M[3] Challenge Fifth Place (Exemplary Team Prize) $5,000
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Team 659

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EXECUTIVE SUMMARY

High-speed rail is a practical means of transportation that offers a rapid, efficient form of travel utilized by countries around the world. Efforts have been made to increase the amount of high-speed railway travel in the United States beyond the Acela Express line on the Eastern coast. One such effort is the High-Speed Intercity Passenger Rail Program (HSIPR), which aims to develop a speed rail network in ten heavily populated metropolitan areas across the country.

We developed five models to assess the following in each HSIPR-identified area: ridership on high-speed rail, cost to construct and operate the railways, Gross Domestic Product, and reduction in oil use due to increased high-speed train usage. To determine the ridership on the high-speed railways, we created a logarithmic model asymptotic to a value proportional to the projected population growth of each region and evaluated projections of riders diverted to high-speed rail from airplane, automobile, and low-speed train transportation. We evaluated construction and operating costs by measuring track construction costs, equipment purchases, contingency costs, soft costs, and right of way costs. We based the GDP model on the multiplier resultant from government infrastructure investment, and the oil reduction model on the reduction of gas use from switching from car and plane transportation to high-speed railways.

We concluded that the California region was the most viable location to construct a high-speed rail facility in terms of operating costs per rider and ridership due to the high population density and the potential for decreased unemployment through government investments. The projected construction cost is $189 billion but has a lower operating cost than any other region, $174 per ride.

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1|INTRODUCTION

1.1|BACKGROUND OF THE SITUATION

The High-Speed Intercity Passenger Rail Program (HSIPR) is one of the most ambitious and far-reaching railroad projects known to date. Initiated by the Federal Railroad Association (FRA) in June 2009, the HSIPR aims to construct an efficient and rapid form of transport between major population centers 100 to 500 miles apart. It also offers environmental benefits from reducing the number of people traveling by car, plane, and bus and improves economic activity between cities (Federal Railroad Administration).

The HSIPR has identified ten potential regions in the United States in which to develop high speed rail corridors. The regions are: California, Chicago, Southeast, Pacific Northwest, Keystone, Empire, Northern New England, Gulf Coast, Florida, and South Central (Federal Railroad Administration).

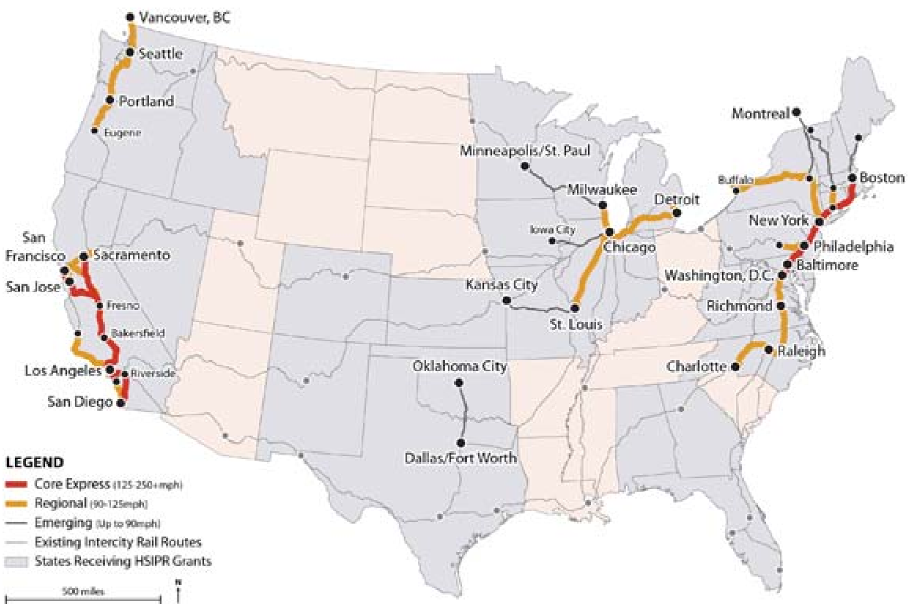

Figure 1: Map of the planned HSIPR system

The HSIPR was formed by passing two pieces of legislation, the Passenger Rail Investment and Improvement Act of 2008 (PRIIA) and the American Recovery and Reinvestment Act of 2009 (ARRA or Recovery Act). Combined with the FY 2010 Department of Transportation Appropriations Act, a total of $10.5 billion was appropriated to supplement projects to construct the high-speed rail systems (Texas Department of Transportation). However, the HSIPR has not been successfully implemented. 39 states, the District of Columbia, and Amtrak have requested over $75 billion in applications, greatly exceeding the funds provided by the government (Texas Department of Transportation), and in November 2011, Congress eliminated funding for high-speed rail entirely (Midwest High Speed Rail Association).

Currently, the Acela Express railway line from Washington D.C. to Boston is the only high-speed railway system in the United States (High-Speed Rail in the United States). The line carried over 3.2 millions passengers in 2010, and over 25 million passengers have traveled on the train since the train's inception in 2000 (Amtrak National Facts). Utilizing a densely populated region and high demand for travel around Boston, the tri-state area, and Washington D.C., Acela has managed to earn a profit over its years of operation (Slate Magazine).

