 Representatives. However, if the number of Representatives were, for example, 525, a number far from a multiple of 50, half of the states would have more Representatives than the other half, despite having no significant difference in population. Furthermore, minor changes in population may rearrange some of the 25 “extra” Representatives, changing representation without significant change in population. To avoid both of these issues, the number of Representatives should stay close to a multiple of 50. This way, changes in representation will, as much as possible, represent actual changes in population and will, as much as possible, be evenly distributed among similarly large states.

Thus, we see that the number of Representatives should be close to some multiple of 50. Which multiple? Practical matters dictate this choice. Due simply to logistics, it would be difficult to accommodate 600 or more Representatives in the House; meanwhile, a small number (such as 400) of Representatives would allow for dangerous collusion. Thus, it seems best that the expected number of Representatives be set at 500 or 550 to allow a comfortable range of variation without putting undue stress on the Capitol building or allowing unlawful practices. For convenience's sake, we shall choose 500.

Apportioning Techniques

Once we have this set the total number of Representatives, we must create bounds within which the number is allowed to fluctuate. Historically, the standard deviation of percent changes in the number of Representatives from year to year is 9.2. 9.2% of 500 is 46 (when rounded to the nearest integer). Therefore, we allow the number of Representatives to fluctuate between 454 members and 546 members. The fluctuation limits allow the transition between changes in the number of House Representatives to run smoothly. The limits prevent the number from changing too drastically. This brings us to a question: How is the final number chosen every ten years? We will do so by using the Census and incorporating demographic statistics calculated during each Census.

The major demographics calculated during the Census are age, population density, population growth, migration, birth rates, death rates, life expectancy, and unemployment. The only demographic that is actually usable in choosing the number of Representatives is population growth. While birth and death rates are representative of state population, they are too subject to fluctuations---fluctuations large enough to change apportioning. Migration and unemployment will depend too much on economic conditions, which would lead to undue fluctuation in apportionment precisely during economic disasters, when it would be most urgent that government be stable and willing to act quickly and efficiently. Life expectancy, age, and race would simply be unethical to discriminate by, not to mention illegal per the Voting Rights Act and Amendment 14. Finally, using population density would be similar to using population growth, except that we would incorporate state size; why incorporate such irrelevant data?

Thus, our method will be based on population growth. Every ten years, the Census Bureau would calculate the state population growth rate and national population growth rate between the previous Census to the current Census. The unsigned difference between the state population growth rate and national population growth rate would then be calculated. The state for which the unsigned difference is smallest would be the first state we use to analyze. The hope is that by using data from the state in which the unsigned difference is closest to zero, the final number of Representatives will remain fairly constant each time the population is recalculated.

The next step is to calculate the previous population of that specific state divided by the number of Representatives it had during the previous decade's Census, a ratio we call the "*Q* ratio." We further define the "*P* factor" to be the current national population divided by the *Q* ratio. If there is an integer that, when multiplied by the *P* factor, equals a number that falls within our range of 454 and 546, the *Q* ratio for this state is used for the entire nation. The *P* factor for the first state analyzed is called the "first degree *P* factor." If the first degree *P* factor does not satisfy this condition, the first state's data is dismissed and we begin analysis on the state with the next lowest unsigned difference. (The *P* factor of the state with the second lowest unsigned difference is known as the "second degree *P* factor," and so on.) The previous steps are repeated until a state is found in which the *P* factor multiplied by some integer falls within the desired range. The hope is to achieve a working result for the *P* factor with the lowest possible degree. Once the working *P* factor is discovered and the *Q* ratio is set, the ratio is divided by the current population of each state and rounded to the nearest whole number to find the number of Representatives for each state. Finally, the number of Representatives for each state is added to give the total number of House Representatives.

