M^3 Challenge Third Place, Cum Laude Team Prize of $10,000

Maggie Walker Governor's School, Team #365
Richmond, VA

Coach: Kristine Chiodo

Students: Susan Margaret Ballentine, William Steadman Farmer, Ashish Ashok Makadia, Cody Tyler Talmadge, Milton Frederick Tyler IV

THE SENSIBLE CENSUS

SUMMARY

As mandated by our Constitution, the United States Government requires an "enumeration" every ten years to determine the number of people in the country so that it can apportion seats of the House of Representatives appropriately, a process we call the Census. Despite the best efforts of the U.S. Government, our nation's census remains inaccurate—the count is always below the true number, which is estimated by the Accuracy and Coverage Evaluation. To make up for those missed in the count, the government has imputed data; however, even this number is inaccurate. Our goal was to compare and devise a method to more accurately predict the actual population of the United States. We were also asked to find a method to fairly apportion the Congressional seats and find a method to redistrict each state while avoiding gerrymandering, which are both processes resulting from the census. In our approach, we looked at each section of the problem separately; although each previous problem impacts the next in a given situation, they can be looked at independently.

For our first model, we decided to use a mark and recapture method, used by the Census Bureau with the Accuracy and Coverage Evaluation (ACE), to readjust the census data. According to our calculations, using a mark and recapture method will yield less error than using the census alone to estimate the population. To test the model, we compared the standard error of the mark and recapture method to the undercount error in the 2000 Census, using data from the 2000 Census and ACE. From data given by the Statistical Research Center of the Census Bureau, the standard error of the ACE was found to be 541,631, about .19% of the population. This error is less than the number of people undercounted in the 2000 Census, 1,331,656 people, about .48% of the population.

We proposed that the best method for reapportionment utilizes the CAWES Model which is based on population size and rate of population increase. In most other apportionment models, seats are distributed evenly among all the states, giving each state a number exactly proportional to their populations. However, this number is a decimal, and because partial seats cannot be distributed, different models have been created to determine how these seats are given out. The CAWES model rounds this decimal number down and then gives seats back; due to the Constitutional requirement, it first gives those states with zero seats one. Then it distributes the leftover seats based on the population growth rates of each state. Our model contains many discrepancies with the currently used model (21 states differ in their apportionments). However, all differences between other models and the CAWES Model differ by only one seat, except Texas which gains two more with our method due to its increasing population.

It is proposed that the best method for redistricting the provincial cake between the Democratic and Republican Parties is the modified Zeph–Landau method. This model stems off the philosophy "I cut, you choose" in order to prevent the unfair distribution of districts and thus gerrymandering. The modified Zeph–Landau method allows states that are not equally party- distributed to have slanted districts, yet not let some districts consistently dominate others. This model could be applied in every state and is an ideal answer to stop gerrymandering.

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INTRODUCTION

Background

According to the Constitution of the United States of America, an "enumeration shall be made within three years after the first meeting of the Congress of the United States, and within every subsequent term of ten years, in such manner as they shall by law direct." A census accounts for the population from each district from each state as well as those individuals overseas given by employing federal departments and agencies from their administrative records.[1] The census does not include private citizens that live abroad. Census presents "information about race, Hispanic origin, age, sex, household type, housing tenure, and other social, economic, and housing characteristics."

Restatement

This paper addresses the following questions: (1) How should the census figures be adjusted for the undercount of population? (2) What are the errors of our solution and how are they calculated? (3) What method should Congress select for apportioning the House of Representatives and how does this method compare to other methods? (4) What recommendations should be made to the states to ensure that Congressional districts are fairly drawn, and how are these recommendations justifiable?

Global Assumptions

1. There will be no loss or gain in states during the time the Census is taken.
2. There will be no sudden unexpected gains or losses in population.