2|ASSUMPTIONS

We will assume the following information throughout our analysis:

- The population of every HSIPR-identified region grows every year at a linear rate equal to the yearly population growth of the United States
- The costs of fuel and building materials are constant across the country

3|MODELING AND FORECASTING RIDERSHIP

3.1|THE MATHEMATICAL MODEL AND HIGH-SPEED RAILWAY RIDERSHIP PROJECTIONS

In order to model the number of people who will ride the high-speed railway system, we first considered all of the types of travelers who would convert to riding on high-speed rail from another form of travel. The major sources of travel that we determined people currently take in the 10 HSIPR regions are by automobile, bus, plane, and low-speed train. We can develop a generic equation to model the number of people diverted to high-speed railway travel:

$D_{total} = D_a + D_b + D_p + D_t,$

where D_a, D_b, D_p, and D_t represent the number of diverted travelers who were previously traveling by automobile, bus, plane, and low-speed train, respectively. Using data provided by the Centerhood for Neighborhood Technology, we found the following numbers of passengers expected to be diverted to high-speed rail from other conventional travel by the year 2025:

Region	2012 Population (millions)	2025 Projected Population (millions)	Number of Diverted Auto Passengers by 2025 (millions)	Number of Diverted Bus Passengers by 2025 (millions)	Number of Diverted Plane Passengers by 2025 (millions)	Number of Diverted Slow-Speed Train Passengers by 2025 (millions)	Total Number of Diverted Passengers by 2025 (millions)
California (HSR)	24.215	26.714	19.237	0.241	17.599	2.52	39.597
Chicago Hub Network	48.833	53.873	2.502	2.945661	8.234	13.894	27.576
Southeast	33.383	36.829	0.579	0.636	1.566	0.141	2.922
Pacific Northwest	16.072	17.731	0.864	0.16	1.504	0.384	2.912

Keystone	11.157	12.309	0.069	0.03156	0.055	0.901	1.05656
Empire	18.938	20.893	2.311	0.053562	1.074	5.478	8.196562
Northern New England	10.333	11.34	0.013	0.00667	0.647	0.247	0.914
Gulf Coast	16.308	17.991	0.727	0.103853	3.583	0.208	4.621853
Florida	13.169	14.528	0.585	0.06882	2.374	0.138	3.16582
South Central	15.124	16.685	1.12	0.288	1.408	0.064	2.88

Figure 2: Projected populations and riders diverted from conventional forms of transportation to high-speed railway for each HSIPR region

Using the number of people expected to be diverted to high-speed rail from each region in the year 2025, we can determine the number of people who join high-speed rail. We assumed that at $t = 0$, where t represents the number of years after 2016, the presumed start date of operation of the high-speed rail system, 0 people will have converted over to high-speed rail, and at $t = 9$ years, the total number of passengers who have converted to high-speed rail will be the specified numbers for each region as shown by the table above.

To model the growth in the total number of passengers who divert to high-speed rail as a function of time, we determined that the growth should initially start out large since many people will initially divert to the service and, over time, become asymptotic to the natural population growth rate. Population predictions for the United States through 2050 are projected to be roughly linear, and the population growth each year is approximately 2.403 million people per year (Negative Population Growth). Since the population of the United States is 302,624,000 people as of 2012 (Negative Population Growth), the growth of the population of the United States will be about 0.794% of the current population of 302,624,000. Assuming that the population growth rate for each HSIPR region is the same as the population growth rate of the United States, then each region's population will increase by 0.794% of its 2012 population each year. Therefore, each model for the total number of people diverted to high-speed rail service will need to be asymptotic to $(0.00794P)*t$, where P is the 2012 population of the HSIPR region, and increase at a decreasing rate. However, since not all of the new population in a region will necessarily convert to rail, so the model needs to be asymptotic to $(0.00794P)*t*r$, where r is the percentage of people who will use high-speed rail service. According to a survey by the American Public Transit Association, would "definitely" or "probably" use high-speed rail ("Tree Hugger"). Therefore, we set r to 0.62 as a correction factor for the asymptote.

For each model, we started with the equation:

$$\frac{dD}{dt} = rp + \frac{k}{t+1},$$

where $\frac{dD}{dt}$ = rate of change that people divert to high rail service in a given region, p the projected population growth rate of the region, t the number of years after 2016, r = 0.62, and k is an arbitrary constant. As t increases, $\frac{dD}{dt}$ approaches rp, or the rate that people divert to high-speed rail service approaches the natural population growth of the region times the percent of people who would use high-speed rail.

Integrating the equation gives us

$$D(t) = rpt + k * \ln(t+1) + C.$$

By setting the constant C to 0, this equation gives us 0 diverted travelers to high-speed rail 0 years after 2016, which is expected. For each region, we then inputted $t = 9$ and the predicted number of diverted travelers in 2025 for $D(t)$ to solve for k. The following models show the total number of passengers who divert to high-speed rail as a function of t years after 2016:

California: $D(t) = 0.1192t + 16.7307 * ln(t+1)$
Chicago: $D(t) = 0.2404t + 11.0366 * ln(t+1)$
Southeast: $D(t) = 0.1644t + 0.6266 * ln(t+1)$
Pacific Northwest: $D(t) = 0.0791t + 0.9554 * ln(t+1)$
Keystone: $D(t) = 0.0549t + 0.2441 * ln(t+1)$
Empire: $D(t) = 0.0932t + 3.1952 * ln(t+1)$
Northern New England: $D(t) = 0.0508t + 0.1982 * ln(t+1)$
Gulf Coast: $D(t) = 0.0803t + 1.6934 * ln(t+1)$
Florida: $D(t) = 0.0649t + 1.1214 * ln(t+1)$
South Central: $D(t) = 0.0745t + 0.9597 * ln(t+1)$