Testing the Method

The following paragraph shows how the Growth Factor Method can be applied to calculate apportionment for the year 2000. The national population in the year 2000 was 281,424,177. The national growth rate for 1990 to 2000 was approximately 11.5%. The state with the smallest unsigned difference was Montana, with an unsigned difference of approximately 0.284. The *P* factor for this state is calculated by dividing the national population in 2000 by Montana's *Q* ratio, which is itself calculated by dividing the population of Montana in 1990 by its number of Representatives. The *Q* ratio for Montana is 803,655, and the first degree *P* factor is 350. Because there is no integer that can be multiplied by 350 to equal a number between 454 and 546, we move on to the state that has the next lowest unsigned difference, Arkansas. The *Q* ratio for Arkansas is 590,560, and the second degree P factor is 477. When 477 is multiplied by 1, it equals 477, which is in our desired range. Because this number falls in the desired range, the 2000 population of each state population is divided by the *Q* ratio of Arkansas and then rounded to the nearest whole number to find the number of Representatives for each state. The number of Representatives from each state is then added to find the total number of House Representatives for the year 2000, which turns out to be 477. This model can be seen in Table 1.

Table 1a. **Apportionment Based on the 1990 Census and 2000 Census**

States	Size of State Delegation in 1990	Apportionment Population in 1990	Apportionment Population in 2000	Population Growth Rate for 1990 - 2000
United States	435	249,022,783	281,424,177	13.01141751
Alabama……………………	7	4,062,608	4,461,130	9.809511526
Alaska………………………	1	551,947	628,933	13.94807835
Arizona……………………	6	3,677,985	5,140,683	39.76900395
Arkansas…………………	4	2,362,239	2,679,733	13.44038431
California…………………	52	29,839,250	33,930,798	13.71196662
Colorado…………………	6	3,307,912	4,311,882	30.35056555
Connecticut………………	6	3,295,669	3,409,535	3.4550193
Delaware…………………	1	668,696	785,068	17.4028258
District of Columbia…….	x	x	x	x
Florida……………………	23	13,003,362	16,028,890	23.26727503
Georgia……………………	11	6,508,419	8,206,975	26.09782806
Hawaii………………………	2	1,115,274	1,216,642	9.089066902
Idaho………………………	2	1,011,986	1,297,274	28.19090383
Illinois……………………	20	11,466,682	12,439,042	8.479872382
Indiana……………………	10	5,564,228	6,090,782	9.463199567
Iowa…………………………	5	2,787,424	2,931,923	5.183961966
Kansas……………………	4	2,485,600	2,693,824	8.377212745
Kentucky…………………	6	3,698,969	4,049,431	9.474586027
Louisiana…………………	7	4,238,216	4,480,271	5.711247374
Maine………………………	2	1,233,223	1,277,731	3.609079623
Maryland…………………	8	4,798,622	5,307,886	10.6127134
Massachusetts………….	10	6,029,051	6,355,568	5.415727948
Michigan…………………	16	9,328,784	9,955,829	6.721615593
Minnesota……………….	8	4,387,029	4,925,670	12.278036
Mississippi………………	5	2,586,443	2,852,927	10.30310739
Missouri………………….	9	5,137,804	5,606,260	9.117825437
Montana………………….	1	803,655	905,316	12.64983108
Nebraska…………………	3	1,584,617	1,715,369	8.251331394
Nevada……………………	2	1,206,152	2,002,032	65.98504998
New Hampshire………..	2	1,113,915	1,238,415	11.17679536
New Jersey………………	13	7,748,634	8,424,354	8.720504801
New Mexico…………….	3	1,521,779	1,823,821	19.84795427
New York…………………	31	18,044,505	19,004,973	5.322772778
North Carolina………….	12	6,657,630	8,067,673	21.17935361
North Dakota……………	1	641,364	643,756	0.372955139
Ohio……………………….	19	10,887,325	11,374,540	4.475066189
Oklahoma……………….	6	3,157,604	3,458,819	9.539353256
Oregon……………………	5	2,853,733	3,428,543	20.14238893
Pennsylvania……………	21	11,924,710	12,300,670	3.152781074
Rhode Island……………	2	1,005,984	1,049,662	4.341818558
South Carolina………….	6	3,505,707	4,025,061	14.81452957
South Dakota……………	1	699,999	756,874	8.125011607
Tennessee……………….	9	4,896,641	5,700,037	16.40708396
Texas………………………	30	17,059,805	20,903,994	22.53360458
Utah……………………….	3	1,727,784	2,236,714	29.45564955
Vermont………………….	1	564,964	609,890	7.952011102
Virginia……………………	11	6,216,568	7,100,702	14.22222036
Washington……………..	9	4,887,941	5,908,684	20.882883
West Virginia……………	3	1,801,625	1,813,077	0.635648373
Wisconsin……………….	9	4,906,745	5,371,210	9.465847522
Wyoming…………………	1	455,975	495,304	8.625253577

