Executive Summary
2012 Moody's Mega Math Challenge

This year's competition featured the most difficult problem to date. The student teams were asked to evaluate and prioritize the locations across the United States for the construction of a new high speed rail system. The teams were given three tasks including the requirement to bring their models together to make a single conclusion for a prioritized list of sites.

The set of tasks could not be fully addressed in the allotted time. That is the nature of modeling activities, though, and one of the primary aspects of this year's event was for students to decide how to simplify the model and make difficult decisions on what aspects to focus their efforts. Student teams that were able to strike the difficult balance between simplifying the physical situation and performing the requisite analysis on their resulting models achieved the most success.

Students are expected to submit papers that are well written. The ability to communicate non-trivial models is as important as the development of the models themselves. The entries are expected to include a complete and well structured introduction as well as a complete description of the model and a clear description of how the team achieved its results.

The entries that made it to the final rounds of the competition included a strong analysis component. The best papers included models with approaches that ranged from quite simple to highly sophisticated. The one thing that set them apart from the rest of the field, though, was the critical analysis of the team's model.

The full commentary includes observations from one judge as well as a description of the judging process. The commentary includes a broad overview of some of the aspects of the entries that were likely to elicit a negative reaction from a judge, and the commentary also includes a list of some of the aspects that were likely to elicit a positive reaction. This year a separate section is included on the use of references and citations.

JUDGING PERSPECTIVE 2012
ALL ABOARD: CAN HIGH SPEED RAIL GET BACK ON TRACK

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1. Introduction

The 2012 modeling competition featured our most difficult challenge. The viability of an ambitious high speed railway project was considered, and three related questions were asked. Each individual question could be a difficult task in the given time limit, and the students were expected to bring them all together into a coherent report.

The reports that the students submit continue to improve in quality. It is always impressive to see what the students are able to achieve in a short time. The response to the Challenge has been tremendous, and the program continues to grow. A large part of the success and growth is due to the work and dedication of our high school advisers. The continued success and improvement depends on our high school advisers, and we are fully aware that they are doing this in the face of continued increasing responsibilities.

This perspective is one way that we try to give the advisers feedback. I will try to give a glimpse into the judging process itself as a way to provide a context as to why the different parts of the students' submission can be more important in certain stages of the judging. Second, I will try to list some of the aspects of different reports that were likely to elicit a negative reaction from a judge reading the report. After that a list of some of the positive aspects that were likely to elicit more positive reactions is given. Finally, a short note is given about the importance of including both references and citations.

2. The Judging Process

The judging process proceeds in three stages. The first stage is the pre-triage stage, the second is the triage stage, and the last stage is a series of rounds dedicated to choosing the papers that will go on to the final presentations at the Moody's Foundation offices in New York City. Here I will describe each stage and try to highlight what matters in each stage with respect to the student's work.

2.1. Pre-Triage

The pre-triage stage happens shortly after the final papers are submitted. This stage is designed to give the organizers of the event a feel for the range of papers that have been submitted. The organizers do

not want to impose their idea of what a good paper should be, but instead let the students dictate how the judges approach the competition. The reason for this is that it can be difficult for the judges to predict what the students will be capable of doing in the short time allotted. The judges are also aware that students are likely to interpret and approach the problem in different ways than the judges are likely to approach the problem. It is not the judges' goal to reward those students who happen to "think like us." The beauty of mathematical modeling is that there are many ways to approach a problem. Two things that set different approaches apart are consistency throughout the paper and the way that a team is able to express itself.

The pre-triage stage is set up so that a small number of judges pick out a number of papers and try to find a wide cross-section of papers. The papers range from those considered to be excellent entries to those considered to be adequate entries.

Once a collection of papers that represents the full breadth of the students' submissions has been identified, they are collected and made available for the judges in the triage round. These papers are used to provide an introduction to the problem and an introduction to the types of approaches that the judges are likely to see as they read through the entries.