Using these models, the predicted high-speed rail ridership numbers for each region from 2016-2036 are as follows:

Region	**Ridership in 2021 (millions)**	**Ridership in 2026 (millions)**	**Ridership in 2031 (millions)**	**Ridership in 2036 (millions)**
California	30.57	41.31	48.175	53.321
Chicago	20.976	28.869	34.206	38.409
Southeast	1.945	3.147	4.203	5.196
Pacific Northwest	2.106	3.08	3.833	4.488
Keystone	0.712	1.134	1.5	1.841
Empire	6.191	8.594	10.257	11.592
Northern NE	0.609	0.983	1.312	1.619
Gulf Coast	3.436	4.864	5.9	6.762

Florida	2.334	3.338	4.083	4.712
South Central	2.092	3.046	3.778	4.412

3.2|IMPLICATIONS OF REDUCED RAIL TRAVEL TIMES FOR RIDERS

In general, the change in rail travel times introduced by the high-speed rail system will be positively received, and high-speed rail would become a significant percentage of means of travel. As shown by the data, over 100 million passengers will be travelling by high-speed rail by 2036, with over 1 million passengers in each of the 10 HSIPR-identified areas. Even current prospects suggest that people are already eager to embrace this new form of travel. According to the survey by the American Public Transportation Association, only 11% of the people surveyed stated that they would not want to use high-speed rail ("Tree Hugger"). A faster rail system that eliminates the inconveniences of driving (traffic and parking), flying (high costs, wait times in airports), and traditional trains (slower speeds) will appeal to travelers across the country.

4|COSTS

4.1|COST TO BUILD

We determined that the costs to build a high-speed rail line are divided into five categories: track structures and stations, vehicles, soft costs, contingencies, and right of way. According to "A Vision for High-Speed Rail in the Northeast Corridor," a proposal by Amtrak for a high-speed rail replacement option for the Northeast Corridor, in 2010 dollars, the cost to construct track structure and stations is estimated to be \$67 billion, vehicles are estimated to cost \$3 billion, soft costs are estimated to be \$21 billion, contingencies are estimated to cost \$13 billion, and right-of-way costs are estimated to be another \$13 billion, for a total of \$117 billion to construct a new Next-Gen High-Speed Rail system for the Northeast Corridor (NEC) (Amtrak 20).

To determine the cost to build tracks and stations for the other ten geographic regions, we constructed a proportion based on the projected costs of the NEC and created a function based on the length of the track:

$$Cost_{structure} = \frac{Cost\ of\ NEC\ structures}{Length\ of\ NEC\ track} \times Length = \frac{\$67\ billion}{457\ miles} \times Length$$

$$= 146608315.1\ \times Length$$

To determine the cost of vehicles for the other ten geographic regions, we constructed a ceiling function based on the projected costs for the NEC function, the number of cars in the NEC Acela system, and the projected number of passengers in each of the geographical regions.

According to an article on railway-technology.com, the Acela system has 20 trains, each with 10 cars ("Amtrak to Purchase High-Speed Rail Fleet"). Assuming the Next-Gen High-Speed Rail system will use the same amount of trains, and since the estimated cost of vehicles for the NEC is \$3 billion, the cost per high-speed vehicle car for our model is

$$Cost\ per\ car = \frac{\$3\ billion\ total}{200\ cars\ total} = \$15\ million\ per\ car.$$

There are 434 people per Acela train; thus our equation for cost of vehicles for each geographic region is

$$Cost_{vehicles} = Cost\ per\ car\ \times \frac{Total\ NEC\ cars}{Total\ NEC\ riders} \times Riders$$

$$= \$15\ million \times \frac{200\ total\ NEC\ cars}{3.2\ million\ riders} \times Riders$$

$$= \$15\ million \times 6.25 \times 10^{-5} \times Riders.$$

We assume that the soft costs for the NEC will remain constant for all geographic regions, and therefore all regions will have soft costs of $21 billion.

We determined that contingencies increase as track length increases, but increase at a decreasing rate. Thus, we decided that in order to achieve this characteristic, the contingency cost function would have to take the form of $Cost_{contingency} = a\ln(length_{track} + 1)$. As mentioned previously, based off the Next-Gen estimates, the contingency cost for 457 miles would be equal to $13 billion, thus $\$13\ billion = a\ln(457 + 1)$. Solving for a, we get the value $a = 2.12 \times 10^8$. The final contingency cost function is therefore

$$Cost_{contingency} = 2.12 \times 10^8 \ln(length + 1).$$

We determined that cost of right-of-way would be affected by the price of land in each geographical region; a region with more expensive land would have higher cost of right-of-way, and similarly, a region with less expensive land would have less cost of right-of-way. In order to compute an average price index of commercial and industrial land in the Northeast Corridor, we averaged the price indexes of Baltimore, Boston, New York, Philadelphia, and Washington, DC, which are all stops along the Acela route. According to the Federal Reserve, these price indexes are, respectively, 108, 168, 216, 134, and 230 (Nichols, Oliner, Mulhall 32). Our calculated price index for the NEC is thus

$$Average\ Price\ Index = \frac{108 + 168 + 216 + 134 + 230}{5} = 171.2.$$

We assume the cost of right-of-way for each region is proportional to that of the NEC. Thus, our full right-of-way cost equation is

$$Cost_{rightofway} = \frac{\$13\ billion}{457\ miles} Length \times \frac{Price\ Index}{171.2}.$$