CENSUS UNDERCOUNTS

Rationale

The Census Bureau's estimate of the population of the United States for April 1, 2000 reported 278 million people living in the United States. The actual census count as announced by the Census Bureau on December 28, 2002 was 281.4 million people. Over three million people had not been accounted for in 2000.[2] In the 2000 Census, "approximately 5.77 million persons had all their characteristics imputed" and were added to the census with imputations. Currently, "undercounts cost more than $26,000 [in lost federal funding] per 1000 people not counted."[3]

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1. http://www.census.gov/prod/2001pubs/c2kbr01-7.pdf
2. http://www.psc.isr.umich.edu/dis/acs/handouts/SupremeCourt.doc
3. http://news.newamericamedia.org/news/view_article.html?article_id=e3183a93b659c0a19ebf4c439f0fe64b

There is much debate over imputations mainly because one party may be favored over another. "In general, poorer neighborhoods have higher undercount rates" because of "limited English proficiency,"[4] and those individuals may sway toward a given party.

There are numerous reasons for the undercount in the 2000 Census, many of which will be repeated in the 2010 Census. One reason that people are missed in the census is because not everyone has a Decennial Master Address File (MAF) ID.[5] Also, the form the census uses only has six people per household; if the household is larger, the extra people can only be accounted for by follow-up interviews and phone calls, which are unreliable. "Some immigrant workers have told ethnic newspapers and radio programs that they fear personal information could be used against them if it is revealed to local authorities—even if the information turns out to be inaccurate."[6] Another problem is that if the method for collecting the census data is not completed by the deadline, people's information will be left out.

To prevent this major undercount, imputations should be taken into account. The U.S. Census Bureau created the Accuracy and Coverage Evaluation (ACE) in order to allow for the possibility of correcting the census results for the measured undercount. Currently there are three imputation types: The Household Size Imputations are used to estimate the number of persons in a household when occupancy is ambiguous. The Occupancy Imputations are used to estimate the population for housing units that do not have sufficient info to classify them as occupied or vacant. The Status Imputations are used to estimate when the Census has insufficient records about whether an address is valid and nonduplicated; the Census then imputes the status of the unit.

Assumptions

1. During the interval between the census taking and the follow-up ACE, the proportions of people that were surveyed in the census and those who weren't remain the same. This means no new people are born, die, immigrate, or emigrate.
2. The two surveys are independent, meaning the chance that a person was counted in the ACE does not change depending on whether or not they were counted in the census.
3. There is sufficient time between the census and the ACE for all "marked" individuals to be randomly dispersed in the population.

Design

After the initial census period, the Census Bureau should conduct a postcensus population estimate, currently called the Accuracy and Coverage Evaluation (ACE), using a mark and recapture method to account for the people missed in the original count. In this mark and recapture method, a second sample should be taken of randomly selected areas within each state. The extent of the overlap should then used to adjust the census population estimates for each state. The formula used for adjustment is:

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4. http://www.lewis.ucla.edu/publications/workingpapers/LACensusUndercount.pdf
5. http://www.census.gov/dmd/www/pdf/Report21.PDF
6. http://feetin2worlds.wordpress.com/2009/03/31/us-census-reaches-out-to-ethnic-media-to-avoid-undercounting-of-minorities/

$$\frac{R}{M} = \frac{C}{N}$$

and therefore, $N = \frac{CM}{R}$

where

N = Population size for each state
M = Total number of individuals surveyed and recorded in the census
C = Total number of individuals surveyed during the ACE
R = Number of individuals surveyed in the census that were then resurveyed during the ACE.[7]

Error Analysis

While the mark and recapture method would improve upon the raw census data, there will still be some error. This can be found using the standard error:

$$SE = \sqrt{\frac{(M+1)(C+1)(M-R)(C-R)}{(R+1)^2(R+2)}}.$$

The standard error can be used to find a 95% confidence interval for the population size, where

$$Population = N \pm (1.96)(SE).$$

This formula gives a range where 95% of the time, when calculated using sample data, the actual value of the population is included.[8]

Testing the Model

In order to find and compare the error of this model with the undercounts of the census, we would need data from both the 2010 Census and ACE. So, since the error for the 2010 Census cannot yet be calculated, to test our model we compared the standard error of the mark and recapture method in the 2000 ACE to the 2000 Census. Using data from the Statistical Research Center of the Census Bureau, the standard error of the ACE was found to be 541,631. This error is less than the number of people undercounted in the 2000 Census, 1,331,656 people.[9] This means that using the mark and recapture method will yield less error than using the census alone.