2.2. Triage

Once the set of papers is identified, the triage rounds begin in earnest. A number of triage sites are assembled in mathematics departments at a variety of colleges and universities across the country. The judges at each site are asked to review the question and a wide range of supporting literature, including the literature made available to teams on the Moody's Challenge website, so that they know and understand the instructions and support that are available to the teams.

Once the pre-triage and other supporting materials are made available, the members of each triage team are encouraged to meet and discuss the problem and the instructions. Soon after, the members of the teams read their assigned papers and give each paper a score. The team members are encouraged to provide any feedback that is deemed appropriate for each paper. In this stage of judging every paper is read at least twice by a practicing academic or professional mathematician. Each paper gets a thorough reading, but the paper is not given as much time as is devoted in the following rounds.

The primary focus during triage is to identify papers that should be given a more thorough reading in a later round. Therefore, if a paper is unclear or borderline, judges give it the benefit of the doubt and indicate that it should be given more attention.

After each paper has been read by two different judges the scores are collected. A small number of papers are given conflicting scores, one high and one low. Those papers are then given a third reading to determine the appropriate action.

In this round of judging it is vital that a paper be well written and well structured. The student teams are expected to make it as easy as possible for the judges to find their sources, their assumptions, their model, and their results. If a team makes a judge work too hard in this round to find a result in a table or to determine how two different parts of the paper relate then that is sure to elicit a negative reaction. Little things, like including a table of contents, can make a big difference for the judges as they read through a paper.

In this round, a high premium is placed on a paper that is well organized and which answers most—if not all—of the problem. Additionally, it is very important in this round that the introduction be well written and complete. This is the team's first impression and can have a larger impact on how the judge perceives the paper than in the later stages of the judging.

A well organized and complete introduction should include a brief overview of the problem. A team should not assume that the reader knows the original problem statement. It should include an overview of the questions. It should include an overview of the paper and the basic approaches used. And finally, the introduction should include a synopsis of the primary results. There should not be any surprise or anticipation as to what the team's findings are.

2.3. Final Rounds

Papers that make it through to the final rounds are read by judges in a group setting at the SIAM offices in Philadelphia. The judges meet there as a way to ensure consistent scorings and that everybody has similar expectations. Each of the judges in this last set of rounds has taken part in triage and has a good idea of the range of approaches.

The judging at SIAM takes place in multiple rounds in which the judges are given more and more time to read each paper. They are expected to read the paper more thoroughly and begin to pay more attention to the “nuts and bolts.”

As these final rounds progress, the judges slowly move from an ideal of deciding which papers need more attention to beginning to think instead of finding the papers that represent the best approach to the problem. By the end of the last round, the judges will have done their absolute best to determine the entries whose teams most deserve to travel to New York City to defend their solution at the Moody’s Foundation's offices.

In these rounds the quality of the writing remains a high priority, but since all of the remaining entries maintain a consistent and high quality of writing and organization, the things that begin to set them apart are the consistency in their mathematics and the modeling. While the results that are found are expected to be correct, it is just as important they are tightly integrated into the results of the other aspects of the different parts of the models.

3. Negative Reactions

Some of the aspects of a paper that are likely to elicit a negative opinion of a team's entry are discussed here. These include the introduction of the paper, the writing, and the team's model. The competition leaves little time for a team to thoroughly address all of these aspects, and it is not uncommon for an entry to have multiple issues. The judges try very hard to be understanding of the limited nature of the event and the difficulty of the problem.

Teams should be aware of these issues though and do their best to minimize their impact, but a team that has multiple negative issues is not likely to go very far in the competition.

3.1. Introduction to the Problem

The introduction to the problem should be on a separate, single page. It should provide a clear overview of the problem, the paper itself, the approach, and the results. This is a difficult task and only a few teams are able to bring all of this together. And while most judges enjoy seeing a paper in which it is clear that the students enjoy the problem and have a sense of humor about the situation, it is highly unlikely that a team that tries to do this in the introduction will have room to address the more important aspects that should be in the introduction.