Combining all of these costs, we get the complete equation for cost of construction:

$$Cost_{construction} = Cost_{structure} + Cost_{vehicles} + Cost_{soft} + Cost_{contingency} + Cost_{rightofway}$$

$$= 146608315.1 \times Length + \$15\ million \times 6.25 \times 10^{-5} \times Riders + \$21\ billion + 2.12 \times 10^8 \ln(length + 1) + \frac{\$13\ billion}{457\ miles} Length \times \frac{Price\ Index}{171.2}.$$

See Appendix A for the individual building costs for each region.

4.2|COST TO MAINTAIN

The cost to maintain the high-speed rails, according to “A Vision for High-Speed Rail in the Northeast Corridor,” is comprised of seven categories: train operations, on-board services, maintenance-of-way, electric traction power, equipment maintenance, station services, and sales and marketing (“A Vision for High-Speed Rail in the Northeast Corridor”). Thus, we model total maintenance costs in the equation

$$Cost_{maintenance} = T + S_o + M + P + E + S_s + S_m$$

with

$T = train\ operations, S_o = on\text{–}board\ services, M = maintenance\text{–}of\text{–}way$

$P = electric\ traction\ power, E = equipment\ maintenance, S_s = station\ services,$

$S_m = sales\ and\ marketing.$

The same article projected annual operational costs for the NEC using the Next-Gen plan as follows:

$T = \$156\ million, S_o = \$139\ million, M = \$122\ million, P = \$178\ million, E = \$307\ million,$

$S_s = \$161\ million, S_m = \$194\ million.$

To calculate *T*, we assumed that train operations are dependent on the number of vehicles in each system. Thus, the model for *T* would take on the form $T = aV$ with *V* being the number of vehicles in the system. We plugged the data for the Northeast into the equation to find the constant *a*:

$$\$156\ million = a \times 200\ vehicles; \quad a = 780{,}000.$$

Similarly, to calculate S_o, we assumed that on-board services are also dependent on the number of vehicles in each system. Thus, the model for S_o would take on the form $S_o = bV$. We plugged in the data for the Northeast into the equation to find the constant *b*:

$$\$139\ million = b \times 200\ vehicles; \quad b = 695{,}000.$$

To calculate *M*, we assumed that maintenance-of-way is dependent on the total length of track in each geographic region. Thus, the model for *M* would take on the form $M = cL$ with *L* being the total length of track in each system. We plugged in the date for the Northeast into the equation to find the constant *c*:

$$\$122\ million = c \times 457\ miles; \quad c = 266{,}958.4245.$$

To calculate *P*, we assumed that electric traction power is dependent on the number of vehicles in each system. Thus, the model for *P* would take on the form $P = dV$ with *V* being the number of vehicles in the system. We plugged the data for the Northeast into the equation to find the constant *d*:

$$\$178\ million = d \times 200\ vehicles; \quad d = 890{,}000.$$

To calculate *E*, we assumed that equipment maintenance is dependent on both number of vehicles in each system and the length of track in each system. Thus, the model for *E* would take on the form $E = eL + fV$. To find the constants *e* and *f*, we split the equation up into two—one finding the percentage of *E* devoted to structural costs, with the form $E_s = eL$, and another finding the percentage of *E* devoted to vehicle costs, with the form $E_v = fV$.

We found E_s, the amount of *E* devoted to structural costs, using the equation $E_s = \frac{Structural\ Cost}{Structural\ Cost + Vehicle\ Cost} \times E$. For the Northeast, this amount was

$$E_s = \frac{\$67\ billion}{\$67\ billion + \$3\ billion} \times \$307\ million \approx \$294\ million.$$

Thus, for the NEC,

$$E_s = eL$$

$$= \$294\ million = e \times 457\ miles; \quad e = 643326.0394.$$

Likewise, to find *Ev*, the amount of *E* devoted to vehicle costs, we used the equation $E_v = \frac{Vehicle\ Cost}{Structural\ Cost + Vehicle\ Cost} \times E$. For the Northeast, this amount was

$$E_v = \frac{\$3\ billion}{\$67\ billion + \$3\ billion} \times \$307\ million \approx \$13\ million.$$

Thus, for the NEC,

$$E_v = fV$$

$$= \$13\ million = f \times 200\ vehicles; \quad f = 65{,}000.$$

Plugging the two constants *e* and *f* into the main equipment maintenance equation $E = eL + fV$, we yield the final model

$$E = 643326.0394 \times L + 65000 \times V.$$

To calculate S_s, we assumed that station services are proportional to the number of stations, which we also assumed to be proportional to the total length of track in each region. Thus, the model for S_s would take on the form $S_s = \frac{16}{457} g \times L$, with the constant $\frac{16}{457}$ calculated because the Northeast has 16 stations in 457 total miles of track. Plugging in the data for the Northeast, we found that constant *g*:

$$\$161\ million = \frac{16}{457} g \times 457\ miles; \quad g = 10062500.$$

To calculate S_m, we assumed sales and marketing is proportional to the total length of track in each system. Thus, the model for S_m would take on the form $S_m = hL$. We plugged in the data for the Northeast to find the constant *h*:

$$\$194\ million = h \times 457\ miles; \quad h = 424507.6586.$$

Therefore, the complete equation for maintenance costs is

$$Cost_{maintenance} = 780{,}000 \times V + 695{,}000 \times V + 266{,}958.4245 \times L + 890{,}000 \times V + 643{,}326.0394 \times L + 65{,}000 \times V + \frac{16}{457} 10{,}062{,}500 \times L + 424{,}507.6586 \times L.$$