[1] The apportionment population for the District of Columbia is disregarded

To test the Growth Factor Method further, we apply it to calculate apportionment for the years 1970, 1980, and 1990. Using historically apportioned state delegation sizes and real population data from the 1960 and 1970 Censuses, we calculate that the total number of House Representatives for the year 1970 is 491. This calculation uses a first degree P factor. Similarly, the number of House Representatives for 1980 and 1990 are 516 and 483, using first and second degree *P* factors, respectively.

We now apply a different way of testing the Growth Factor Method. We must be sure that our method can be applied consecutively. From our previous test, based on actual data from 1960 and 1970, we have already calculated the apportionment for each state in 1970. We now use this data, along with real Census data from 1970 and 1980, to calculate apportionment for 1980. From this calculation, we find that the total number of House Representatives for 1980 is 516, using a first degree *P* factor. Applying the method twice more, using our new apportionment data each time, we calculate values for 1990 and 2000. These values are 483 (using a second degree *P* factor) and 507 (using a fifth degree *P* factor), respectively. This last application may seem a bit inaccurate at first; however, the fifth degree *P* factor occurred with an unsigned difference that still remained below the value of 1. Thus, we see that our method remains accurate even after multiple consecutive applications.

We justify the use of historical data simply by noting that the geographical population dynamics currently in place in the US are similar to countless changes in history. For example, the current movement to the West is similar to migrations to the South and North over previous decades[10]; thus, historical data gives us an opportunity to test precisely that which our model aims to solve.

Conclusion

From the tests previously presented, it is clear that using the Growth Factor Method on historical records produces results that are nearly synonymous with those generated by using the method on previous Growth Factor Method results. This illustrates how the Growth Factor Method is sustainable. The number of Representatives fluctuates slightly with each new Census, that is, enough to accommodate changes in the relative populations of the states but not so much as to significantly disrupt the overall structure of the House of Representatives.

This method is far superior to the apportionment method currently used today, the Method of Equal Proportions, which proceeds as follows: The population of each state is

[10] For example, the 2000 Census noted decreases in Rust Belt city populations from 1990; or consider the Great Migration from the South to the Northeast between 1920 and 1970.

divided by a normalizing factor, and the state with the highest resulting value receives the 51st Representative. The normalizing factor for a state is the geometric mean of the state's current and next seats. In other words, if we call the current number of seats that a state has "*n*," then the normalizing factor is the square root of the product of n and $n+1$. (Clearly, once a state is assigned a Representative seat, its normalizing factor increases, so the state moves down on the list.) This process then continues, with the state with the current highest resulting value receiving the next Representative seat, until all 435 Representatives have been assigned.

This suggested method is more accurate than the current method because it takes state population and population growth rate into account, whereas the geometric mean method simply takes state population into account and disregards growth rate. By accounting for growth rate, the new apportionment method should prove to be more accurate in the decades between Censuses. Since we take into account decennial growth rate and attempt to model the state Representative count after it, we in effect use an exponential approximation to population size, whereas current Representative apportioning methods use constant approximations. This should prove to be far more accurate, at least over the ten-year periods for which it will be used.