Additionally this is the team's first impression. Any misspellings or a lapse in grammar presents the judge with an immediate indication of the quality of the paper. This is not fair nor is it desirable, but that is the unfortunate reality.

3.2. Existing Models Without Additional Insights or Results

There are a great deal of resources associated with high speed rail and this particular program. A number of teams were able to find these resources and make use of them. This is encouraged and seen as an excellent way to start the modeling process.

Teams that were able to find these materials and make use of them were likely to make a more positive reaction in the triage round. Teams that were not able to go beyond the current material or bring things together in a novel way were not likely to get far in the subsequent rounds. When the models were more closely examined, the papers more thoroughly read, and sources examined it was easy to identify those teams that simply repeated results from a source.

3.3. Assumptions With Little or No Justification

This year’s Challenge was a difficult problem with three difficult tasks. Each aspect of the problem required a number of assumptions. For example, a large number of teams assumed that the cost of travel is directly proportional to the amount of time required. This is a valid and appropriate

assumption, but we do expect that space in the report be given to explicitly stating the assumption with some justification as to why this is a reasonable assumption.

As another example, the problem required that the population of the given regions be predicted, that assumptions be made about the people who might ride the trains and how that changes over time, as well as a number of other assumptions. The team members should explicitly point out what their assumptions are and demonstrate that they are aware of the good and the bad aspects of those assumptions.

As a further example, a number of teams based their calculations for the cost of building the railways on the cost associated with the Acela line in the US Northeast while others based the costs on those found in China, Japan, or Europe. All of these are reasonable approaches, but the teams should discuss why they made their choices and why they are appropriate.

3.4. (Too) Incomplete Solutions

Each year the judges go through the problem and carefully note what tasks have been requested. The expectation is that the teams try to address each problem and that certain tasks will be given more time and effort than others. Even if a team decides that a task cannot be adequately addressed, we do expect that some minimal discussion of the task be included.

This year was an exception, however. The problem this year was harder than those in previous years, and very few teams were able to address all of the questions in a meaningful way. For example, the task associated with quantifying the impact on energy dependency was the task most likely to be given a small amount of attention. Choosing how to deemphasize a particular task is a difficult decision, and the judges made every effort to be understanding in this regard.

The primary difficulty was in deciding how to balance the papers that chose different aspects to emphasize. In the end this boiled down to a matter of judgment, and the primary factor was the level of discourse on the items that were included in the team's report.

3.5. Describing the Model

The modeling approaches for most of the tasks varied from relatively simple models to extremely complicated models. One example is that for ridership. Some teams used simple exponential growth models for the populations to determine ridership while other teams found complicated models in which the percent of people using the rail changed in time. Both models are appropriate approaches to the problem.

The difference in how a judge reacts to a model is based on how the team motivates and describes the model. For example, a team that uses a relatively simple exponential growth model with a percentage of the population but provides an appropriate discussion as to why this is a reasonable model is more likely

to make a positive impact than a team that uses a highly sophisticated model found in a source but simply states it with little motivation or analysis. The background research is important, but judges also want to know how deeply the team thought about the problem and how they were able to translate their ideas into a mathematical description of the problem.

Another example is the way the team calculated the cost for the construction. The vast majority of teams used a simple linear model based on the length of the tracks, and this is entirely appropriate. Other teams made use of detailed models using costs associated with the geography of the area and the amount of additional support services required for the track. Teams that did a good job of motivating and describing these additional costs made a strong positive impact, but teams that merely threw those extra costs in with little justification made the opposite impression.

3.6. Tables and Figures With Inadequate Discussion

One aspect of this modeling problem is that the students had to make a number of decisions and assumptions about the areas and populations that would make use of the new railways. The teams had to find large amounts of information about the numbers of people, the costs of materials, and the times involved for construction, as well as the travel itself. The vast majority of teams organized this information in a number of tables and figures as a way to simplify the presentation.