Simplified, this equation is

$$Cost_{maintenance} = 2{,}430{,}000 \times V + 1{,}687{,}089.715 \times L$$

5|DEPENDENCE ON FOREIGN ENERGY

5.1|CALCULATIONS

The implementation of high-speed trains in the United States will reduce our dependence on foreign oil because the oil used by the trains is negligible in comparison to that used by cars and short-distance plane rides. The United States imports 11.8 million barrels of oil per day (“How Dependent Are We on Foreign Oil?”). Annually this equals 133.5 billion gallons of oil. The reduction in oil consumption as a result of adding high-speed rail lines is modeled by the following equation depending on the number of car riders that are diverted to high-speed rails and the number of plane riders that are diverted to high-speed rails:

$$\Delta Gallons\ Consumed = Number\ of\ Car\ Riders \left(\frac{Average\ Distance\ Traveled\ per\ Year}{Fuel\ Efficiency}\right) + Number\ of\ Plane\ Riders \left(\frac{Average\ Distance\ Traveled\ per\ Year}{Fuel\ Efficiency}\right)$$

The average fuel efficiency for cars in America was 15 mpg in November 2011 (“Political Calculations”). The average vehicle occupancy, however, was 1.7 (Burns). Therefore the fuel efficiency used in the model will be 25.5 passenger mpg. The average fuel efficiency for a commercial airliner is 27.8 passenger mpg (“Why Acela”). The average distance traveled per year divided by the fuel efficiency equals the gallons used per year per person. Multiplying the people diverted from using planes and cars give the total gallons saved per year. It is assumed that the average distance traveled per year by a car rider using cars is equal to the average distance an American travels by car each year. The same is assumed true for plane riders. Therefore the average distance traveled per year by the car rider equals 8100 miles (Burns). The average distance traveled per year by the plane riders equals 1055 miles (“Beverton Oregon: Carbon Calculator”).

6|RANKINGS

To rank the ten different regions, we considered six factors: operating cost per ride of the high speed rail, GDP of the region, energy saved for the region, number of riders, construction cost, and unemployment rate of the region.

6.1|OPERATING COST

Operating cost for the ten different regions was calculated in Section 4.2 of this report. The operating costs per ride for the different regions were calculated as follows: $175 for the California Region, $311 for the Pacific Northwest Region, $237 for the South Central Region, $444 for the Gulf Coast Region, $267 for the Florida Region, $328 for the Southeast Region, $271 for the Chicago Hub Region, $311 for the Keystone Region, $213 for the Empire Region, and $862 Northern New England Region.

6.2|CALCULATIONS

The implementation of high-speed trains in the United States will reduce our dependence on foreign oil because the oil used by the trains is negligible in comparison to that used by cars and short-distance plane rides. The United States import 11.8 million barrels of oil per day ("How Dependent Are We on Foreign Oil?"). Annually this equals 133.5 billion gallons of oil. The reduction in oil consumption as a result of adding high-speed rail lines is modeled by the following equation depending on the number of car riders that are diverted to high-speed rails and the number of plane riders that are diverted to high-speed rails:

6.3|GDP

Another one of the factors used to rank the regions was projected additional GDP for the ten regions due to the construction of the high speed rails. Based off an estimate by Mark Zandi, chief economist for the Society for Industrial and Applied Mathematics, it was assumed that 1.59 is the multiplier due to increase in government expenditures in infrastructure. In order to determine the Marginal Propensity to Consume we use the relationship:

$$Multiplier = \sum_{k=0}^{\infty} (Marginal\ Propensity\ to\ Consume)^k.$$

Therefore, assuming the Marginal Propensity to Consume is a number between 0 and 1,

$$1.59 = \sum_{k=0}^{\infty} R^k,$$

$$1.59 = \frac{1}{1-R} \rightarrow 1.59 - 1.59R = 1 \rightarrow 1.59R = .59 \rightarrow R = \frac{.59}{.159} \approx .371.$$

The calculated value for the Marginal Propensity to Consume is approximately .371, meaning that for every dollar a consumer receives they will spend 37.1 cents and save the rest. Next, assuming this R value is correct for the population, a 99.9% confidence interval can be created for the R value of a sample size N, which will represent the number of dollars invested in the project by the government. In order to maintain accuracy the following calculations will be performed using the exact fraction (.59/1.59), where N equals total government spending in dollars :

$$Standard\ Deviation = \sqrt{\frac{R(1-R)}{N}},$$

$$Confidence\ Interval = R \ \pm Zscore_{.9995} \times (Standard\ Deviation),$$

$$R = \frac{.59}{1.59} \pm 3.291 \sqrt{\frac{\frac{.59}{1.59}(1 - \frac{.59}{1.59})}{N}} \ ,$$

$$R = \frac{.59}{1.59} \pm \frac{3.291}{1.59} \sqrt{\frac{.59}{N}},$$

$$Lower = \frac{.59 - 3.291\sqrt{.59/N}}{1.59}, \quad Upper = \frac{.59 + 3.291\sqrt{.59/N}}{1.59}.$$