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7. http://www.pitt.edu/~yuc2/cr/main.htm
8. http://www.neiu.edu/~jkasmer/Biol380/Labs/mark&.htm
9. http://www.census.gov/srd/papers/pdf/rrs2006-03.pdf

APPORTIONMENT

Rationale

To understand our choice for apportionment, we must first understand a bit about the basics of apportionment. The term "standard divisor" refers to the ratio of the total population to the number of seats, while the "standard quota" for a state refers to the population of the state divided by the standard divisor. This produces a number which will be used to calculate the number of seats for a state; a state's "upper" and "lower" quotas are the closest integers above and below this number.

In choosing a method, we faced a dilemma: choose a method that would allow for paradoxes when population, the number of seats, or the number of states changed, or choose a method that would violate the quota rule, where each state must receive either their upper or lower quotas. Unfortunately, both cannot be satisfied; the Balinski–Young impossibility theorem says that both the paradoxes and the violated quota rule cannot be avoided.

Assumptions

1. No new states will be added to the union; therefore, the *New States* paradox (adding a new state but keeping the number of representatives constant resulting in an increase in representatives for an existing state) would never occur.
2. The total number of seats in the House of Representatives will remain constant. Since the number of seats is not increasing, the *Alabama* paradox (a state losing a seat when the total number of seats increases) will never occur.
3. Each state must have at least one representative.

Design

For apportionment, the CAWES model is proposed to determine the number of representatives each state receives in the House of Representatives. Because we have limited the number of seats and number of states, the only paradox worth noting is the population paradox, where if one state has an increase in population at a faster rate than others, it may still lose seats. The CAWES model accounts for the population paradox and does not violate the quota rule, which states that the apportionment always allocates only lower or upper bounds of the quota.

The CAWES Model attempts to avoid as many of the paradoxes as possible. By its definition, it avoids breaking the quota rule. All states are given the lower quota, and then one more is added only to states that qualify, giving them their upper quotas—nothing will ever fall outside these bounds. As for paradoxes, our model avoids the population paradox by awarding extra seats to those that are increasing at the fastest rate. None of the other paradoxes are possible in this scenario either, because neither states nor seats are being added in our model. This is because none have been changed since the sixties, and none are planning on changing anytime soon. A bonus of our model is that it takes into account not only the present, but also the future. By predicting the rate of change of each state, we can better represent the state not only at the time of the census, but also between censuses.

Variables:

- P_{total} = Total population of the United States (in 2010)
- P_{2005} = the Census projected population for 2005
- P_{2015} = the Census projected population for 2015
- M = Number of seats in the House of Representatives
- P_i = Population of an individual state (where $1 \leq i \leq 50$)
- SD = Standard divisor
- Q_i = Standard quota for each state
- LQ_i = Lower quota for each state
- Y = Seats given to states with an LQ_i of zero
- S = Surplus number of seats
- R_i = Number of representatives each state receives
- r_i = rate of population increase for each state

Model

First, the total population of the United States is divided by the number of seats in the House of Representatives to calculate the standard divisor:

$$SD = \frac{P_{total}}{M}.$$

Each state's population is divided by the standard divisor to calculate the standard quota, or the number of representatives each state is expected to receive:

$$Q_i = \frac{P_i}{SD}.$$

The lower quota of each state is calculated by rounding the quota down to the nearest whole number. Those states with a lower quota of zero automatically get one seat added so that each state will have at least one representative. Next, the surplus is calculated by subtracting the sum of all the lower quotas and those given to states with a zero for a lower quota from the total number of representatives (435):