Furthermore, since the approximation to state populations should be more accurate over periods of ten years, changes to Representative apportionment should be more gradual. In the past, there have been instances of states losing and then gaining Representatives year after year. Due to a combination of a numerically more stable size for the House and a better apportionment method, these states should instead stay at a relatively constant number of Representatives. Similarly, states should change in smoother steps; for example, theoretically instead of gaining no Representatives one year and three the next, a state would earn one during the first and two during the second. This would lead to more stable government and a more efficient Congress due to fewer transfers of and changes in direction.

Part III: How should states ensure that Congressional districts are fairly drawn?

At present, in 38 states, redistricting is controlled by some combination of the state legislature and the governor. As a result, they are often divided opportunistically to benefit the parties and/or persons in charge in a process known as gerrymandering.[11]

[11] http://www.search.com/reference/Redistricting#United_States

In his dissertation, Micah Altman set out five criteria, now widely cited in the literature as the five points to address when deciding congressional districts.[12] If a certain algorithm for determining congressional districts could be shown to address these concerns more adequately than another, we will consider the former algorithm superior to the latter (for our purposes, we will consider gerrymandering an "algorithm"). The five criteria are as follows:

1. *Equal Population.* Congressional districts are to be nearly equal in their population counts.
2. *Contiguous.* All parts of the district are connected to all others.
3. *Compact.* The average distance per person to a district's center of population is minimal.
4. *Fair contest.* No district's elections should be needlessly one-sided.
5. *Representation.* Communities of common interest are grouped into the same district and minorities are not diluted (i.e., split among several districts).

[12] http://maltman.hmdc.harvard.edu/dispdf/dis_5.pdf

With these criteria in mind, we will consider the relative merits of three algorithms:

1. The current practice of gerrymandering, where redistricting is done entirely by humans.
2. A computer algorithm developed by Ivan Ryan known as "split-line districting."
3. The same computer algorithm but with a modification of our own intended to address some criticisms of the original.

First, consider gerrymandering. It is documented that the current process of gerrymandering creates contiguous districts that are population-balanced but not at all compact (see Figure 1), addressing concerns 1 and 2, but not 3. On the other hand, gerrymandering, by definition, is the antithesis of concern 4: Rather than balancing contests, gerrymandering makes them more one-sided.[13] As a byproduct, gerrymandering tends to address concern 5, since a politician will ensure that his supporters fall into his district. In fact, so-called *affirmative gerrymandering*, which followed the 1965 Voting Rights Act, was viewed in a positive light since it deliberately grouped minorities together.[14]

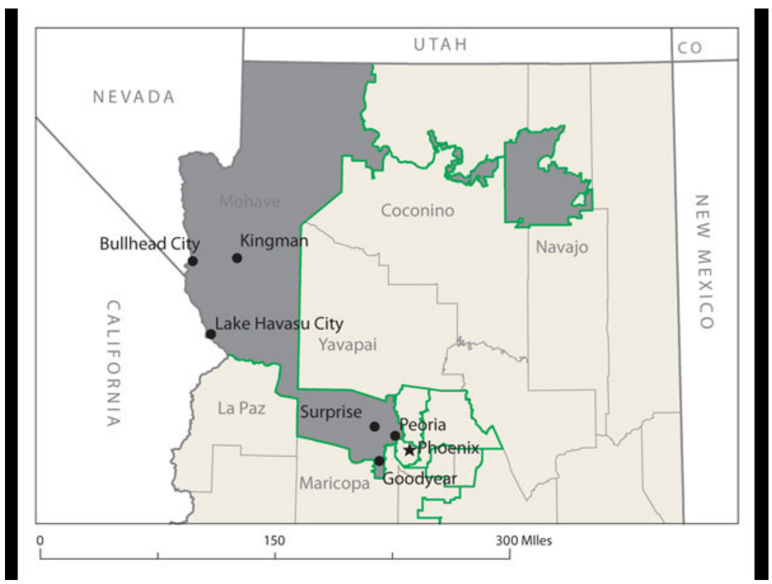

Figure 1: Arizona's second congressional district, demonstrating the highly noncompact regions gerrymandering can produce.