When using tables and figures to present information it is vital that the teams let the reader know where the information can be found, how to read the information, and what aspects deserve special attention. For example, a number of teams assumed that the judges could sort out the tables on their own and would know how to read them. Every team organizes their tables in different ways and each team chooses to emphasize a slightly different aspect of the table. It can be quite difficult for judges to sort out what a team thinks is important.

Teams that did a good job of discussing what is in the tables and explaining how to read them made life much easier for the judges, especially in the earlier rounds. Teams that simply put the information in a table or figure with little discussion were more likely to elicit a negative reaction from a judge.

3.7. Inappropriate Regression

Finally, the use of data to construct a model solely via regression is not appropriate. The model should be based on some basic physical principle. For example, distance can be equated with money through the relationship that the time multiplied by the speed is distance, and as we all know *time is money*.

Simply looking at data and then deciding what the model is by itself is not an adequate way to derive the model. This can be a good first step in trying to organize thoughts and gaining insight, but it in the end there has to be a rational justification for why a relationship exists. Once the model is derived then it is appropriate to estimate parameters using regression.

4. Positive Reactions

Here we focus on aspects of the entries that were more likely to result in a positive reaction from the judges. These range from some of the issues discussed above to additional issues of the analysis of a model including sensitivity analysis.

4.1. Introduction to the Problem

The introduction is the judge's first view of the team's efforts. This first impression has a bigger impact in the earlier rounds of the competition but has a smaller contribution in the later rounds. The writing is expected to be clean and direct.

The teams should not assume that the reader knows the problem statement or the specific tasks that are required. The introduction should provide a brief overview of the problem, a brief overview of the paper itself as a large-scale outline of the paper, a brief overview of the mathematical approaches used, and finally the introduction should include a brief synopsis of the results. Also, the introduction should be on a single page of paper.

This is a difficult task, and there are few teams that are able to do all of this on a single page. The few teams that are able to achieve this make an immediate strong, positive impression. This greatly increases the probability that a judge will have a more positive attitude when going on to read the rest of the paper.

4.2. Good Writing

Again, this was a difficult challenge. The teams had a large number of tasks. Few teams were able to provide good solutions for every aspect of the problem, and the judges made every effort to understand this and balance the issue. The teams had an extra burden to justify their decisions and try to focus on the positive aspects of what they were able to accomplish.

Those teams that were able to concisely present their findings and do so in a well ordered and well structured document were more likely to ease the burden on the judges as they read the papers. Especially in the earlier rounds, a judge that could easily understand a paper was more likely to react positively and recommend that the paper be given additional consideration. If a judge can immediately make sense of the difficult concepts that a team is trying to convey then the team conveys a sense that the whole of their activities represents a careful and conscientious approach.

4.3. Simplifying the System

As noted, this year's challenge was difficult, and very few teams were able to address every aspect of the challenge. This offered an opportunity to consider another important aspect of the modeling process. The system had to be simplified, and this is a necessary task in every modeling activity. Knowing

what to simplify and change can be a difficult problem in itself, but a clever simplification can still offer tremendous insight into the problem.

As an example, one of the highest-rated teams submitted an answer to a different problem. Their choice was to simplify the regions into a few small urban areas and then ask what that implied about the usage of high speed rail. The smaller population resulted in a solution that resulted in a negative response, that no rails should be built. The team went on, however, to focus on one of the most complete analyses of their system. The insights and analyses that the team put together resulted in a paper that very few teams were able to match.

4.4. Simple Models Tied Together

As noted, each of the tasks in this challenge was difficult. Any one task by itself could easily consume more than the allotted time, and every team had to make difficult decisions about where to focus their efforts. In this year's event a number of highly successful teams put together relatively simple models for each aspect of the problem. These models included a straight-forward exponential model for the population and linear models for the cost of construction in relation to the length of the rail lines.