$$S = M - \sum_{i=1}^{50} LQ_i - Y.$$

Then, the rate of population increase for each state in 2010 is also calculated from the census projected population rates from 2005 and 2015 such that

$$r_i = \frac{P_{2005} + P_{2015}}{10}.$$

The top S number of states with the highest rate of population increase in 2010 will also receive one additional seat unless that state's LQ was originally zero. Thus those states that have a population increasing more than others will receive extra seats and therefore prevent the population paradox. If done correctly, the total number of representatives should equal the number of representatives to be divided:

$$\sum_{i=1}^{50} R_i = M = 435.$$

Results

State	2005 Pop.	2010 Pop.	2015 Pop.	Standard Quota	Rate of Change	Lower Quota	2010 Predicted Apportionment
Alabama	4447100	4527166	4596330	6.68	0.0999	6	7
Alaska	626932	661110	694109	0.98	0.0999	0	1
Arizona	5130632	5868004	6637381	8.65	0.1003	8	9
Arkansas	2673400	2777007	2875039	4.10	0.0999	4	5
California	33871648	36038859	38067134	53.15	0.0998	53	54
Colorado	4301261	4617962	4831554	6.81	0.0989	6	7
Connecticut	3405565	3503185	3577490	5.17	0.0997	5	5
Delaware	783600	836687	884342	1.23	0.0997	1	1
Florida	15982378	17509827	19251691	25.82	0.1006	25	26
Georgia	8186453	8925796	9589080	13.16	0.0996	13	14
Hawai'i	1211537	1276552	1340674	1.88	0.1	1	1
Idaho	1293953	1407060	1517291	2.08	0.0999	2	3
Illinois	12419293	12699336	12916894	18.73	0.0998	18	18
Indiana	6080485	6249617	6392139	9.22	0.0998	9	9
Iowa	2926324	2973700	3009907	4.39	0.0998	4	4
Kansas	2688418	2751509	2805470	4.06	0.0998	4	4
Kentucky	4041769	4163360	4265117	6.14	0.0998	6	6
Louisiana	4468976	4534310	4612679	6.69	0.1001	6	6
Maine	1274923	1318557	1357134	1.94	0.0998	1	1
Maryland	5296486	5600563	5904970	8.26	0.1	8	9
Massachusetts	6349097	6518868	6649441	9.61	0.0997	9	9
Michigan	9938444	10207421	10428683	15.05	0.0998	15	15
Minnesota	4919479	5174743	5420636	7.63	0.0999	7	8
Mississippi	2844658	2915696	2971412	4.30	0.0997	4	4
Missouri	5595211	5765166	5922078	8.50	0.0999	8	9
Montana	902195	933005	968598	1.38	0.1003	1	1
Nebraska	1711263	1744370	1768997	2.57	0.0998	2	2
Nevada	1998257	2352086	2690531	3.47	0.0997	3	4
New Hampshire	1235786	1314821	1385560	1.94	0.0997	1	2
New Jersey	8414350	8745279	9018231	12.90	0.0997	12	12
New Mexico	1819046	1902057	1980225	2.81	0.0999	2	2
New York	18976457	19258082	19443672	28.40	0.0998	28	28
North Carolina	8049313	8702410	9345823	12.83	0.0999	12	12
North Dakota	642200	635468	636623	0.94	0.1006	0	2
Ohio	11353140	11477557	11576181	16.93	0.0999	16	17
Oklahoma	3450654	3521379	3591516	5.19	0.1	5	5
Oregon	3421399	3596083	3790996	5.30	0.1003	5	6
Pennsylvania	12281054	12426603	12584487	18.33	0.1001	18	18
Rhode Island	1048319	1086575	1116652	1.60	0.0996	1	2