Second, consider the split-line algorithm. The split-line algorithm works as follows: First, the algorithm determines the shortest line that will divide a state into 2 parts of equal population and draws this line. Then, this process is repeated on the two

[13] http://en.wikipedia.org/wiki/Gerrymandering; http://www.allaboutvoting.com
[14] http://www.allaboutvoting.com

subdivided sections. While there are more subtleties to deal with in special cases, the demonstrated net result is a series of convex districts in which the population is equal and whose total perimeter is nearly minimal (see Figure 2). Again, concerns 1 and 2 are addressed. Thanks to the convexity of the regions in question, concern 3 is clearly addressed more effectively than in plan 1, though a provably optimal algorithm for compactness is far more difficult to implement.[15] Concern 4 is not explicitly addressed, but one should expect that, in either case, the average political climates of the districts will reflect the political climate of the state as a whole. In case 1, the districts are polarized in their political leanings, while in case 2 the districts' political leanings will be pseudo-randomly distributed around the mean. Statistically speaking, this means that one would expect the political leanings per state to have a smaller standard deviation in case 2. This analysis suggests that there will be more close contests when plan 2 is implemented. The only exception to this rule would be states that are *exceptionally* polarized *on average*, which is exceedingly rare. Finally, concern 5 is not addressed at all.

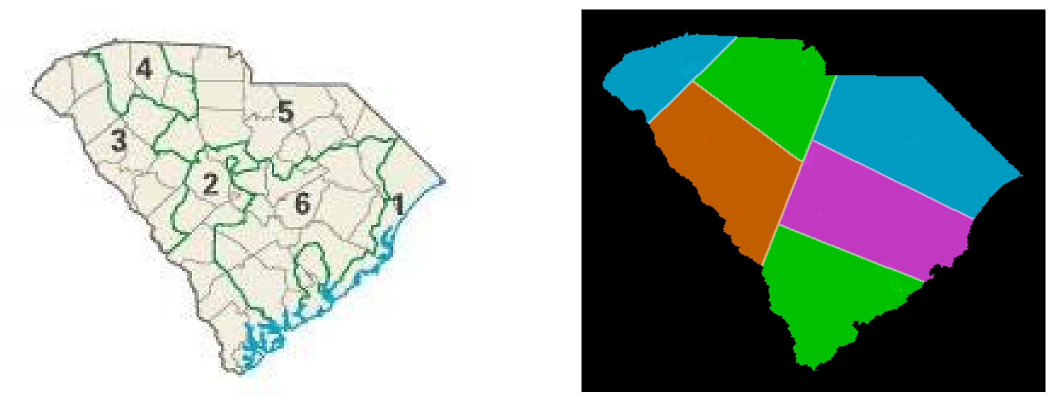

Figure 2: An example map produced by split-line districting.

Before presenting our own solution, it would be worthwhile to analyze the pros and cons of either existing solution. According to Altman, 37 states *require* concern 2 and 24 “require” concern 3, though only two of them have a rigorous definition of it. Furthermore, plan 1 is dubious for its neglect of concerns 3 and 4, while plan 2 is dubious for its neglect of concern 5. Our proposal is to modify plan 2 as follows.

Before the split-line algorithm is implemented, state legislatures are allowed to partition the blocks into pairwise-disjoint sets of at most *N* elements. The split-line algorithm is implemented with the added criterion that no line may divide a set of blocks.

[15] http://bolson.org/dist/

The intent is that this will allow a state legislature to group together blocks that define some community of interest, or preserve together the blocks containing a minority so as not to dilute it. If N remains small, the algorithm continues to address concerns 3 and 4 effectively. On the other hand, as N grows large, concern 5 is addressed more effectively. Thus, the real difficulty is determining an optimal value of N to balance these opposing tendencies. Unfortunately, implementing the split-line algorithm accurately is beyond the scope of this paper. Even so, it is the authors' belief that an appropriate amount of simulations on a state-by-state basis could lead to discovering a value of N that would work; in any case, the modifications to the algorithm presented in plan 2 are slight.

All in all, we believe our algorithm is potentially an improvement over previous algorithms, but more testing would be required to ensure that it works.