Therefore to determine the lower and upper estimates for GDP increase as a function of government expenditures we can input these lower and upper estimates for the Marginal Propensity to Consume into the multiplier equation:

$$Lower\ Multiplier = \sum_{k=0}^{\infty} \left(\frac{.59 - 3.291\sqrt{.59/N}}{1.59} \right)^k = \frac{.159}{.59 - 3.291\sqrt{\frac{.59}{N}}},$$

$$Upper\ Multiplier = \sum_{k=0}^{\infty} \left(\frac{.59 + 3.291\sqrt{.59/N}}{1.59} \right)^k = \frac{.159}{.59 + 3.291\sqrt{\frac{.59}{N}}}.$$

Thus, the GDPs for the regions were calculated as follows: $408 billion for the California Region, $221 billion for the Pacific Northwest Region, $147 billion for the South Central Region, $357 billion for the Gulf Coast Region, $183 billion for the Florida Region, $207 billion for the Southeast Region, $997 billion for the Chicago Hub Region, $103 billion for the Keystone Region, $247 billion for the Empire Region, and $313 billion for the Northern New England Region.

6.4| ENERGY SAVED

Energy saved by all the travelers diverted to the high speed rails from planes, cars, and buses was calculated in Section 5 of this report. The amount of fuel energy saved in the ten different regions in the year 2020 was calculated as follows: 187 million gallons for the California Region, 14 million gallons for the Pacific Northwest Region, 14 million gallons for the South Central Region, 18 million gallons for the Gulf Coast Region, 16 million gallons for the Florida Region, 11 million gallons for the Southeast Region, 55 million gallons for the Chicago Hub Region, 0.8 million gallons for the Keystone Region, 18 million gallons for the Empire Region, and 4 million gallons for the Northern New England Region.

6.5| NUMBER OF RIDERS

The number of riders in 2020 was calculated in Section 3 of this report, and the numbers for the different regions are as follows: 53.3 million for the California Region, 4.49 million for the Pacific Northwest Region, 4.41 million for the South Central Region, 5.24 million for the Gulf Coast Region, 4.71 for the Florida Region, 4.19 for the Southeast Region, 38.4 for the Chicago Hub Region, 1.84 for the Keystone Region, 11.6 million for the Empire Region, and 1.62 for the Northern New England Region.

6.6| CONSTRUCTION COST

Total construction cost for the ten different systems was calculated in Section 4.1 of this report. The calculated construction costs are as follows: $213 billion for the California Region, $106 billion for the Pacific Northwest Region, $67 billion for the South

Central Region, $188 for the Gulf Coast Region, $86 billion for the Florida Region, $103 billion for the Southeast Region, $546 billion for the Chicago Hub Region, $41 billion for the Keystone Region, $118 for the Empire Region, and $159 for the Northern New England Region.

6.7| UNEMPLOYMENT RATE

We found the unemployment rates of the ten regions from US Bureau of Labor Statistics. Based on this source, the unemployment rates are as follows: 11.2% for the California Region, 8.75% for the Pacific Northwest Region, 7.4% for the South Central Region, 8.21% for the Gulf Coast Region, 9.9% for the Florida Region, 8.7% for the Southeast Region, 8.9% for the Chicago Hub Region, 7.7% for the Keystone Region, 8.2% for the Empire Region, and 5.95% for the Northern New England Region.

Weighting Value	Criteria	California	Pacific NW	South Central	Florida	South East	Chicago	Keystone	Empire	Gulf	North New England
4	Operating Cost/Ride	1	6	3	4	8	5	6	2	9	10
2	Change in GDP	2	6	9	8	7	1	10	5	3	4
3	Oil Saved	1	7	6	5	8	2	10	3	4	10
1	Usage	1	6	7	5	8	2	9	3	4	10
1	Construction Cost	9	5	4	3	9	10	1	6	8	7
5	Unemployment Rate	1	4	9	2	5	3	8	7	6	10
	Total (weighted)	26	88	104	65	112	55	124	71	96	145
	Rank	1	5	7	3	8	2	9	4	6	10

Our weighting methodology encompasses the relative rankings of increased GDP (as calculated above), oil saved, train usage, construction cost, operating cost per ride, and unemployment rates. The factors are rated according to the values in the corresponding leftmost column, with highest weight being most important, and, per each factor, the ten locations are ranked from most deserving (1) to least (10). The lower the total score, the more deserving the state.

Explanation of weightings of factors: The unemployment rates of each region were calculated using the particular unemployment rates of states in each region, weighted according to state population. This was chosen as our most important factor in determining which region is most deserving of the high-speed rails system because the overall purpose of this strategy is an economic investment. Through Okun's law, we can empirically theorize that the additional GDP contributed to each state through the plan would lessen the unemployment, and therefore, the states with the highest current unemployment would benefit most immediately from the plan. The second and third most important factors were operation cost per ride and foreign oil saved, respectively, because the plan must be both economically viable (i.e., train tickets must be at a reasonable level) and environmentally viable (thereby reducing dependence on gasoline). The factors of construction cost, usage, and GDP were weighted less critically because, though important, they tend to favor the bigger regions and longer tracks.