South Carolina	4012012	4239310	4446704	6.25	0.0998	6	7
South Dakota	754844	771803	786399	1.14	0.0999	1	1
Tennessee	5689283	5965317	6230852	8.80	0.0999	8	8
Texas	20851820	22775044	24648888	33.59	0.0999	33	34
Utah	2233169	2417998	2595013	3.57	0.0998	3	4
Vermont	608827	630979	652512	0.93	0.1	0	1
Virginia	7078515	7552581	8010245	11.14	0.0999	11	11
Washington	5894121	6204632	6541963	9.15	0.1002	9	10
West Virginia	1808344	1818887	1829141	2.68	0.1	2	2
Wisconsin	5363675	5554343	5727426	8.19	0.0998	8	8
Wyoming	493782	507268	519886	0.75	0.0999	0	1
TOTAL	280849847	294955998	308405796				435

Figure 1: The CAWES Model with the predicted values for 2005, 2010, and 2015.

States that gain more than one seat (using the CAWES Model as opposed to the currently used Huntington–Hill model):

- Texas

States that gain one seat:

- Arizona
- Florida
- Hawai'i
- Maine
- Nebraska
- New Mexico
- North Carolina
- Tennessee
- West Virginia

States that lose one seat:

- Arkansas
- California
- Idaho
- Maryland
- Michigan
- Minnesota
- Missouri
- North Dakota
- Ohio
- Oregon
- Washington

As we can see, most of the data fits the current trend that most of the population is shifting from the North and East to the South and West. Note that just because a state loses a seat from the current model to the CAWES model does not necessarily mean that that state is losing citizens. Rather, they are either growing at a slower rate than the rest of the union or they have been over-represented in the past.

CONGRESSIONAL DISTRICTS

Rationale

When the demographics of an area change, redistricting occurs so that the population is equally distributed among the districts and each representative represents roughly the same percentage of the state. Currently for thirty-six states, the state legislature has the primary responsibility for creating a redistricting plan for its given state. In some states this is subject to approval by the state governor. Five states redistrict under the direction of a commission. In two states, state legislatures must approve redistricting plans proposed by independent bodies. Seven states have only a single representative for the entire state.

Gerrymandering refers to redistricting in which electoral districts are deliberately modified to give advantage to a particular group or candidate; this is usually done by linking similar areas, that contain a particular group of people, in a contorted or unusual shape. This can be prevented through legislation. Currently certain types of gerrymandering are illegal, such as those that deliberately isolate a certain race. Gerrymandering is a problem because it does not reflect the actual will of the entire population through true democracy.

Assumptions

1. There are two opponents in redistricting, the Republicans and the Democrats.
2. The districts should be fairly drawn and should prevent gerrymandering.

Design

The redistricting process could be executed fairly if each state were to use the our modified version of the Zeph–Landau method for distributing population size, which is based on the philosophy of "I cut, you choose" used when dividing a cake. Thus, we recommend the modified Zeph–Landau method for reapportionment.

This method works by dividing the state into 25 sections such that all the sections have an equal number of people. For example, see Figure 2.

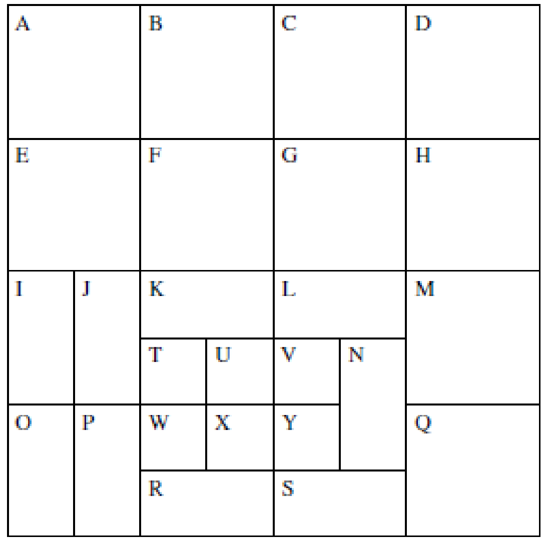

Figure 2: An example of a state being divided by population before redistricting occurs.[10]

Next, a random line is drawn dividing the state in two (see Figure 3). The placement of this line does not matter.