What set the different approaches apart was how they put the models together into a system whose composite was more complicated than the individual parts. The task of tying things together into a system and exploring the inter-dependencies can offer insight into how the different parts of the system depend on one another. By keeping the individual models simpler and then focusing on the complex interactions, a number of student teams were able to explore more complex analyses and come to nontrivial conclusions.

4.5. Honesty

The difficult nature of this year's problem resulted in teams having to decide which parts of the problem to emphasize. Almost every team had some part of the problem that they did not adequately address, and the judges understood this. Each year the judges make every effort to find the balance in comparing different group's entries.

This year was more difficult because few teams were able to meet every criterion. One thing that stood out was the teams that were honest about their decisions. Rather than trying to repeat some analysis performed in another source, many teams tried to indicate the aspects they were able to address and the aspects they were not able to address. Teams that tried to pretend otherwise were more likely to elicit a negative reaction, while teams that tried to convey a sense of the balance that they attempted to reach were more likely to receive a more favorable reaction.

4.6. Analysis

The papers that made it to the very last rounds of the competition were the ones that offered more complete solutions in the sense that they had a number of models for each aspect of the problem and demonstrated an awareness of how the models were connected to one another as a system. The very best papers were the ones that took those systems and explored the implications.

The teams took a critical look at their models and did their best to tear them apart. Once a model was constructed the next question was "what does the model imply?" not just "what are the results for the model?" For example, a team can put together a model that combines a wide range of aspects to be considered that results in a final cost and a set of benefits, but the team should then go on and ask what are the parts of the model that need more attention and what parts of the model have the greatest impact on their results. These are the strengths and weaknesses of a model and also the sensitivity of a model.

4.6.1. Strengths and Weaknesses

A critical examination of a model should include an explicit discussion on what parts of the model offer the best insights into a particular aspect of the situation, and also which parts require more attention.

For example, a team that makes use of a model based on an exponential model for the population should recognize that it is a simple model that is robust for a stable population that has good data to support the estimation of the various parameters. At the same time, the model may be lacking if the population includes a large transient component or the geographic area is not well defined. An explicit acknowledgment of these issues demonstrates that a team has given thoughtful concern to the nature of the model and what it represents.

4.6.2. Sensitivity

Another crucial aspect of a model is its sensitivity. Every year this is an issue that is discussed between the judges, and it makes a large difference in the final rounds. Teams should ask basic questions about how the model is impacted for changes in the assumptions or parameters. Going back to the example of the population, a team can explore what happens to their results if the estimate for the initial population changes by some small amount. Additionally, the team can ask what happens if the growth rate varies by some small amount.

The exploration of the sensitivity offers vital insights into how well the results of the model can be trusted. If a small change in the rate of growth results in a difference in the final rankings then that is an indication that the model itself should be more carefully explored. It should be noted that in this context of a very tight time constraint,

a model that is sensitive is not a strike against the model, but it is a point in favor of a team that carefully considers the model itself and is careful in its interpretation of its results.

5. References and Citations

Every year there has been some issue with the use of references and citations. The use of references has improved every year. In this latest round the majority of teams did make proper use of references, and it is good to see this improvement. It is expected that every submission has a list of references. This is usually put at the end of the paper and includes a list of all resources utilized by the team.

Citations on the other hand are the marks within the text that indicate which reference was used for a specific idea. This can be in any format including APA style or footnotes. The primary concern is that the citations be present and be consistent. A document that has a reference list but no citations provides little clue to the reader how the ideas were used.

6. Conclusions

This year's event featured one of the most difficult problems given in the Moody's Mega Math Challenge. The student teams had to make difficult decisions on what parts to focus their efforts and how to convey their results. The student teams were asked to put together an entry that featured good, well-organized writing as well as a good, structured mathematical model.

The models did not have to be intricate or overly complicated, but the teams were expected to perform a critical analysis of their models. Teams that were able to make clever decisions on which aspects to simplify were able to bring together models that featured rich connections between the different aspects. More importantly, they were able to demonstrate that they thought critically about the model itself and the implications of the results of the model.