Results: Our ranking system determined that California is the most deserving state due to a combination of its usability, low cost per ride, high unemployment rate, savings on gas, and increased GDP. California was followed by Chicago and Florida, with Keystone and North New England as the least deserving due to their low usability and high costs per ride.

7|CONCLUSION

While the initial startup cost of producing high-speed railroads is a deterrent to committed investment, constructing high-speed railway systems under the HSIPR provides convenient and relatively inexpensive transportation that can save time for millions of citizens. The models we have presented can be tested against high-speed railway systems in Europe and Japan as well as the Acela Express in the United States, determining their effectiveness.

8|APPENDIX A

	California	Chicago Hub Network	Southeast	Pacific Northwest	Keystone
Number of Vehicles Needed at Year 20	3,333.00	2,401.00	263.00	281.00	116.00
Length of Track in Miles	800.00	3,000.00	480.00	466.00	105.00
Number of Diverted Auto Passengers at Year 20	25,904,384.54	3,484,905.69	831,385.70	1,332,416.39	120,239.82
Number of Diverted Bus Passengers at Year 20	730,175.52	4,102,858.02	521,231.45	24,674.38	54,996.65
Number of Diverted Plane Passengers at Year 20	23,698,667.33	11,468,710.40	2,248,618.31	2,319,391.50	95,843.34
Number of Diverted Slow-Speed Train Passengers at Year 20	3,393,411.08	19,352,230.06	202,461.80	592,185.06	1,570,088.12
Total Number of Diverted Riders at Year 20	53,320,991.55	38,409,176.34	4,195,697.76	4,490,736.74	1,841,167.93
Annual Train Operations Cost	$2,599,740,000.00	$1,872,780,000.00	$205,140,000.00	$219,180,000.00	$90,480,000.00
Annual On-Board Services Cost	$2,316,435,000.00	$1,668,695,000.00	$182,785,000.00	$195,295,000.00	$80,620,000.00

Annual Maintenance-of-Way Cost	$213,566,400.00	$800,874,000.00	$128,139,840.00	$124,402,428.00	$28,030,590.00
Annual Electric Traction Power Cost	$2,966,370,000.00	$2,136,890,000.00	$234,070,000.00	$250,090,000.00	$103,240,000.00
Annual Equipment Maintenance Cost	$733,649,531.25	$2,086,898,045.71	$325,933,090.17	$318,115,482.48	$75,144,271.96
Annual Station Services Cost	$158,501,531.73	$594,380,743.98	$95,100,919.04	$92,327,142.23	$20,803,326.04
Annual Sales and Marketing Cost	$339,606,400.00	$1,273,524,000.00	$203,763,840.00	$197,820,728.00	$44,573,340.00
Annual Total Operational Cost	$9,327,868,862.98	$10,434,041,789.69	$1,374,932,689.21	$1,397,230,780.71	$442,891,528.00
Operational Cost per Rider	$174.94	$271.65	$327.70	$311.14	$240.55

	Empire	Northern New England	Gulf Coast	Florida	South Central
Number of Vehicles Needed at Year 20	725.00	102.00	328.00	295.00	276.00
Length of Track in Miles	463.00	750.00	1,000.00	354.00	245.00
Number of Diverted Auto Passengers at Year 20	3,268,295.18	23,033.39	823,586.37	870,736.35	1,715,710.96
Number of Diverted Bus Passengers at Year 20	75,749.21	11,817.90	117,650.50	102,434.32	441,182.82
Number of Diverted Plane Passengers at Year 20	1,518,887.50	1,146,354.00	4,059,023.35	3,533,552.32	2,156,893.78

Number of Diverted Slow-Speed Train Passengers at Year 20	7,747,174.81	437,634.37	235,634.07	205,404.47	98,040.63
Total Number of Diverted Riders at Year 20	11,591,858.09	1,619,424.35	5,235,894.30	4,712,127.46	4,411,828.18
Annual Train Operations Cost	$565,500,000.00	$79,560,000.00	$255,840,000.00	$230,100,000.00	$215,280,000.00
Annual On-Board Services Cost	$503,875,000.00	$70,890,000.00	$227,960,000.00	$205,025,000.00	$191,820,000.00
Annual Maintenance-of-Way Cost	$123,601,554.00	$200,218,500.00	$266,958,000.00	$94,503,132.00	$65,404,710.00
Annual Electric Traction Power Cost	$645,250,000.00	$90,780,000.00	$291,920,000.00	$262,550,000.00	$245,640,000.00
Annual Equipment Maintenance Cost	$345,395,393.08	$488,946,779.28	$664,559,896.19	$247,022,478.11	$175,687,491.71
Annual Station Services Cost	$91,732,761.49	$148,595,186.00	$198,126,914.66	$70,136,927.79	$48,541,094.09
Annual Sales and Marketing Cost	$196,547,204.00	$318,381,000.00	$424,508,000.00	$150,275,832.00	$104,004,460.00
Annual Total Operational Cost	$2,471,901,912.57	$1,397,371,465.28	$2,329,872,810.85	$1,259,613,369.90	$1,046,377,755.80
Operational Cost per Rider	$213.24	$862.88	$444.98	$267.31	$237.18