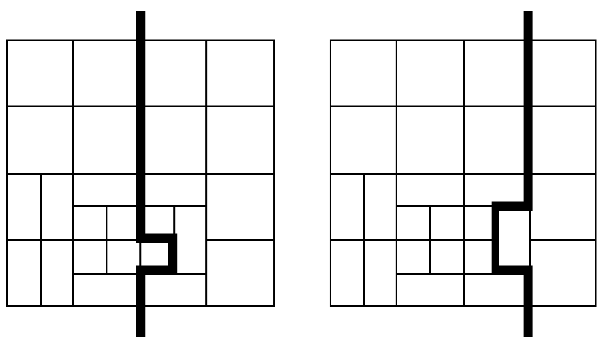

Figure 3: An example of two possible dividing lines.[11]

Step 1: The Republican Party can divide piece #1 of the split into *k* districts and have Democratic Party divide piece #2 into *n* - *k* districts (where *n* is the total number of districts allocated by the reapportionment). The resulting wins and losses for each party, based on the known voting records for these districts, are recorded for this division.

Step 2: Then, the Democratic Party divides piece #1 of the split into *k* districts and have the Republican Party divide piece #2 into *n* - *k* districts. These resulting wins and losses for each party, based on the known voting records for these districts, are recorded for this division.

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10. http://www.eecs.berkeley.edu/~landau/papers/redistrictFZsubmitted.pdf
11. http://www.eecs.berkeley.edu/~landau/papers/redistrictFZsubmitted.pdf

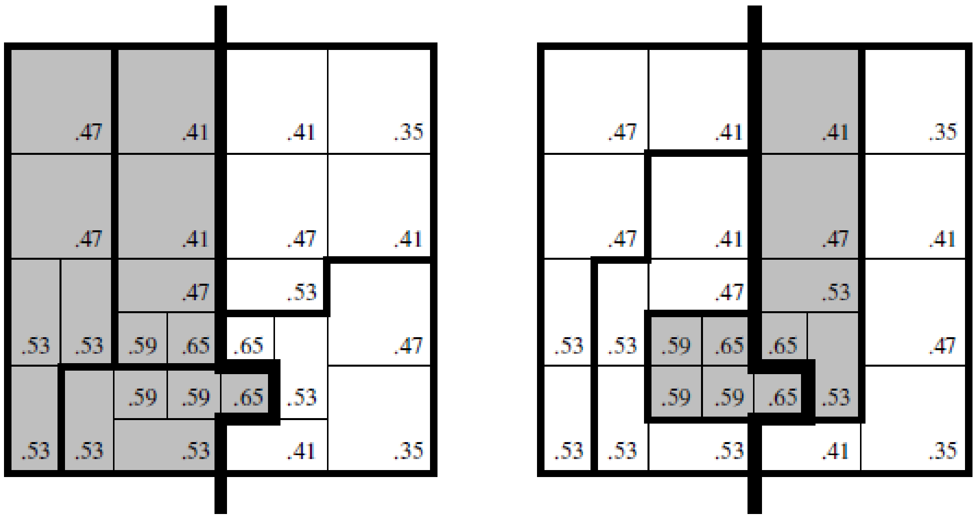

Figure 4: The numbers in each section represent the ratio of Republicans to Democrats based on polls. The first diagram shown is Step 1, in which the Republican Party divides the left piece and the Democratic Party divides the right piece. The second diagram is Step 2, in which the Democratic Party divides the left piece and the Republican Party divides the right piece.[12]

After repeating these procedures multiple times with different dividing lines, a table is constructed with this gathered data as shown below. The table is ordered by ascending wins for one of the Steps. There will be a point at which the favorable step for that party switches between Step 1 and Step 2. This is the point in which the number of representative districts is about equal toward each party.

