Judge Perspective 2011
Colorado River Water: Good to the Last Acre-Foot

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1. Overview

Water is a vital and scarce resource in the American West. This was recognized early during the settlement of the American West, and a number of policies have been implemented to deal with the problems associated with the scarcity of water. One important policy has been to recognize the importance of the different water sheds and to ensure that water is shared amongst the states that span the region. The Colorado Compact of 1922 was one action used to ensure that water is shared in an equitable manner.

The students who took part in the 2011 Moody's Mega Math Challenge were asked to focus on Lake Powell, which is a focal point for the water flowing out of the upper basin of the American West. Student teams were asked to model the water levels in Lake Powell based on a range of possible inflows. The teams were also asked to address three questions.

Before discussing the details of each question we first provide an overview of the process of how the submissions are judged. Next, an overview is given for the modeling of the reservoir. Then each of the three questions is discussed in turn: the economic impact, sensitivity of the model, and recommendations.

2. Judging Process

The judging proceeds in two main steps. The first is a triage round in which every paper is read by at least two different professional mathematicians. (The judging corps is a composite of university professors and industrial mathematicians.) The primary goal in the triage round is to identify papers that require a more detailed reading.

Papers that do not receive a consistent set of scores in this initial round are read by a third judge. The judges try to err on the side of caution, and an entry that addresses all of the questions, has a reasonable model, and is well written is likely to be read in later rounds. For this initial round the summary is important, and the overall writing has an elevated importance.

For the second step, a smaller number of judges get together and take turns reading the papers. This step consists of a number of rounds in which the judges are given a longer amount of time to read each paper as the rounds progress. As a consensus begins to emerge for an individual submission, a paper may continue in the readings or it may be removed from further consideration.

The papers that remain in the last rounds are read by almost every judge, and during the final rounds a judge is given thirty to sixty minutes to read a paper. As the rounds progress, the judges are given time to focus on the details of each paper. The further a paper goes in the last rounds the more the details matter, and it becomes more important that the paper provide a more complete model with a consistent analysis.

3. Mathematical Model

The teams were asked to develop a mathematical model to estimate the current drought's impact on Lake Powell. Almost all of the teams interpreted this requirement to find an estimate on the amount of water stored in the lake. The types of models ranged from complex differential equations to simpler difference equations.

Neither approach was considered better than the other, but the main difference between teams was the other factors that were considered. Almost all teams considered inflow from the Colorado River and outflow at the Glen Canyon Dam. A number of teams also considered evaporation. A smaller number of teams considered seepage into the surrounding aquifer.

The majority of teams handled the inflow based on a percentage of the given average inflow rate. Some teams examined the historical data and instead tried to extrapolate into the future based on the past data. Given that the problem explicitly discussed the high variation in projected inflow rates and asked to examine different inflow rates, teams that simply used regression for projecting the inflow were expected to include strong arguments as to why this would be appropriate, which is a difficult case to make.

The second decision that teams had to make was how to model the outflow. Again, teams that simply extrapolated based on a regression of past data were expected to make a case as to why this is an appropriate approach. In this situation, though, a good argument can more easily be made. Many teams were able to find historical data that indicated that the outflow rate is relatively constant in time for the majority of years on record. Other teams found other data sources that indicated a strong correlation between the inflow and outflow rates, and developed a model based on the projected inflow rates. As long as the team provided appropriate citations and good arguments both approaches were considered appropriate.

Models made of relatively simple difference equations, a model that calculates the next year's water volume based on the previous year’s total, were considered appropriate models for this situation, and no preference was given compared to models made of differential equations. In this particular case, the teams that chose to use difference equations had an advantage in that it was easier to include the more complex terms accounting for evaporation or bank seepage.

As an example, some teams modeled the losses due to evaporation by using a percentage of the total amount of water in the reservoir. Some teams were able to create complex models that made use of the surface area of the reservoir. These approaches were much easier to integrate into a difference equation as opposed to a differential equation.

The primary difference in the best entries came down to the analysis that the teams performed on the models. For example, the vast majority of teams simply looked at the projected water levels for the three given inflow rates. The more successful teams went further and found water levels for the full range of inflows using the top and lower bounds given in the problem statement to determine the extremes. Also, some teams tried to determine the minimum flow rate necessary to maintain the current water levels.

In creating a mathematical model, the model itself is important. It is just as important, however, to perform a wide range of analyses to explore the implications of the model.

4. The Three Questions

In addition to the development of a model for the content of Lake Powell, the student teams were given three additional aspects of the problem to explore. These aspects included the economic impact of the drought, the sensitivity of the resulting model to different inflow rates, and the provision of recommendations for how to maintain the capacity of Lake Powell. Each of those aspects is examined in more detail in the subsections that follow.

4.1 Economic Impact

The question as to the economic impact of the drought is a difficult and far ranging question. The teams addressed this issue in a number of different ways. No one approach was considered stronger than another, but the primary consideration was whether or not the team was able to make the connection between their economic predictions and the model that was developed to describe the capacity of the lake. Also, a team's economic results were expected to be consistent with the team's predictions on the future water levels within the lake.

4.2 Sensitivity

The second question focused on the sensitivity of the mathematical model. In any mathematical model the sensitivity of a model to small changes in any parameter is a crucial question. In some situations a slight difference in the value of a parameter can lead to very large differences. In the case of the water levels at Lake Powell, a critical question is whether or not a small difference in the inflow rate could lead to a very different conclusion.

The vast majority of entries made little reference to this important aspect of the problem. A small number of teams examined the differences in their projections if the inflow rate differed by a small amount, and those that did had an immediate advantage. Additionally, the few teams that examined the projections if the flow rate fluctuated a small amount over the five years also made a strong, positive impression.

The idea of the sensitivity of the model is an aspect of the modeling process that is given a good deal of attention by the judges every year. Mathematical models are developed by making some basic assumptions that help simplify the process. In some instances a small deviation away from those assumptions or a small change in the assumed value of a parameter can result in a large difference in the final result. Student teams are always expected to perform some basic validation that explores how the results change if there is a small change in the model or its parameters.

This exploration can be as simple as changing the value of one parameter a small amount and then calculating new results. A simple exploration of the model with respect to small changes is something that few teams do, and including this in a submission will make a strong impression.

4.3 Recommendations

The third question was to include some recommendations on how to reduce water usage. This is another very broad question, and student teams supplied a wide variety of responses to this part of the problem. Some teams examined a number of general methods to reduce water usage, and other teams focused on one particular way that water is used.

Teams that were able to interpret and tie their results to their original mathematical model had an immediate advantage. For example, if a team decided to focus on agricultural use of water and provide specific reduction amounts, they could then go back to their original model and find a new prediction for the water levels in Lake Powell. Comparing the new results is a clear and explicit way to compare the impact of their recommendations with the present situation. It also demonstrates that they understand how to use and manipulate their model to aid in the use of making projections.

5. Conclusions

The 2011 Moody's Mega Math Challenge required student teams to project the water capacity of Lake Powell in the United State's Mountain West. The region is going through and extended drought and the use of water is a critical issue that impacts people across the entire region. The teams were asked to address three related questions.

The vast majority of teams did a tremendous job and submitted insightful entries for the contest. The teams that provided a model, performed a detailed analysis of their model, and then went on to address the three additional questions made the strongest impressions on the judges. The very top papers were those in which the team was able to bring together a set of projections based on the different questions that were tied together by the original mathematical model for the water capacity in the lake.