	California	Chicago Hub Network	Southeast	Pacific Northwest	Keystone
Structure Cost	$117,286,652,080.00	$439,824,945,300.00	$70,371,991,248.00	$68,319,474,836.60	$15,393,873,085.50
Vehicle Cost	$49,995,000,000.00	$36,015,000,000.00	$3,945,000,000.00	$4,215,000,000.00	$1,740,000,000.00

Contingency Cost	$1,417,402,520.78	$1,697,420,579.23	$1,309,283,861.26	$1,303,021,802.63	$988,649,087.95
Right-of-Way Cost	$22,896,685,000.00	$47,355,262,890.00	$6,220,986,114.00	$11,420,926,300.00	$2,337,854,557.00
Soft Cost	$21,000,000,000.00	$21,000,000,000.00	$21,000,000,000.00	$21,000,000,000.00	$21,000,000,000.00
Total Construction Cost	$212,595,739,600.78	$545,892,628,769.23	$102,847,261,223.26	$106,258,422,939.23	$41,460,376,730.45
Total Construction Cost	$212,595,739,600.78	$545,892,628,769.23	$102,847,261,223.26	$106,258,422,939.23	$41,460,376,730.45
Lower Multiplier Estimate	$1.59	$1.59	$1.59	$1.59	$1.59
Upper Multiplier Estimate	1.5900087145	1.5900054383	1.5900125293	1.5900123265	1.5900197337
Lower Change in Real GDP	$338,025,373,313	$867,966,311,010	$163,525,856,763	$168,949,582,696	$65,921,180,856
Upper Change in Real GDP	$338,029,078,638	$867,972,248,497	$163,528,433,947	$168,952,202,271	$65,922,817,167
Error Range	$3,705,324.96	$5,937,486.55	$2,577,183.78	$2,619,574.21	$1,636,310.23
Government Consumption	$212,595,739,601	$545,892,628,769	$102,847,261,223	$106,258,422,939	$41,460,376,730
Multiplier Effect	$125,431,486,375	$322,076,650,984	$60,679,884,132	$62,692,469,544	$24,461,622,281
Unemployment Rate in 2012	0.112	0.089	0.087	0.0875	0.077
Gallons of Oil Saved from Diverted Auto Passengers Annually	101,585,821.71	13,666,296.81	3,260,336.06	5,225,162.33	471,528.72
Gallons of Oil Saved from Diverted Plane Passengers Annually	85,247,004.79	41,254,353.96	8,088,555.06	8,343,134.90	344,760.20

Total Gallons of Oil Saved from Diverted Passengers Annually	186,832,826.49	54,920,650.76	11,348,891.12	13,568,297.23	816,288.92

	Empire	Northern New England	Gulf Coast	Florida	South Central
Structure Cost	$67,879,649,891.30	$109,956,236,325.00	$146,608,315,100.00	$51,899,343,545.40	$35,919,037,199.50
Vehicle Cost	$10,875,000,000.00	$1,530,000,000.00	$4,920,000,000.00	$4,425,000,000.00	$4,140,000,000.00
Contingency Cost	$1,301,655,525.07	$1,403,737,998.17	$1,464,656,013.21	$1,244,888,971.37	$1,167,130,285.62
Right-of-Way Cost	$16,617,210,990.00	$20,936,010,960.00	$14,788,134,730.00	$7,793,679,319.00	$4,723,230,536.00
Soft Cost	$21,000,000,000.00	$21,000,000,000.00	$21,000,000,000.00	$21,000,000,000.00	$21,000,000,000.00
Total Construction Cost	$117,673,516,406.37	$154,825,985,283.17	$188,781,105,843.22	$86,362,911,835.77	$66,949,398,021.12
Total Construction Cost	$117,673,516,406.37	$154,825,985,283.17	$188,781,105,843.22	$86,362,911,835.77	$66,949,398,021.12
Lower Multiplier Estimate	$1.59	$1.59	$1.59	$1.59	$1.59
Upper Multiplier Estimate	1.5900117134	1.5900102117	1.5900092479	1.5900136729	1.5900155292
Lower Change in Real GDP	$187,099,512,750	$246,171,735,577	$300,160,212,486	$137,315,849,012	$106,448,503,201
Upper Change in Real GDP	$187,102,269,443	$246,174,897,643	$300,163,704,116	$137,318,210,646	$106,450,582,526
Error Range	$2,756,692.91	$3,162,066.14	$3,491,630.57	$2,361,634.38	$2,079,324.94
Government Consumption	$117,673,516,406	$154,825,985,283	$188,781,105,843	$86,362,911,836	$66,949,398,021

Multiplier Effect	$69,427,374,690	$91,347,331,327	$111,380,852,458	$50,954,117,993	$39,500,144,843
Unemployment Rate in 2012	0.082	0.0595	0.0821	0.099	0.074
Gallons of Oil Saved from Diverted Auto Passengers Annually	12,816,843.83	90,327.01	3,229,750.49	3,414,652.37	6,728,278.27
Gallons of Oil Saved from Diverted Plane Passengers Annually	5,463,624.11	4,123,575.53	14,600,803.43	12,710,619.84	7,758,610.71
Total Gallons of Oil Saved from Diverted Passengers Annually	18,280,467.94	4,213,902.54	17,830,553.92	16,125,272.21	14,486,888.99

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