	Step 1: Part 1+Part 2 = total	Step 2: Part 1+Part 2 = total
Line 1	1+1=2	1+3=4
Line 2	1+1=2	1+2=3
Line 3	3+0=3	1+1=2
Line 4	4+0=4	1+0=1

Figure 5: This figure shows an example of several divisions (referred to as Line 1, Line 2, etc.) and the resulting Republican Party win distribution. For example, Figure 3 would be the Line 3 division which in Step 1 gives Republicans 3 wins while Step 2 gives Republicans 2 wins. In this example, the Republican Party would change from Step 2 to Step 1 between Line 2 and Line 3.

The change between the steps serves as a point in which the party distribution is about equal. This leaves four possible rearrangements of the distribution. For example, in Figure 5, Republicans can have:

12. http://www.eecs.berkeley.edu/~landau/papers/redistrictFZsubmitted.pdf

- Step 1 and Line 2, leaving Republicans with 2 districts.
- Step 1 and Line 3, leaving Republicans with 3 districts.
- Step 2 and Line 2, leaving Republicans with 3 districts.
- Step 2 and Line 3, leaving Republicans with 2 districts.

The modified Zeph–Landau method would then look to the party distribution for Democrats and Republicans in that state, and the party with the most supporters would get the most districts in that state. For example, in the above scenario, the given state is majority Republican and thus the Republicans would end with 3 districts. This would narrow the choices down to:

- Step 1 and Line 3, leaving Republicans with 3 districts.
- Step 2 and Line 2, leaving Republicans with 3 districts.

Between these two choices, the result would be randomly chosen.

The reason the modified Zeph–Landau method is used rather than the Zeph–Landau method is because the latter method does not account for states that lean heavily toward a party but assumes that a gerrymandering-free state is one that is equally Republican and equally Democrat. The modified Zeph–Landau method, however, allows states that are not equally distributed to have slanted districts, yet not so slanted that the districts consistently dominate one another.

Testing. This model could be applied in every state. Because there is no current system to prevent gerrymandering other than legislative agreement, the modified Zeph–Landau method would be ideal.

CONCLUSIONS

Our population estimates error using the mark and recapture method was estimated at approximately half a million people, far less than the 1.3 million people uncounted by the Census. While our method is not perfect, it is far more accurate than using the Census alone.

The Huntington–Hill method is currently used to apportion the seats in the House of Representatives to the states. This method has a major problem—it can violate the quota method, causing a state to get far more or fewer states than would logically be expected. Our CAWES model does not have this problem; we assign all states at least the minimum number of seats they would expect to receive and then assign the remaining seats based on the rate the population of each state is increasing. Since the states with the fastest increasing population are more likely to "deserve" seats before the next apportionment, we decided to assign the extra seats to these states.

Compared to the current Huntington–Hill method of apportionment, our CAWES model takes into account not just the current population, but also the predicted future population. This provides for a better representation of the state into the future rather than just at the present. It also doesn't severely damage the amount of seats according to the current model that predicts the 2010 data.

We used a modified version of the Zeph–Landau method to redistribute states. This stems from the philosophy "I cut, you choose" and makes both parties work together in order to divide the districts evenly among the parties. In this way, one district can never consistently dominate the others.

The models proposed are the best that we recommend but have flaws nevertheless. The mark and recapture method is a statistical analysis and therefore will always contain some degree of error. For example, if the time between surveying is too great, then the demographics may have changed, and therefore the two surveys would not be comparable. The CAWES model becomes potentially problematic upon introduction of the Alabama and New States paradoxes, but since new states or new seats cannot be added, this is not a problem. Also, since the seats are allocated based on the rate of population change, the Population Paradox should also not be a problem. The modified Zeph–Landau method does not work for more than two parties and takes longer than the current methods of redistricting because it requires every district to be reevaluated after every Census.

Another idea for a possible future model to imputate the census is to analyze past birth, death, immigration, and emigration data on a state-by-state basis to calculate the states' net population changes. As long as good records are kept, this would allow for a census to be approximated any year and therefore assist in census imputation. This model would not account for illegal immigrants.

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